The Origin of the Sexagesimal System: The Interaction of Language and Writing

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The origin of the sexagesimal system has been much debated, but all past theories have neglected the linguistic evidence of the ancient cuneiform lexica. The problem of origin is twofold: 1) the origin of counting with sixty as a base and 2) the origin of sexagesimal place notation. The first problem is linguistic and anthropological in nature and must be studied through the ancient lexica. The second can be elucidated by a combined analysis of the Sumerian number words and the symbols used to represent them. Such an analysis indicates that sexagesimal place notation arose from an interaction between the numerational framework of the Sumerian language and the symbols used to write those numbers, but the sudden appearance of place notation about 2050 B.C. indicates that the final step toward the creation of place notation was an act of conscious invention.

Readers of this journal may well ask themselves what the sexagesimal system has to do with written language. In fact, a little known but important episode in the history of writing revolves around the sexagesimal system of numeration, and it provides an interesting example of the mutually dynamic relationship between writing and language. For most people, the word sexagesimal brings to mind the division of the circle and the hour, but these are developments subsequent to those with which we are concerned here. Sexagesimal, in the mathematical sense of the word, refers to a system of numeration or numerical notation which rises in powers of sixty. In this paper I use the term sexagesimal to denote the system of numeration employed by the ancient Sumerians—a people who flourished in Southern Iraq during the fourth and third millennia B.C.—in spite of the fact that it does not conform precisely to the mathematical sense of the term.

The Sumerians spoke a language which has no known linguistic affinities with any other language. Our knowledge of Sumerian is dependent upon the fact that the Sumerians, who seem to have been the inventors of writing as a system, used clay and a reed stylus as

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their writing materials. The durable character of the clay which received the imprint of the stylus has resulted in the survival of many thousands of documents written in this language, in the script which has been dubbed "cuneiform" by modern scholars because of the characteristically wedge-shaped markings left by the stylus on the clay. Of only slightly less importance for our understanding of Sumerian is the fact that a Semitic people, speaking a language called Akkadian, adopted the Sumerian script with which to write their own language. Since the Sumerian script employed large numbers of logograms (word-signs, formerly referred to as ideograms), the Semitic peoples of the Tigris-Euphrates Valley developed an elaborate set of lexica, giving—in the more complex series—the sign, its Sumerian pronunciation, and its meaning in Akkadian. It is upon these "lexical series," as they are called by the modern Assyriologist, that one must largely depend for a knowledge of the Sumerian language and its system of numeration. In Figure 1 I have presented the basic outlines of this system of numeration, insofar as one may reconstruct the system from the evidence of the ancient lexica.[1]

Controversy over the origin of the sexagesimal system has revolved around two major points: the origin of the use of sixty as a base and the origin of the system of place notation attested in mathematical texts from the eighteenth century B.C. to the end of cuneiform writing in the first century A.D. Even in antiquity scholars began theorizing about the first point. Theon of Alexandria (4th century A.D.) argued that the Babylonians used the number sixty as a base because of its divisibility. This argument has also been repeated in modern times, but the nineteenth and early twentieth century favored explanations drawn from geometry, astronomy, the length of the solar year, and various combinations of these.[2] There was not a single scholar among this earlier group of theorizers who had sufficient knowledge of the Sumerian language and the documentary evidence—which only became available in large quantity in the twentieth century—to warrant such speculation, and, as is often the case when people debate something about which they have no firsthand knowledge, one groundless theory produced another to support it.

The work on Babylonian mathematics done in this century by François Thureau-Dangin and Otto Neugebauer has had the effect

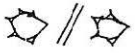
Number	*Number Word*	*Etymological Meaning*	*Cuneiform Notation*
1	diš	unknown	
2	min	unknown	
3	eš	many/much	
4	limmu	unknown	
5	ia	unknown	
6	aš	five, a single one	
7	imin	five, two	
8	ussu	unknown	
9	ilimmu	five, four	
10	u	unknown	
20	niš	unknown	
30	ušu	ten threes	
40	nimin	two twenties	
50	ninnu	two twenties, ten	
60	ĝeš	uncertain	
3600	šar	everything	
216000	šargal	big everything	
12960000	šargal šunutaga	big everything which hand cannot touch	

