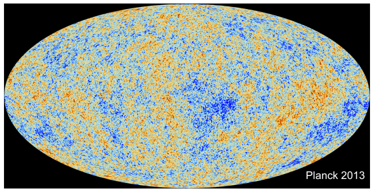

Pure-B Filtering by Messenger Method

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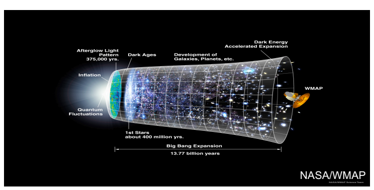
Abstract

Our work implements a method of reconstructing B-mode only maps from noisy CMB data using a Wiener Filter[1]. We have shown the effectiveness of this method by applying it to simulated CMB maps. B-mode signal in the CMB can help us to probe the structure of the early universe and potentially provide evidence to support or deny the theory of inflation.

Introduction

The early universe was a hot dense plasma where photons could not easily travel. All the photons in the early universe bounced around and entered thermal equilibrium with their surroundings until 380,000 years after the big bang, when the universe had expanded and cooled down enough for protons and electrons to combine to make neutral Hydrogen. At this point, the universe became transparent and the photons were free to move about. These photons make up what we call the Cosmic Microwave Background (CMB). The CMB gives us a thermal picture of what the early universe looked like. We can also see from the CMB how all the light in the early universe was polarized, and we have observed the CMB to consist mostly of E-modes.

Cosmic inflation theory postulates that there was a period in the evolution of the universe in which extremely fast expansion occurred. This expansion would have sent gravitational waves through space, which would leave a B-mode signature in the CMB. By looking for B-polarized light in the CMB, we can establish evidence for or against inflation theory.

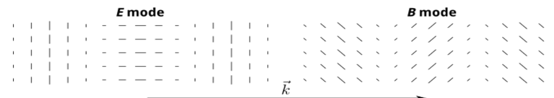


Maps, Spherical Harmonics, and Power Spectra

There are a few different ways we can choose to represent CMB data. The most straightforward way is to represent it in the pixel domain, with a map. We can also transform this map into the spherical harmonic domain using algorithms from the python package Healpy[4]. Another common way to represent data is with a power spectrum. This is found by taking the absolute value of each spherical harmonic coefficient, squaring it, and for each ell value, averaging across m-values. In practice, we end up binning power spectra into bins that go over a range of 35 ells. This allows us to see average properties of the power spectra.

Pure-B Wiener Filter

The basic idea of our work is to apply a Wiener filter to a CMB data set. The Wiener filter serves as a method of noise filtering in data sets. A Wiener filtered map is defined to be $m_{wf} = m\frac{S}{S+N}$ where S represents the signal covariance, N represents the noise covariance of the data set and m represents the noisy data. If signal and noise are both Gaussian (which seems to be the case for the CMB), the Wiener filter maximizes the a-posteriori probability of a map given m, S, and N. Any kind of light has what we call an E-mode and a B-mode component (shown in figures below). To find just the B-mode signal in the CMB, we treat the E-mode part of the signal covariance as if it was noise covariance[2] so the wiener filter equation becomes $\frac{S_B}{S_B+S_E+N}$. We cannot write down the filtering matrix directly, however, because the signal and noise covariance matrices are sparse in difference bases.



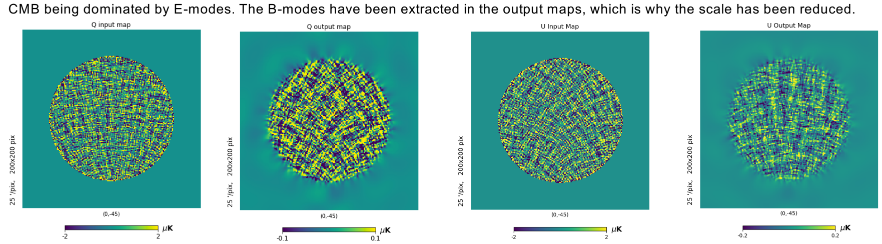
Messenger Method

One way to get around the issue of not being able to write down the filtering matrix is to introduce a "messenger field", t, such that the covariance of the messenger field, T, is proportional to the identity matrix. This allows us to write down T in any basis, and thus we can write the equation for a wiener filtered map, s, in an iterative manner as such:

$$(\overline{N_C}^{-1}+(\lambda T)^{-1})t = \overline{N_C}^{-1}d + (\lambda T)^{-1}s$$

