The Research Connection

Visible Language is broadening its role as a disseminator of often inaccessible information. It is becoming a vehicle for the exchange of information on research and experiment in design schools. Where are visible language innovations, insights, technological developments, teaching experimentation, and perceptual explorations taking place? We intend (with your co-operation) to find out. We are beginning a census of design schools and design departments with reference to their programs for research and experiment in visible language. It is our plan to collect information and develop a taxonomy which will relate similar and complementary design research. *Visible Language* will report on work recently done or in progress as well as selected research monographs—functioning as a clearinghouse for designers involved in similar research projects.

The value of such an undertaking became clear last summer as I participated in the Edugraphic conference sponsored by ICOGRADA at the University of Alberta in Canada. I was specifically involved in the workshops on research and experiment in visual design. Many of the workshop participants desired better access to research and improved contact possibilities with those interested in similar areas of research.

The *Visible Language* census is designed to fill this void. During the coming months, you will receive a census form soliciting information. It is your collective participation that will make this census a significant resource. Any inquiry regarding this project can be directed to: Visible Language, Prof. Sharon H. Poggenpohl, The Institute of Design, 3360 South State Street, Chicago, Illinois 60626 USA.

The Quipu as a Visible Language

Marcia Ascher and Robert Ascher

The Inca are often cited as a civilization "without writing." But writing is more than a record of language sounds placed upon familiar materials. The media of the Inca were devices made of cotton cords that are called quipus. This introduction to the quipu is based upon a recent study of most of the world's known quipus now spread throughout three continents and concentrates on what we infer to be the way the physical elements of quipus are combined to create a symbolic structure; i.e., the representation of numbers, the expression of N-dimensional arrays, and hierarchical configurations. A discussion of the connections between the quipu and civilization includes: (1) cotton as a material which carried its own message for the Inca; (2) reflections of the quipu in non-media domains of Inca civilization; and (3) the purpose of writing in early civilization.

Around five thousand years ago a major change in human history was well underway. Compared to the preceding several million years, a new kind of human organization had emerged more or less simultaneously at several places on earth. The term civilization is applied to the phenomena, and it is used with reference to its local manifestations: we speak, for example, of Chinese civilization or of Egyptian civilization.

It is one thing to discuss civilization in sweeping terms and quite a different matter to describe exactly what it is. One approach is to compose a list of things that characterize early civilizations, such as strong centralized authority, massive building programs, and occupational specializations. Almost all lists show "writing" in a position very close to the top.

The problem is that some human organizations that appeared to be very much like civilizations in all other respects did not "have writing." What are we to make of a people who built a road system that ran thousands of miles linking cities and villages,

Visible Language, IX 4 (Autumn 1975), pp. 329-356

Marcia Ascher's address: Department of Mathematics, Ithaca College, Ithaca, NY 14850.

maintained store houses, irrigation canals, and mines, and indeed kept records, all without writing? The Inca, who are at issue here, continue to be one of the few cases that do not fit the list makers' definition.

In the past the solutions to the problem of a civilization "without writing" have been to neatly skirt the issue, or to deny the civilization. Skirting around it, for example, is done by Rowe (1946): "The fact is that the Andean peoples possessed substitutes for writing which were so satisfactory that they probably never felt the need for anything more elaborate." The strength of feeling of those who hold the alternate opinion is evident in the following response to the above: "Such a conclusion [that proposed by Rowe] is in praise of intellectual castration; it belies the whole of culture history. For of all discoveries and inventions, writing gave us the continuity of that we are pleased to call civilization . . ." (Von Hagen, 1961). The main "substitute" for writing that Rowe refers to is a device called a quipu. This, for the moment, can be described as a device made out of cotton cords.

Most opinion favors the first solution illustrated by the citation from Rowe: that is, (a) writing is implicitly retained as a mark of civilization; (b) the Inca were a civilization even if they did not "have writing" because (c) they had something else, namely quipus. It is interesting that this solution is reached from very different starting positions. In the context of orthodox Marxist analysis, for example, Valćarel (1965) asks why it is that the Inca did not write when Marxist-Leninism has demonstrated that all civilizations follow the same road. His answer is that the Incas had, in particular, quipus, and that these devices when fully deciphered will resolve the issue.

It seems that the favored solution results in a paradox. Our approach began with the suspicion—a suspicion that quickly grew into a working hypotheses—that the ordinary conception of writing is too restrictive. If the Inca "substitute" is a visible language, the paradox vanishes. At the least, quipus (and other similar devices) should force us to reconsider what is meant by writing.

Some authors—most recently Pirsig (1975)—go so far as to claim that the sacredness attached to the spoken and written word is culture-bound to western civilization. In our view, writing is clearly not a record of speech only. A glance at a musical score, a chemical journal, or a mathematics text, is sufficient to demonstrate the point. Quipus were seen by westerners in the sixteenth century. These people wrote chronicles, but they did not understand what they beheld. They thought that quipus were writing—that is, written speech. When it was finally determined that quipus did not record the sounds of language (Locke, 1932), any value attached to them disappeared. It is as if language written down, say in alphabetic letters, is so closely associated with writing that other forms of visible language do not count: this is the western bias that turns everything else into a "substitute."

What follows is a discussion of the quipu organized around two issues. In the first, focus is on selected aspects of the way physical elements are combined to create a symbolic structure. This is followed by a discussion of the connections between the quipu and civilization. Many issues of a more than elementary nature are left out; further, the issues that are discussed are not treated exhaustively. In the end, the reader should be in a better position to appreciate the quipu as a visual language medium.

Preliminaries

Basic quipu description. A quipu has a main cord from which knotted cords are suspended. When laid flat on a table, with the main cord placed horizontally, the majority of suspended cords fall one way (downward) but sometimes a few are directed upward. Closer examination shows that not all cords are attached to the main cord; some are attached to these attachments. The attachment is tight so that all cords have a fixed position on whatever cord they are hung. The cords are made of colored cotton. Some rare exceptions are made of colored wool. The knots on each cord are formed into clusters; that is, a few knots, then a distant space with no knots, then a few more knots, etc. The knots are of three different types.

Figure 1 is a schematic of a minimal quipu. (1) The *main cord* has a color, say brown (B). The ends of this cord are free and can be distinguished from each other by the manner in which they are finished. Hence, uniformly, directionality can be attributed to the

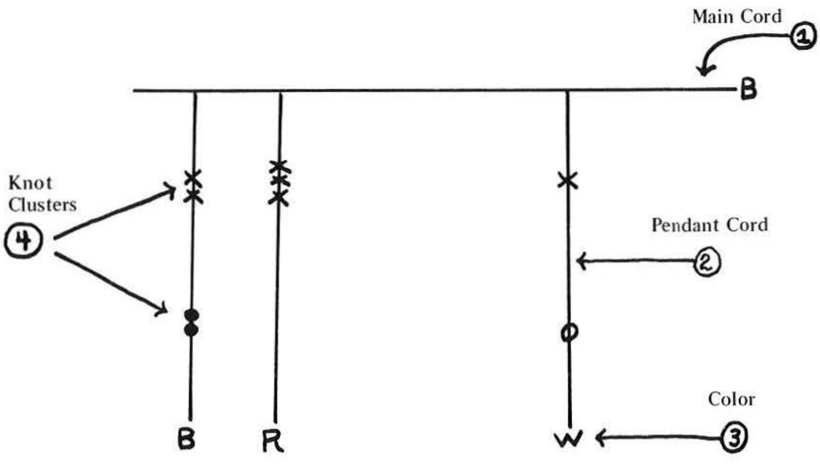
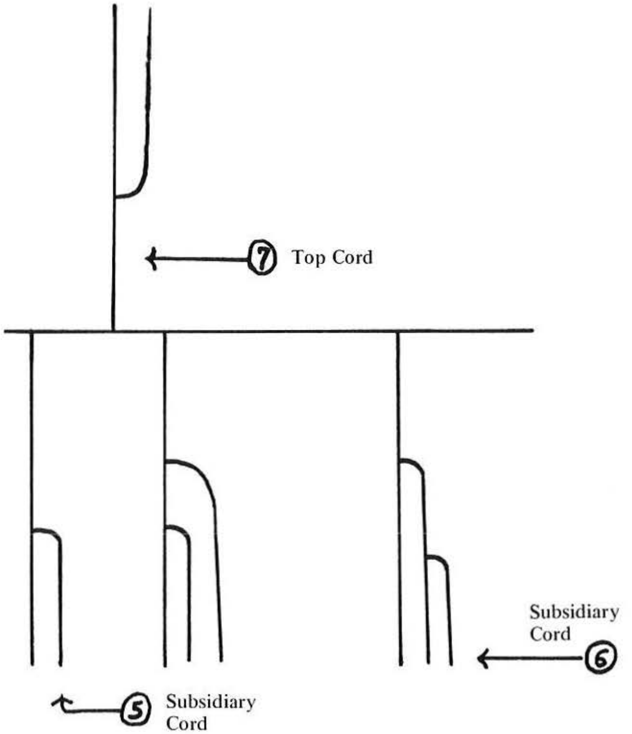