Figure 1. Sumerian numeration and notation. The number 60 (ĝeš) is multiplied by 2, 3, 4 . . . 10; then 60 times 10 (ĝešu) is multiplied by 2, 3, 4, 5; 60 times 10 times 6 is called šar (i.e., 60^2), which forms the basis for the next stage of numeration. The number šar is multiplied by 2, 3, 4 . . . 10 to arrive at 10 šar (šaru). The next stage is uncertain. It is either: šar times 10 multiplied by 2, 3, 4, 5, (and 6), or šar times 20, 30, 40, 50, (and 60) to arrive at 60^3 (šargal). Multiples of šargal and šargal—šunutaga are not specifically attested. They presumably follow the pattern of šar.

of bringing this barren speculation to an end by providing abundant documentation for the true character of Babylonian mathematics.[3] Both of these scholars advanced their own theories about the origin of the sexagesimal system; Neugebauer's, being the most recently expounded,[4] appears to have the widest vogue among scholars interested in the history of science. These theories merit brief consideration, since they rest in part upon documentary evidence but nevertheless both result in false conclusions.

The Origin of Sixty as a Base

Basic to the understanding of the origin of the Sumerian system of numeration is the recognition that numbers are an integral part of a language and cannot be separated from a discussion of the language as a whole.[5] Thureau-Dangin, as a philologist, quite rightly sought the origin of the use of sixty as a base within the framework of the language itself. The solution to the problem, wrote Thureau-Dangin, was that the Sumerian word for sixty was identical with the word for one. Unfortunately, this identification rests upon the misinterpretation of a passage in one of the ancient lexical series and is therefore invalid.[6] Neugebauer, on the other hand, has maintained since 1927 that the sexagesimal system owed its origin to an original decimal system interacting with the process of weighing and measuring.[7] This theory contains several crucial philological errors and false assumptions regarding the Sumerian metrological system too complicated to go into here, but the basic error is a methodological one: the failure to recognize the linguistic nature of the problem. This error results in the over-emphasis of the graphic representation of the numerals to the exclusion of the evidence offered by the language itself. There is, in fact, no linguistic evidence which supports the idea of a "decimal core" in Sumerian numeration.

As one may see by a glance at Figure 1, Sumerian numeration certainly employs the numeral ten as a *multiplier*, but to refer to this use as a "decimal system" in any sense of the word is to use the word system in a peculiar manner and to ignore the basic linguistic facts. Let us look for a moment at these facts to which I refer. Although ten functions in Sumerian as a multiplier, it is never *multiplied*, as in our decimal system. The assumption, drawn from the system of notation, that ten plays a prominent role in primary Sumerian numeration

(one through sixty) cannot be supported by evidence from the language. Perusal of the chart of numerical notation and etymologies presented in Figure 1 will reveal that the system of notation does not by any means reflect the etymological meanings of the number words. Thus, niš, "twenty," *cannot* be analysed as "two tens." The word ušu, "thirty," has the remarkable etymology "ten threes," *not* "three tens," as one might suppose on the basis of the notation. The word for forty is "two niš," for fifty "two niš, ten"; ğeš, "sixty," is again a word of uncertain etymology. In fact, Sumerian numeration up to sixty has rather more a vigesimal than a decimal character.

Sumerian numeration seems to have begun—as most primitive systems of counting do—by numbering the fingers, one through five. Then one started over again and added up to ten—though eight is anomalous and of uncertain origin. This process was apparently repeated on the toes until one arrived at twenty. Then, beginning with the hands once more, one counted to twenty again by the same method and arrived at forty, which was called "two twenties." Then, starting back on the hand again, one counted to fifty, calling this stage "two twenties, ten," and, continuing on, one counted again on the feet until one reached sixty. This stage should have been—in keeping with the vigesimal stages "two niš" and "two niš, ten"—"three niš." This would be in Sumerian, theoretically, *nišeš, but such an etymology cannot be made to conform with the form ğeš, since the phoneme which we conventionally transcribe /ğ/ is represented in standard cuneiform orthography by syllabic signs beginning with /g/ or /m/, whereas the word niš is only attested spelled with a sign beginning with /n/. The best policy is to leave the question of etymology open for the present.[8]

The Sumerians, having arrived at sixty, now began to count over again, and this process resulted in the formation of the sexagesimal system. The problem is: why did they begin to count over at sixty? This is the crux of the matter, and unfortunately we are not in a position to answer it with certainty so long as the etymology of the word for sixty remains uncertain. The clue to the riddle is probably to be sought in the role played by the numeral three. A unique role for three is indicated not only by the word ušu, "ten threes," for thirty, but also by an alternative system of numeration, in which three plays a central role.

