

∞ *Computer-Aided Analysis of Handwriting Movements*

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The paper attempts to demonstrate that writing movements — once recorded on a suitable xy-digitizer — can be processed and analyzed by means of a computer for many different purposes. This method is used in the authors' laboratory to study the human writing apparatus and, to a smaller extent, the reading of handwriting. From the writing movement certain time and frequency functions are derived and the kinds of information that may be obtained from these functions are briefly indicated. Some feasible practical applications are discussed, and the paper concludes with a suggestion on a new kind of writing instruction.

1 *Introduction*

At Nijmegen we are analyzing handwriting to discover by which mechanism the human writing apparatus is able to perform the complex movements required for this task. We are particularly interested in the development of graphic abilities in children and in the nature of grapho-motor dysfunction in certain groups of patients with well-specified neurological disturbances. We are also involved in the investigation of reading handwritten words. It will become clear that there is some relation between those two topics when viewed from the methodological side of our study.

When studying handwriting, one may merely look at a static sample of produced handwriting, as when one inspects a piece of somebody's writing. The fact that we are interested in motor control, however, implies that we have to study the actual movements being performed during the production of handwriting. Handwriting is brought about by the combined activity of muscle systems in hand, arm, and shoulder. A complete description would imply that all this muscle activity should be recorded, and that the relative movements of every limb of the human writing apparatus should likewise be registered. We have recognized that it is not

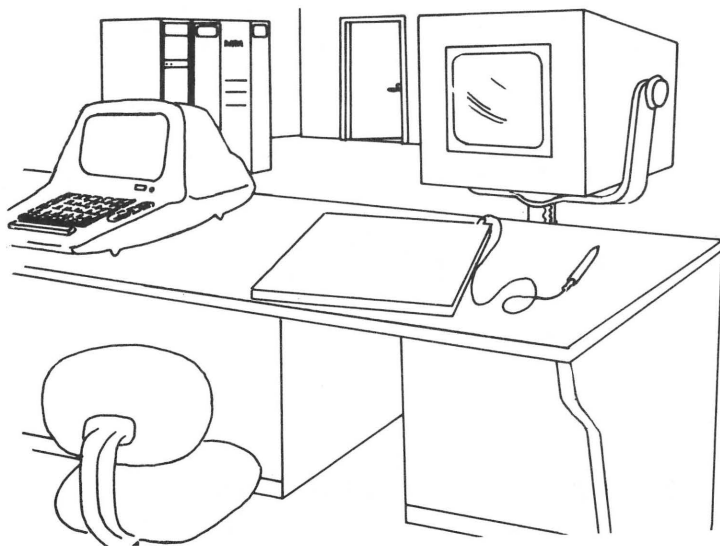


Figure 1. Arrangement of the xy-digitizer with computer and display.

feasible to record all these components. Since the aim of the handwriting act is to produce a well defined static writing trace, it seems reasonable (at a certain level of analysis) to confine ourselves to looking at how the stylus tip moves over the paper.

The purpose of the present article is to show that, by using a digital computer, the movement of the pen may provide us with extremely valuable information for the investigation of motor control in the self-paced movements involved in handwriting. We also wish to indicate that it is possible by means of the computer to transform the graphic product in various ways that are useful in the study of the reading of handwriting.

2 Apparatus

To record handwriting in our laboratory we use a so-called xy-digitizer, in the shape of a flat board, which is covered by a sheet of ordinary writing paper (Figure 1). The subject writes with an electronic pen that is connected to a control unit by a flexible wire. The pen is ordinary in appearance and has a simple ball-point refill. The position of the tip of the stylus on the xy-digitizer is registered in terms of the horizontal (x) and vertical (y) distance