Figure 1. Schematic of a minimal quipu showing (1) main cord, (2) pendant cords, (3) color designation, and (4) knot clusters.

Figure 2. Schematic of a quipu with subsidiary cords and top cord in addition to main cord and pendant cords.



main cord. (2) *Pendant cords* are suspended from the main cord. They have a distinct position on the main cord. Notice that a larger space separates the second and third than separates the first and second. (3) A *color* is associated with each pendant cord, say brown (B), red (R), white (W). (4) *Knot clusters* appear on the pendant cords. Here x, •, o denote the three different knot types. The clusters are defined by the spaces between them as well as the knots in them. Some pendants have no knots.

In Figure 2 additional cords have been included. (All, of course, have colors and most have knots as described above.) (5) *Subsidiary cords* are attached to the pendant cords. A pendant cord can have more than one subsidiary as on the second cord; or, as on the third cord (6), *subsidiary cords* are attached to other subsidiaries. (7) *Top cords* are directed upward (as compared to pendants which are considered downward). Top cords too can have subsidiaries.

Quipus vary considerably in size and elaboration. The quipu in Figure 1 is minimal in that it contains the most basic elements and few cords. Small quipus have about 8 or 10 pendants, most have between 40 and 150 pendants, and large ones can contain over 1,000 pendants. The cords vary in length from about 15 cm. to 75 cm., but on any individual quipu most of the pendants are about the same length. Many quipus, but not all, have subsidiaries. An individual quipu can have as few as 1 subsidiary or, as in Figure 2, more subsidiaries than pendant cords. Some quipus have a few top cords on them. These are the most common elements; other elements appear on individual quipus, or on a few quipus. Figure 3 is a photograph of a quipu showing cords and knots.

Recent History. In the belief that quipus were ungodly, the western conquerors of the Incas undertook to destroy quipus by burning. From the point of view of the Incas, it was godly to bury with the dead the trappings they used in life. Hundreds of years after the Inca state had passed, westerners started to dig up the graves. This time the quipus were given, sold, or otherwise found their way to private collections or to major museums in North America, South America, and Europe. In every case, quipus that are known to us come from graves.

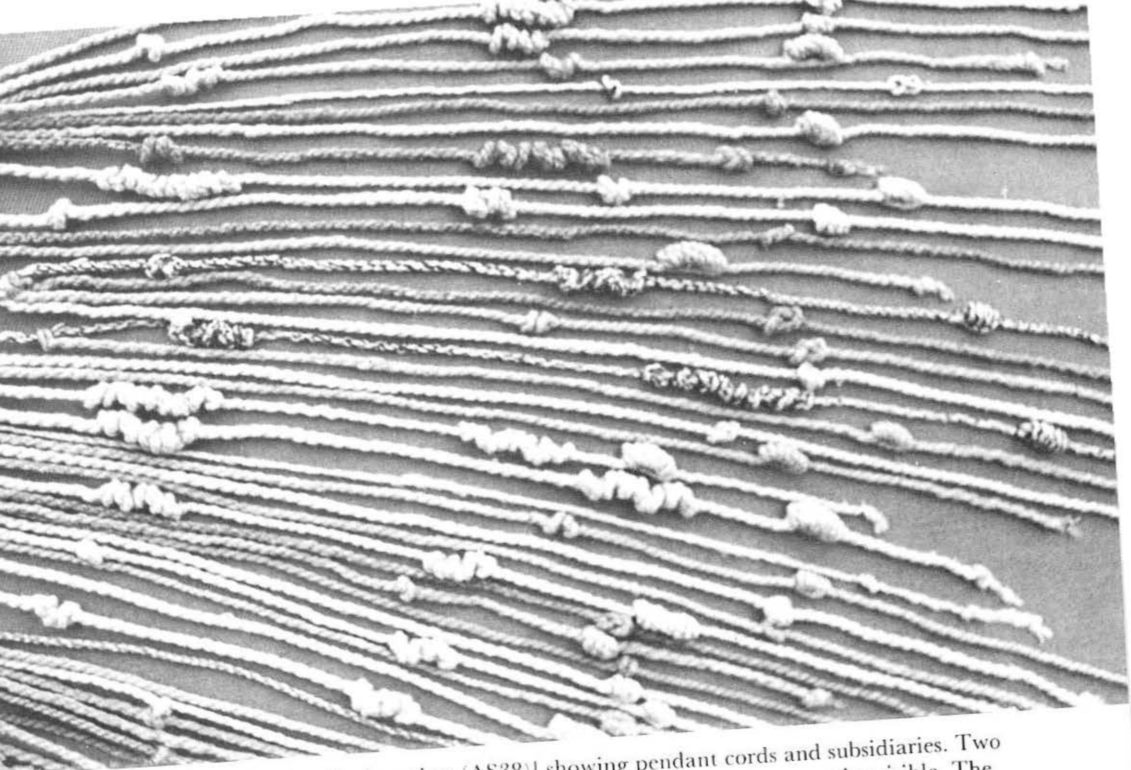
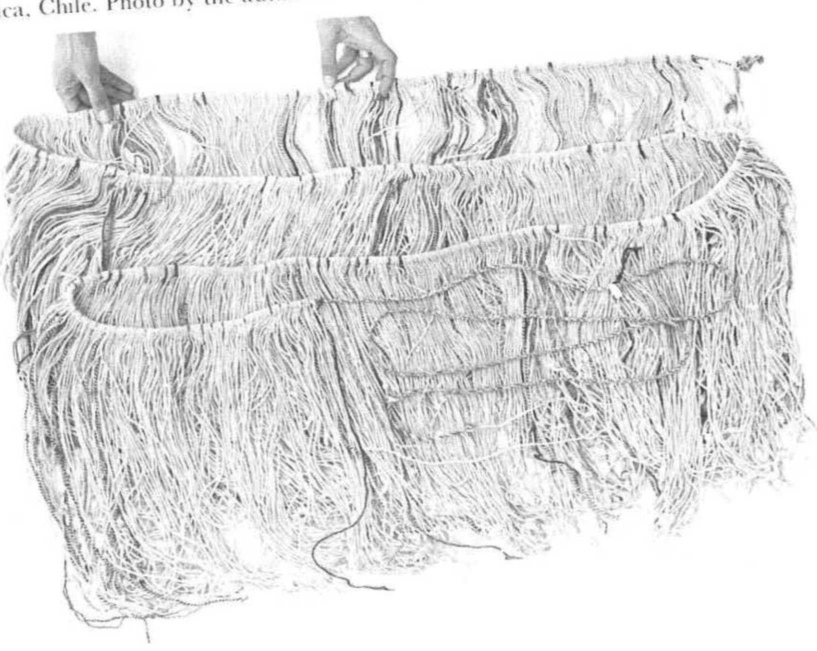


Figure 3. Detail of a quipu (AS38)[1] showing pendant cords and subsidiaries. Two of the three major knot types—single knots and long knots—are also visible. The quipu is in the collection of the Museo Nacional de Anthropologiá y Arqueologiá, Lima, Peru. Photo by the authors.