This system of numeration is definitely Sumerian, but it differs radically from the normal system employed in the main dialect. It is attested, oddly enough, only in one copy of a text from the Neo-Babylonian period, some 1500 years after Sumerian ceased to be widely spoken. In this system, the word for three is peš. Four is represented by peš bala, "three passed." Five is peš bala ge, "three passed, one." Six is peš bala gege, "three passed, one, one," and seven is peš peš ge, "three, three, one."[9] As a consequence of this, I am inclined to connect, in some fashion, the word for sixty and the word for three and to see the stage sixty in Sumerian numeration originating from a vigesimal system of counting of the fingers and toes interacting with the special word three, which seems to be identical with the third person plural morpheme of certain forms of the Sumerian verb. This is a solution which is not susceptible of proof at the present time, but it at least conforms to the facts in a way that none of the other theories have done.

The Origin of Place Notation

The Sumerian system of sexagesimal place notation antedates that invented by the Hindus by some two thousand years. It is a fully developed place system of notation like that of our decimal system, with the exception that the absolute value of the numerals (indicated in our system by the decimal point) is not indicated, and a sign for zero is lacking. The absence of these two factors caused Neugebauer to regard place notation as the product of gradual evolution out of the terminology and notation used in the system of weight metrology.[10] There can be little doubt that the standard system of notation provided the necessary conditions for the creation of the place system, but the transition from the old to the new system is brought about by the act of conscious creation. This is vividly illustrated by a text published over fifty years ago—the significance of which has not been recognized—but before we can address ourselves to this piece of evidence, we must give a brief account of the graphic developments which lie behind the invention of place notation.

Around the end of the fourth millennium B.C., the Sumerians invented the system of writing, which in the course of the third millennium developed into what we call cuneiform script. This script did not always possess the wedge-shaped character which has resulted in

its modern name. Originally the script was quite linear, being drawn upon the clay by the tip of the stylus. This writing procedure was especially necessary for executing the graphs containing curved lines, but already in the archeological stratum Uruk IVa (radio-carbon date: 2815±85 B.C.), most of the straight lines appear to be made by simply pressing the edge of the stylus into the clay to form the line. In the course of time, broadening of the tip of the stylus resulted in the wedge-shaped script called cuneiform. The transition from linear to cuneiform script is more or less fully accomplished by about 2600 B.C.[11]

Unlike the rest of the language, the Sumerian numerals were never written in a linear script, so far as we know. The Sumerians used two distinct reeds to execute the graphs in their system of numerical notation. One of these reeds was identical with the standard stylus, but in making numerals the end opposite to the usual writing end was employed, as though we, writing on a soft material with a pencil, should turn the eraser end of the pencil around and push it into the material to make number symbols. The number signs made with the standard reed were those for one and ten, and this method of making ten is reflected in the Akkadian name, *gigurû* (from Sumerian *gigura, "turned reed"), for the sign used to represent ten, long after the sign ceased to be made in this way. The other reed used to write numerals seems to have been originally twice the diameter of the standard reed. This double-diameter reed was used to make the signs for sixty and 3600.

The signs made by these two styli are illustrated in Figure 2. The signs for one and sixty are executed by holding the round ends of the styli at an angle of about forty-five degrees to the tablet and pressing

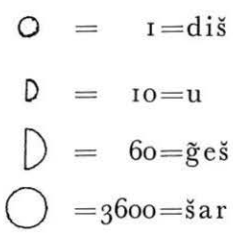


Figure 2. Archaic (rounded) Sumerian numerals.

them into the clay. The signs for ten and 3600 are made by holding the styli perpendicular to the tablet and pushing the round ends into the clay, the small one (the normal stylus, also used to make one) to make ten, and the larger one (also used to make sixty) to make 3600. It has sometimes been assumed that the signs for sixty and 3600 could be thought of as a "big one" or a "big ten," but actually these signs have nothing in common but their superficial resemblance and their character as numerals. The Sumerians could have written their numerals in linear script just as easily as the rest of their language, but the inventor or inventors of the script chose not to do this, for the obvious reason that the notation produced by the round ends of the styli stood out vividly from the linear script and made the process of totaling and book-keeping much easier. Indeed, from the standpoint of visual communication, the early notation with "rounded" or "curvilinear" numerals is certainly superior to the later cuneiform notation.