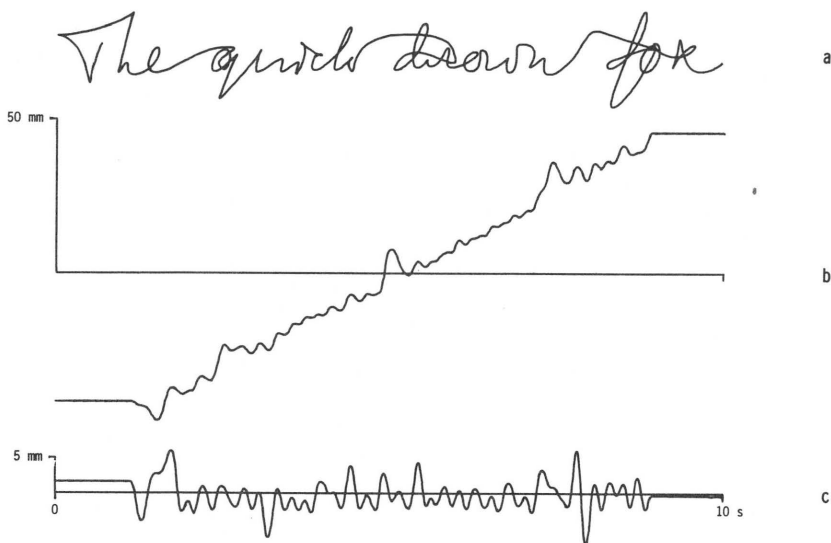


Figure 2. Sample of a handwriting signal as represented in the computer, depicted in two ways: the static writing trace (a), and the x and y coordinate as a function of time (b and c, respectively).

from a fixed point. These distances, or coordinates, may be sampled with a frequency as high as 200 per second and with an accuracy of better than 0.2 mm. It will be clear that a writing movement — even if it lasts only a few seconds — usually contains a very large amount of information. Therefore, the coordinates are fed to a laboratory computer to store the data and to process them. A written word consisting of only a few letters occupies about 2,000 characters in the computer if no data reduction is applied.

These x and y coordinates, which vary with time, will be referred to as the writing signal. The latter comprises all the static and dynamic information of the writing movement that we are going to use in our analyses. The set of positions defined by the simultaneously obtained x and y coordinate values yields the static writing result (Figure 2a). Of course, the writing movement includes also the up and down movements of the pen. Because one may regard these movements above the paper also as part of the writing movement, we can project them onto the two-dimensional writing plane. The dynamic aspects, which can be made visible by plotting the x and y coordinate separately against time, are also retained (Figure 2b, c).

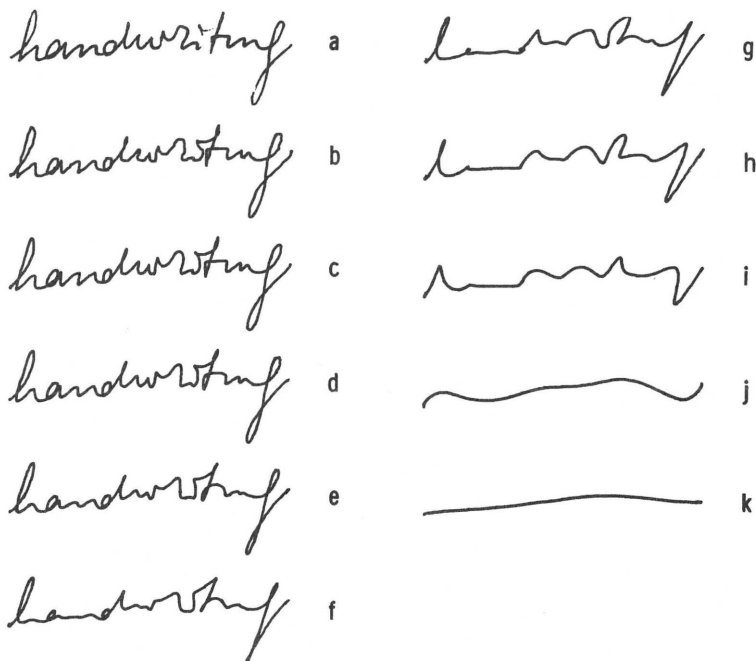


Figure 3. The original writing trace (a) compared with the raw writing trace as stored and displayed by the computer (b) and compared with the writing signal after filtering the raw signal according to low-pass cut-off frequencies of 10, 7, 6, 5, 4, 3, 2, 1, 0.5 Hz respectively (c to k).

