

A Method of Quantifying Order in Typographic Design

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A description is given of a method to compare in quantitative terms the degree of order of two typographically designed pages. A distinction is made between two kinds of order: the order of a system (systemic order) and the order of arrangement. The Shannon formula was used for measuring the degree of order of the two designs—an old version and its redesign. It shows that a judgment based on optical evidence can be supported by a mathematical-empirical judgment. Possible consequences for an aesthetics based on information theory are drawn. A design heuristic is sketched, and the possibilities as well as limits of an algorithmization of design decisions are discussed.

Introduction: Mathematical Aesthetics

Any attempt to express in mathematical terms the physical correlate of percepts comes under grave suspicion of being academic in intention and conservative in result. And this suspicion is aggravated when statements about the pattern of the stimulus (objective description) are confused with those about its aesthetic properties (subjective evaluation).

The mathematicization of aesthetics, or more specifically of visual aesthetics, has resulted in two main doctrines: that of proportion and that of distribution. The first is concerned with the quantitative relationship between elements (e.g., the ratios of distances and gray-scale values). The second is concerned with the distribution of elements within configurations. (In neither type of aesthetics based on mathematics can the explicative and generative aspects of a design be separated. The principle of arrangement underlying the design of a facade should both explain the pleasure it affords and illustrate the method by which it was created.)

The doctrine of proportion has elected geometry as its patron; that of distribution draws upon statistics. Notwithstanding the substantial

differences between these two aesthetic approaches in their mathematical apparatus, the properties they explore are very similar. They are syntactic properties; i.e., properties which consist entirely of intersign relationships. The semantic dimension has so far remained inaccessible to mathematical aesthetics, and in the light of this it might be pertinent to ask how historicity can ever be put into numbers. It might be said in response that what is typical in a work can be expressed in typical numerical values. Undoubtedly. But the idea that a number—apart from its function as a numerical tag—has any expository force or can convey information about a work is still open to doubt.

Seeking a mathematical explanation of the beautiful is rooted in the desire to rationalize preferences. This is a wish which cannot be entertained by anyone who realizes that beauty is more complex than mathematics and that it is not advisable to mix two different universes of discourse which are not reducible to each other. Not to accept this advice is to risk the criticism that one is celebrating accuracy in trivia and cultivating bad platonism by elevating a stock, as it were, of transhistoric forms to a glorified museum status. But although beauty is not immanent in mathematics, this does not mean that it cannot be codified by mathematical processes. The heuristic value of mathematics for creative design clearly exceeds its epistemological value. Mathematics must be conceived as providing design with a series of instruments for the conscious and controlled generation of forms, rather than as a source of explanations for aesthetic phenomena. Mathematics is more successful in making things than in explaining the beauty of what is made. It is no longer possible to offer methodological excuses for the meagreness of the objects upon which mathematical aestheticians exercise their skills by arguing that one has to start with the simple before proceeding to the complicated. There is no lack of simplicity about geometrical patterns, upon which designs are based. Traverse surveys and vase contours are among the objects for which it is difficult to claim a key position in aesthetic experience.

In its application to aesthetic problems, information theory has been rightly charged with using a great deal of paraphernalia and having little to show for it. Nonetheless, it must be admitted that a statistical theory of the physical substrates of information has had a fruitful influence on the development of aesthetics, if only in the sense that modern terminology has enabled us to bring familiar phenomena into

sharper focus. Yet it remains in servitude to classical aesthetics so long as it sees the most pressing problem of aesthetics as the formulation of a theory of beauty. A system of axioms would first have to be replaced by an empirically based theory of preferences. Only then would aesthetics be liberated from its conservative shackles.

All this underlines the wisdom of exercising great caution in drawing parallels between expressions of preference and the mathematical categorization of the stimulus pattern. Another qualification is necessary. Earlier works by the author (1965, 1967) in which Shannon's complexity formula was applied to design problems must, in the present state of knowledge, be relativized. There are two reasons for this:

1) It has not yet been determined at what number of elements the statistical formula becomes operable—at 10, 50, 100, 500, or 1000 elements.

2) No one has yet thought out the theoretical consequences arising from the fact that qualitatively different formal elements—e.g., straight line and circle—are compared with respect to the effort required for their description and generation.

The Creation of Order

There is a platitude which might be very well applied to the activities of the designer—namely, that a state of order is preferable to a state of disorder. For, among many other things, designing means creating order, means putting structure into an array of objects or signs, means reducing disorder and arranging elements into a whole that makes sense. To be sure, the creation of order is the daily business of the designer; but so far there has been (a) no design heuristic as a set of operational rules which say how order is to be created, and (b) no procedures by which the varying degree of order displayed by different design solutions can be quantified with accuracy.

