THEORETICAL THERMAL CONDUCTIVITY EQUATION FOR UNIFORM DENSITY WOOD CELLS[1]

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(Received May 2006)

Abstract. The anisotropy of wood creates a complex problem requiring that analyses be based on fundamental material properties and characteristics of the wood structure to solve heat transfer problems. A two-dimensional finite element model that evaluates the effective thermal conductivity of a wood cell over the full range of moisture contents and porosities was previously developed, but its dependence on software limits its use. A statistical curve-fit to finite-element results would provide a simplified expression of the model's results without the need for software to interpolate values. This paper develops an explicit equation for the values from the finite-element thermal conductivity analysis. The equation is derived from a fundamental equivalent resistive-circuit model for general thermal conductivity problems. Constants were added to the equation to improve the regression-fit for the resistive model. The equation determines thermal conductivity values for the full range of densities and moisture contents. This new equation provides thermal conductivity values for uniform-density wood material using inputs of only oven-dry density and moisture content. An explicit method for determining thermal conductivity of uniform density wood cells has potential uses for many wood applications.

Keywords: Resistive-circuit modeling, wood cell, thermal conductivity, moisture content, heat transfer, cellular structure, finite element modeling, anisotropy.

INTRODUCTION

The structure of wood has a significant effect on its heat transfer process as when drying lumber, heating logs in veneer mills, or hot-pressing wood composites. Therefore, for optimum wood heating, whether from mature or small-diameter trees, the fundamental heat transfer properties are needed to accurately predict process conditions.

Anisotropy of wood is due to wood fiber's radial, tangential, and longitudinal orientation (Fig 1) and the structural differences between the development of earlywood and latewood bands for each annual ring (Fig 2). In two previous papers (Hunt and Gu 2006; Gu and Hunt 2006), a finite element (FE) model was presented that solved for the heat transfer coefficient of a wood cell structure over the full range of porosities (or density) but without any moisture effect.

Moisture has a significant effect on the heat-transfer coefficient, but to measure this effect is not easy. Early in the 1940s, MacLean (1941) pointed out that conductivity of wood at various moisture contents, as determined under steady-

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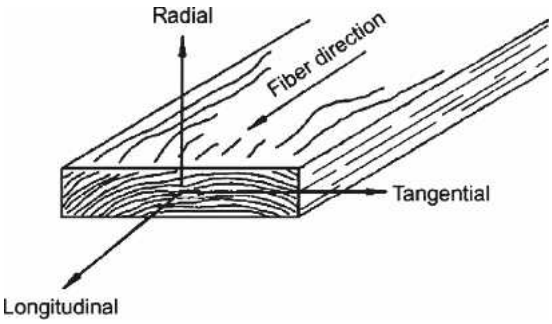

FIGURE 1. Three principal axes of wood with respect to fiber direction and growth rings.

state conditions, does not represent true conductivity of wood under the original moisture-distribution conditions. This change from the original uniform to steady-state redistributed moisture is a result of the process to conduct the experiment that causes moisture to migrate and results in slight errors in measurement. Therefore, we believe a better value of thermal conductivity of wood can be obtained by theoretical modeling because modeling with fundamental principles does not involve moisture redistribution errors associated with physical testing. The FE model results were compared with MacLean's data in a previous paper (Gu and Hunt 2006), and showed fair agreement for average wood densities. However, when extrapolated beyond the empirical equation's data set, MacLean's equation does not approach those of pure substances on either end of the density scale. A new set of equations of heat transfer will help determine thermal conductivities over the full range of moisture contents and densities that could be used to better understand the wood–water–density relationships in the heat-transfer processes.

Previous models using a cellular structure of wood have been developed (Hart 1964 and Siau 1995) to describe thermal conductivity. They used an electrical resistive modeling technique to describe thermal conductivity effects for a unit cell. Equivalent electrical resistive-circuit thermal conductivity models have been extensively developed for many nonwood applications. These steady-state one-dimensional equations are well established and found in fundamental heat transfer textbooks. Hart's model describes thermal conductivity only in the cell-wall material but does not include any effect of air, vapor, or free water in the cell lumen. Siau also uses a resistivity-circuit model and does include vapor effects in the lumen. He also limited his investigation to moisture content (MC) below the fiber saturation point (FSP) and porosities greater than 25%, thus not dealing with free water in the lumen. No other model was found in the literature on wood cellular thermal conductivity that covered the full range of porosity or density and moisture contents for the wood cell.