Figure 4. The largest known quipu (AS69). Collection of Percy Dauelsberg, Arica, Chile. Photo by the authors with the permission of Mr. Dauelsberg.



Cotton quipus survived in graves where conditions of preservation were favorable; that is, in the desert area between the Andes to the west and the Pacific to the east. The specimens vary widely in condition: some fall apart upon touch; others are as fresh as yesterday's purchase of cord. With some exceptions, quipus were deliberately separated from other items in the graves; when the grave site is given, it is usually identified in vague terms such as by the name of a nearby city.

Prior to starting our research, about 60 quipus had been published in 14 separate articles and monographs (Ascher and Ascher, 1969). In 1972 we added nine descriptions of quipus to the literature (Ascher and Ascher, 1972). Then in 1974 we began to search for all the remaining quipus in the world. So far, we have located about 475 specimens that are in 30 different places spread across three continents.[2] As of now, we have studied 400 specimens and about half of these are individually recorded in schematics, drawings, photographs, tables, and words (Ascher and Ascher, 1975).[3] Any interested person, we believe, will be able to use our primary descriptions to test independently or formulate his own notions.

Included in the new data are the largest specimens in the world. For example, the specimen shown in Figure 4 contains 1800 pendant cords. Also in this group are quipus that are hung from carved wood frames (Figs. 5 and 6). Our research turned up several instances where the context of the quipu recovery can be reconstructed. Instead of the generality that quipus are found in graves, we now have retraced the association of some particular quipus with fragments of pottery, pieces of cloth, baskets, decorated bags, and other objects. Perhaps of greater importance, we know of cases where a group of quipu can be assigned to a specific quipu maker. The generalizations that follow are based upon all of this work.

The Symbolic Structure

In the preliminaries, a quipu was described as consisting of cords that have knots, colors, and particular positions in a spatial configuration. On most of the quipus the knot clusters form a symbolic representation of numbers. Once each cord is read as a number, a

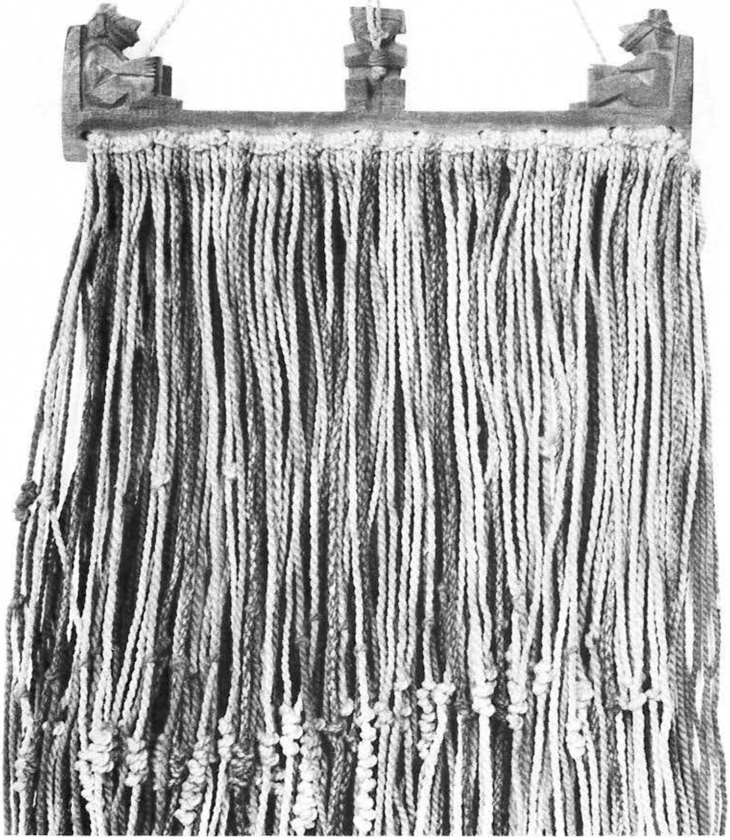
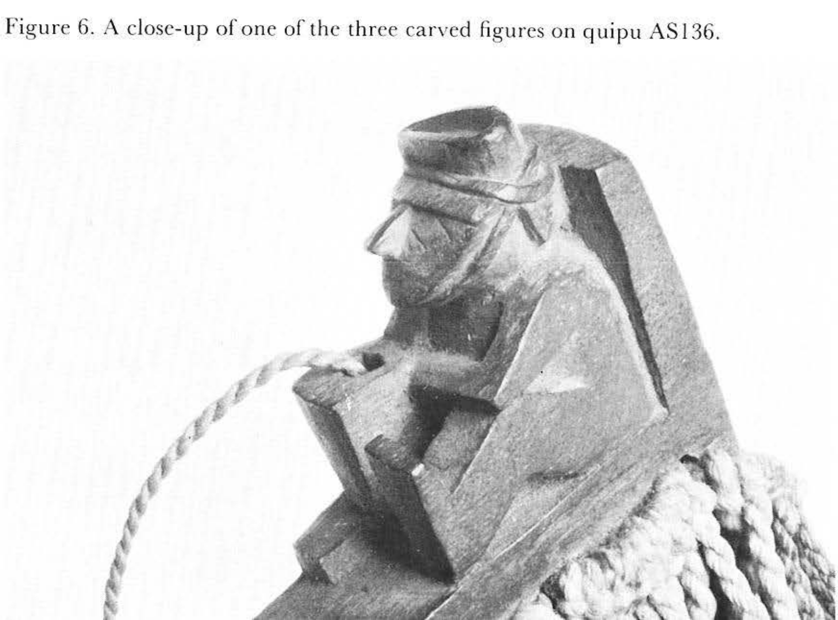


Figure 5. An unusual quipu (AS136) in that it is suspended from a carved wood bar. The lower part of the pendant cords are not shown in the photograph. Collection of Museum für Völkerkunde, Berlin, Staatliche Museen Preussischer Kulturbesitz, Dept. Amerikanische Archäologie. Photo by the Museum at the request of the authors.

Figure 6. A close-up of one of the three carved figures on quipu AS136.



quipu can be viewed as a spatial array of differently colored numbers. Since it is a carefully constructed array, the configuration which is defined by the color and placement of the cords also constitutes part of the overall symbolic system of the quipus. Two types of configurations are included here for close scrutiny. They have been selected because of their frequent appearance and because their logical underpinning cannot be ignored when such specimens are studied further. Both are amenable to being expressed in familiar terms keeping in mind that it is only the logic that is analogous. In both cases we focus first, through example, on the logic with which we are concerned and then on the expression of this logic using the quipu elements that have been introduced. The discussion of two such different configurations, and the fact that these can be combined to form one more complex configuration, should convey that the quipu is not a single-purpose device.

Since the configurations to be discussed both depend on cords being read as numbers, we first look at the way numbers are represented symbolically and some ideas involved in such a representation.

A positional number system. If, in answer to the question "How many?", 5 fingers are held up, or 5 pebbles are amassed, or 5 scratch marks are made, the respondent is merely carrying out a matching process by placing 2 sets of objects in 1-1 correspondence. More abstraction is shown by developing a name or symbol to represent the quantity 5. Also, the use of a special name or symbol for each quantity indicates that the concept of order among quantities exists. Because they are on strings, the number representation on quipus have frequently been erroneously classed with the less sophisticated one-to-one string records. The knots used on the quipu are, instead, a symbolic representation. The representation they use is, in fact, basically the same as ours: it is a base 10 positional system.