In comparing the order of two designs, one usually judges on visual merits. And in practice that is perfectly adequate. Yet a mathematical correlate for such psychophysical judgments would certainly be welcome, however inaccessible this aspect of design may appear to the popular mind. Two psychologists may be quoted as writing, with slight modifications, that it is evidently more difficult to measure order than to manipulate it (Attneave and Arnoult, 1966).

There follows a report of an attempt to measure the difference in

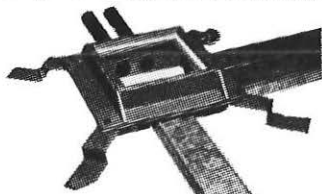
Bauteile für Unterflur-Installation (System B)



Lichte Weite mm	Höhe mm	Bodendose					
		ohne Bodenplatte		mit Bodenplatte			
		Bestell-Nr. 1)	1 Stück Preis 2) netto DM kg		Bestell-Nr. 1)	1 Stück Preis 2) netto DM kg	
Für Blechkäbtle 1)							
129x129	25 35	SVP3 019 SVP3 029	28.70 29.—	0,8 1	SVP3 119 SVP3 129	30.80 31.20	1,1 1,3
180x180	25 35	SVP3 039 SVP3 049	35.— 35.50	1,4 1,6	SVP3 139 SVP3 149	37.— 37.50	2,1 2,3

Gegenrahmen

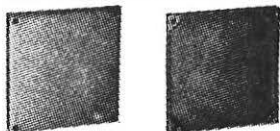
Zur Aufstockung der Bodendosen bei Estrichhöhen über 50 mm, Sie werden mit der Flanschseite nach oben unter die Bodendose gesetzt. In den Gegenrahmen sind die Einführungsöffnungen für die Käbtle vorgestanzt.



Für Boden- dosen mit lichter Weite mm	Höhe mm	Gegenrahmen 1)					
		ohne Bodenplatte		mit Bodenplatte			
		Bestell-Nr.	1 Stück Preis 2) netto DM kg		Bestell-Nr.	1 Stück Preis 2) netto DM kg	
129x129	25 35	SVP3 219 SVP3 229	13.— 13.20	0,7 0,86	SVP3 319 SVP3 329	15.— 15.20	1,1 1,2
180x180	25 35	SVP3 239 SVP3 249	16.40 16.70	0,96 1,1	SVP3 339 SVP3 349	18.60 19.20	1,7 1,9

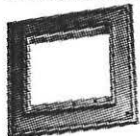
Verschlussdeckel für Bodendosen

Die Verschlussdeckel können sowohl mit der glatten als auch mit der Hohlseite nach oben aufgeschraubt werden. Die nach oben liegende Hohlseite läßt sich mit dem gleichen Bodenbelag ausfüllen, den der Raum hat.



Für Bodendosen mit lichter Weite mm	Bestell-Nr.	1 Stück	
		Preis DM	netto kg
129x129	SVP3 41	9.50	0,18
180x180	SVP3 43	14.80	0,35

Übergangsstück für Bodendosen

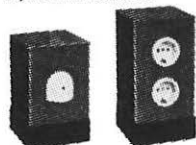


Das Übergangsstück ist erforderlich, wenn eine Zapfsäule auf eine Bodendose mit der lichten Weite 180x180 mm montiert werden soll.

Das Übergangsstück kann sowohl mit der glatten als auch mit der Hohlseite nach oben aufgeschraubt werden. Die nach oben liegende Hohlseite läßt sich mit dem gleichen Bodenbelag ausfüllen, den der Raum hat.

Bestell-Nr. SVP3 42, 1 Stück Preis DM 20.50, netto 0,68 kg.

Zapfsäulen aus Stahlblech



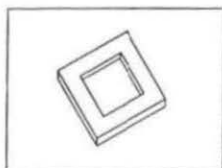
Die Zapfsäulen nehmen die Installationsgeräte (z. B. SCHUKO-Steckdosen) und die Schwachstrom-Klemmenleisten auf. Eine Zwischenwand Innerhalb der Säule trennt die Starkstromseite von der Schwachstromseite. Als SCHUKO-Steckdosen sind Geräte mit Zentraleplatte vorgesehen. An Stelle der SCHUKO-Steckdosen können auch andere Installationsgeräte, z. B. Kraft-Steckvorrichtungen eingebaut werden. Näheres auf Anfrage. Um das Anschließen der Schwachstromleitungen zu erleichtern, kann für die 2- und 3-etagigen Zapfsäulen ein Kippbügel geliefert werden, mit dem sich die Klemmenleisten nach oben oder unten ausschwenken lassen.