The thermal conductivity models developed by Hart and Siau were one-dimensional, whereas the FE model previously described (Hunt and

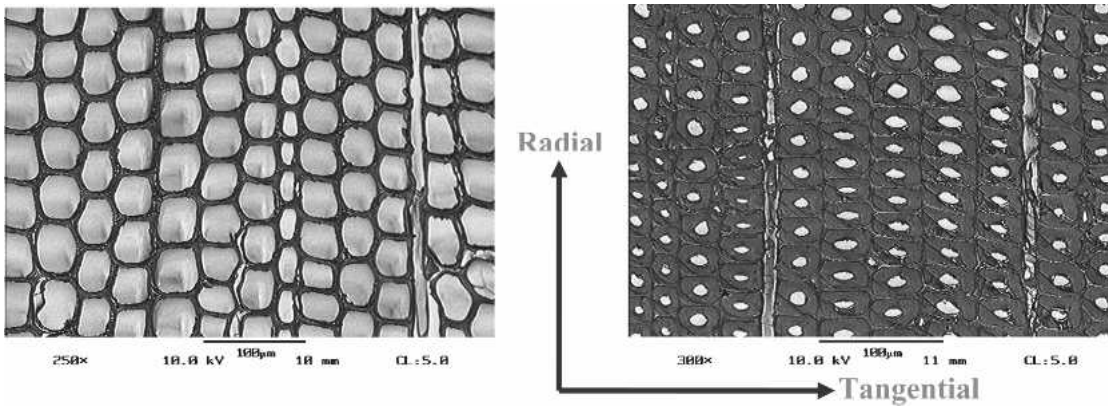

FIGURE 2. Microscope images of softwood structure in earlywood (left) and latewood (right) regions.

Gu 2006; Gu and Hunt 2006) uses the FE approach to incorporate two-dimensional analysis. We believe the FE modeling approach may be a better predictor of cellular thermal conductivities for the full range of cellular density and moisture contents because the heat transfer in a wood cell involves complex two- and three-dimensional flow and should be studied using a method capable of handling these complex characteristics. For through-the-thickness analysis, a two-dimensional model can provide a better understanding of the wood–water relationship in the heat transfer process.

FINITE ELEMENT MODEL—DISCUSSION

The material properties used for input variables in the FE model for the cell-wall substance, bound water in the cell wall, air, water vapor, and free water in the lumen, are listed in Table 1.

The results from the FE modeling studies (Hunt and Gu 2006; Gu and Hunt 2006) are plotted as a function of wood cell oven-dry density (Fig 3). For each line shown, 0% porosity (an impossible case, but shown for theoretical purposes) is on the right end, and 90% porosity is on the left end, with increments of 10% porosity plotted between these two extremes. All the thermal conductivity values from the FE model are based on the property values at 30°C. Values at other temperatures can be reestimated by the model or using the simple K-Temp relationship given by Siau's Eq (5.23) (1995).

Thermal conductivity values at four moisture content (MC) conditions are plotted in Fig 3: 1) 0% MC (oven-dry); 2) 30% MC in the cell wall (FSP); 3) cell lumen filled with 50% free water; 4) a fully saturated lumen (the maximum MC condition). For this model, the FSP is assumed to be nominally 30% but could range 25–35% based on extrapolated adsorption data from the dry-condition or even as high as 40% from never-dried wood (Stamm and Smith 1969). These plots show that when wood is not fully saturated by water (ie certain amount of free water and vapor in lumen), there is a significant increase in thermal conductivity as density increases (porosity decreases). But when the cell lumen is fully filled with free water (the maximum moisture content of wood at that porosity or density), the thermal conductivity decreases as density increases. At fully saturated conditions, the thermal conductivity through the water dominates the thermal conductivity effect through the wood cell structure. Thus the lower the density (or the higher the porosity), the more water in the wood lumen, the higher the effective thermal conductivity for wood. Theoretically, the maximum thermal conductivity of a fully saturated cell approaches that of water (0.61 W/m-K) as porosity approaches 100%. While at any nonfully saturated conditions, low thermal-

TABLE 1. *Physical and thermal properties.*

	Symbol	Material properties in the cellular model		
		Thermal conductivity (W/m·K)	Density (kg/m^3)	Specific heat (J/kg·K)
Cell-wall substance (0% MC)[1]	k_{CW}	0.410	1540	1260
Air in the lumen (0% MC)[2]	k_{air}	0.026	1.161	1007
Bound water in cell wall[3]	k_{BW}	0.680	1115	4658
Saturated cell wall (FSP)[4]	k_f [7]	0.489	1415	2256
Water vapor in cell lumen[5]	k_V	0.018	0.734	2278
Free water in cell lumen[6]	k_{FW}	0.610	1003	4176

Note:
1. Property values for cell-wall substance at 0% MC was obtained from Siau's book (1995).
2. Air property values for air were obtained from Incropera and DeWitt (1981).
3. Density of bound water was obtained from Siau (1995). Thermal conductivity and specific heat of bound water was obtained based on water properties and assumption of the linear relationship with density.
4. Property of saturated cell wall was obtained by rule of mixture. FSP, fiber saturation point.
5. Property values of water vapor were obtained from Ierardi (1999).
6. Property values of free water were obtained from Incropera and DeWitt (1981).
7. The *kf* is constant when MC is over FSP, but changes with MC below FSP.