3 *The Raw Writing Signal, Noise, and Filtering*

The way in which the writing movement is represented in the computer differs in two ways from the original writing movement. First, the position of the pen (i.e., the coordinates) are specified only at certain moments in time. From signal analysis theory we know, however, that the position at all intermediate moments can be estimated exactly, provided that the sampling frequency is high enough. Second, because the accuracy of measurement by the digitizer is limited, the coordinate values are only an approximation; positions of the pen tip intermediate with respect to the resolution of the digitizer are rounded-off to the nearest position that can be identified by the digitizer. The raw writing signal looks jagged relative to the trace left by the pen (Figure 3a, b). This form of distortion we call noise. Noise will often interfere when the signal is processed further. The effects of noise must therefore be reduced by some preliminary processing.

In order to remove noise from the writing signal we must define suitable criteria for separating these. We may state that writing takes place with slow movements relative to the sampling frequency, which implies that the writing signal should be smooth. The contribution of noise results in unpredictable differences between the coordinates as sampled and the true coordinates. Noise appears as fast waves (i.e., high frequencies) in the writing signal. The effects of noise may be reduced by removing these fast waves, which obviously do not occur in real writing movements; this is called smoothing, or low-pass filtering. This type of filtering is commonly encountered in acoustics where it can be done by means of hardware filters. We will perform an analogous manipulation with the writing signal by means of a computer algorithm, which makes use of the Fourier transform. The Fourier transform decomposes the writing signal into a set of sine waves with different amplitudes and frequencies. Plotting the amplitude of all sine waves against their frequency yields the frequency spectrum. By applying the Fourier transform to our writing signal, no information is lost; if we apply the inverse transformation afterwards, we once again obtain the original writing signal. Low-pass filtering, now, is concerned with the following three stages: applying the Fourier transform, setting the high-frequency components to zero, and applying the inverse Fourier transform. Beside the fact that extremely efficient Fourier transform algorithms (called fast Fourier transform) for computers are available, this method possesses some attractive aspects which make fast processing possible. First, other calculations (like determining velocity or acceleration) can be done together with low-pass filtering. Second, the Fourier transform can handle two independent time functions (e.g., x and y coordinates) simultaneously. The described method of removing noise from the writing signal is, of course, just one method among various other methods available, each with their own advantages.

It is difficult to determine a priori at which frequency one should filter; in fact, it depends upon the specific situation. The resemblance between Figure 3a and Figure 3c suggests that we obtain a good approximation to the writing movement if we apply a low-pass filter which removes frequencies higher than 10 Hz. Of course, filtering out frequencies which are important in writing movements will cause distortions (Figure 3d, e, etc.). Filtering at a very low frequency will even lead to the complete removal of all

letters from the writing signal (Figure 3k). What is left is merely the slow displacement movement.

4 *Spectrum*

We shall now pay some further attention to the frequency spectrum or, for short, the spectrum.¹ The question as to what frequency components are produced by the writing movement and what components are produced by noise can be answered in a more analytical way by using the spectrum.

From the spectrum of an example of handwriting we can see that most of the frequency components produced by the writing movement are found between 1 and 6 Hz (Figure 4). That these frequencies are essential for easily readable writing may be directly inferred from Figure 3e, f, etc. It appears, moreover, that in some cases one can discern significant frequency components of approximately 10 Hz. If one does not want to remove essential parts of the writing signal one should use a filter cut-off frequency of 10 Hz or higher.

There is also a difference between the horizontal and the vertical component of the writing movement. The vertical component of the writing movement does not comprise slow movements. In other words, nearly immediately after an upstroke, a downstroke will follow since all letters should be aligned according to the base line. As a consequence, if we determine the spectrum of the vertical component of the writing movement, the part of the spectrum between 0 and 0.5 Hz is nearly zero (Figure 4d). We can use this information when we want to estimate the horizontal progression movement according to which writing takes place, including the small vertical variations of the base line. For this purpose we simply filter the complete writing signal with a low-pass filter frequency of about 0.5 Hz (Figure 3k).