1) Die Einführungsöffnungen für die Käbtle bzw. für Stahlrohre werden werkseitig hergestellt. Der Bestellung ist eine Skizze beizufügen.
2) Zusätzlich Preis für werkseitig hergestellte Einführungsöffnungen. Preis je Öffnung DM 0.80.

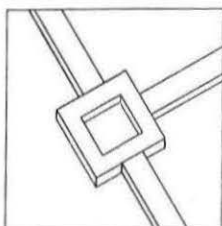
Figure 1. Old version of a catalogue page.

**Beuteile für Unterflur-Installation
(System B)**

3.
30

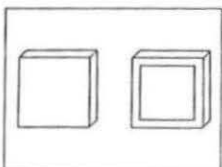


Lichte Weite mm	Höhe mm	Bodendose ohne Bodenplatte		mit Bodenplatte			
		Bestell-Nr. 1)	1 Stück Preis *) DM	netto kg	Bestell-Nr. 2)	1 Stück Preis *) DM	netto kg
Für Steckkanäle 1)							
129 x 129	25	SVP3 018	28,79	0,8	SVP3 118	36,80	1,1
	35	SVP3 029	28,--	1	SVP3 129	31,20	1,3
180 x 180	25	SVP3 039	35,58	1,4	SVP3 139	37,--	2,1
	35	SVP3 049	35,--	1,6	SVP3 149	37,50	2,3



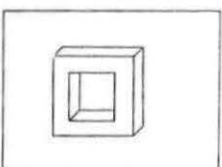
Gegenrahmen
Zur Aufstockung der Bodendosen bei Estrichhöhen über 50 mm. Werden mit Flanschsohle nach oben unter die Bodendose gesetzt. Im Gegenrahmen vorgezante Einführungsoffnungen f. Kanäle

Für Boden- dosen mit lichter Weite mm	Höhe mm	Gegenrahmen 1)		mit Bodenplatte			
		Bestell-Nr.	1 Stück Preis *) DM	netto kg	Bestell-Nr.	1 Stück Preis *) DM	netto kg
129 x 129	25	SVP3 219	13,--	0,7	SVP3 319	15,--	1
	35	SVP3 229	13,29	0,96	SVP3 329	18,20	1,3
180 x 180	25	SVP3 239	18,40	0,96	SVP3 339	18,60	1,7
	35	SVP3 249	18,79	1,1	SVP3 349	18,20	1,9

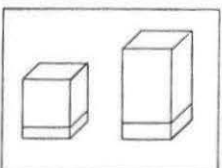


Verschlussdeckel für Bodendosen
können mit der glatten oder mit der Hohlseite nach oben aufgeschraubt werden. Die nach oben liegende Hohlseite läßt sich mit dem gleichen Bodenbelag ausfüllen, den der Raum hat.

Für Bodendosen mit lichter Weite mm	Bestell-Nr.	1 Stück	
		Preis DM	netto kg
129 x 129	SVP3 41	8,50	0,18
180 x 180	SVP3 43	14,80	0,25



Übergangstück für Bodendosen
Das Übergangstück ist erforderlich, wenn eine Zapfsäule auf eine Bodendose mit der lichten Weite 180 x 180 mm montiert werden soll. Bestell-Nr. SVP3 42, 1 Stück Preis DM 20,50, netto 0,68 kg.



Zapfsäulen aus Stahlblech
Die Zapfsäulen nehmen die Installationsgeräte (z. B. SCHUKO-Steckdosen) und die Schwachstrom-Klemmenleisten auf. Eine Zwischenwand innerhalb der Säule trennt die Starkstromseite von der Schwachstromseite. Als SCHUKO-Steckdosen sind Geräte mit Zentralplatte vorgesehen. An Stelle der SCHUKO-Steckdosen können auch andere Installationsgeräte, z. B. Kraft- Steckvorrichtungen eingebaut werden. Näheres auf Anfrage. Um das Anschließen der Schwachstromleitungen zu erleichtern, kann für die 2- und 3-stufigen Zapfsäulen ein Kippbügel geliefert werden, mit dem sich die Klemmenleisten nach oben oder unten aus-schwenken lassen.

1) Die Einstüßöffnungen für die Kanäle bzw. für Stahlblech werden wasserdicht hergestellt. Der Bestellantrag ist eine Skizze beizulegen.
2) Zusätzlich Preis für werkseitig hergestellte Einstüßöffnungen. Preise je Öffnung DM = 80.

Figure 2. Redesign of the catalogue page according to the rules of the design manual. (Half-tone prints are represented schematically.)

order between two typographic designs of a printed page. Figure 1 shows the original design; Figure 2, the redesign according to the style manual which will be discussed more fully below.

