THE INFLUENCE OF WOOD SPECIMEN GEOMETRY ON MOISTURE MOVEMENT DURING DRYING

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ABSTRACT

The influence of the geometrical shape of a wood specimen on the dynamics of drying under isothermal conditions is investigated in this research. Polynomials describing the dependence of half-drying time on the ratio of the area to the perimeter of the transverse section of extremely long specimens of northern red oak *Quercus rubra*, are presented for drying from above the fiber saturation point. This paper describes the conditions of usage of a two-dimensional moisture transfer model in contrast to the one-dimensional model for accurate prediction of the drying process. The dependence of halfdrying time on the ratio of the volume to the surface of the specimens in the shape of a rectangular parallelepiped has been investigated using a three-dimensional moisture movement model. Polynomials describing that dependence are presented.

Keywords: Wood drying, diffusion, modeling.

INTRODUCTION

During drying, moisture evaporates from the surface of a piece of wet wood to the surrounding air. When the moisture concentration in the outer layers decreases, the moisture begins to move from the wetter interior to the drier surface.

The wood drying problem can be treated as a diffusion problem based on Fick's second law. A model of the drying process, under isothermal conditions, can be expressed through the diffusion equation with initial and boundary conditions. The initial condition expresses an initial moisture concentration in the specimen, and the boundary condition describes the surface evaporation. Several authors have developed methods to solve the moisture diffusion problem. Newman gave a solution in 1931 (Newman 1931). Some modifications of the method, as well as new techniques, were developed later. Drying models based on Fick's second law have been successfully used to predict a wood drying process. Several authors have used a wood drying model to solve the inverse problem, i.e., to determine diffusion and surface emission coefficients. Literature surveys can be found in Rosen 1978,

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1987; Söderström and Salin 1993; Liu and Simpson 1997.

Since the moisture transfer in a piece of wood is three-dimensional (3-D), the wood drying model should involve all three space directions to accurately predict the drying process. The two-dimensional (2-D) moisture transfer model accurately describes the wood drying process for extremely long sawn boards (thickness and width are much smaller than length). Respectively, the one-dimensional (1-D) model of moisture movement accurately describes the wood drying process only for extremely long and wide boards (thickness is much smaller than length and width). However, even 1-D analysis can be successfully used to predict the drying process for a relatively short and narrow specimen if four of six surfaces are heavily coated in order to reduce the moisture transfer from those four surfaces.

This paper deals with the influence of the specimen geometry on the drying dynamics under isothermal conditions. Advantage of the 2-D model over the 1-D model when predicting the drying process of an extremely long specimen is investigated. Usage of the corresponding 3-D model is also considered. Only specimens without any surface coating were analyzed.

The influence of the geometrical shape of a wood specimen on the dynamics of drying is investigated in this paper. Polynomials, describing the dependence of halfdrying time on the ratio of the area to the perimeter of the transverse section of the extremely long specimen of northern red oak, are presented for drying from above the fiber saturation point. The 3-D model has been used to determine the dependence of halfdrying time on the ratio of the volume to the whole surface of the specimens in the shape of a rectangular parallelepiped, and polynomials describing the dependence are presented.

DRYING MODEL

In a three-dimensional formulation, the moisture movement, under isothermal conditions, in the direction normal to the surface of a symmetric wood piece of thickness $2a$, width $2b$, and length $2c$ can be expressed through the following diffusion equation with variable coefficients:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D_x(u)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y(u)\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z(u)\frac{\partial u}{\partial z}\right)$$

$$(0 < x < a,\ 0 < y < b,\ 0 < z < c,\ t > 0); \qquad (1)$$

here u is moisture content, t is time, x, y, z are space coordinates measured from the surface of the board to the center, and $D_x(u)$, $D_y(u)$, $D_z(u)$ are the concentration-dependent diffusion coefficients in the space directions x, y, z, respectively. The initial condition ($t = 0$) is

$$u = u_0 \qquad (0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c) \qquad (2)$$

The boundary conditions that describe symmetry and surface evaporation ($t > 0$) are

$$\frac{\partial u}{\partial x} = 0\ (x = a), \qquad \frac{\partial u}{\partial y} = 0\ (y = b), \qquad \frac{\partial u}{\partial z} = 0\ (z = c) \qquad (3)$$

$$D_x(u)\frac{\partial u}{\partial x} = S(u_e - u) \qquad (x = 0) \qquad (4)$$

$$D_y(u)\frac{\partial u}{\partial y} = S(u_e - u) \qquad (y = 0) \qquad (5)$$

$$D_z(u)\frac{\partial u}{\partial z} = S(u_e - u) \qquad (z = 0) \qquad (6)$$

where S is the surface emission coefficient and u_e is the equilibrium moisture content (EMC) with the ambient air climate.