A base X positional system means that the value of each position is dependent on a different power of the base X. In our number system, each consecutive position, moving toward the left, is one higher power of ten. For example:

$9432 = (9 \times 1000) + (4 \times 100) + (3 \times 10) + (2 \times 1)$

In our system, only 10 different symbols (0, 1, 2, . . ., 9) are needed. The placement of the symbols and the knowledge that the base is 10 is all that is needed for the representation of any value. To use, for example, a base 6 positional system, 6 different symbols are needed and each consecutive position is a higher power of 6. Using the symbols (0, 1, 2, . . ., 5), the base six numbers 231, 154, and 11 correspond to what in base 10 are symbolized as 91, 70, and 7.

$$\mathit{2} \times 36 + \mathit{3} \times 6 + \mathit{1} \times 1 = \mathit{9} \times 10 + \mathit{1} \times 1$$
$$\mathit{1} \times 6^2 + \mathit{5} \times 6 + \mathit{4} \times 1 = \mathit{7} \times 10 + \mathit{0} \times 1$$
$$\mathit{1} \times 6 + \mathit{1} \times 1 = \mathit{7} \times 1$$

In the history of number representation, non-positional systems and positional systems with other bases have been used. European adoption of the Hindu-Arabic base 10 positional system happened sometime in the fourteenth century. Prior to that, the non-positional Roman numbers were in use.

In order to appreciate the significance of the invention of a positional system, briefly consider the non-positional Roman system. There are distinct symbols for 1, 5, 10, 50, 100, 500, 1000 (I, V, X, L, C, D, M); these are added or subtracted to arrive at other values. When the symbols are placed in descending value moving toward the right, they are additive (VII=5+1+1). When a smaller valued symbol precedes a larger one, it is subtractive (IX=10−1). The subtractive property is usually only used with single symbols (i.e., 90=XC while 80=LXXX rather than XXC) that are close valued (i.e., 95=XCV rather than VC) but not necessarily (i.e., 1900=MCM rather than MDMCD). It can be observed that the order of magnitude of numbers is not related to the size of their representation. For example, M is greater in value than XXXVIII. In the positional system, the size and value are related: 3724 is greater than 899. Another important contrast is that in the positional system, no new symbols need be introduced as the order of magnitude grows. For example, try to write the equivalent of 4,200,000,000 in Roman numbers. Apparently, although we rarely see them now, horizontal strokes above a number were used to indicate thousand-fold and vertical strokes on the sides to indicate hundred thousands.

CDXX = 420 | $\mid\text{XLII}\mid = 42000000$
$\overline{\text{XLII}} = 42000$ | $\mid\text{MMMMCC}\mid = 4200000000$

Most importantly, the use of the positional representation simplifies arithmetic as the symbols are manipulated in a formal repetitive way to arrive at results. In a positional system, in order to add 2 values, the rightmost positions are lined up, and starting at the right, addition is done position by position. If the sum of the 2 digits in a position is less than 10 (or whatever base is used), the value is recorded and one moves on to the next position. If the sum is greater than 10, the excess above 10 is recorded, and a 1 is added into the next position. Thus, to add any 2 values, the only knowledge needed is the sum of any 2 digits. This is clearly not the case with a non-positional system where the relative values of the symbols and their specific placement must be considered:

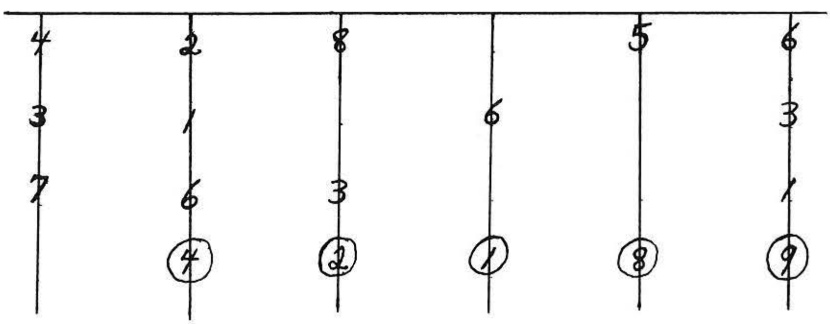
CCXLVII	DCCXXVII
+CCCXII	+DXLVI
DLIX	MCCCLXXIII

Although the greater flexibility and simplicity of a positional system makes it preferable, the choice of base is completely arbitrary. For example, most modern computers work in a base 2 (binary) system because of the ease of electronic representation and manipulation when only 2 different symbols are involved. Many people working directly with computers become adept at the use of a base 8 (octal) system not only because of its ease of conversion to base 2 but also because of its more concise representation of values [Ex. 110011 (base 2) = 63 (base 8) = 51 (base 10)].

The quipu maker used a base 10 positional system. From the above discussion, it should be clear that although this is most familiar to us, the Inca use of this representation was neither necessary or to have been expected. On each string there are clusters of knots. Each cluster contains 1 to 9 individual knots. Each consecutive cluster position, moving from the free end of a cord to where it is attached to another cord, is one higher power of 10.

A crucial concept in the development of a positional system is the concept of "zero." The concept of zero is a big idea in mathematics and still causes problems for schoolchildren. The number 407 is quite different in value from the number 47: a symbol for "none" or "nothing" is placed in the second position in order to have the 4 fall in the third position. The concept of zero can be divided into two parts: first—the understanding that positions containing nothing contribute to the value of a number; second—that a special symbol is necessary for the representation of "nothing." In the absence of a special symbol, there must be means of distinguishing positions; nevertheless some ambiguity can remain.

The quipu uses the idea of zero with no special symbol for zero. Instead, two tactics are used to make the representation clear. One is the type of knots used in the units cluster differs from the type of knot used in the rest of the clusters. The knot type is standardardized: long knots and figure eight knots for units, single knots for the others. Secondly, the cluster positions are aligned from string to string, and so a space left on one string becomes apparent when related to the others. No problem is found when reading the following set of numbers (transcribed with our symbols for 1-9; circled for units; no zero symbol; and increasing powers of 10 from bottom to top):



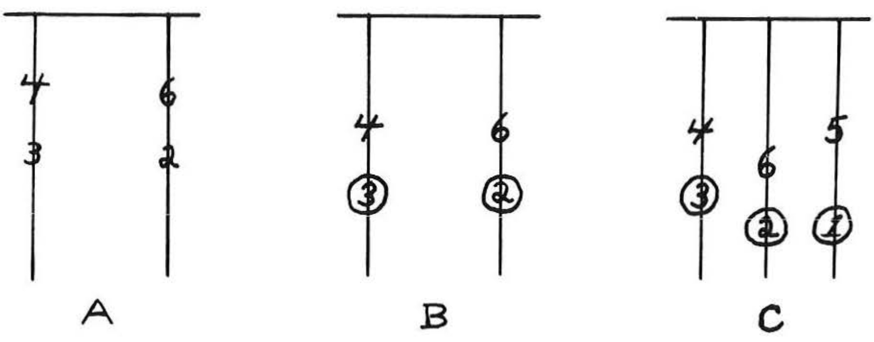
While for the most part the lack of a symbol for zero causes little ambiguity in the reading of quipus, two types of problems do arise. One is inherent in the symbolic representation, and the other is due to the fact that, as with any handwritten document, we are dependent on the "penmanship" of the quipu maker. The first problem is this: if *all* the numbers on a quipu lack knot clusters in the same position, there is no way of knowing that the position exists. For example, if an entire quipu consisted of cords carrying

430 and 620, it would look like Figure 7A. This is indistinguishable from 4300 and 6200. The missed position could also be between clusters. For example, Figure 7B could represent 43 and 62 or 403 and 602 if these were the only values present. The second problem is that ambiguity arises when knot clusters have not been carefully aligned. For example, the numbers in Figure 7C might be 43, 62, 51; or 403, 62, 501; or even 43, 62, 501.

If there were no means of corroborating this numerical interpretation, it would remain only a hypothesis. The evidence that convinces us comes from the juxtaposition of the strings. When we read each cord as a base 10 number and further examine the colored spatial array of these numbers, we find some numerical relationships that cannot be due to chance. The simplest of these is that on several quipus, a single cord dangles from the end of the main cord and when all the pendants are interpreted as base 10 numbers, and the single end cord is similarly interpreted, it is found that the end cord number is the sum of all the other numbers.