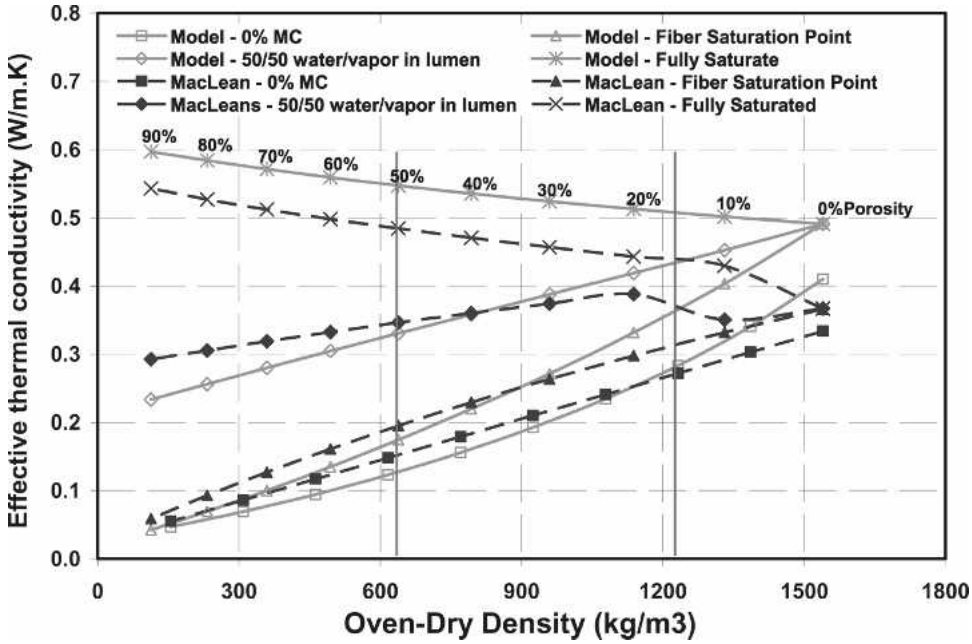

FIGURE 3. Thermal conductivity values predicted from the FE model plotted vs oven-dry density of a wood cell and compared with MacLean's (1941) empirical equations estimates.

conductive water vapor in the cell lumen has a dominant effect on K_{eff}-density relationship. Thus a cell with an increasing amount of water vapor results in decreasing thermal conductivity.

Plots extrapolated from MacLean's (1941) two empirical equations are also shown in Fig 3. One equation is for MC <40% and the other for MC >40% MC. Most of MacLean's test data were measured using wood blocks having a density between 600 and 1200 kg/m^3. In Fig 3, MacLean's equations are extrapolated beyond the data test conditions and show that they do not adequately describe wood-cell thermal conductivity over the full range of densities and moisture conditions. The differences and similarities of the two modeling approaches are discussed in a previous paper (Gu and Hunt 2006).

The 2D FE model can analyze the geometrical description of the cell, including the interior radius of the lumen as part of the heat transfer effects, which is not possible at the cellular level with other models or average-density empirical models. The FE results plotted for selected conditions are useful for understanding general trends, but for practical everyday use it would require interpolation between plotted figures or tabled data to determine the thermal conductivity at any other condition between plotted values. The usefulness of the FE model would be better if an independent equation could be developed that fits results. This study focuses on developing an explicit equation for thermal conductivity by curve fitting an electrical-resistive-circuit equation to the FE model results (Gu and Hunt 2006) based on the cell-wall structure.