In the calculations, discussed below, a corresponding model in 1-D and 2-D formulation was used as well. This model can be derived from the model expressed by Eqs. (1)–(6) by ignoring the corresponding space coordinates.

Closed mathematical solutions are not usually possible when analytically solving the differential equations with a variable diffusion coefficient and complex boundary conditions; therefore the mathematical model represented by Eqs. (1)–(6) was solved numerically. The finite-difference technique has been used for the discretization of the model (Samarskii 1983). That technique allows us to effectively solve the differential equations with variable diffusion coefficients and complex boundary conditions. We introduced a nonuniform discrete grid to increase the efficiency of calculations. Since moisture evaporates from the surface of a piece of wet wood, an exponentially increasing step of the grid was used in all space directions from the surface of the wood piece to the center, while a constant step was used in t direction.

The moisture transfer model (1)–(6) was used to simulate the drying of specimens of northern red oak (*Quercus rubra*). It was assumed that the diffusion coefficient is constant above fiber saturation point (fsp) (Hunter 1995; Yokota 1959), and it is equal to the diffusion coefficient at the fsp value. Though the radial and tangential diffusion coefficient may be different, the transverse (in the x and y directions) diffusion below fsp for red oak was represented by

$$D(u) = Ae^{-5280/T}e^{Bu/100} \qquad (7)$$

where T is the temperature in Kelvin, u is percentage of the moisture content, A and B are experimentally determined coefficients (Simpson 1993). It was assumed that fsp is 30% for the red oak. It was also assumed that longitudinal diffusion in wood is faster than the transverse diffusion (Siau 1984):

$$D_x(u) = D_y(u) = D(u), \qquad D_z(u) = CD(u) \quad (8)$$

where C is an experimentally determined coefficient.

RESULTS OF CALCULATIONS AND DISCUSSION

The experimental moisture content values for red oak by Simpson and Liu (1997) were used for numerical analysis. Simpson and Liu presented these drying test data to us in a numerical form. Experimental drying conditions were 43°C at 84% relative humidity (16.2% EMC, u_e in Eqs. (4)–(6)). There were two air velocities: 1.5 and 5.1 m/s. The size of specimen (sawn board) was 102 by 305 by 29 mm, i.e., $2a = 29$, $2b = 102$, $2c = 305$ in Eqs. (1)–(3). The average initial moisture content was 82.5%. Values of the coefficients A, B in Eq. (7) and S in Eqs. (4)–(6) were found (Simpson and Liu 1997) for both air velocities.

We investigated the influence of geometry of the transverse section of a specimen on drying dynamics when the specimen is extremely long. At first, we considered the influence of the ratio of the width to the thickness of the specimen. In this case, the drying model in 2-D formulation was used for numerical simulation of drying.

The fraction of total moisture content in a specimen can be defined as

$$E(t) = (\bar{u}(t) - u_e)/(u_0 - u_e) \qquad (9)$$

where $\bar{u}(t)$ is the calculated average moisture content value, which can be determined by numerical integration of the finite difference solution.

Let $t_{0.5}$ be the time when the drying process reaches the medium, called halfdrying time, i.e. $E(t_{0.5}) = 0.5$ (Choong and Skaar 1969). Let k be a dimensionless ratio of the width to the thickness of a wood specimen. The transverse section of the specimen was modeled as a rectangle with various values of the ratio k ($k = 1, \ldots, 100$) keeping the thickness equal to $2a = 29$ mm in Eqs. (1)–(3). In each board geometry case, the drying until the time $t_{0.5}$ was simulated to determine dynamics of the drying. The results of calculations are depicted in Fig. 1. Curves drawn through all calculated values of halfdrying time are

$$t_{0.5}(k) = 113.7\exp(-0.91/k), \quad v = 1.5 \text{ m/s} \quad \text{(solid line in Fig. 1)} \qquad (10)$$

$$t_{0.5}(k) = 87.7\exp(-0.94/k), \quad v = 5.1 \text{ m/s} \quad \text{(dashed line in Fig. 1)} \qquad (11)$$

