# ELASTO-PLASTIC FRACTURE MECHANICS OF WOOD USING THE J-INTEGRAL METHOD

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ABSTRACT

Linear-elastic fracture mechanics has been applied extensively to wood. However, wood is not a perfectly linear-elastic material, particularly at elevated temperatures and moisture contents. The applicability of the J-integral method to wood was therefore investigated. Douglas-fir (*Pseudotsuga menziesii*) and Pacific madrone (*Arbutus menziesii*) were tested. Opening mode (Mode I) fracture tests at two loading rates were conducted in a test chamber with controlled environment at temperatures from 70 to 140 F and moisture contents from 12% to green. The critical elasto-plastic fracture toughness ($J_{Ic}$), critical strain energy release rate ($G_{Ic}$), and fracture toughness ($K_{Ic}$) were calculated. Species, moisture content, temperature, and loading rate were all found to have a significant effect on the three fracture parameters. The relationship between any two of these parameters was also established by empirical equations, making it possible to estimate $J_{Ic}$ from existing values of $K_{Ic}$ and $G_{Ic}$. Since the degree of plasticity was as high as 60% for Pacific madrone and 40% for Douglas-fir at elevated temperature and high moisture content conditions, a large amount of strain energy capacity might be held in reserve (would not be accounted for) when the linear-elastic fracture mechanics theory is applied. Consequently, the J-integral method is more appropriate to characterize the actual fracture strength of wood.

*Keywords:* J-integral, fracture, fracture mechanics, elasticity, plasticity, fracture toughness, strain energy release rate.

## INTRODUCTION

Wood as a natural product is distinctly orthotropic in its elastic properties and approaches brittle behavior in some forms of loading. Failure phenomena for wood were extensively studied in the past few decades based on the concept of linear-elastic fracture mechanics (LEFM). The LEFM concept assumes brittle behavior, i.e., a perfectly linear-elastic stress and strain field at the initiation of crack propagation. This requires that any region of plastic deformation at the crack tip be confined to a very small area.

Like most engineering materials, wood is not perfectly linear-elastic, particularly at elevated temperatures and high moisture contents. The applicability of the LEFM theory at such severe conditions has been questioned by some researchers (Mindess and Bentur 1986; Soube et al. 1985). However, very little effort has been made to investigate this using an elasto-plastic approach. One elasto-plastic fracture mechanics (EPFM) theory that has been widely applied to metals is the J-integral method. Based on

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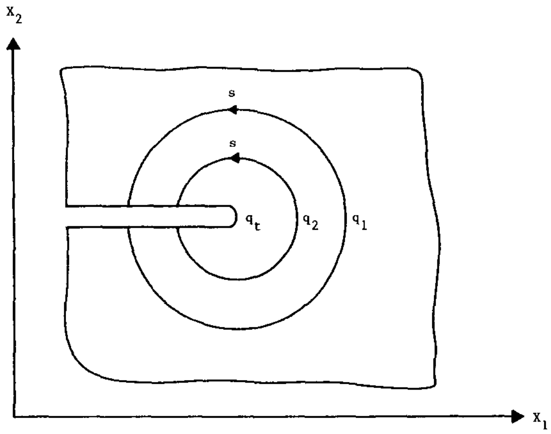



FIG. 1. Contours surrounding the notch tip in the $X_1$-$X_2$ plane.

energy conservation laws, the J-integral is a path independent, one-parameter method, characterizing the elasto-plastic stress and strain field near a crack tip without directly involving the complex stress-strain states in its vicinity.

The main objective of this study was to investigate the applicability of the J-integral method to wood for opening mode fracture, and to examine the effects of species, moisture content, temperature, and loading rate on the J-integral by quantitatively determining the degree of plasticity of wood at various environmental conditions. This study was intended to provide insight into the concept of elasto-plastic fracture mechanics in wood. The refinement of fracture criteria made possible by incorporating the theory of elasto-plastic fracture mechanics can provide for more efficient utilization of wood by recognizing the reserve of load resistance represented by a significant component of plastic deformation.

## THEORY

A detailed literature review on LEFM has been published by Patton-Mallory and Cramer (1987). Consequently, only the theory of the J-integral method of EPFM will be briefly introduced here.

The J-integral concept was first derived by Rice (Rice 1968; Rice and Rosengren 1968) and was proposed for use as a fracture criterion for elasto-plastic materials by Begley and Landes (1972). According to the theory of elasticity, for a traction free notch that has a round tip with its flat surface lying parallel to the $X_1$-axis, as shown in Fig. 1, the J-integral is defined by Eq. (1) for any arbitrary open contour $q_1$ starting and ending at the lower and upper flat notch surfaces, respectively (Rice 1968):

$$J = \int (w\, dx_2 - T_i u_{i,1}\, ds) \tag{1}$$

where w represents the strain energy density, $T_i$ the traction vector, $u_{i,1}$ the partial derivative of the Cartesian displacement vector with respect to $X_1$, and ds the arc length along q.