Two Types of Order

Order is a relational concept. It does not vary with magnitude. To be able to speak of the order of a configuration or an ensemble, we need several elements which have a defined relationship one to the other. In typography order is mainly a question of dimensional relationships within groups of elements (e.g., the proportion of picture sizes and line lengths) and the distribution of these elements on a page and in a printed work. In this connection a distinction must be made between two types of order:

1) The order of the system. From this standpoint the typographic items (main headings, blocks of composition, number of pages, illustrations, tables, and footnotes) are regarded as elements of a system. This order is a function of the data needed to describe all the items occurring—i.e., the dimensional classes and the frequency with which the dimensions occur. A hypothesis may be formulated that the smaller the number of dimensional classes and the greater the number of the elements contained in these classes, the higher will be the degree of order of the whole; for order depends to a large extent on the repetition of elements.

2) The order of arrangement. This type of order is based on the relational connections between the items within a configuration (or, in terms of information theory, within a supersign). This order is a function of the horizontal and vertical reference lines on a page and the frequency with which the corners of the items fall on these lines.

Both types of order, although distinguishable analytically, overlap and influence each other. Their interaction determines the relative simplicity of a typographical page and the ease or difficulty with which it can be taken in at a glance.

Determination of the Degree of Order

In order to allow a comparison between the old and new versions of the catalogue page and to reveal the structure or skeleton of the designs, a number of conditions had to be observed and an operation performed on the items:

1) The type of items used was to remain constant—e.g., a table of the old version corresponds to a table of the new version.

2) The information content of both versions was to remain constant—e.g., the information contained in a footnote in the old version was to be taken over into the new version. Without impairing intelligibility, the quantity of composition in one block was slightly compressed in order to accommodate the text in the space available. The redundancy cushion allowed this compression.

3) The parallel arrangement of the items with respect to the edges of the page was to be maintained—i.e., items were neither turned over nor tilted.

4) The items were to be inscribed in rectangles corresponding to their real or maximum possible extents, which were not to overlap another item (Figs. 3 and 4). All items are similar in having four right angles. The similarity increases to the extent that other elements—vertical and/or horizontal sides—are equal.

First the dimensions of width and height of the items of both versions were compared. To this end it was assumed that the dimensional order—i.e., in this case the system order of the typographical items appearing on a printed page—is a function of the information needed to generate these items from the dimensions of the sides. The question was therefore raised: how often does the side dimension i, j, \dots, n occur? In this way the number of width classes and height classes and their relative frequency can be determined. The following particulars were established.

Old version: 19 widths divisible into 9 classes; 19 heights divisible into 14 classes (Fig. 5a).

New version: 20 widths divisible into 3 classes; 20 heights divisible into 5 classes (Fig. 5b). (The higher number of items in the new version is due to the fact that a text set as a single column in the original version was set as a double column in the redesigned version.) To determine the complexity of the system expressed here as an order, we used the Shannon formula:

$$C = -N \sum_{i=1}^{i=n} p_i \text{ld} p_i$$

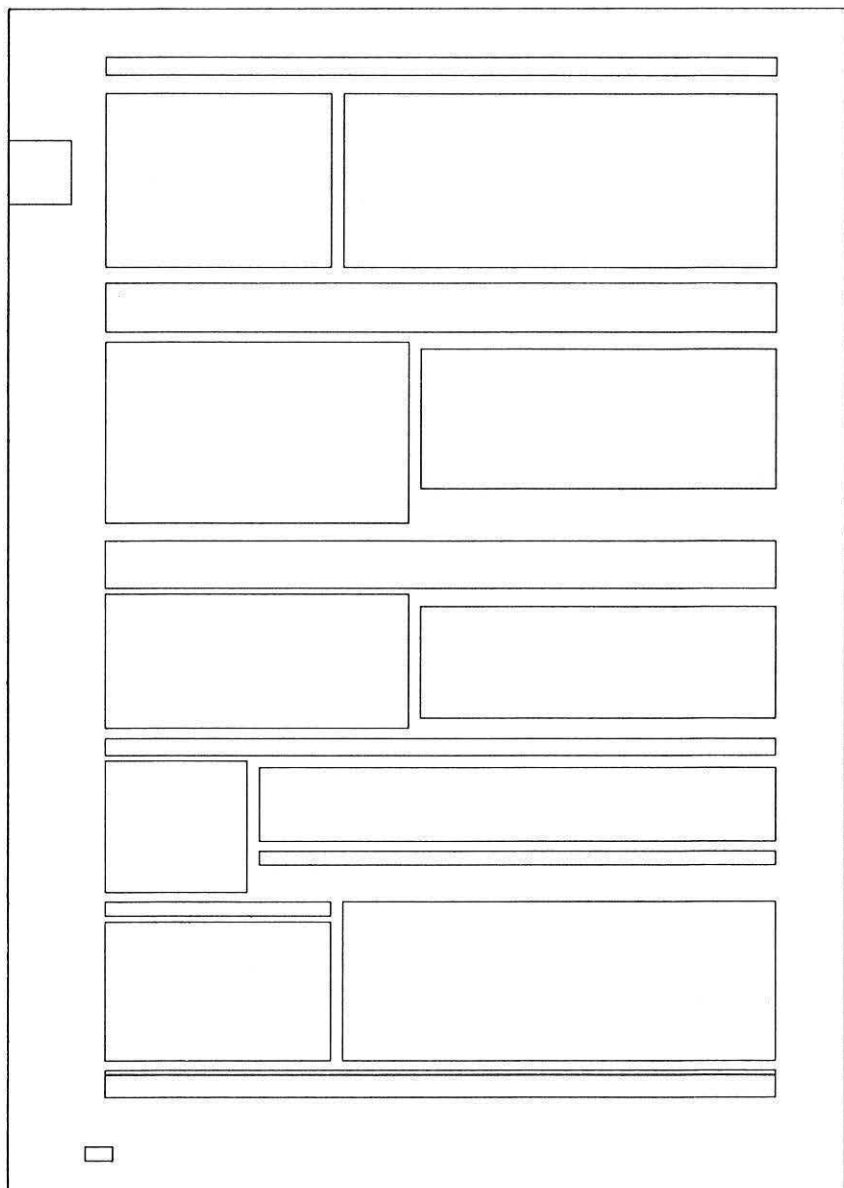


Figure 3. Contour lines around items of the old version.

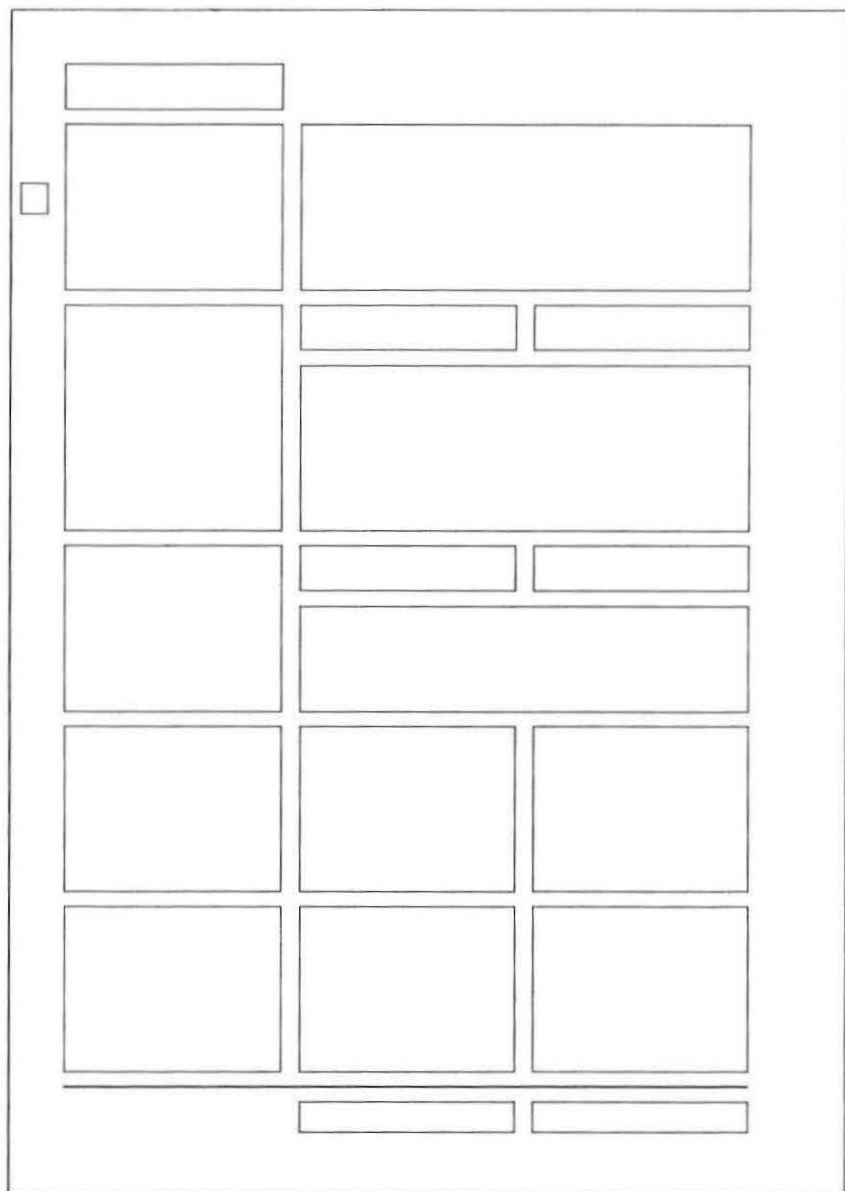


Figure 4. Contour lines around items of the redesigned version.