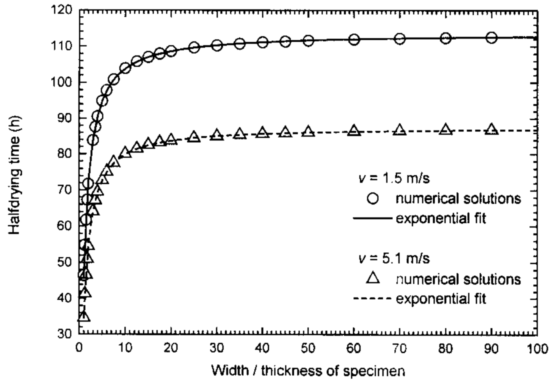

FIG. 1. Dependence of the halfdrying time on the ratio of the width to the thickness of a specimen keeping the thickness 29 mm. The white circles and triangles show the calculated values of the halfdrying time for the air velocities v 1.5 and 5.1 m/s, respectively. The solid and dashed lines are exponential functions fitted to these values (Eqs. 10–11).

The relative thickness of the lumber appears to be important for the drying dynamics. The halfdrying time $t_{0.5}$ notably increases for small values of the ratio k ($k < \approx 10$), and it stays almost unchanged for large values of k ($k > \approx 30$). While drying, the moisture from within the board moves to the surface of the board, where it evaporates into the airstream. The moisture content near the surface is lower than at the center and it is particularly significant at the corner of the board (Ferguson and Turner 1995, 1996). The influence of the moisture content at the corners on the average moisture content within the whole specimen is significant for narrow plates. Let t^* be a limiting value of halfdrying time, extremely increasing the width of the specimen, i.e., $t^* = t_{0.5}(\infty)$. The value of t^* can be calculated from Eqs. (10) and (11) or by using the corresponding 1-D model. In addition to the thickness equal to $2a = 29.0$ mm in Eqs. (1)–(3), we calculated t^*, $t_{0.5}(10)$ and $t_{0.5}(30)$ for two more thicknesses: 58.0 mm and 14.5 mm. The calculation showed that in all these cases $t_{0.5}(10) = (0.91 \pm 0.01)\ t^*$ and $t_{0.5}(30) = (0.97 \pm 0.005)t^*$ for both air velocities.

We used the values of the coefficients A and B in Eq. (7) and S in Eqs. (4)–(6), which were determined by Simpson and Liu (1997) using a new technique that requires both experimental and the finite difference solution values. From Fig. 1 it can be noticed that usage of the 1-D model to determine the surface and internal moisture transfer coefficients for a rather narrow specimen ($k < \approx 10$) can result in inexact values of the coefficients. Since the 1-D model accurately describes the process of drying only for very wide and long plate, Simpson and Liu (1997) used a 1-D moisture transfer model to predict drying time. The sawn boards, used in the real experiments, were relatively narrow (k = 102 mm/29 mm $\approx$ 3.52). However, the edges and ends of each specimen were heavily coated to reduce moisture transfer from these coated surfaces. Figure 1 shows that it was well justified to coat the edges in case of usage of the 1-D model for specimens with $k \approx 3.52$. Thus, a 1-D model should be used to simulate wood drying only if a specimen is relatively long and wide ($k > 10$) or the corresponding surfaces of the specimen should be heavily coated. Let us recall that in our calculations it was assumed that specimens were not coated.

The investigation of the influence of the ratio of the width and thickness of the specimen on drying dynamics was then generalized. The transverse section of the specimen was modeled as a rectangle changing the thickness and width in both directions. The results of calculation showed that the halfdrying time greatly increases if the area of the transverse section of the specimen increases, and the wide board dries faster than the narrow one with the same area of the transverse section. The results of calculation based upon the idea that the halfdrying time depends mainly on the ratio of the area to the perimeter of the transverse section of the specimen. Let l be the ratio of the area to the perimeter of the transverse section of the specimen (mm), i.e., $l = ab/(a + b)$ for a sawn board of thickness $2a$ and width $2b$. Note that the square has the minimal perimeter among all the rectangles having the same area, and $a > l$, $b > l$ for any specimen having rectangular transverse sec-

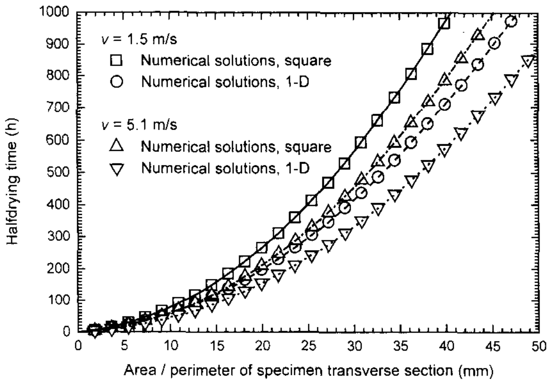

FIG. 2. Dependence of the halfdrying time on the ratio of the area to the perimeter of a specimen. The white squares and circles show the calculated values for the air velocity $v = 1.5$ m/s, and triangles show them for $v = 5.1$ m/s. The solid, dash-dot, dashed, and dot lines are polynomials fitted to these values (Eqs. 12–15).