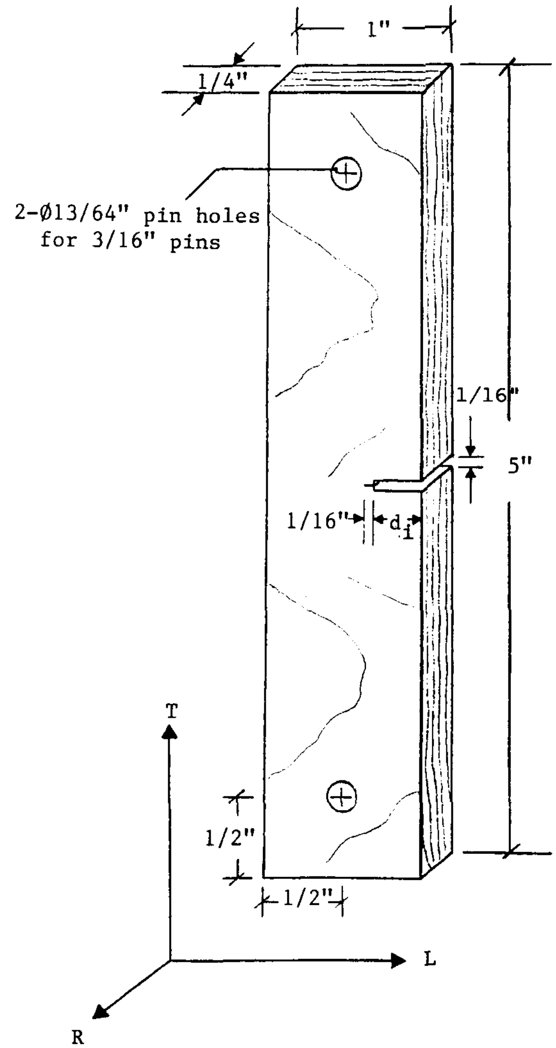
Since the contours of any two arbitrary paths (e.g., $q_1$ and $q_2$ in Fig. 1) must total zero (Rice 1968), it is necessary that the values of the J-integral along two such paths are equal to each other (as traction and $dx_2$ are zero in the segments along the flat notch surface, the contribution of these segments to the J-integral equals zero). Accordingly, the J-integral is a path independent contour integral. This characteristic of the J-integral not only provides a unique solution for determining J values away from the notch tip region where highly nonlinear plastic deformations prevail, but also enables the direct measurement of the J-integral simply by choosing a proper path q.

The invariant J-integral is analogous to $K_c$ in LEFM because a crack will start propagating when the former reaches a critical value, $J_c$. The limit-state value of $J_c$ for Mode I loading configuration, $J_{Ic}$, is a material parameter. It was shown by Rice and Rosengren (1968) that the J-integral was equivalent to the potential energy release rate with respect to the crack length, a, in the absence of body forces and surface tractions:

$$J = -(1/B)U_{,a} \tag{2}$$

where B is the thickness. For linear-elastic materials in a two-dimensional Cartesian system, the potential energy, U, is conventionally given as an energy per unit thickness. In this case J is therefore equivalent to the strain energy release rate G in LEFM. For elasto-plastic materials, however, the J-integral does not define the elastic energy release rate at the crack tip due to plastic flow in that region. Consequently, J is then no longer equivalent to G.

There is little information available on the direct application of the J-integral to wood. The $J_{Ic}$ value in the TL system of European spruce was reported to be about 1.14 lb/in. (Anonymous 1984). In studies of mixed mode (Mode I and II) fracture, Morlier and Valentin (1982) showed that $J_c$ of European pine had a constant value of about 1.66 ± 0.17 lb/in., regardless of crack length and loading mode.




Note: $d_1$=3/16", $d_2$=1/4", and $d_3$=5/16"

FIG. 2. Specimen configuration.

## EXPERIMENTAL PROCEDURE

Two species, Douglas-fir (*Pseudotsuga menziesii*) and Pacific madrone (*Arbutus menziesii*), were tested in this study. Test specimens were prepared from three straight grain, clear, air-dried, flat-sawn parent boards for each species. Single-edge notch (SEN) specimens, as shown in Fig. 2, were used with dimensions of 5 by ¼ by 1 inch in the tangential, radial, and longitudinal directions, respectively.

Initially, three nominal moisture contents (NMC) of 12%, 18% and green, and four temperature levels of 70, 95, 120, and 140 F were planned to give a combination of twelve test conditions. However, preliminary tests showed that it was very difficult to obtain the 140 F and 18% NMC condition without severe moisture condensation occurring in the test chamber. Consequently, the 18%-140 F condition had to be eliminated, leaving a total of eleven conditions. For each condition, two loading rates of 0.02 and 0.002 in./min, and three net notch depths of ¼, 5/16, and 3/8 in. (Fig. 2) were used. A total of 396 specimens (2 species × 2 loading rates × 3 notch depths × 3 boards × 11 test conditions) were therefore tested.

After cutting, the specimens for 18 and 12% NMC were immediately stored in separate controlled environmental rooms for MC conditioning at 70 F. The remaining specimens were submerged under fresh water at room temperature to simulate green wood. Upon reaching moisture content equilibrium, specimens at a particular test condition were tightly wrapped in plastic and placed in an Aminco-Aire Climate System chamber overnight to bring the specimens to temperature equilibrium before testing. The Aminco chamber was preset at the test temperature and the relative humidity required to achieve either 12 or 18% of wood at that temperature, depending on test conditions.

Prior to testing, one specimen at a time was removed from the Aminco chamber and had its sawn notch tip extended by $^1\!/_{16}$ in. using a prefabricated razor-blade notching device to give a net notch depth of exactly $^1\!/_4$, $^5\!/_{16}$, or $^3\!/_8$ in. A U-shaped strain-gage clip gage with a gage length of $^1\!/_2$ in. was attached to the specimen edge across the notch to measure the crack opening displacement (COD) to the nearest 0.001 ± 0.0001 in. The specimen and gage were then promptly positioned in the jaws of an Instron Table Model testing machine using steel pins of $^3\!/_{16}$-in. diameter. The specimen and the jaws were completely enclosed in a mini-chamber. The chamber was physically connected to the Aminco system through air ducts, and its dry and wet bulb temperatures were kept the same (±1 F) as in the Aminco chamber with the aid of thermofoil mini-heaters.

During testing, load and COD were continuously recorded by a data acquisition system and stored in a personal computer for further analysis. Upon fracture, the specimen was promptly removed from the test chamber and its MC and specific gravity based on oven-dry weight and volume (SG) were determined using the oven-dry method.

It was observed after testing that the specimens for the 18% NMC condition had obtained an average MC of only 15% due to malfunction of a humidity sensor in the 18% room. This unexpected situation resulted in somewhat unevenly distributed data within the MC range studied. As a partial remedy to this problem, three additional MC levels between 15% and green at 70 F were also included, giving a total of fourteen test conditions with 504 specimens.