Another feature of the spectrum is the possibility of detecting temporal periodicity of the writing movement. If, for instance, a

1. For the sake of accuracy we must distinguish between spectrum and energy spectrum. The spectrum still contains all the information of the writing signal. However, when we are only interested in the relative strengths of the different frequency components, we may use the energy spectrum, which lacks specific information about the phases of the individual frequency components. In this section, spectrum must be interpreted as energy spectrum. Furthermore, the energy spectrum is calculated from the velocity time function — instead of from the position time function — in order to prevent the very low frequency components from becoming predominant.

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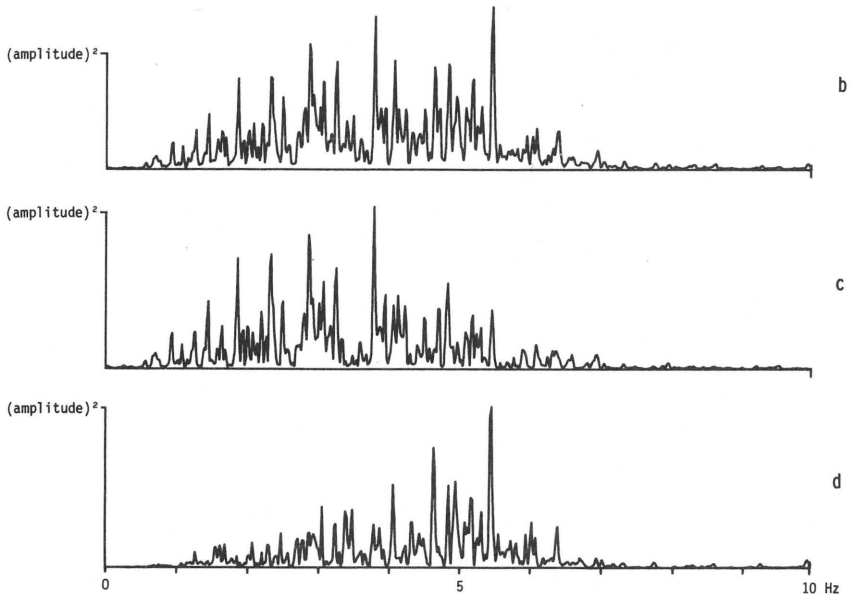


Figure 4. From a sentence of normal writing (a) the frequency spectrum is depicted of the two dimensional movement (b) and for the horizontal and vertical component of the writing movements separately (c and d, respectively). (See footnote).

writing movement contains a stable rhythm or if the writer has some specific tremor, one may expect a strong peak at the corresponding frequency. In general, the spectrum of real writing movements does not contain very high peaks, which indicates that the various strokes in the writing movement take a variable amount of time — depending upon the specific demands of every stroke — or that the writing movement is performed in a rhythm which varies.

5 *Velocity*

For certain purposes one might be interested in the velocity with which the pen moves over the writing surface. Velocity is calculated by determining the change of the position of the pen per unit of time. Velocity, in complex and variable tasks such as handwriting, is itself again a function of time. Velocity must be calculated for changes in the x and y coordinates separately, thus yielding the velocity components in horizontal and vertical directions respectively (Figure 5d, e). Together, the two velocity components define the velocity vector. Absolute velocity or speed can be calculated by determining the size of the velocity vector (Figure 5f).

There is a large amount of data which can be calculated using velocity as a function of time. Imagine that we want to decompose a sample of handwriting into smaller elements. If, for instance, two letters are connected horizontally by a concave-up linking stroke, one way to separate these two letters is to choose the lowest point of the linking stroke. This can be done most accurately by using the vertical component of the velocity over that connection. At its lowest point the vertical velocity component in the linking stroke changes sign from negative to positive.

Often the experimenter wants to know exactly when a writing movement starts, stops, or is disrupted. For that purpose we use absolute velocity as a function of time. In normal, smooth handwriting absolute speed within a word is seldom close to zero for more than 30 ms. We found, however, that this duration may be prolonged drastically, depending upon the demands of the next pen stroke. If, for instance, the subject wants to make the next stroke in a new direction as straight as possible (which is not very common in normal writing) a pause as long as 100 ms is made prior to producing the straight line.