tion. The ratio l equals $a/2$ (thickness/4) for the board having the square transverse section, and a (thickness/2) is the limit value of the ratio l extremely increasing the width of the board of thickness $2a$. We have investigated these two extreme cases of the geometrical shape of the transverse section of the specimen:

a) the transverse section is a square changing the side (l = thickness/4);

b) the transverse section is an extremely wide rectangle changing the thickness ($l \rightarrow$ thickness/2).

The 2-D model was used in case (a), and the 1-D model was used to predict halfdrying time in case (b). The results of calculations are presented in Fig. 2. In all the cases, curves drawn through the halfdrying time values are second-order polynomials. The polynomial fits for specimens having the square transverse section (a) are the following:

$$t_{0.5}(l) = 2.37l + 0.549l^2, \qquad v = 1.5 \text{ m/s}$$

(solid line in Fig. 2), (12)

$$t_{0.5}(l) = 1.40l + 0.459l^2, \qquad v = 5.1 \text{ m/s}$$

(dash-dot line in Fig. 2). (13)

The polynomial fits for the extremely wide specimens (b) are the following:

$$t_{0.5}(l) = 1.99l + 0.396l^2, \qquad v = 1.5 \text{ m/s}$$

(dashed line in Fig. 2), (14)

$$t_{0.5}(l) = 1.17l + 0.331l^2, \qquad v = 5.1 \text{ m/s}$$

(dot line in Fig. 2). (15)

The polynomial fits for these extreme cases form a sector for each air velocity. In this paper, the sector is considered as a plane region bounded by two curves. As shown in Fig. 2, these two sectors have a nonempty intersection. Additional calculations showed that the values of the halfdrying time lie in the sector for any specimen, having the rectanglar transverse section. This property is valid for both air velocities: 1.5 and 5.1 m/s. In particular, Fig. 2 and Eqs. (12)–(15) show that the halfdrying time $t_{0.5}$ varies between 297 and 402 h for the air velocity 1.5 m/s and between 236 and 322 h for the air velocity 5.1 m/s for specimens with $l = 25$ mm. Samples of specimens with $l = 25$ mm are sawn boards having transverse section as 100 by 100 mm, 75 by 150 mm, 60 by 300 mm, 55 by 550 mm, etc.

The investigation of the influence of the geometry of a specimen on drying dynamics was extended to 3-D moisture movement. The specimen was modeled as a rectangular parallelepiped changing the thickness, width, and length in all directions. Let m be the ratio of the volume to the entire surface area of the specimen (mm), i.e., $m = abc/(ab + bc + ac)$ for a sawn board $2a$ by $2b$ by $2c$. The cube has the minimal surface area among all the rectangular parallelepipeds having the same volume. The ratio m equals $a/3$ (thickness/6) for the board of the shape of the cube, and a (thickness/2) is the limit value of the ratio m extremely increasing the width and length of the board (plane) of thickness $2a$. Thus, we chose the following two extreme cases:

c) the specimen is a cube changing the side (m = thickness/6);

d) the specimen is an extremely wide and

TABLE 1. *The values of B_1 and B_2 (Eq. 16) fitting the polynomials to the calculated vales of the halfdrying time for specimens in the cubic shape for two values of* C *(Eq. 8); v denotes the air velocity.*

	v = 1.5 m/s		v = 5.1 m/s	
Coefficient	C = 2.5	C = 5.0	C = 2.5	C = 5.0
B_1	2.75	3.02	1.66	1.86
B_2	0.421	0.295	0.352	0.245

long plate changing the thickness ($m \rightarrow$ thickness/2).