## DATA ANALYSIS

### *Fracture toughness*

The fracture toughness parameter of LEFM, $K_{Ic}$, was determined from the following equation (Brown and Srawley 1966):

$$K_{Ic} = Y(P_c/WB)(a)^{1/2} \tag{3}$$

where $P_c$ is the critical load at fracture, a is the crack length, B and W are the thickness and width of the specimen, respectively, and

$$Y = 1.99 - 0.41(a/W) + 18.70(a/W)^2 - \\ - 38.48(a/W)^3 + 53.85(a/W)^4 \tag{4}$$

Although Eqs. (3) and (4) were originally derived for isotropic materials, it has been shown that under certain conditions they could be applied to orthotropic materials as well (Walsh 1972; Sih et al. 1965).

### *Critical J-integral*

In evaluating $J_{Ic}$, the numerical-analytical method proposed by Landes and Begley (1972) was employed with some modifications for application to wood. This required numerical integration of the area under the load vs. COD curves to obtain the strain energy U, establishing plots of U vs. crack length a, and finally obtaining plots of J vs. COD. A typical J-COD curve, as shown in Fig. 3 for madrone specimens tested at 70 F, 12% NMC (actual 11.3%), and 0.02 in./min loading rate, is a combination of parabolic and linear functions.

The general relationship between J and COD, as illustrated in Fig. 4 (Bucci et al. 1972), is parabolic for perfectly linear-elastic materials and linear for perfectly plastic materials. For real materials, the J-COD curve is parabolic at small COD, indicating linear-elastic behavior. At large COD where the critical load is being approached and large plastic deformations are present, J-COD becomes linear with a slope parallel to that of perfectly plastic materials. It is clearly seen in Fig. 3 that wood also shows both linear-elastic and plastic behavior even at 70 F and 12% MC conditions.

Two-step fitting of the J vs. COD data was performed using numerical iteration techniques and the principle of the sum of squares of the difference. The iteration initially assumed a fully parabolic function. The upper portion of the curve was then replaced by a linear function, and the linear portion lengthened stepwise until the sum of squares of the error for the new fit was larger than that for the previous function. Once the functions best characterizing the parabolic-linear J-COD curve had been obtained, $J_{Ic}$ was obtained by

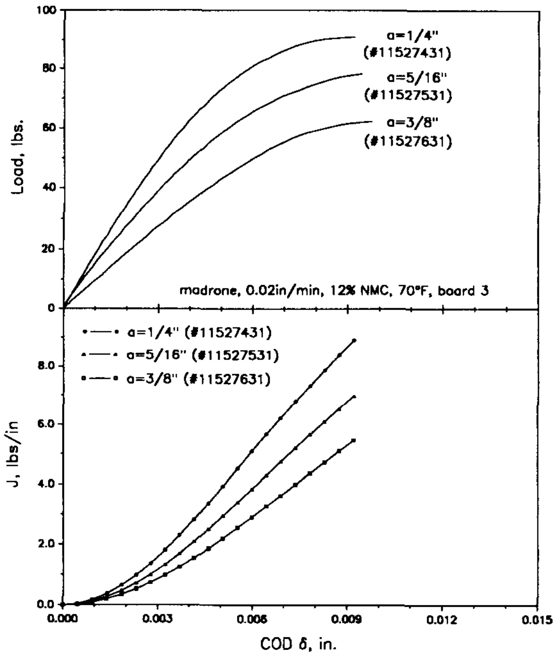



FIG. 3. Typical load vs. COD and J vs. COD plots for madrone specimens.

substituting the critical COD value, $\delta_c$, (the COD value corresponding to the maximum load on the load vs. COD curve of each specimen) in the relation.

### *Critical strain energy release rate*

The critical strain energy release rate, $G_{Ic}$, was calculated using the following equation (Irwin and Kies 1954):

$$G_{Ic} = (P_c^2/2B)C_{,a} \tag{5}$$

where $P_c$, B and a are as defined in Eq. (3), and $C_{,a}$ is the partial derivative of the spring compliance C with respect to crack length a. The compliance and crack length data were fitted to an exponential function as given by Eq. (6):

$$C = \exp(ma + n) \tag{6}$$

where m and n are constants. Since the proportional limit at extreme environmental conditions is usually not well defined, the transition point between the parabolic and linear portions of the J-COD curve was used to determine the compliance of the specimens.

After $J_{Ic}$ and $G_{Ic}$ were determined, $J_{Ic}^p$ ($=J_{Ic} - G_{Ic}$) and the degree of plasticity (DOP), as defined in the following equation, were obtained.

$$\text{DOP}\ (\%) = 100(J_{Ic} - G_{Ic})/J_{Ic} \tag{7}$$

## RESULTS AND DISCUSSION

Average calculated results of $J_{Ic}$, $G_{Ic}$, $K_{Ic}$, and DOP together with gravimetrically determined MC and SG are shown in Table 1. Each value represents an average of nine specimens (3 boards × 3 notch depths).

Analyses of variance were used to determine the effects of loading rate, temperature, MC, and parent board on the fracture parameters. These results, which used MC as a covariate, are shown in Table 2. In these analyses, the

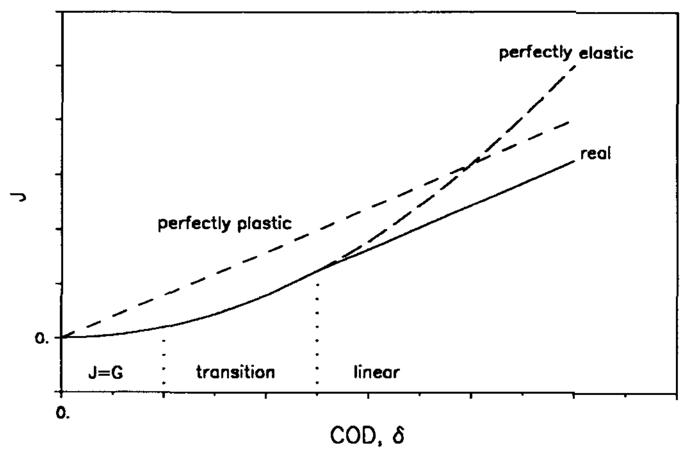



FIG. 4. Schematic relationships between J and COD for perfectly elastic, perfectly plastic, and real materials.