N-dimensional arrays. From the color and spatial arrangements of the cords, logical relationships between numbers can be inferred. Before dealing with quipus, let us focus on the ideas of matrices, subscripts, and dimensionality. Suppose we have our standard media, namely pencil and paper, and are given the following information: the costs of 4 different automobiles sold in Detroit in

Figure 7. Quipus with ambiguous numbers. (Digits represent knots in a cluster, circle indicates units position).



	1973	1974	1975
car 1	2841	2963	2900
car 2	3442	4013	4207
car 3	3380	3365	3162
car 4	2964	3268	3574

Figure 8. A two-dimensional array.

Figure 9. A three-dimensional array.

	DETROIT			NEW YORK			LOS ANGELES		
	1973	1974	1975	1973	1974	1975	1973	1974	1975
car 1	2841	2963	2900	2763	3605	3120	2941	2990	3002
car 2	3442	4013	4207	3084	3712	4113	3540	4089	4300
car 3	3380	3365	3162	2961	3364	3248	3470	3378	3251
car 4	2964	3268	3574	2964	3486	3447	3012	3300	3600

1975 are 2900, 4207, 3162, and 3574 dollars. These costs can be recorded as a one dimensional array: (car 1, car 2, car 3, car 4) = (2900, 4207, 3162, 3574). If we were given the cost of each of the cars for several years, a two dimensional array (Fig. 8) could be used to record them and retain both the distinction as to the car and the year. This two dimensional array can be described as a matrix containing elements a_{ij} where i = 1, 2, 3, 4 and corresponds to rows (cars) and j = 1, 2, 3 and corresponds to columns (years). Hence, $a_{13} = 2900$; $a_{42} = 3268$; etc.

Let us add another descriptor: suppose we are given the cost of each of the 4 cars, for each of the 3 years, for each of 3 cities. To record the costs and retain the 3 distinctions, another spatial dimension, in addition to rows and columns, is needed. Instead, to remain on this 2D page, it might be more readable to resort to 3 tables side by side (Fig. 9).

The elements in our 3D array can be described as a_{ijk}

i = 1, 2, 3, 4 corresponds to row (car)

j = 1, 2, 3 corresponds to column (year)

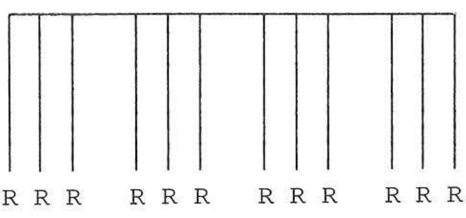
k = 1, 2, 3 corresponds to table (city)

Whether actual spatial dimensions are associated with each descriptor (as in the 1D and 2D examples), or whether arbitrary and ingeniously arranged tables are used when more descriptors are involved, the format of the array is intended to convey the relationship between the values. The subscript notation conveys only the structure and not the values themselves. The 3D example conveys that there are 3 descriptors (hence 3 subscripts i, j, k), 2 of them with 3 states each (hence j can equal 1, 2, or 3; k can equal 1, 2, or 3) and the other with 4 states (hence i = 1, 2, 3, 4). The correspondence of subscript i to row, j to column, and k to table, gives each value in the original array an identity in terms of the different states of the descriptors (ex. $4207 = a_{231}$). It is important to understand that the subscript notation describes the *classificatory relationship* of the values to each other rather than any algebraic or numerical relationship.

When we find that this subscript notation provides a convenient form of description for a quipu, we infer that we are dealing with a collection of numbers similar to the above—that is, values recorded in several dimensions. Since the quipu consists of a horizontal main cord where each pendant along the cord represents a number, it is basically one dimensional. However, through the use of color and spacing, 2, 3, and 4 dimensional arrays in fact occur on actual quipus.

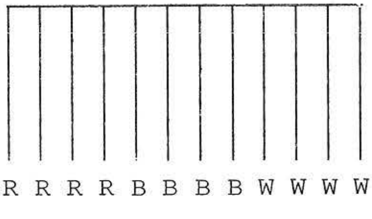
Consider the following arrangements (where R denotes Red, B denotes Blue, W denotes White, G denotes Green); all are two dimensional arrays of the kind commonly found on quipus.

(1)

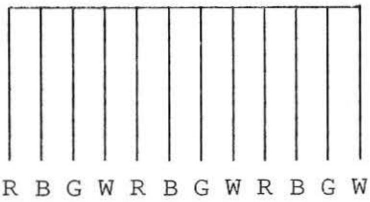


a_{ij} i = 1, 2, 3, 4 (corresponds to groups set off by spacing)
j = 1, 2, 3 (corresponds to position in group)

(2)

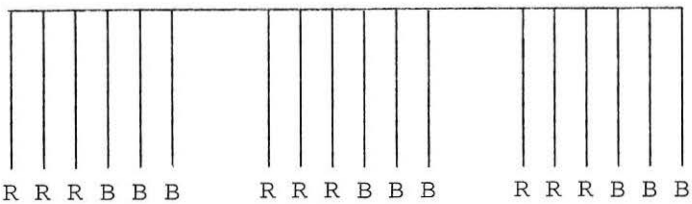


a_{ij} i = 1, 2, 3, 4 (corresponds to position in group)
j = 1, 2, 3 (corresponds to group, group set off by repetition of color)

(3) 

a_{ij} i = 1, 2, 3, 4 (corresponds to color)
j = 1, 2, 3 (corresponds to group, group set off by repetition of color)

Similarly, three dimensional arrays are formed such as:



a_{ijk} i = 1, 2, 3 (position in subgroup)
j = 1, 2 (subgroup, set off by color)
k = 1, 2, 3 (group, set off by spacing)

In order to have a great variety of colors available, the quipu maker created new colors by combinations of his basic colors. Using 3 standard weaving techniques to combine colors, 2 basic colors could be used to designate 6 different states and with 3 colors as many as 23 different states could be designated. Another separation technique, in addition to color and spacing, is the use of special markers. The markers can be pendant strings with no numbers on them that stand out because they are much longer, much shorter, or differently colored. Alternately, some conspicuous flag is found: for example, several short strings are wrapped around the main cord and tied together so that they form a cotton bump.

On actual quipus, separation and identification techniques are combined to yield many different arrays. Some arrays are:

AS99 a_{ij} i = 1, 2, . . . , 16
j = 1, 2, 3

AS115 a_{ij} i = 1, . . . , 7
j = 1, . . . , 8

AS114 a_{ijk} i = 1, . . . , 16
j = 1, . . . , 10
k = 1, 2

AS121 a_{ijkm} i = 1, . . . , 10
j = 1, . . . , 7
k = 1, 2, 3
m = 1, 2

The notation permits a compact statement. But in the brevity, one should not lose the sense of complexity, for to do so would be to miss the power of the quipu as an expressive symbolic device. Consider, for example, AS114 above. This is a three-dimensional array containing 320 pendants; 160 of the pendants are formed into 16 groups of 10 pendants each. All the pendants in a group are united by sharing the same color; and each group is distinguished from the one next to it by being a different color. Then, separated by a large space on the main cord, there are another 160 pendants. These also are formed into 16 groups of 10 pendants each by color. The color pattern on the second part of the quipu is the same as on the first part. Hence, $k=1, 2$ corresponds to part set off by space and repetition of color pattern; $i=1, \ldots, 16$ corresponds to group set off by color; and $j=1, \ldots, 10$ corresponds to position in group.

After it has been observed that a given quipu is an N-dimensional array, numerical or algebraic relationships between values can be sought. When relationships exist between the values and are consistent for some entire category, we feel confident that it is more than a spurious observation because it depends on both the form and the content of the array. Look once more at the 2D example (Fig. 8). An examination of the values shows that:

$$a_{43} > a_{32} > a_{13}$$

Since this has no consistency according to category, it is valid but not of any particular interest. However, the statement that

$$a_{2j} > a_{ij} \text{ for all } i \neq 2, \text{ all } j$$

says that each year car 2 cost more than each of the other cars. Looking next at our 3D example (Fig. 9), we see that, in fact,

$$a_{2jk} > a_{ijk} \text{ for all } i \neq, \text{ all } j, \text{ all } k$$

that is, each year in each city car 2 cost more than each of the other cars. This must be considered more than chance.