The 3-D model (1)–(6) was used to simulate drying in case (c). Case (d) is equivalent to case (b), which was investigated above, and, in this case, $m = l$. The 3-D model requires a value of the dimensionless coefficient C in Eq. (8). We used two different values of this coefficient, 2.5 (Siau 1984) and 5.0 (Liu 1998). The polynomial fits to the calculated values of the halfdrying time for the cubic specimens (c) can be expressed by

$$t_{0.5}(m) = B_1 m + B_2 m^2, \qquad (16)$$

where B_1 and B_2 are coefficients. The values of B_1 and B_2 are presented in Table 1. The polynomial fits for case (d) are expressed by Eqs. (14) and (15) assuming $m = l$. The polynomial fits for extreme cases (c) and (d) form a sector for both values of the coefficient C, 2.5 and 5.0, and both air velocities, 1.5 and 5.1 m/s. We have established that, for any specimen, the values of the halfdrying time lie in a sector of the shape of a rectangular parallelepiped for both air velocities and both values of the coefficient C, provided $c \geq a$ and $c \geq b$ in Eqs. (1)–(3). The last condition for the values of a, b, and c is important because of the assumption that the longitudinal diffusion is faster than the transverse one. In particular, according to Table 1 and Eqs. (14)–(16), the halfdrying time $t_{0.5}$ varies between 260 and 297 h for the air velocity 1.5 m/s, and between 200 and 236 h for the air velocity 5.1 m/s assuming that $C = 5.0$ for specimens with $m = 25$ mm. Samples of specimens with $m = 25$ mm are sawn boards 150 by 150 by 150 mm, 100 by 150 by 300 mm, 60 by 400 by 1200 mm, etc.

CONCLUSIONS

The two-dimensional and three-dimensional moisture transfer models can be successfully used to investigate the influence of the geometrical shape of a wood specimen of northern red oak on the dynamics of drying from above the fiber saturation point under isothermal conditions.

The investigation showed that the two-dimensional model guarantees better prediction than the one-dimensional one for the drying process in the case of the extremely long specimen if the ratio of the width to the thickness of the specimen is less than 10. Otherwise the edges of the specimen should be heavily coated in order to reduce moisture transfer from these two surfaces.

The halfdrying time can be expressed as the second-order polynomial of the ratio of the area to the perimeter of the transverse section of the specimen for two extreme cases of the geometrical shape of the section: a square and an extremely wide rectangle. The values of the halfdrying time, for any specimen in the shape of a rectangle in the transverse section, lie in a sector, formed by these two polynomials. A similar pair of polynomials of the second-order can express the dependence of the halfdrying time on the ratio of the volume to the whole surface of the specimens in the shape of a rectangular parallelepiped.

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REFERENCES

CHOONG, E. T., AND C. SKAAR. 1969. Separating internal and external resistance of moisture in wood drying. Wood Science 1(4):200–202.

FERGUSON, W. J., AND I. W. TURNER. 1995. A comparison of the finite element and control volume numerical solution techniques applied to timber drying problems below the boiling point. Int. J. Num. Methods Eng. 38(3): 451–467.

———, AND ———. 1996. A control volume finite element numerical simulation of the drying of spruce. J. Comput. Phys. 125:59–70.

HUNTER, A. J. 1995. Equilibrium moisture content and the movement of water through wood above fibre saturation, Wood Sci. Technol. 29:129–135.

LIU, H. 1998. Development of a simulation model for drying deformation in radiata pine boards. Ph.D. thesis, Lincoln University. New Zealand.

LIU, J. Y., AND W. T. SIMPSON. 1997. Solutions of diffusion equation with constant diffusion and surface emission coefficients. Drying Technol. 15(10):2459–2477.

NEWMAN, A. B. 1931. The drying of porous solids: Diffusion and surface emission equations. Trans. Amer. Chem. Eng. 27:203–220.

ROSEN, H. N. 1978. The influence of external resistance on moisture adsorption rates in wood. Wood Fiber Sci. 10(3):218–228.

———. 1987. Recent advances in the drying of solid wood. Adv. in drying (Ed. Mujumdar). Hemisphere 4: 99–146.

SAMARSKII, A. 1983. Difference scheme theory [in Russian]. Nauka, Moskow. 610 pp.

SIAU, J. F. 1984. Transport processes in wood. Springer-Verlag, New York, NY. 245 pp.

SIMPSON, W. T. 1993. Determination and use of moisture diffusion coefficient to characterize drying of northern red oak (*Quercus rubra*). Wood Sci. Technol. 27:409–420.

———, AND J. Y. LIU. 1997. An optimization technique to determine red oak surface and internal moisture transfer coefficients during drying. Wood Fiber Sci. 29(4):312–318.

SÖDERSTRÖM, O., AND J. G. SALIN. 1993. On determination of surface emission factors in wood drying. Holzforschung 47(5):391–397.

YOKOTA T. 1959. Diffusion of sorption-water through the cell wall of wood. Mokuzai Gakkaishi 5(4):143–149.