TABLE 1. *Average results for Pacific madrone and Douglas-fir.*

| | Pacific madrone | | | | | | Douglas-fir | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tmp °F | MC % | SG | $J_{Ic}$ lb/in. | $G_{Ic}$ lb/in. | $K_{Ic}$ psi (in.)$^{1/2}$ | DOP % | MC % | SG | $J_{Ic}$ lb/in. | $G_{Ic}$ lb/in. | $K_{Ic}$ psi (in.)$^{1/2}$ | DOP % |
| Loading rate = 0.02 in./min | | | | | | | | | | | | |
| 70 | 11.3 | 0.59 | 7.55 | 4.91 | 494.5 | 33.7 | 13.3 | 0.44 | 1.96 | 1.61 | 272.2 | 17.4 |
| 70 | 14.0 | 0.58 | 9.41 | 5.05 | 437.9 | 45.2 | 15.3 | 0.45 | 1.93 | 1.52 | 248.4 | 19.6 |
| 70 | 19.5 | 0.56 | 6.54 | 3.58 | 354.7 | 45.3 | 18.6 | 0.44 | 1.87 | 1.34 | 232.0 | 28.1 |
| 70 | 21.6 | 0.56 | 5.90 | 3.26 | 331.9 | 44.7 | 19.7 | 0.44 | 1.59 | 1.14 | 236.9 | 28.4 |
| 70 | 27.7 | 0.53 | 4.35 | 2.50 | 255.8 | 42.6 | 22.9 | 0.45 | 1.77 | 1.17 | 230.5 | 33.8 |
| 70 | 147.6 | 0.54 | 3.98 | 2.04 | 241.7 | 48.4 | 144.3 | 0.44 | 2.00 | 1.33 | 189.9 | 34.2 |
| 95 | 11.0 | 0.59 | 9.02 | 5.14 | 481.3 | 42.1 | 12.7 | 0.45 | 1.80 | 1.43 | 276.7 | 19.2 |
| 95 | 13.9 | 0.58 | 6.85 | 3.38 | 365.0 | 50.4 | 15.7 | 0.45 | 1.62 | 1.13 | 233.1 | 22.6 |
| 95 | 148.6 | 0.53 | 3.10 | 1.77 | 210.0 | 41.4 | 135.1 | 0.45 | 1.61 | 1.07 | 176.7 | 32.5 |
| 120 | 11.0 | 0.60 | 9.54 | 4.86 | 454.8 | 49.4 | 13.0 | 0.44 | 2.24 | 1.70 | 258.9 | 22.5 |
| 120 | 14.2 | 0.59 | 7.39 | 3.82 | 310.3 | 47.5 | 16.0 | 0.46 | 1.99 | 1.40 | 221.4 | 28.9 |
| 120 | 145.6 | 0.53 | 2.98 | 1.58 | 170.3 | 45.6 | 133.9 | 0.43 | 1.33 | 0.83 | 139.6 | 37.5 |
| 140 | 11.4 | 0.59 | 8.93 | 4.38 | 351.5 | 50.3 | 13.5 | 0.45 | 2.55 | 1.74 | 237.8 | 31.2 |
| 140 | 147.7 | 0.52 | 2.82 | 1.61 | 161.6 | 42.3 | 131.3 | 0.44 | 1.04 | 0.65 | 122.5 | 35.4 |
| Loading rate = 0.002 in./min | | | | | | | | | | | | |
| 70 | 11.0 | 0.59 | 7.60 | 4.41 | 476.1 | 41.1 | 13.1 | 0.45 | 2.10 | 1.45 | 255.3 | 29.2 |
| 70 | 13.8 | 0.58 | 6.90 | 3.67 | 380.5 | 45.4 | 15.1 | 0.45 | 1.43 | 0.92 | 214.9 | 35.3 |
| 70 | 19.3 | 0.55 | 4.73 | 2.47 | 320.6 | 47.7 | 18.7 | 0.44 | 1.92 | 1.30 | 221.5 | 32.5 |
| 70 | 21.9 | 0.56 | 5.60 | 2.99 | 307.6 | 46.6 | 19.3 | 0.43 | 1.52 | 1.04 | 222.9 | 31.8 |
| 70 | 27.2 | 0.55 | 4.81 | 2.48 | 228.2 | 48.6 | 23.2 | 0.44 | 1.94 | 1.29 | 207.7 | 33.4 |
| 70 | 151.0 | 0.51 | 3.29 | 1.82 | 209.3 | 44.2 | 141.7 | 0.44 | 1.53 | 1.03 | 181.4 | 32.8 |
| 95 | 10.8 | 0.59 | 7.85 | 4.21 | 453.8 | 46.0 | 12.6 | 0.44 | 1.77 | 1.16 | 247.8 | 34.0 |
| 95 | 13.8 | 0.59 | 7.99 | 3.97 | 327.5 | 50.0 | 15.6 | 0.45 | 1.91 | 1.30 | 222.9 | 31.5 |
| 95 | 145.3 | 0.53 | 2.94 | 1.62 | 181.4 | 44.0 | 135.6 | 0.44 | 1.45 | 0.90 | 147.1 | 37.8 |
| 120 | 11.1 | 0.59 | 8.29 | 4.08 | 373.4 | 50.4 | 13.3 | 0.44 | 2.24 | 1.48 | 234.1 | 32.2 |
| 120 | 14.8 | 0.58 | 6.06 | 2.39 | 252.8 | 60.0 | 16.0 | 0.45 | 1.85 | 1.18 | 183.3 | 35.7 |
| 120 | 143.0 | 0.53 | 2.74 | 1.49 | 142.7 | 44.8 | 134.0 | 0.43 | 1.11 | 0.66 | 121.8 | 40.1 |
| 140 | 11.7 | 0.58 | 5.97 | 2.70 | 295.9 | 54.6 | 13.0 | 0.45 | 1.60 | 1.01 | 201.3 | 35.7 |
| 140 | 142.9 | 0.52 | 2.34 | 1.34 | 137.5 | 42.4 | 121.5 | 0.43 | 1.06 | 0.67 | 111.7 | 36.2 |