Why then, one may ask, did the Sumerians abandon a superior system of visual communication for an inferior one? The answer to this question is to be sought in the exigencies of the writing process. The use of round numbers passes out of existence as the need for more economical expression is felt. Round numerals took up a disproportionately large part of the available writing space on the tablet. In the earlier documents, the thing which is being counted and its intended use are represented in such an abbreviated manner that it is still impossible for us to understand much more than what the object is. As time went on the script became more precise, which enables us to understand what is being done with the objects enumerated, but with precision came the need for more room to write and therefore reduction of the space available for numerals. Finally also, the cumbersome process of inverting the stylus to make the numerals one and ten and of changing to another reed to make sixty and 3600 was abandoned, and numerals came to be written with the same tip of the stylus that one used to write the rest of the language.

This was not a process which happened very quickly. The rudiments of standard Sumerian numerical notation can be discerned in tablets from Jemdet Nasr,[12] a site in northern Babylonia, which belongs to a time period somewhat later than the earliest examples of systematic writing. The earliest examples of writing as a system come

from the archeological stratum Uruk IVa (2815±85 B.C.),[13] and the Jemdet Nasr tablets may belong to the twenty-eighth century B.C. It is not until the so-called Presargonic period (ca. 2500–2350 B.C.) that one begins to find evidence of the use of protocuneiform numerals in certain cases in place of round numerals, and it is another two centuries before round numerals are abandoned for cuneiform notation, corresponding in general to that shown in Figure 1. This last change is itself a gradual one and seems to have extended over a period of about a generation at the end of the reign of Shulgi, king of Ur (ca. 2094–2047), and the first part of the reign of his son, Amar-Su'ena (ca. 2046–2038).

The result of this process was that the sign for ten came to be written by a sideways thrust of the stylus into the clay to produce the sign shown in Figure 1. This is really the evolution of a practice already discernible in the Presargonic period, which represented ten by simply holding the standard writing stylus perpendicular to the tablet and pushing the normal writing tip of the stylus into the clay instead of inverting it as previously done.[14] The round sign for 3600, originally written with the double-diameter round reed, was represented in the new notation by four or five cuneiform wedges arranged in a more or less circular shape, as shown in Figure 1. The sign for sixty, however, came to be written with the standard stylus, in the same manner as the sign for one. At first, the sixty is written larger than the one, but from the beginning, the distinction between these two signs was so small, they could only be easily distinguished by their position in the script.

From the standpoint of unequivocal visual linguistic communication, this type of notation has serious drawbacks, since it is sometimes impossible for us to tell, lacking the context which must have made doubt impossible for the Sumerians, whether one or sixty sheep, cows, or workmen are intended by a single vertical wedge. The Akkadians, who inherited the Sumerian system of writing, also felt this problem and solved it by writing their word for sixty after those signs which stood for sixties rather than ones. Oddly enough, it was precisely this defect, this lack of distinction stemming from unplanned development of the script, which was to play a fundamental role in the invention of sexagesimal place notation and, therefore, indirectly in the development of Babylonian mathematics. But, in spite of this acci-

dent, sexagesimal place notation may never have been invented, had it not been for the role played by the system of weight metrology.

The system of weight metrology is probably the youngest of the three primary systems: length, capacity, weight. It is also the system which most closely reflects the sexagesimal character of Sumerian numeration. The basic units of this system were, from smallest to largest: the barleycorn, the shekel, the mina, and the talent. The notation used for the three larger units during the latter part of the Third Dynasty of Ur (after ca. 2050 B.C.) is illustrated in Figure 3. It will be observed that these three units stand in a perfect sexagesimal relation to one another, for there are sixty shekels in the mina, and sixty minas in the talent. Higher multiples of the talent are counted and written in the normal system illustrated in Figure 1. The composition of the shekel departs from the strict sexagesimal pattern in that it is composed of three sixties of barleycorns instead of one sixty, for reasons which are too complicated to go into here.[15] The mina was the central unit of the system, and it was possible, at least in certain cases, for the Sumerians to express fractions such as one-sixth by the term "ten shekels." The signs in the bottom row of Figure 3 easily lend themselves to the mathematical interpretation: 60^1, 60^0, 60^{-1}.