RESISTIVE CELL MODELS

An equation or equations that would describe the thermal conductivity results and could be used independently of any FE computer program or used for other heat and mass transfer calculations would be ideal. A typical polynomial regression of the FE data was initially tried but did not fit well because of the complex interaction of density and moisture content. Building on the ideas of Hart and Siau mentioned earlier,

a resistive-circuit model that would have inputs of both density and moisture content was tried. In setting up the resistive circuit, two basic approaches were evaluated. The first approach assumes parallel-circuit (Fig 4a) heat flux paths across the unit cell, and the other approach assumes a series-circuit (Fig 4b) heat flux path. The assumptions made in developing the physical model of the cell structure are listed in the Appendix. The purpose for using these two approaches was to determine which might have a closer relationship to FE model results. In the parallel-circuit flow approach, the cell was divided into independent parallel paths (dotted horizontal lines), Fig 4a, that were modeled by the resistive-circuit shown. The top resistance path, R1, represents the cell wall plus any bound water (Y1) over the cross-section area L-a for the full length of the cell wall, L (Fig 4a). The middle resistance path, R2, represents the series

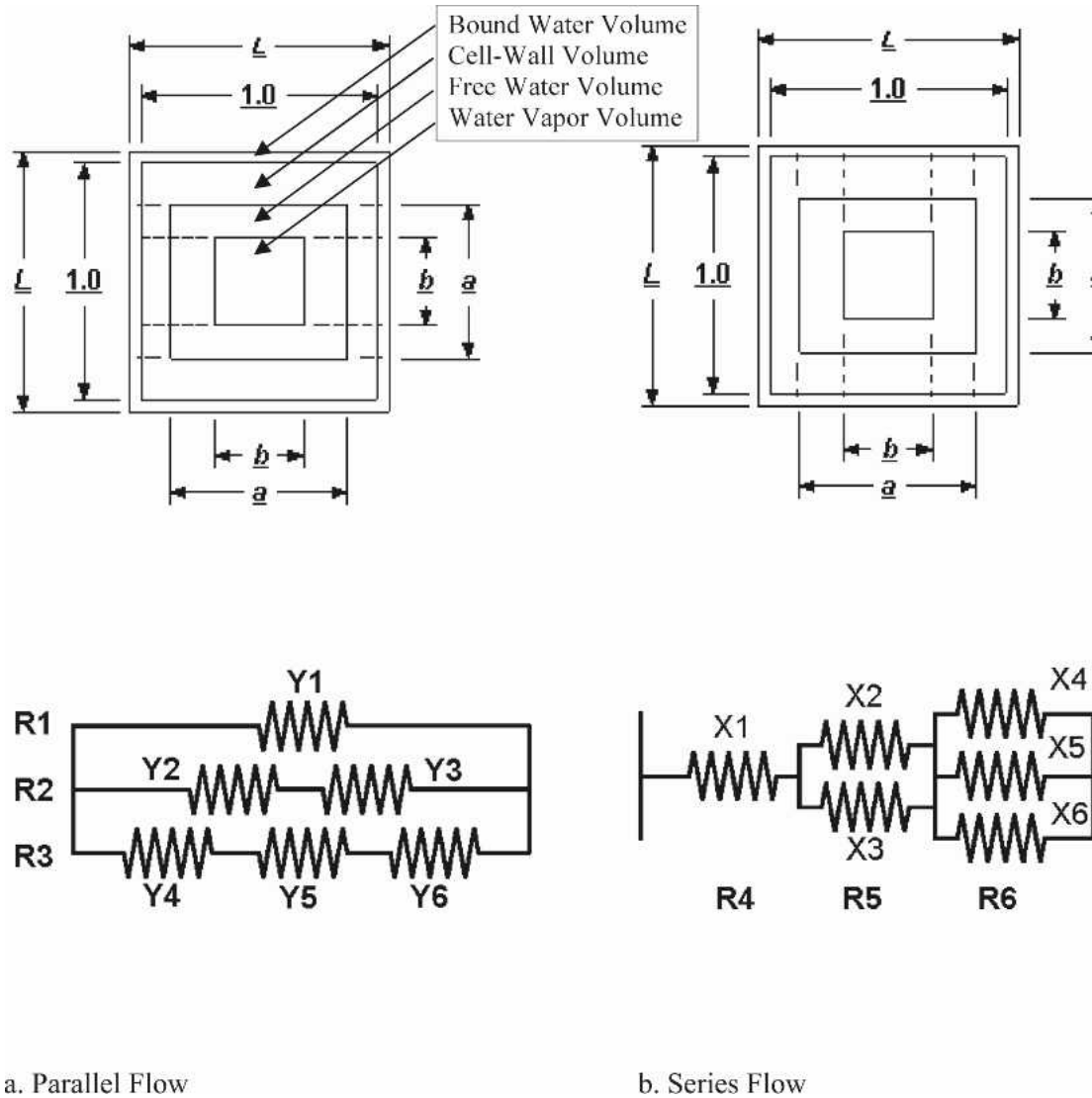

FIGURE 4. Two electrical resistive-circuit models used to describe heat flow through a wood cell: a. parallel flow and b. series flow.

path of the cell wall and any bound water (Y2) plus free water (Y3) across the cross-section area of a-b for the full length of the cell, L. The bottom resistance path, R3, represents series path of the cell wall and any bound water (Y4) plus free water (Y5) plus water vapor (Y6) across the cross-section area of b for the full length of the cell, L.