MC for green specimens was assumed to be the strength intersection MC of 25 and 24% for madrone and Douglas-fir, respectively (Forest Products Laboratory 1987), although the actual average MC was well over 100%. All main effects were statistically significant at the 5% $\alpha$-significance level for Douglas-fir. The same result holds for madrone with limited exceptions (e.g., the board main effect). None of the interactions of main effects were significant at the 5% level for both species. For convenience, the higher loading rate of 0.02 in./min will be denoted as HR, while the lower rate of 0.002 in./min will be referred to as LR in the following discussion.

### *Critical J-integral*

There were no published $J_{Ic}$ data available for madrone or any other hardwood. However, the average $J_{Ic}$ value of 1.96 lb/in. for Douglas-fir tested at 70 F, HR, and 13.2% MC (Table 1) was comparable to the $J_c$ value of 1.66 lb/in. reported for European pine (Morlier and Valentin 1982). This assumes that Douglas-fir resembles European pine more closely than European spruce, the only other species for which a value has been published.

$J_{Ic}$ vs. MC plots obtained from tests run at 70 F are shown in Fig. 5. Similar but less detailed results were obtained at the other tem-

TABLE 2. *Significance of main effects in analysis of variance.*

| Parameter | Level of significance | | | |
|---|---|---|---|---|
| | Rate | Temperature | MC | Board |
| Madrone | | | | |
| $J_{Ic}$ | <0.0001 | <0.0001 | <0.0001 | 0.0951 |
| $G_{Ic}$ | <0.0001 | <0.0001 | <0.0001 | 0.1455 |
| $K_{Ic}$ | <0.0001 | <0.0001 | <0.0001 | 0.0852 |
| DOP | 0.0194 | 0.0026 | 0.0778 | 0.5011 |
| Douglas-fir | | | | |
| $J_{Ic}$ | 0.0120 | 0.0300 | <0.0001 | <0.0001 |
| $G_{Ic}$ | <0.0001 | 0.0002 | <0.0001 | <0.0001 |
| $K_{Ic}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| DOP | <0.0001 | 0.0009 | <0.0001 | <0.0001 |

perature. $J_{Ic}$ decreased with increasing MC for both species. However, madrone was more sensitive to MC change (steeper slope of the regression line) than Douglas-fir. The $J_{Ic}$ value for madrone, based on regression results, decreased 48% (from 8.64 to 4.50 lb/in.) when the MC increased from 12 to 25% (green) at 70 F and HR. For Douglas-fir, the reduction was 6% (from 1.92 to 1.80 lb/in.) on the same basis.

This outcome would not be expected as a matter of course because $J_{Ic}$ is related to the total strain energy which, in turn, is governed by the critical load as well as COD. Increases in MC can be expected to reduce the critical load but also to increase the critical COD as wood becomes more ductile at high MC. Therefore, the relationship between $J_{Ic}$ and MC depends the relative changes in critical load and COD when MC varies.

For madrone, the combined decrease in critical load and increase in critical COD due to increasing MC resulted in a significant decrease in total strain energy, thereby greatly reducing $J_{Ic}$. On the other hand, for Douglas-fir, such changes were small, particularly when the temperature was below 120 F. Therefore, the MC effect on $J_{Ic}$ for Douglas-fir was marginal when temperatures were lower than 120 F.

Typical $J_{Ic}$ vs. temperature plots for both species are shown in Fig. 6. It is evident from this figure that temperature consistently showed a less prominent effect on $J_{Ic}$ than MC. It should be noted that the slopes for all conditions are very flat, indicating that $J_{Ic}$ is not very sensitive to temperature changes up to 140 F. Therefore, the opposite trends for HR and LR shown in Fig. 6 should not be overemphasized. In fact, overall results indicated that $J_{Ic}$ was mostly linearly related to temperature with a negative slope, particularly at high MC levels. Between species, madrone was more temperature-dependent than Douglas-fir, paralleling results for MC dependence.

Although no statistically significant interactions were found, Figs. 5 and 6 indicate that $J_{Ic}$ of madrone was more dependent on MC and temperature at HR than at LR. This can be attributed to the relative changes in critical load and COD, as previously discussed. By contrast, patterns of rate dependence were less apparent for Douglas-fir.

Based on the test results, the relationship between $J_{Ic}$, MC and temperature can be formulated as follows:

$$J_{Ic} = A \exp(BX) \tag{8}$$

where A and B are constants, $X = (T - 243)[1 - \exp(MC/C)]$, C is a sensitivity index determined by statistical analysis, T is temperature expressed in degrees Kelvin, MC is moisture content expressed in percentage, and the re-