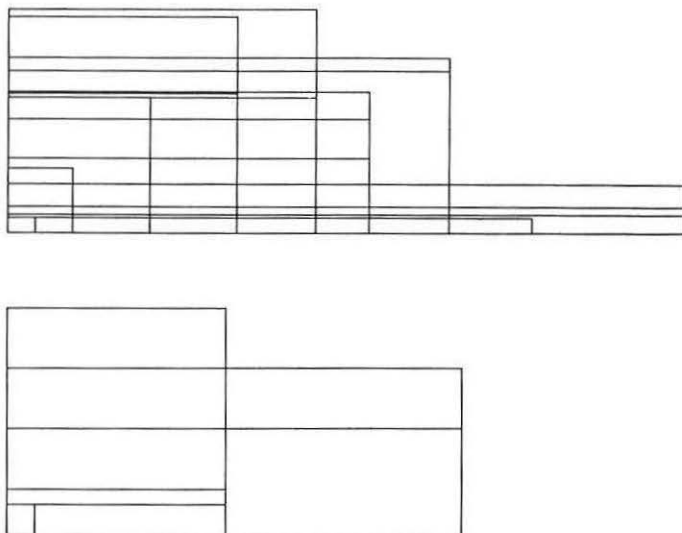


Figure 5. The “systems order” of the elements. Comparison between the old solution (5a) and the redesign (5b).

The data inserted in this formula led to the following results :

Old Version : widths 53 bits, heights 70 bits.

New Version : widths 18 bits, heights 37 bits.

The resulting ratio of total bits for the old versus the new version is 123 to 55, or in percentage, 100 to 45. It is clear that the new version is some 55% simpler than the old, which in this context means better ordered. As already explained, this value refers to the degree of order of the ensembles compared together, to the character of the typographic components as elements of a system, but not to the arrangement of the items on the printed page.

For the second type of order the hypothesis similarly holds good that the order of distribution is a function of the information needed to create this arrangement of the items on the printed page. The position of an item is determined by two coordinates. The coordinate frame originates in the left upper corner of the printed page. The location of, say, an illustration is determined by specifying the distance of its left upper corner from the top edge and from the left edge. If items are “suspended” on a horizontal reference line—“clothesline typography” as it is called—the corresponding y-dimension occurs several times ; i.e., its frequency

increases. The same holds true of the vertical reference lines and the corners of the items located on them.

The following data were determined for the distribution order:

Old version: 19 vertical distances from the top edge divisible into 17 classes; 19 horizontal distances from the left edge, divisible into 6 classes (Fig. 6).

New version: 20 vertical distances from the top edge, divisible into 10 classes; 20 horizontal distances from the left edge, divisible into 4 classes (Fig. 7). Introducing these into the complexity formula we obtain the following results:

Old version: $90 \text{ bits} + 37 \text{ bits} = 127 \text{ bits}$

New version: $63 \text{ bits} + 35 \text{ bits} = 98 \text{ bits}$.

This is a ratio of 127:98. Converted into a percentage (100:77), this shows that the distribution order of the new as compared with the old version has increased by 23%. This state of affairs is illustrated by the smaller dispersion of the corners of the new version as can be seen from the diagrams (Figs. 6 and 7). In terms of gestalt psychology this lower value corresponds to the law of "common destiny": the corners of the items of the new version form imaginary rectangular patterns which are clearly visible to the eye.

On the assumption that the overall order of the printed page results from the degree of order of the items and the distribution of the items, the two values may be added. It may be deduced from the ratio 250:153 (or 100:61 in percentage) that the degree of order in the new version is some 39% higher than in the old. Judgment based on optical evidence confirms that one of the versions is more ordered than the other. The mathematical-empirical judgment determines by how much one version is more ordered than the other.

A Design Heuristic

Design solutions may be of two kinds: they may produce a result which is fixed once and for all, or they may involve the planning of design guidelines within whose framework a series of results may be subsequently produced. In the second case the design planner marks out the area of decision for the executing designer. This type of design activity might be described as pre-design, its purpose being to routinize or semi-routinize the maximum possible number of design decisions. The degree of semi-routine—i.e., the degree to which the area of decision is