Similarly, the cell can also be divided into flow paths that can be modeled by a series resistive circuit, shown as material grouped by vertical dotted lines as shown (Fig 4b). The first resistance path, R4, represents the cell wall plus any bound water (X1) over the cross-section area L for part of the length of the cell wall, L-a (Fig 4b). The middle resistance path, R5, represents the parallel path of the cell wall and any bound water (X2) plus free water (X3) across the cross-section area of L for the partial length of the cell, a-b. The end resistance path, R6, represents parallel path of the cell wall and any bound water (X4) plus free water (X5) plus water vapor (X6) across the cross-section area of L for the partial length of the cell, b. The two approaches are described in more detail in the Appendix and the resulting thermal conductivity, K_{eff}, is the inverse of effective resistance, R_{eff}, Eq (1).

$$K_{\text{eff}} = \frac{1}{R_{\text{eff}}} \tag{1}$$

Comparison of resistive models with FE model results

Both the parallel and series approaches were evaluated at the six moisture conditions described above. A reduced set of results is plotted in Fig 5 (for clarity) and compared with the matching FE model curves. The two resistive-circuit models give slightly different results. The parallel heat flow model predicts lower K_{effp} values than the FE model, and the series flow model predicts a slightly higher but closer thermal conductivity, K_{effs}. Both resistive models converge to pure thermal conductivity values at either extreme of the density range. On one end where porosity has 0% pure cellulose alone or in combination with the bound water, thermal con-

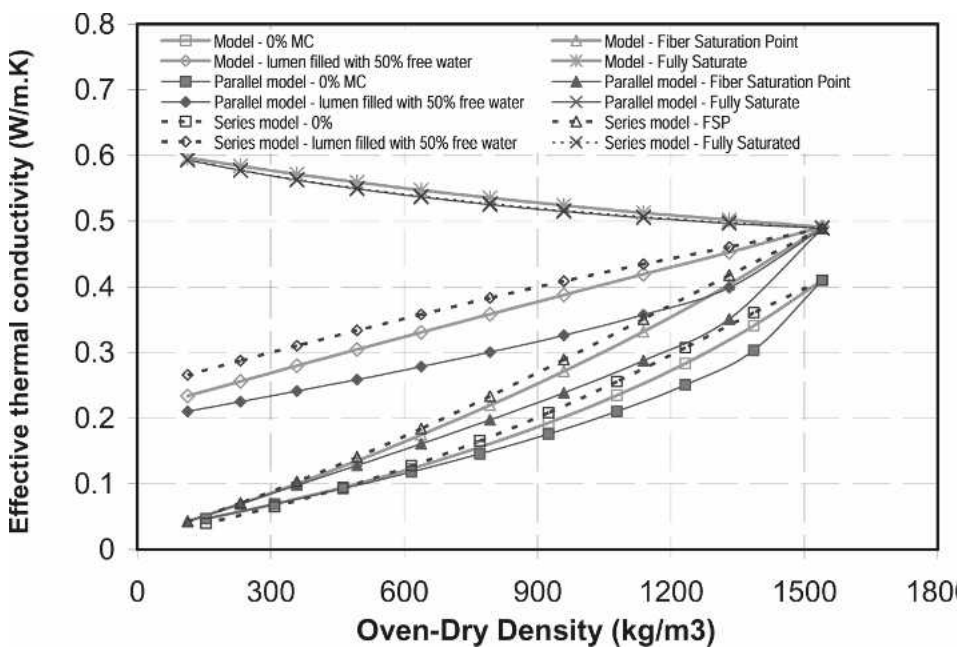

FIGURE 5. Comparison of thermal conductivity values for the FE model, parallel electrical resistance model, and series electrical resistance model.

ductivity approaches that of cellulose with or without moisture. On the other end where porosity approaches 100% (no cell-wall material), thermal conductivity approaches that of 100% air, 100% water vapor, and 100% fully saturated free water at each of the MC conditions, respectively. This indicates that the resistance model equations are predicting near realistic thermal conductivities for the MC conditions.