Thus one may see the system of weight metrology and its notation as the immediate background to the invention of the place notation system, but the existence of this prototype system must not be allowed to obscure the striking departures from standard notation found in the new place system. The system of place notation cannot be said simply to "evolve" out of standard notation employed in the weight system: it is to be regarded as the product of conscious human invention.

The new features of the place systems are decisive in favor of invention as opposed to evolution. These new features are: (1) abandonment of all number signs except those used in writing the numbers one to sixty, which reduces the number of symbols used to two: that for one and that for ten; (2) the use of this notation, in a manner analogous to our decimal notation, to write the smallest fractions on up to the highest integers; (3) abandonment of the standard signs used to write four, seven, eight, nine, and forty (as shown in Figure 1) for a new notation with better visual contrast (as shown in Figure 4). This system, for which a gradual evolution out of the metrological system

Number	*Talent*	*Mina*	*Shekel*
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
20			$\frac{1}{3}$ mina
30			$\frac{1}{2}$ mina
40			$\frac{2}{3}$ mina
50			$\frac{5}{6}$ mina
60			

Figure 3. Notation of weight units.

has been posited,[16] is shown fully developed in a text which would appear to date from the fifth year of Amar-Su'ena (ca. 2042 B.C.), king of Ur.[17] Moreover, it appears on this tablet as an instrument of calculation, in which the weights of certain commodities written in the old system of notation are restated in the new place system, and thus *evolution* out of the metrological system is excluded as the sufficient cause for sexagesimal place notation.

sign NI, with which the word niš is spelled in the lexica, is never used, to my knowledge, to represent initial /ǧ/.

9. I have discussed this in *Sumerian Numeration and Metrology*, pp. 28–32.

10. In *Vorgriechische Mathematik* (1934), p. 108, Neugebauer wrote:
"But origin [of the place notation system] out of the originally concrete weight notation also gives us immediately place notation. This is nothing but an omission of specifically naming the units of measure. If this place notation owed its origin to conscious mathematical reflection, it would be impossible to imagine how one could have failed to introduce a zero sign for lacking places."

11. A major problem in attempting to deal with graphic history in the first half of the third millenium B.C. is the lack of any certain point of chronological reference. For dating examples of writing before ca. 2500, we are still dependent upon stylistic considerations. Where there is a clear order of development, the style of writing is useful in providing a relative chronology, but this does not tell us how much time elapsed between each development. The dates used in this paper, though more or less those used by scholars in the field, must be regarded as strictly provisional.

12. See Stephen Langdon, *The Herbert Weld Collection in the Ashmolean Museum: Pictographic Inscriptions from Jemdet Nasr* (Oxford Editions of Cuneiform Texts 7, 1928), esp. no. 100.

13. The oldest corpus of texts is that published by Adam Falkenstein, *Archaische Texte aus Uruk* (Ausgrabungen der Deutschen Forschungsgemeinschaft in Uruk-Warka 2, 1936).

14. This practice also accounts for the diverse shapes of the sign for ten, which include three-sided and four-sided forms. This also tells us, incidentally, what the tip of a stylus looked like. For references to the various sign forms, see Anton Deimel, *Die Inschriften von Fara I: Liste der archaischen Keilschriftzeichen* (1922), no. 821.

15. I have treated this in *Sumerian Numeration and Metrology*, pp. 208–211.

16. *Cf.* note 10. This was also the position of Thureau-Dangin; *cf. Osiris*, VII (1939), 110–111.

17. Clarence E. Keiser, *Selected Temple Documents of the Ur Dynasty* (Yale Oriental Series 4, 1919), no. 293. The date is not absolutely certain, because the name of the king is not given in the date formula, and the formula itself is abbreviated. An alternative date might be the second year of Ibbi-Sin, fifteen years later.

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