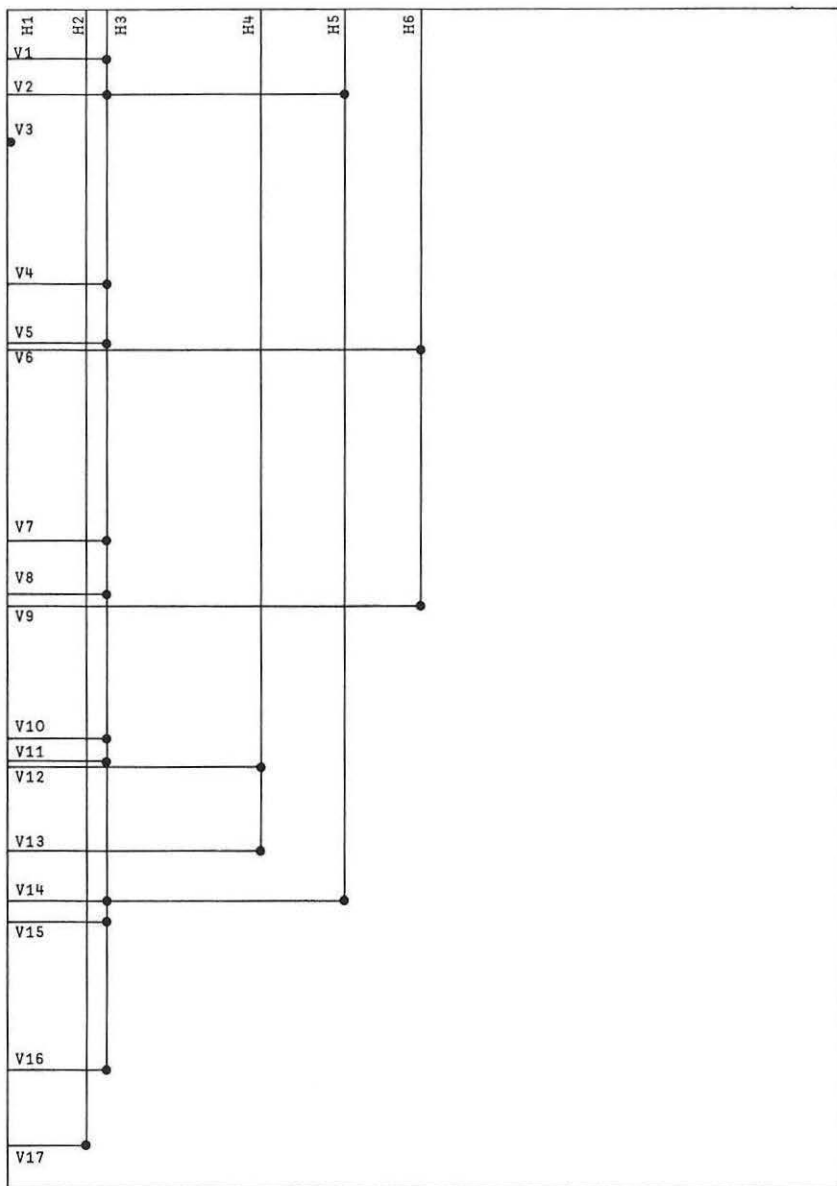


Figure 6. Distribution of location points along horizontal and vertical measures of the old version.