Differences between the two circuit approaches are due to how they represent the equivalent thermal system. Both circuits as well as the FE model assume that uniform temperature exists at either side of the cell. The parallel circuit assumes that horizontal paths through the cell do not interact with the adjacent flow path but are configured as independent paths, R1, R2, and R3, across the entire cell. These resistive circuits are then combined into one final resistance value using the parallel circuit equation, Appendix Eq (A1). If any one resistance path has significantly lower or higher resistance, it will significantly affect the final total value and hence effective thermal conductivity values. The cell conditions where air and water vapor represent most of the cross-sectional area in the lumen. Because they have significantly lower thermal conductivity, the calculated total effective thermal conductivity is below that of the FE results. The parallel flow also does not account for any flow across parallel boundaries.

For the series circuit, the theory assumes uniform temperature at the boundaries of each combination of resistances R4, R5, and R6. Heat flow is conducted uniformly through a section of similar geometry and material until it encounters a change (lumen without or with water). Then another equivalent resistive parallel circuit path is calculated but only across that new change. This assumption is incorrect, but recalculation of resistances at each change of cell configuration may account for some "two-dimensional" flow by changing equivalent resistances at midcell, thus slightly increasing or decreasing the effective thermal conductivity. Two-dimensional flow assumes that the path may have X and Y vector components that allow for "flow" around a higher resistance component of the cell structure. These resistive circuits are then combined into one final resistance value using the series circuit equation, Appendix Eq (A14).

Neither resistive model accurately describes the 2D thermal conductivity, but the series circuit (Fig 4b) better represents the FE thermal conductivity characteristics. The series model was thus used to develop a curve fit equation that can be explicitly used to determine thermal conductivity for all MC conditions of the cell wall without having to interpolate between values or reprogram an FE program to a particular set of conditions.

Curve-fit thermal conductivity equation

The series resistive-circuit model parameters (represented below by constants C4, C5, and C6) were estimated by nonlinear curve-fitting the FE model data (K_{eff}) to the inverse of the resistive model data ($1/R_{eff}$) at the same density and MC conditions. One set of parameters was developed for all MC conditions from 0% to a fully saturated lumen. The fundamental equation for the series resistive circuit, Eq (2), was used to determine the parameters.

$$\begin{aligned} R_{\text{eff}} &= C4R4 + C5R5 + C6R6 \\ &= C4X1 + C5\,\frac{X2X3}{X2 + X3} \\ &\quad + C6\,\frac{X4X5X6}{X4X5 + X4X6 + X5X6} \end{aligned} \tag{2}$$

where

R4 is the horizontal flow through the cell wall (with bound water) along the thickness of the cell wall (*L-a*) (Fig 4b).

R5 is the parallel horizontal flow through the saturated cell wall and free water (depending on moisture content) along the thickness of the free water ($a-b$) (Fig 4b).

R6 is the parallel horizontal flow through the saturated cell wall, free water, and water vapor along for the thickness of the water vapor (b) (Fig 4b).

X1, X2, X3, X4, X5, and X6 are calculated resistance values, see Appendix for details.

Though the FE model is deterministic in nature, nonlinear least squares was used to obtain parameter estimates, which are derived iteratively by minimizing the squared error between the calculated FE K_{eff} and the fitted conductance $\hat{K}_{eff} = ((\hat{1})/(R_{effs}))$. These were fit with the statistical package S-PLUS 6.1 (Insightful Corporation 2001). Fits were evaluated based on the residual plots and error statistics for the fitted data, see Table 2.

Thermal conductivity equations

The wood cell resistive variables X1, X2, X3, X4, X5, and X6 of Eq (2) can be described in terms of the geometrical terms shown in Fig 4 and the fundamental thermal conductivity values in Table 1. Simplifying and substitution of terms in Eq (2), (details can be found in Appendix) one equation can describe the effective thermal conductivity for a wood cell within all ranges of moisture contents and densities, Eq (3).

$$K_{eff} = \frac{(L-a)K_f}{(a-L)\left(\left(\frac{a}{L}-1\right)C4 + K_f\left(\frac{(a-b)C5}{(a-L)K_f - aK_{fw}} + \frac{bC6}{(a-L)K_f + (b-a)K_{fw} - bK_v}\right)\right)} \quad (3)$$

The geometric variables a, b, L, (Fig 4) and thermal variables K_f (Table 1) in Eq (3) are not constants but are dependent variables that are derived from the oven-dry density (ODD), ρ_{od}, of a wood cell and its MC. To determine the effective thermal conductivity, K_{eff}, the following equations need to be determined.

TABLE 2. *Parameter estimates for the series resistive model fit.*

	Estimate	Standard error
MC 0% to fully saturated		
C4	1.00825	0.00663
C5	0.9938	0.00792
C6	1.07389	0.00911

RMSE = 0.0088

First, based on the ODD of the cell wall, dry porosity P_d can be determined from Eq (4).

$$P_d = \frac{\rho_{cw} - \rho_{od}}{\rho_{cw} - \rho_{air}} \quad (4)$$

(Detail derivation of all the equations in this section can be found in the Appendix).