Another function to be mentioned here is the cumulative length of a curve drawn (Figure 5g). This time function can be found by integrating absolute velocity. The time needed to draw a certain pattern appears to be relatively independent of the total length of its trajectory, which is proportional to its size. In an earlier study performed by us, an increase of writing size by a factor of 16 resulted in an increase of the time required by a factor of only 1.6 approximately.

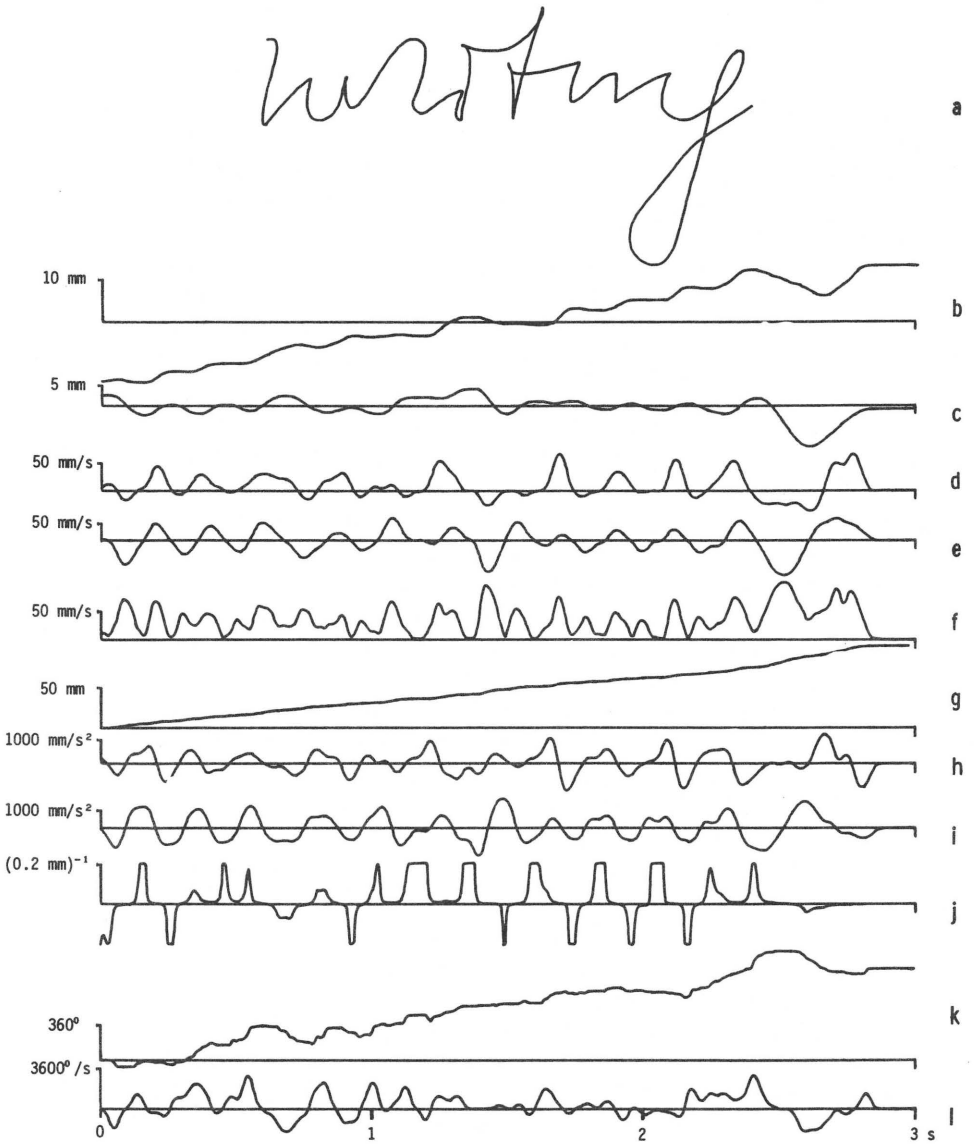


Figure 5. Overview of all time functions discussed in this paper: writing trace (a), x coordinate (b), y coordinate (c), x velocity (d), y velocity (e), absolute velocity (f), cumulative curve length (g), x acceleration (h), y acceleration (i), inverse curve radius (j), running angle (k), and angular velocity (l).