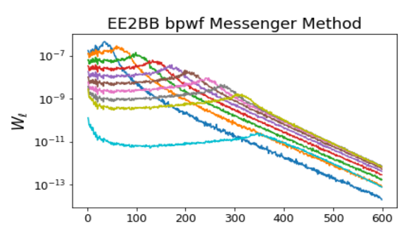
$$(S_B{}^{-1} + (\lambda T)^{-1})s = (\lambda T)^{-1}t$$

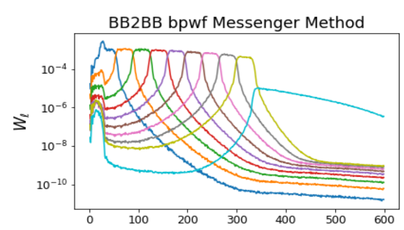
where $\overline{N_C} = N_C - T$, λ is a scalar which we use to speed up convergence, and d is the input data. The first equation above is solved for t in the pixel basis and the second is solved for s in the spherical harmonic basis. Iteratively solving these equations for s while decreasing lambda from some large value down to 1 allows us to filter out both noise and E-modes so that the algorithm's output map is just the B-mode signal from our data. Shown below are a set of input and output Q and U maps. The input maps look like E-modes due to the true CMB being dominated by E-modes. The B-modes have been extracted in the output maps, which is why the scale has been reduced.



Simulations

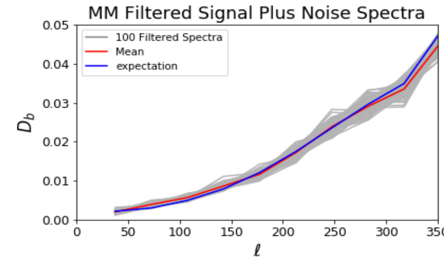
In order to find the E to B leakage, we simulated E-mode only CMB maps containing power at a given ℓ and measured how many E-modes had leaked to B-modes. B-mode only maps were simulated for BB2BB band power window functions. This work is summarized in the plots below. These band power window functions tell us, for a particular bin, how much power is allowed through the filter at each ℓ. So, multiplying the EE2BB band power window functions by the theoretical EE CMB power spectrum, then summing over all ℓ's, gives us the expected EE2BB leakage in each bin from the CMB. The ratio of BB2BB band powers to EE2BB band powers tells us about the purity of the filter.



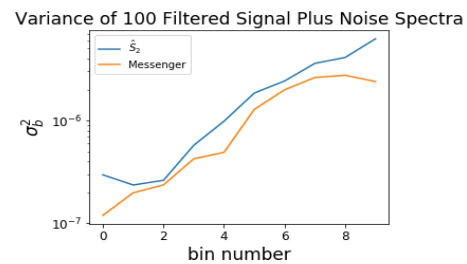


Results

The output binned power spectra from 100 simulated data sets are shown in a plot below along with the expected power in each bin, and the mean of the simulations. The expectation was calculated by multiplying the theoretical BB power spectrum by the BB2BB band power window functions and then summing over ells. The fact that the mean of the filtered simulations matches the expectation very well tells us that the BB2BB band power window functions have been calculated correctly. The spread of the gray lines in the figure gives us the error bar on our measurement.



After running the simulated data sets through our filter, we found that the messenger method approach reduced the variance on our measurement of the B-mode signal when compared to a previous pure-B estimator named the pure-$\hat{S}_2$ estimator[3]. The results of this are shown in the plot below. A lower variance on our measurement amounts to shrinking the error bar on our BB power spectrum estimate. We expect a lower variance on the messenger method estimator because the wiener filter by construction minimizes the chi-squared statistic, which is the same as minimizing variance. The pure-$\hat{S}_2$ estimator is constructed to throw out all E-modes, including any ambiguous E-modes. So, this estimator will end up throwing out some B-modes which should not be thrown away, which will increase the variance.



Next Steps

We have demonstrated the effectiveness of this method as applied to simulations maps. There is still some work to be done, however, before our code can be used as a data pipeline. The code must be packaged up in a manner so that it can be easily used with maps at arbitrary frequencies with arbitrary mask geometries in an efficient manner.

References

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