If the wood cell has any moisture, then the volume percent of bound water in the cell wall needs to be determined using Eq (5). For MC $\leq$ 0.3, $V\%_{bw}$ can be determined by Eq (5) assuming MC_f @ MC, where MC_f is the moisture content in wood fiber cell wall and MC is the overall moisture content of wood. However, for MC > 0.3, MC_f stays the same after reaching the FSP; thus $V\%_{bw}$ is a constant and calculated at 0.293. If the FSP is other than the assumed 30%, then Eq (5) can still be used to determine $V\%_{bw}$ by changing MC_f . The same assumptions still apply, that once the FSP has been reached, the volume of bound water within the cell wall does not increase as the overall MC increases.

$$V\%_{bw} \cong \frac{MC_f\rho_{cw}}{MC_f\rho_{cw} + \rho_{bw}} \quad (5)$$

With the addition of moisture into the cell, cell lumen a^2 is assumed to remain constant, but the outside dimension L increases until fiber cell wall is saturated at MC = 0.3. The porosity value also changes from the dry porosity value to a new wet porosity value, P_w, determined using Eq (6).

$$P_w = \frac{(1 - V\%_{bw})P_d}{1 - V\%_{bw}P_d} = \frac{a^2}{L^2} \quad (6)$$

The variable L can then be determined either by rearranging the terms in Eq (6) in combination with Eq (4) or by using Eq (7) and inserting values determined for P_w and MC_f.

$$L^2 = \frac{\rho_{bw} + MC_f\rho_{cw}}{\rho_{bw} + MC_fP_w\rho_{cw}} \quad (7)$$

Then b can be determined. For MC < 0.3 there is no free water in the lumen and b = 0 (Fig 4). However, for MC above 0.3, there is assumed free water evenly distributed around the inner lumen that extends into the middle of the lumen dimension b. This dimension can be determined by Eq (8) and variable values a^2, $V\%_{bw}$, and P_w from the previous equations and constants from Table 1.

$$b = \sqrt{\frac{a^2\{(1 - P_w)[V\%_{bw}\rho_{bw} - (1 - V\%_{bw})MC\rho_{cw}] + P_w\rho_{fw}\}}{P_w(\rho_{fw} - \rho_v)}} \quad (8)$$

The thermal conductivity of the cell wall with absorbed bound water changes depending on the MC_f and can be determined using rules of mixtures as defined by Eq (9).

$$K_f = K_{cw}(1 - V\%_{bw}) + K_{bw}V\%_{bw} \quad (9)$$

Above FSP (for MC > 0.3) the cell wall is fully saturated with bound water and MC_f stays unchanged at 30%. Using the rule of mixtures, the fiber cell-wall thermal conductivity is determined to be K_f = 0.4891; Table 1, for MC_f = 0.3, and $V\%_{bw}$ of 0.293 as calculated earlier (Eq (5)).

Using Eqs (4) through (9), we can calculate K_{eff} values as Eq (3), and plot as a function of both ODD and MC. A more thorough development of the equations is provided in the Appendix. Figure 6 shows an improved fit with the FE values for the calculated resistive-series circuit equation with the addition of the appropriate parameters.

Other wood cell conditions

While the conditions outlined in this paper are 30°C and FSP = 30%, other conditions may need to be evaluated. We believe this equation can be used to determine thermal conductivity estimates for other wood cell conditions as a function of temperature or FSP. It is possible to input new values for the cell-wall material. Bound water, free water, and water vapor could be determined and the equations used to estimate thermal conductivity. A more rigorous effort