Figure 10 is the same 2D example but it has been augmented by (1) an additional bottom row, and (2) an extra value placed at the lower right. Before jumping to the conclusion that the number of states of a descriptor has been increased, look at the values that were appended. The elements in the new row (row 5) consist of the sums of their respective columns. The extra value is the grand

	1973	*1974*	*1975*	
car 1	2841	2963	2900	
car 2	3442	4013	4207	
car 3	3380	3365	3162	
car 4	2964	3268	3574	
	12627	13609	13843	40079

Figure 10. The two-dimensional array shown in Figure 8 augmented by an additional row and an extra value.

total. Similarly, another column would have been added if row subtotals were to be included. Values which are the sum of other values are now included, but *the array has in no way turned into a calculating device and there is no way of knowing how or where the calculation took place.* It cannot even be stated with certainty whether summing of the 4 values in a column led to the 5th value, or whether the 5th value was subdivided into the 4 values above it. Nevertheless, we can conclude that the concept of addition is involved. Also, the grand total could be viewed as the sum of all 12 values in the original 2D array or as the sum of the 5th row in the augmented array. In either case, the subtotals are not simply a step on the way to the grand total since they have been retained as independent pieces of information.

Just as in our paper and pencil example, one set of subtotals could have been associated with rows and another set with columns, so the different subtotals on N-dimensional quipu arrays adhere to their dimensional indicators. Thus we find that on a quipu such as arrangement (1) above, the subtotal of each of the 4 groups is associated with the group by the inclusion of an additional pendant in each group. When a sum cord is included with a group, it is spatially distinguished by being tied to the main cord so that it falls upward rather than downward! On a quipu such as (3), if the subtotals associated with each of the colors were included, they would constitute an additional group containing each of those colors. Finally, a grand total when present is anomalous, as it was in our paper and pencil example, and would usually be spatially distinguished by simply dangling from the end of the main cord.

To conclude our discussion of the N-dimensional arrays, we note

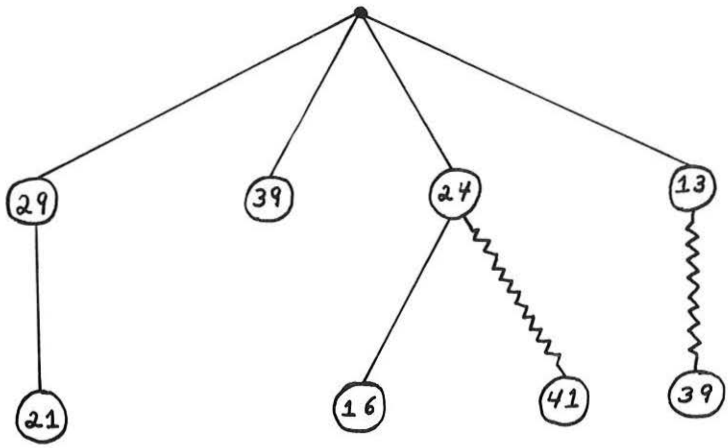


Figure 11. A tree diagram with two levels.

many one dimensional arrays have grand totals, several two dimensional arrays contain upward directed subtotals or an end group of subtotals, and some three dimensional arrays contain both groups that are subtotals of other groups as well as an end group of subtotals that include them. Regularities consistent throughout categories vary in type with one of the most unusual being a fixed proportionality.

Trees and hierarchies. We turn now to a discussion of a second, completely different type of array. Before looking at their expression as a quipu configuration, attention is given to the ideas of tree diagrams and hierarchies.

Suppose we start once again with our standard media of pencil and paper and are given the following information: 8 people (Mr. A and his son; Mr. B; Mr. C and his son and daughter, and Mr. D and his daughter) went fishing on one boat and caught 29, 21, 39, 28, 16, 41, 13, and 39 fish respectively. We want to record the number of fish caught by each family on the boat but retain the identification by generation and sex. Using the natural hierarchy of generation, we record the catch of each father and below that (connected by a straight line for a son and a wiggly line for a daughter) the catch of his children (Fig. 11).

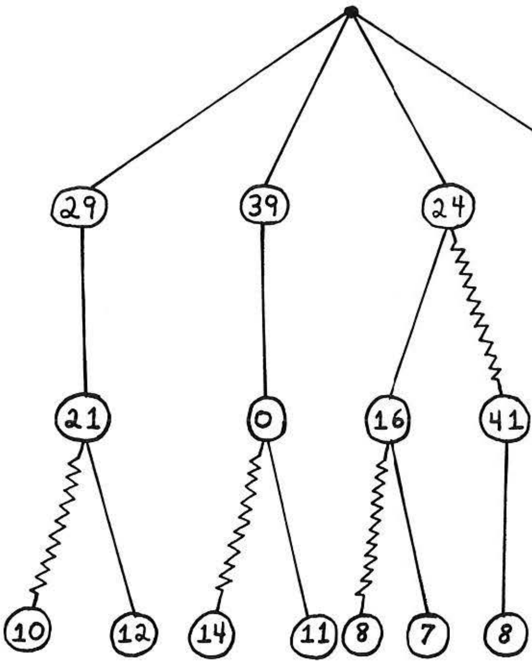


Figure 12. A tree diagram with three levels.

This diagram has 2 levels; the first corresponds to the fathers and the second to the children. As one moves from each value in the first level to the second level, it is possible to branch into 2 values. Although Mr. B is alone so his family is only represented by 1 level and Messrs. A and D each have only 1 child with them, the diagram as a whole has 2 levels and 2 possible branches to the second level.

Let us increase the number of people that were on the boat: Mr. A's son brought his son and daughter, Mr. B brought his son's son and daughter, Mr. C's son brought his son and daughter, and Mr. C's daughter brought her son, who caught 10, 12, 14, 11, 8, 7, 8 fish respectively. To include this information, another level is added to the diagram. All the information has been recorded on a tree diagram (Fig. 12).

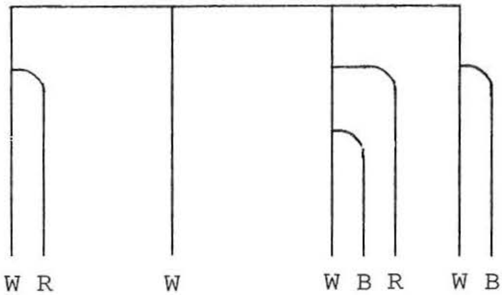
This diagram has 3 levels with 2 possible branches from each value on the first level to the second level and 2 possible branches from each value on the second level to the 3rd level. (Notice that although Mr. B's son was not present, he has been included with a catch of 0 in order to place his children in the appropriate generation.)

With a tree diagram, it is easier to look for regularities consistent across levels or consistent with respect to branches. One can observe, for example, that each of the values on level 1 is greater than the sum of its connected values on level 3; namely, each father caught more fish than all his grandchildren combined.

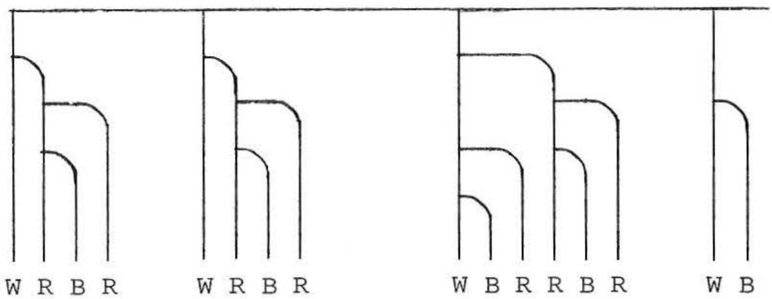
This problem, as it is stated, can easily be extended to have more fathers on the boat (more starting points), or additional generations (more levels). If it is extended by including more than 1 son and daughter per person, there would be more branches per level, but we would have to decide whether to use the same style connector for each son (or daughter) or to use different style connectors depending on some other characteristic (say, age). Also, the problem has the same transitions from each level to the next, which can easily be modified by, for example, letting the children but not the adults bring friends.