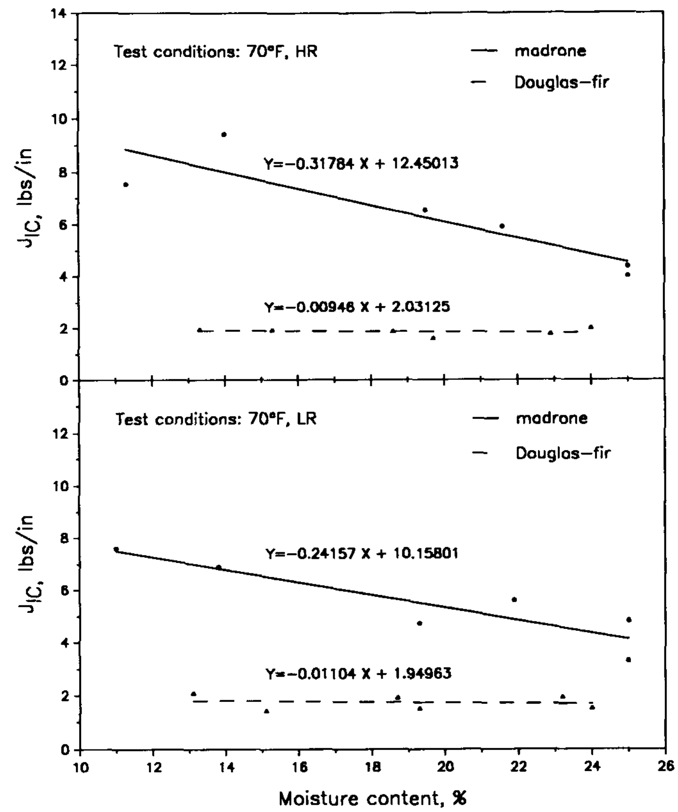



FIG. 5. $J_{Ic}$ vs. MC plots for madrone and Douglas-fir at 70 F. HR and LR represent loading rates of 0.02 in./min and 0.002 in./min, respectively.

sulting $J_{Ic}$ is in lb/in. The constants A, B, and C for Eq. (8) are listed in Table 3.

*Critical strain energy release rate*

Published data for $G_{Ic}$ are very sparse. However, the average $G_{Ic}$ value of 1.61 lb/in. (70 F, 13.3% MC, and HR) for Douglas-fir obtained in this study is comparable to the published value of 2.03 lb/in. (10% MC, 72 F, and 0.005 in./min loading rate) (Schniewind and Pozniak 1971).

The relationships between $G_{Ic}$ and MC, temperature, or loading rate have the same pattern as for $J_{Ic}$. Overall results indicated that the MC effect on $G_{Ic}$ for madrone was significant at all temperatures studied. For Douglas-fir, this effect became prominent only when temperature was 120 F or higher, but it should be recalled that no statistically significant interactions were found for any of the data. The $G_{Ic}$ value for madrone decreased 53% (from 5.10 to 2.38 lb/in.) when the MC increased from 12 to 25% at 70 F and HR. For Douglas-fir, the reduction was 28% (from 1.61 to 1.16 lb/in.). Clearly, the relationships between $G_{Ic}$ and MC, temperature, or loading rate were similar to those for $J_{Ic}$. In fact, $J_{Ic}$ and $G_{Ic}$ should be exactly the same for a perfectly linear-elastic material, as previously discussed. Consequently, Eq. (8) was equally applicable to $G_{Ic}$. The constants characterizing this relationship are given in Table 4.

*Critical stress intensity factor*

There is a wealth of published data for $K_{Ic}$. For Douglas-fir tested at about 70 F and 12% MC, published $K_{Ic}$ values range from 237 to 341 psi $(\text{in.})^{1/2}$ (Johnson 1973; Schniewind and

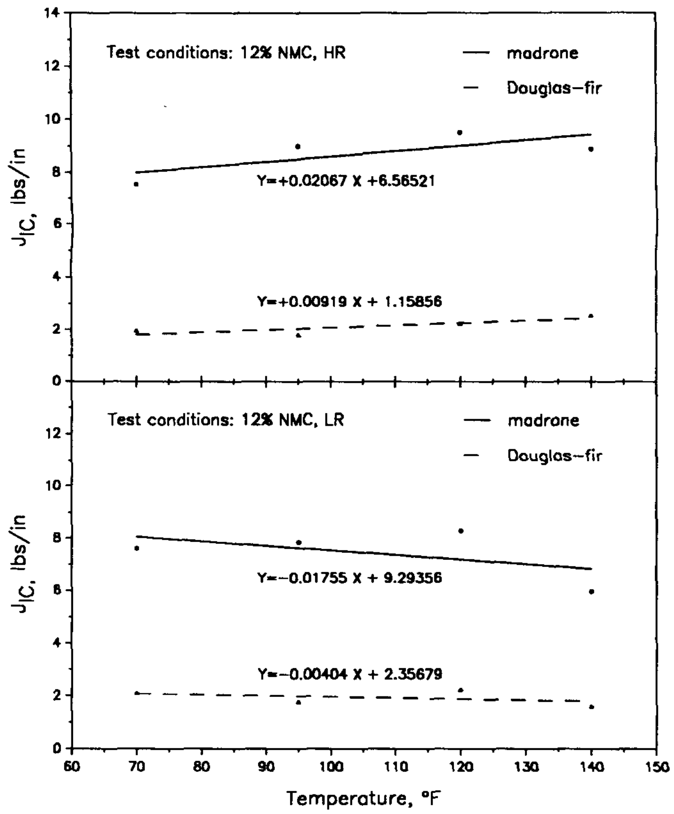



FIG. 6. Typical $J_{Ic}$ vs. temperature plots for madrone and Douglas-fir. HR and LR represent loading rates of 0.02 in./min and 0.002 in./min, respectively.

Pozniak 1971). The average $K_{Ic}$ value for Douglas-fir of 272 psi (in.)$^{1/2}$ (13.2% MC, 70 F, and HR) obtained from this study is therefore in excellent agreement with these data. For madrone, the average $K_{Ic}$ value of 495 psi (in.)$^{1/2}$ (11.1% MC, 70 F, and HR) was lower than the only published $K_{Ic}$ value of 561 ± 36 psi (in.)$^{1/2}$ (Schniewind et al. 1982). However, since the SG of madrone used in this study was also lower (0.58 vs. 0.66), and since $K_{Ic}$ should increase with increasing SG, such a difference was to be expected.