6 *Acceleration*

For certain purposes, acceleration as a function of time is of particular interest. Acceleration is calculated by determining the change of the x and y velocity of the pen per unit of time, thus yielding the acceleration components in horizontal and vertical directions respectively (Figure 5h, i). Acceleration may be called deceleration when acceleration and velocity have opposite directions, i.e., when the movement is slowing down.

The occurrence of acceleration indicates that a force has been applied to the pen. The net force applied is the sum of muscle force and the friction forces of the pen tip and the hand. One might hypothesize that friction helps to stop the pen moving in a certain direction and that muscle force is applied to start the pen moving in another direction. Indeed, in slow writing one can sometimes conclude from the acceleration pattern that the force burst starts just after the friction force has almost stopped the movement. In normal handwriting, however, this effect can be seen only in a few strokes (e.g., see Figure 5d, i). As a first approximation one may say that in normal handwriting the acceleration pattern may be equated to the net force pattern produced by the muscles, because friction levels are in general low as compared to the force levels that apply.

The acceleration pattern often looks very repetitive even though the original writing contains different letters and therefore lacks such a repetitive appearance. The various letters are produced by only tiny variations in duration and amplitude of the individual positive and negative phases in the x and y acceleration and by variations in phase between x and y acceleration.

7 *Functions Related to Form Characteristics*

There is also a class of functions that expresses certain structural features, or form aspects, of a sample of handwriting. We shall look at two simple examples of these functions, to which we refer as curve radius and running angle (Figure 5j, k). One can imagine that a writing trace is represented by a concatenation of small segments of circles, each of them having a specific radius. This radius is, in fact, continuously changing with time. The radius at a certain moment in time can be calculated from the momentaneous velocity and acceleration (or from the running angle, see below). The time function thus calculated we have called the curve radius.

The curve radius may be positive or negative, depending upon whether the curve makes a counterclockwise or a clockwise turn (Figure 5j). For practical purposes the inverse curve radius is plotted so that a small radius is represented by a big deviation from the zero line. A large radius, which always occurs at transitions between counterclockwise and clockwise turning direction, will be represented by a point close to the zero line. In Figure 5j we also restricted the range of values in the graph of the inverse radius because the exact values of the high peaks — representing very small curve radii at the points where the movement reverses — are not of great interest. An attractive application of the curve radius is to use it as a means of reducing the number of data points necessary for displaying a natural looking sample of handwriting, which is in general less easily obtained if the data points are connected by straight lines. For that purpose we approximate the curve radius function by a step function having constant levels at the time intervals during which the curve radius is approximately constant. In other words, we fit the writing trace with segments of different circles, each segment having a certain radius and length of arc.

We now come to our second example function: the running angle. If a writer makes, for instance, one complete loop in counterclockwise direction, the total sum of length of arc covered, irrespective of the curve radius, will be 360 degrees. The cumulative sum of length of arc — which we have called running angle — will thus increase by 360 degrees after each complete counterclockwise loop. Analogously, a complete loop in a clockwise direction will cause the running angle to decrease by 360 degrees. When only a part of a loop is produced, the running angle will change by the amount of the length of arc covered. The running angle is, in fact, a continuous time function which can be calculated from the velocity (Figure 5k). In normal handwriting a counterclockwise curvilinear movement seems to be more frequent than a clockwise one. This may be demonstrated, for example, by showing that the running angle often has a counterclockwise value for a specimen of handwriting, like the one depicted in Figure 5. An application of the running angle procedure may be found in one of the tasks discussed in the paper by Thomassen and Teulings in this issue.