The important feature of any tree diagram is that each item at the bottom can be traced upward in one and only one way passing through each level exactly once as you proceed upward. In drawing a tree diagram, it is crucial that the possible ways to proceed from one level down to the next lower level are clearly defined. Hence, they are most adaptable to describing hierarchical organizational structures whose organizing principle is who reports to whom, or for describing processes with distinct stages which are structured in time.

What we call tree diagrams seem to find a direct expression in the format of quipus; it is as if both contemporary western symboling and Inca symboling approach each other at this point. On many pendant cords there are suspended 1 or more other cords from which there are suspended yet other cords, and so on. With paper and pencil, different symbols to distinguish the branches are needed. In the fish problem, only 2 connectors were used; one represented by a straight line and the other by a wiggly line. The quipu is much less restrictive as different colors are used to distinguish branches. By viewing this array as a tree diagram, we emphasize that cords must be considered as part of a hierarchy where each pendant or subsidiary carries on identification by its level and branch within that hierarchy. For example, a quipu similar to the 2 level tree diagram (Fig. 11) could look like:



and one comparable to the 3 level tree diagram (Fig. 12):



If, in our example, there were a second boat with 4 families, this hypothetical quipu representation might have adjoined to the first group of pendants another group of 4 pendant cords with similarly Red and Blue colored subsidiaries attached to the pendants and to each other. It could then be described as a two dimensional array

a_{ij} $i=1, 2$ (group)
$j=1, 2, 3, 4$ (position in group)

where each pendant is a starting point in a 3 level hierarchy with 2 branches possible from each value from level to level.

An upward directed cord associated with each group could carry the group subtotal. In our example, such a total could be for the entire boat and could be on one pendant alone. Alternatively, the total could be on a pendant with 2 levels of subsidiaries, thus expressing the boat total but keeping the generational distinction;

or, it could be on a pendant with 2 levels of subsidiaries with 2 branches on each, thus keeping both the generational and sex distinctions. On actual quipu there are some group subtotal cords that retain the branches or levels, some that lump them, and some that sum only the pendants and ignore the subsidiaries.

To conclude our discussion of hierarchical arrays, we cite a specific quipu example. The example is chosen to impress the reader with the logical complexity that can be expressed on a quipu—for obvious reasons it cannot be transcribed here. On one quipu the pendants carry up to 5 levels of subsidiaries with as many as 10 branches from each value on some levels.

Media and Civilization

In *Tristes Tropiques,* Lévi-Strauss (1955) tells what happened when the leader of a Brazilian tribal people learned about writing. He takes the story as a point of departure for a discussion of writing in early civilization. The discussion begins, as expected, with the repetition of the self-comforting thought that writing is the most important criterion for civilization, the one that is most worth retaining on the lists of what characterizes civilization. Suddenly, all of the pleasant connotations associated with this thought are thrown into doubt; the author recalls that no sooner did the tribal leader find out about writing then he seized upon it as a means to enhance his prestige and authority. Lévi-Strauss thereupon draws the following inference: "the use of writing for disinterested ends, and with a view to satisfaction of the mind in the fields of the sciences or the arts is a secondary result of its invention—and may even be no more than a way of reinforcing, justifying, or dissimulating its primary function." And what is the primary function? According to Lévi-Strauss, "the primary function of writing as a means of communication, is to facilitate the enslavement of human beings." He points out that writing makes its appearance along with oppressive political systems. And that is why, says Lévi-Strauss, writing goes together, for example, with massive architecture: to accomplish the architectural feats of the ancient world, thousands of people had to be assembled and they were taxed to the limits of their strength. Incidentally, Lévi-Strauss insists that the case of the Inca confirm his hypotheses: they were unstable, they "decomposed" quickly; however politically oppressive the Inca were, the absence of writing supposedly prohibited them from maintaining their position of power.

The last comment can be dismissed as an unsuccessful attempt by yet another author to deal with the "but they have no writing" problem. As for the rest, the evidence can be interpreted as leaning toward support for the notion advanced by Lévi-Strauss. The content of the bulk of early media is statistical and economic; presumably, those in authority needed these data to exercise control. What was wanted was a device to keep track of essential information. In the Andes, Baudin (1961) contends, "The statistical reports of pre-Columbian Peru enabled the Inca and the higher officials to know what the economic condition of the empire was and to act accordingly." The device they used was the quipu; elsewhere other devices were used.

Theoretical support for the Lévi-Strauss opinion comes from what Max Weber (1952), discussing bureaucracies in general, calls the concept of "official secrets." "Bureaucratic administration," says Weber, "means fundamentally the exercise of control on the basis of knowledge." Those in authority, again citing Weber, "acquire through the conduct of office a special knowledge of facts and have available a store of documentary material peculiar to themselves."; all of this "is a product of the striving for power." Now in early civilizations, including the Inca, everything surmisable about scribes and quipu-makers, and everything that is known about the information they recorded, suggests that a privileged appendage of the elite were recording "official secrets" peculiar to themselves and to those whom they served. Writing for what Lévi-Strauss calls "disinterested ends" was indeed thousands of years in the future; in that future, something more than quipus might be required.

A way to characterize a civilization, other than holding up a checklist, is to define its particular style. The search for the style of a civilization requires the identification of basic symbols. The seeker, we think, is left with a sense of having missed the point that comes in the wake of a part by part analytic approach. But the Gestalt, the unity, the underlying principle, is often elusive. We are indebted to A. L. Kroeber (1963) for reminding us of the origins of the word style: "The etymology is from *stylus*, the pointed rod used for writing on wax by Greeks and Romans. The sense here is metaphorical, as we speak of an inspiring or poisoned pen, of a fluent or a bold hand. A man's style originally was his characteristic, idiosyncratic manner of writing: possibly at first with emphasis on the shapes of his letters, his handwriting, certainly later with reference rather to his choice and combination of words." The icon of Inca Civilization is the quipu; we return here to the original association of the word style with media.

As already stated, quipus are made out of cotton shaped into cords. Cotton cords are light and not especially durable; in this regard, they are more like the papyrus of Egypt than, say, the clay of Babylonia. One theory has it that heavy durable media go with early empires that stress time, and lighter, less permanent materials go with empires that stress space (Innes, 1950). However, this may be, the view from inside Inca Civilization (as contrasted with the "outside" comparative view) points to cotton as the obvious material.

Cotton had been brought under cultivation for the first time in human history along the Pacific coast within sight of the Andes thousands of years prior to the appearance of the Inca. The long period of cotton cultivation and use had important consequences that are directly relevant to the present study. Gayton (1961) puts it succinctly: "The head start, so to say, which textiles had in ceramics, metal, and mural decoration may have established a lasting priority for the textile art as the primary means of visual communication." But there is even more to it than that: over the millenia, and before the emergence of the Inca, cloth took on a peculiar significance for Andean peoples. Commenting on this, Murra (1962) writes: "There is nothing strange in the political use of prestige objects; the novelty consists in the discovery that, in the Andean area, the artifact of greatest prestige and thus the most useful in power relations was cloth." Thus, in the cotton quipu, the Inca had a material that carried its own extraordinary message.