Relationships between $K_{Ic}$ and MC at 70 F are shown in Fig. 7. Results at other temperature levels were similar but less detailed. Overall results indicated that $K_{Ic}$ was linearly related to MC with a negative slope and had the least variability among the fracture parameters studied. The latter can be explained by noting that $K_{Ic}$ is only a function of critical load, while $J_{Ic}$ and $G_{Ic}$ depend on COD as well. Between species, madrone was more MC dependent than Douglas-fir.

Typical plots showing the relationships between $K_{Ic}$ and temperature are given in Fig. 8. Overall results showed that $K_{Ic}$ was linearly related to temperature with a negative slope. Between species, madrone was more temperature-dependent than Douglas-fir. $K_{Ic}$ of madrone is therefore more sensitive to MC and temperature than that of Douglas-fir. Similar effects of MC and temperature, and similar differences between a hardwood (buna) and two softwoods (sugi and hinoki) were noted by Schniewind et al. (1982).

As noted from Table 1, $K_{Ic}$ decreased with decreasing loading rate for both species within the entire MC and temperature range studied.

TABLE 3. *Constants for predicting $J_{Ic}$.*

| Condition | No. of observ. | C* | A* | B* | S** |
|---|---|---|---|---|---|
| Madrone | | | | | |
| HR | 126 | 5 | 8.59668 | 0.0000947 | <0.0001 |
| LR | 126 | 10 | 8.71739 | 0.0013781 | <0.0001 |
| Douglas-fir | | | | | |
| HR | 126 | 7 | 2.09699 | 0.0002017 | <0.0001 |
| LR | 126 | 22 | 2.32494 | 0.0043611 | <0.0001 |

* A, B, and C are the constants given in Eq. (8).
** S represents the level of significance of the regression.

This result is due to decreases in critical load with decreasing loading rate, most likely because of the rheological characteristics of wood. The ratio of $K_{Ic}$ values at LR to HR ranged from 0.81 to 0.96 for madrone, and 0.83 to 0.96 for Douglas-fir. All ratios for both species were less than unity, indicating a consistent loading rate effect on $K_{Ic}$.

### *Degree of plasticity*

The DOP, as defined in Eq. (7), is an index characterizing the amount of strain energy reserved (i.e., not accounted for) in the material when the linear-elastic theory is applied. The reserved energy for Yugoslavian fir was reported to be about 20% at 70 F and 10% MC (Zakic 1988). This value is similar to the DOP value of 17% for Douglas-fir determined in this study (13.3% MC, 70 F and HR). The overall DOP values, as noted from Table 1, varied from 34 to 60% for madrone and 17 to 40% for Douglas-fir in the MC and temperature range studied. These values show that a significant amount of strain energy capacity is not accounted for when the $G_{Ic}$ of LEFM is used as a fracture criterion for wood, particularly at elevated temperatures and high MC conditions.

Between species, the DOP value for madrone was higher than that for Douglas-fir when compared on the same basis. For both species, DOP increased with increasing MC and temperature, and decreasing loading rate.

### *Interrelationships between fracture parameters*

Relationships between $J_{Ic}$ and $K_{Ic}$ for both species within the temperature, MC, and loading rate range studied were obtained by regressing $J_{Ic}$ on $K_{Ic}^2$ using all 252 observations (per species). The results are shown in Eqs. (9) and (10) for madrone and Douglas-fir, respectively.

For madrone:

$$J_{Ic} = 0.0000263125K_{Ic}^2 + 3.05371847 \quad (9)$$

For Douglas-fir:

$$J_{Ic} = 0.0000153978K_{Ic}^2 + 1.02266123 \quad (10)$$

Both Eqs. (9) and (10) were statistically significant at the 0.01% level, but $R^2$ values were

TABLE 4. *Constants for predicting $G_{Ic}$.*

| Condition | No. of observ. | C* | A* | B* | S** |
|---|---|---|---|---|---|
| Madrone | | | | | |
| HR | 126 | 8 | 5.16839 | 0.0006869 | <0.0001 |
| LR | 126 | 52 | 5.81336 | 0.0294292 | <0.0001 |
| Douglas-fir | | | | | |
| HR | 126 | 9 | 1.74667 | 0.0007239 | <0.0001 |
| LR | 126 | 34 | 1.70810 | 0.0104704 | <0.0001 |

* A, B, and C are the constants given in Eq. (8).
** S represents the level of significance of the regression.

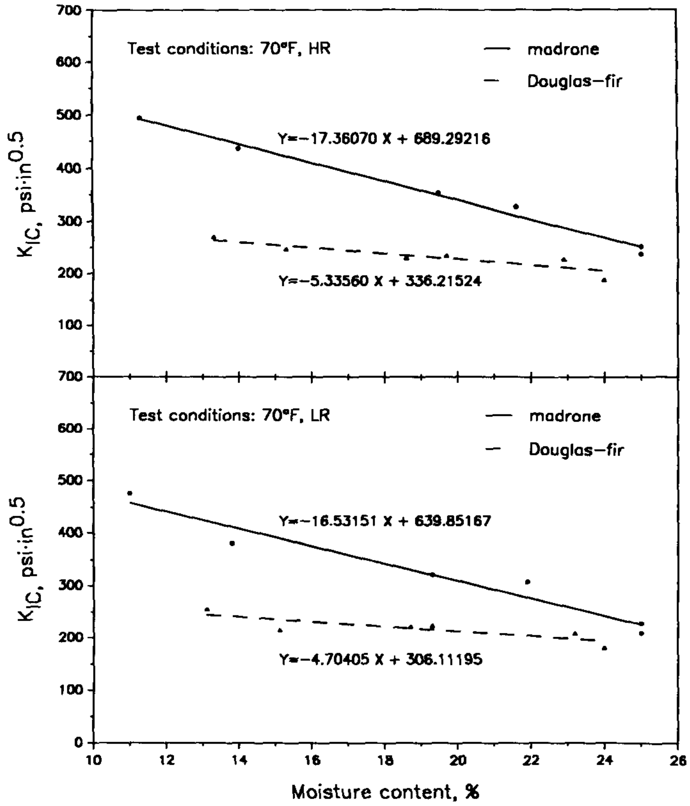



FIG. 7. $K_{Ic}$ vs. MC plots for madrone and Douglas-fir at 70 F. HR and LR represent loading rates of 0.02 in./min and 0.002 in./min, respectively.