As with any time function, we can calculate the time derivative of the running angle. This function is termed the angular velocity

(Figure 5l). The peaks appear to reach a constant maximum and minimum value for the counterclockwise and clockwise movements, respectively. This is in agreement with the interpretation in terms of the maximum angular velocity, which is limited for counterclockwise and clockwise circular movements. As we found in an earlier study, the maxima are often somewhat higher for the counterclockwise than for the clockwise direction.

8 *Transformations of the Handwriting Product*

For certain psychological experiments (e.g., on factors involved in the reading of handwritten words) various manipulations of the handwriting product are of interest. The simplest manipulation is a linear axes transformation. This allows us, for example, to change the slant of a sample of handwriting. There are algorithms available which determine the slope of a person's handwriting. These, combined with linear axes transformation, enable us to change the handwriting of one person into the same person's handwriting with another person's slope or to alter the handwriting of many different writers, each with their own slope, such that they have a single uniform, standard slope (Figure 6a, b).

A different manipulation allows us to vary line width as a function of certain parameters. One may, for instance, choose the width to be proportional to the overlap between the pen movement direction on the one hand and the downward direction on the other. The latter manipulation adds extra "natural" appearance to a piece of handwriting (Figure 6c).

9 *Some Practical Applications*

There are a number of obvious, or at least feasible, applications of the algorithms developed for the purpose of our analysis of handwriting. Certain dynamic properties of the writing movement can be used to verify whether the producer of a signature is its real "owner." Such a method should work well because these dynamic properties are totally absent in the static result of handwriting, so that they cannot be forged as easily as the mere appearance of the signature can. One of the main problems in identifying the producer of a signature is how to reduce the amount of information in the writing movement to those parameters that, on the one hand, have maximum constancy for the same signature produced under widely varying conditions by the same person and that, on the other, show maximum variability when that signature is attempted

handwriting a

handwriting b

handwriting c

Figure 6. Two examples of the visual transformation of a writing trace (a): a linear axes transformation, where the new vertical axis is chosen parallel to the slant, while the horizontal axis is unchanged (b), and variation of line width (c).

by other persons. Suitable parameters may be the relative durations involved in producing certain elements in writing movements, since it is known that these relations are fairly constant within one person irrespective of overall speed or size variations. Methods like this have been developed, using the pen pressure pattern rather than the writing signal.

Further possible applications include the development of training procedures employing feedback on the writing movement. A simple example of these is the task that may be called track reproduction. One can imagine the experimenter writing a certain track on the xy-digitizer. This track is recorded and stored by the computer. Subsequently, the subject is asked to trace the track accu-

rately. While the writing movement by the subject is being performed, the computer determines, for example, the time needed for any part of the track and the instantaneous distance of the pen from the original track. If the subject takes too much or too little time on a certain part of the track or if the distance is larger than a predetermined value, a buzzer may sound. Thus, the subject is given feedback on the speed and the accuracy of his attempt. He may in the same way be taught to write rhythmically when the same or similar parts are repeatedly written. The maze running test used as an assay of spatial learning ability is, in fact, a rigid precursor of the above procedure. Other examples of providing feedback which can likewise be achieved by real-time processing are, for example, concerned with transformed information of a writing movement with or without delay.

More ambitious applications for instruction purposes are concerned with handwriting as a liberal, expressive means of communication. As such, handwriting is directly tied neither to strict starting and finishing positions nor to template-like shape prescriptions, but instead it allows large amounts of freedom within certain structural or "topological" bounds. A grapheme or a word may be written in as many varied ways as can be recognized. If the purpose of writing instruction is to produce readable but also rapid, natural, and individual handwriting, one must admit that to regard learning to write as a kind of tracking task is totally inadequate. If for the purpose of writing instruction, descriptions of letters and perhaps letter combinations were prepared in terms of their structural properties such as loops, crossing points, and ratios, programmes could be made available to give beginning writers the appropriate immediate feedback, irrespective of the individual size, slope, curve radius, etc., which constitute precisely the personal features of their own handwriting. This is presumably a better basis for the further development of handwriting as a personal and efficient means of communication than is any uniform set of letters, each of which is practised to a criterion of maximum fit to the standard.