In every culture, there are special objects that stretch to regions far beyond the particular purpose for which they were intended. These objects sum up areas of meaning for which there may be no

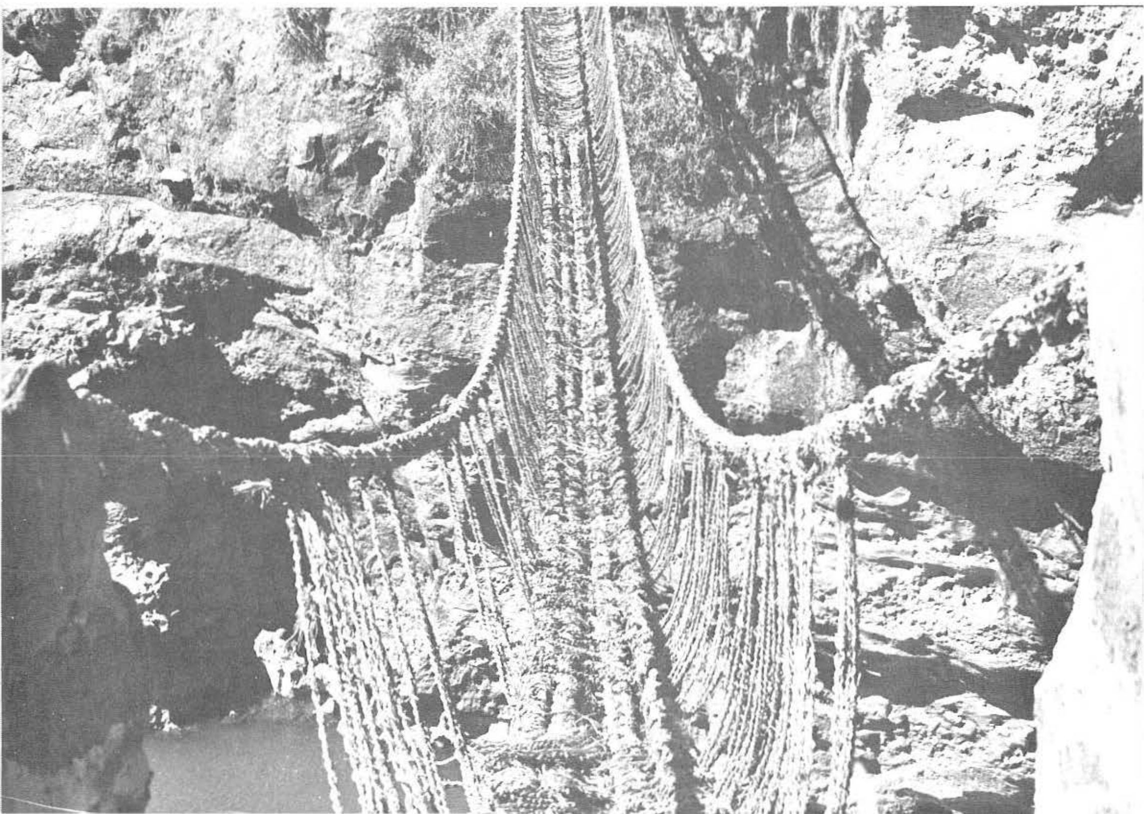


Figure 13. This quipu (AS33) is in the collection of the Peabody Museum of Archaeology and Ethnology, Harvard University. Photo by the authors.

verbal counterparts. They usually are common and replaceable, but they are not always inexpensive, as witness the American automobile. Sometimes they are keys to the culture for the outsider; for the insider, in particular children in the process of learning, they are indispensable cues. They may be fixed and rather permanent as the Medieval cathedral, or they can be impermanent and leave no trace, like an igloo.

We think that the quipu is such a special object. There is no way to prove this; but we can give a few examples that form part of the basis for our intuition, and thus conclude our essay. When the main cord of a quipu is freely held, one sees a smooth parabola bounding the tops of a series of vertical lines (Fig. 13). This shape is repeated in the hanging bridges of the Andes (Fig. 14). Suspended from the sides of rivers like giant quipus, these bridges were made, like quipus, from plant material and they are similar to quipus in construction. If the quipu as a special object is like a theme it should be repeated visually and thus underscored: the bridge is one of several instances that meets this requirement. Another requirement for being a special object is the ability of the object to express fundamental cultural notions. This is easy to show in the case of the quipu and the ceque system. The ceque system was the all important social organizational basis for the Inca state; it guided marriage, work, myth and ceremony, and so on (Zuidema, 1964). Its complexity is too great to detail here; however, it can be formally described as a division of the whole into 4 quarters, and within the quarters, a further division into 3 sets of 3. Using standard elements, a quipu expressing the formal structure of the ceque system can be constructed as follows:

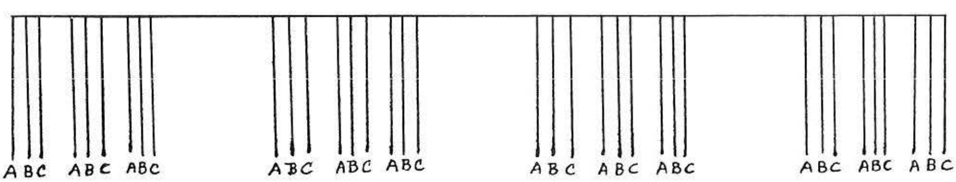


Figure 14. Until recently, this bridge hung over the Apurimác River in Peru. It was replaced by a metal bridge in 1968 (Gade, 1972). The photograph is here through the courtesy of Professor Daniel W. Gade and by permission from the *Annals* of the Association of American Geographers, *62*, 1972.

1. References to actual quipus are followed by a letter and number designation. We identify all published quipus by this sytem: the letters refer to the author; the number to each quipu in chronological order of publication by author. For example, AS100 refers to the one hundredth quipu published by the authors of this article. For more details, see Ascher and Ascher, 1972.
2. Very many people in several countries cooperated with our efforts to locate and study quipus. Instead of thanking only a few people here, we will, instead, include everyone in a forthcoming final publication.
3. The Wenner-Gren Foundation for Anthropological Research provided partial financial support for our work.

REFERENCES

Ascher, Marcia, and Robert Ascher. Code of ancient Peruvian knotted cords (quipus). *Nature*, 1969, *222*, 529-533.

——, Numbers and relations from ancient Andean quipus. *Archive for History of Exact Sciences*, 1972, *8*, 288-320.

——, *Code of the quipu: data book*. 1975, ms.

Baudin, Louis. *A socialist empire: the Incas of Peru*. New York: D. Van Nostrand, 1961.

Gade, Daniel W. Bridge types in the central Andes. *Annals of the Association of American Geographers*, 1972, *62*, 94-109.

Gayton, A. H. The cultural significance of Peruvian textiles: production, function, aesthetics. *Kroeber Anthropological Society Papers*, 1961, *25*, 111-128.

Innes, Harold. *Empire and communications*. Oxford: University of Oxford Press, 1950.

Kroeber, Alfred L. *Style and civilization*. Berkeley and Los Angeles: University of California Press, 1963.

Lévi-Strauss, Claude. *Tristes tropiques*. Paris, 1955.

Locke, L. Leland. The ancient Peruvian abacus. *Scripta Mathematica*, 1932, *1*, 37-43.

Murra, John V. Cloth and its function in the Inca state. *American Anthropologist*, 1962, *64*, 710-727.

Pirsig, Robert M. *Zen and the art of motorcycle maintenance*. New York: Bantam Book, 1975.

Rowe, John. The Inca culture at the time of the Spanish conquest. In J. H. Steward (ed.), *Handbook of South American Indians, the Andean Indians*, Vol. 2. Washington D.C.: Smithsonian Institution, Bureau of American Ethnology, *143*, 1946. Pp. 1-147.

Valcárcel, Gustavo. *Perú. Mural de un pueblo: apuentes Marxistas sobre El Perú prehispanico*. Lima: Editora Press Nuevo, 1965.

Von Hagen, Victor W. *Realm of the Inca* (revised edition), New York: The New American Library, 1961.

Weber, Max. The essentials of bureaucratic organization: an ideal-type consstruction. In Robert K. Merton, *et. al.*, (eds.), *Reader in bureaucracy*. Glencoe, Illinois: The Free Press, 1952. Pp. 18-27.

Zuidema, R. T. *The ceque system of Cuzco*. Leiden: E. J. Brill, 1964.