only 0.51 for madrone and 0.37 for Douglas-fir. It is clear from these equations that madrone should have a higher $J_{Ic}$ value than Douglas-fir given the same $K_{Ic}$ value. These equations can be used to obtain estimates of $J_{Ic}$ from $K_{Ic}$ values, provided that temperature, MC, and loading rate are within the range investigated in this study.

Empirical equations for $J_{Ic}$ and $G_{Ic}$ were also obtained using the same regression techniques, and are given in Eqs. (11) and (12) for madrone and Douglas-fir, respectively:

For madrone:

$$J_{Ic} = 1.77331857G_{Ic} + 0.36686280 \quad (11)$$

For Douglas-fir:

$$J_{Ic} = 1.28140109G_{Ic} + 0.20936420 \quad (12)$$

Both equations were also significant at the 0.01% level, but here $R^2$ was 0.84 for madrone and 0.82 for Douglas-fir. The difference in coefficients of determination is not so surprising if it is considered that J and G are both based on energy considerations, while K values are based on maximum load. Again, these equations show that madrone should have a higher $J_{Ic}$ value than Douglas-fir at the same $G_{Ic}$ value, indicating greater plasticity effects in the hardwood. It is also important to note that since both $G_{Ic}$ and $K_{Ic}$ decrease with increasing temperature and MC, $J_{Ic}$ will decrease as well. By combining the above equations, $G_{Ic}$ can also be predicted from $K_{Ic}$ and vice versa.

## CONCLUSIONS

The following conclusions can be drawn from the results of this study:

1. Data analysis procedures used in this study, including numerical integration, two-step curve fitting, and parameter estimation

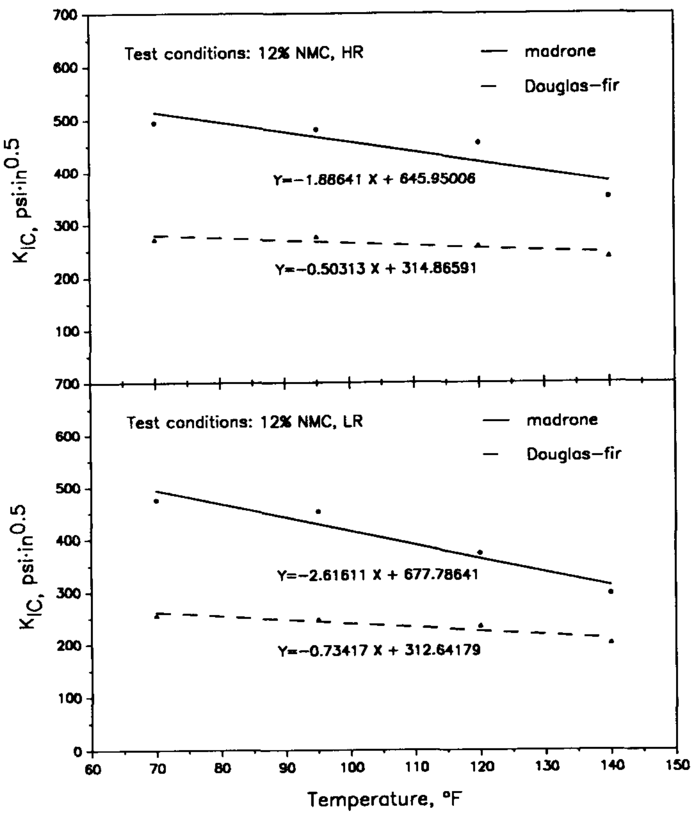



FIG. 8. Typical $K_{Ic}$ vs. temperature plots for madrone and Douglas-fir. HR and LR represent loading rates of 0.02 in./min and 0.002 in./min, respectively.

were appropriate. The same procedures could be applied to other EPFM studies of wood.

2. Species, MC, temperature, and loading rate were all found to have a significant effect on $J_{Ic}$, $G_{Ic}$, and $K_{Ic}$.

3. Pacific madrone had higher $J_{Ic}$, $G_{Ic}$, and $K_{Ic}$ values than Douglas-fir at comparable conditions. Pacific madrone was more sensitive to MC and temperature variations than Douglas-fir.

4. For both species, $J_{Ic}$ and $K_{Ic}$ decreased linearly with increasing MC and temperature, and with decreasing loading rate. However, temperature had less effect than MC.

5. $J_{Ic}$ and $G_{Ic}$ of both species could be predicted using MC and temperature as variables (Tables 3 and 4).

6. $K_{Ic}$ was the least variable among the fracture parameters studied.

7. DOP was as high as 60% for Pacific madrone, and 40% for Douglas-fir, indicating that a large amount of total strain energy was reserved (not accounted for) when the LEFM theory was applied.

8. Relationships between any two of the fracture parameters ($J_{Ic}$, $G_{Ic}$, and $K_{Ic}$) could be established by empirical equations, the best relationship being found between $J_{Ic}$ and $G_{Ic}$. The equations may be used to estimate $J_{Ic}$ from existing $K_{Ic}$ or $G_{Ic}$ values although the correlation is not high.

9. The results of this study demonstrate the applicability of the J-integral method to describe opening mode fracture in wood. As shown by the high DOP of wood at various conditions tested in this study, the J-integral method is a more appropriate method to characterize the actual fracture strength of wood than methods relying solely on linear elastic behavior.

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