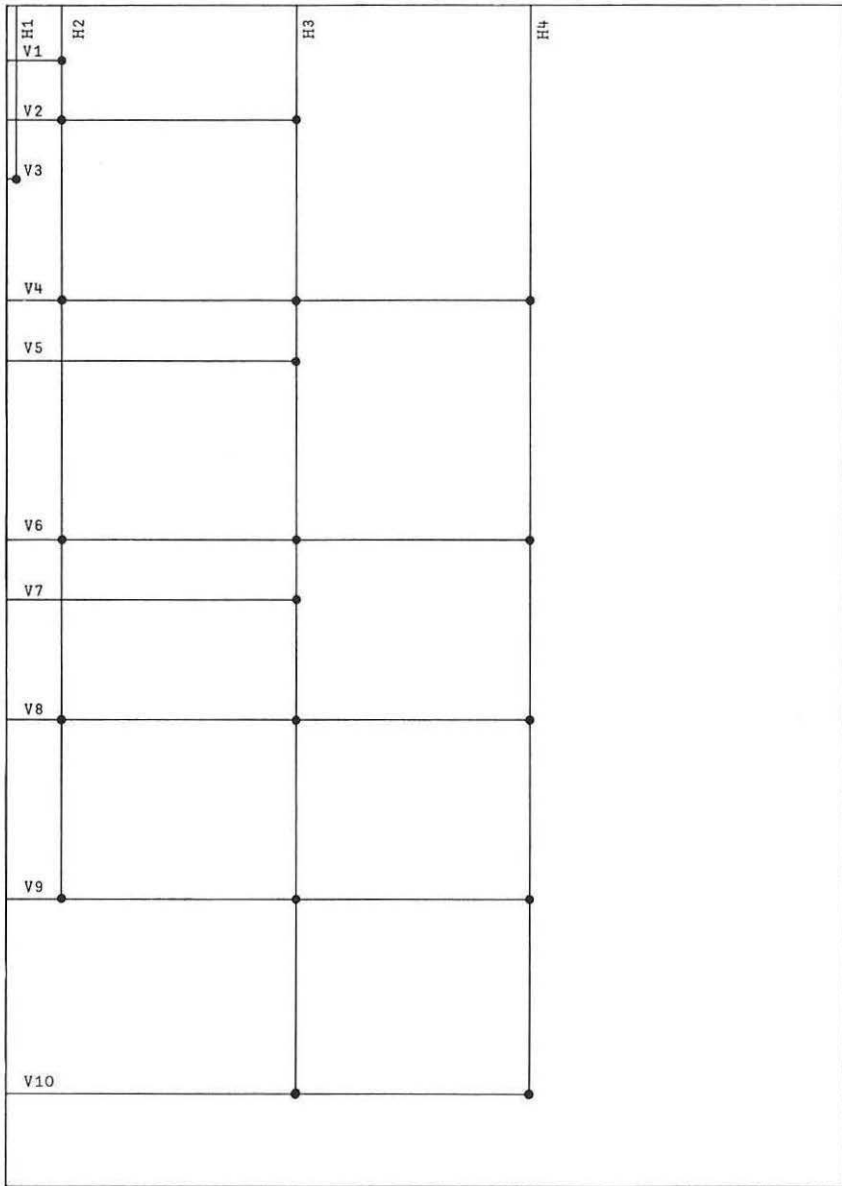


Figure 7. Distribution of location points along horizontal and vertical measures of the new version.

	Form of typeface	Size of typeface	Character of typeface (e.g., bold)	Kind of typeface (e.g., compress)	Inclination of typeface (e.g., italics)	Upper-case (capitals)	Extended composition	Tone background	Inversion	Colored printing	Orientation (e.g., vertical line)	Empty space, one line	Empty space, n lines	Leading up to one line	Stroke	Separation by drawn line	Underlining	Fixed position	New line	Indentation	Surrounded by lines	
Titles																						
Level 1		x										x						x				
Level 2		x										x										
Level 3												x										
Level 4															x						x	
Text																						
Standard																						
Special									x													
Innumeration																					x	
Formula																					x	
Paragraph											x											
Period																					x	
Footnote		x														x						
Reference		x																				
Caption																					x	
Name																						
Merchandise																						
Producer																						
Logotype			x																			
Index																						
Product group ..		x																			x	
Page number		x																			x	
Table																						
																						x
Line drawing																						
																						x
Halftone illustration																						

Figure. 9. Matrix of typographical variables (means) and typographical items (functions). The lines standard text, name of merchandise, and producer's name are empty because the selected size of the typeface served as point of reference.

proviso that although the graphic-typographic system makes it easier to achieve a multiplicity of variations in the individual pages of the work, it cannot replace the executing designer. Moreover, it must be conceded that any problems cropping up cannot be foreseen in all their extent and for this reason cannot be planned for.

The design manual—which is a modest work of its kind—contains just 100 verbalized instructions formulated on two basic axioms: the axiom of economy of means and the axiom of formal coherence.

Work on the manual proceeded in three phases. In phase 1 it was decided which elements went into the system. Method employed: listing. In phase 2 the elements were compiled into element groups. Method employed: testing the dependence of the elements, the functional relationship of the elements. In phase 3 the rules were formulated in the light of the two axioms stated above. One of the familiar rules of thumb (heuristics) used in this connection is illustrated in the following example. The problem is to know which width of type area will go into whole or half cicero measures (cicero = 12 points didot, 13 English-American points) when the type is set in two or three columns with 1 cicero between the columns. Constraints are: paper width 21 cm., minimum fore-edge, maximum back, maximum width of type measure. Ideal solution: 41 cic. (20/1/20 cic.; 13/1/13/1/13 cic.). Solution selected: 38 cic. (18.5/1/18.5; 12/1/12/1/12).

Aesthetics and Order

So far nothing has been said about aesthetic preferences, although aesthetic considerations have naturally had a part to play in the formation of the maxim system. The conclusion that the new version, in comparison with the old, is more beautiful is not one that is susceptible to logical validation. But, as far as the designer is concerned, it may be assumed that the concept of order and the concept of beauty often coincide. To claim that the concepts are synonymous or to subsume the concept of beauty under that of order is possible within the terms of an aesthetic which succeeds in explaining the descriptively analyzed data. Assuming that the responses of a number of text subjects are favorable—i.e., assuming that the new version is felt to be better ordered—an aesthetic theory might adduce the mathematically categorized objective correlate of the subjective judgments as a verification of the hypothesis “designed beauty is a function of designed order.”

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