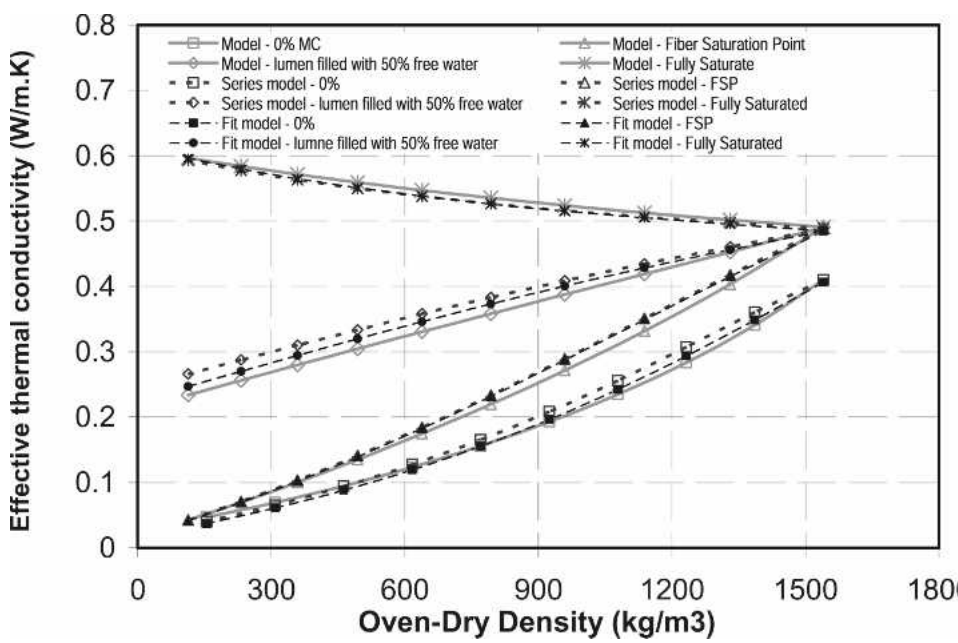

FIGURE 6. The calculated resistive-series circuit equation with the appropriate parameters improves the fit with the FE values.

could include equations that characterize the changes to density and thermal conductivity values for the cell-wall material, bound water, free water, and water vapor described as a function of temperature. These could then be inserted into the appropriate equations listed above. The trends for the new thermal conductivity estimated values should follow the same basic trends for the FE analysis. However, if a more rigorous approach is needed and thermal conductivity for extreme temperature conditions is wanted, then it may be necessary to rerun the FE model to verify the shape of the curves and adjust the fit parameters.

A simple approach to determining the estimated changes to wood cell thermal conductivity would be to use Siau's K-Temp relationship Eq (5.23) (1995). It assumes a simple relationship with temperature.

It is also possible that for some wood cells or conditions, the FSP is not 30%. In the same way, the equations can be used to estimate thermal conductivity values where FSP is higher or lower. Calculations and plots of results were examined (not shown) to determine the shape of the thermal conductivity curves for FSPs of 20 and 40%. The FE thermal conductivity estimates followed the same curves as those shown, only shifted slightly up or down from that of FSP at 30%. The curves still approached the pure substance values at 0 or 100% porosity conditions, but based on the new set of conditions.

SUMMARY AND CONCLUSIONS

One explicit equation was developed that matches the FE model data so that thermal conductivity information on a cell level can be determined across the full range of density and moisture conditions without the need for FE software. Having thermal conductivity values of actual cellular characteristics of the wood can help in studying the heat transfer effects in wood boards where earlywood and latewood densities (porosity), ring orientation, growth rate, and earlywood/latewood ratio are significantly different. The thermal conductivity equation or values developed in this study are for uniform density wood cells only. We will model and report in the next paper effective thermal conductivity values for wood boards across multiple bands of high/low density or earlywood/latewood bands in the structure.

The equation can also be used to determine estimated thermal conductivity values at temperatures other that 30°C by either using temperature-dependent relationships of individual input variables or by using the equation for a simple K-Temp relationship given by Siau's Eq (5.23) (1995).

Similarly, the equation can be used to determine estimated thermal conductivity values where FSP is other than 30%. By changing the volume percent of bound water, $V\%_{bw}$, (Eq (5)) and recalculating the values, the equation will calculate values that follow the same general trends and approach "pure" substance thermal conductivity values at either 0 or 100% cell-wall porosity.

If significant material property changes were determined or unusual conditions existed. it may be necessary to reevaluate thermal conductivity using a finite element approach and then new parameters determined for those specific conditions. The benefit here is that an equation has been developed that is independent of any specific software and could be easily used to help estimate the thermal conductivity of wood cell material.

NOMENCLATURE

a = lumen dimension
b = vapor dimension inside the lumen
C = constant parameter
K = thermal conductivity
L = full width and height of the cell
P = porosity
R = thermal resistance
V% = volume percent
Subscript $_{air}$ = pure air

Subscript $_{CW}$ = pure cell-wall substance (no water)
Subscript $_{d}$ = dry
Subscript $_{eff}$ = effective
Subscript $_{f}$ = fiber or cell wall with bound water
Subscript $_{FW}$ = free water
Subscript $_{od}$ = oven-dry
Subscript $_{V}$ = water vapor
Subscript $_{w}$ = wet
Subscript $_{BW}$ = bound water
ρ = density

REFERENCES

GU H, HUNT JF (2006) Two-dimensional finite element heat transfer model of softwood. Part III. Moisture content effect on thermal conductivity. Wood Fiber Sci 39(1): 159–166.

HART CA (1964) Theoretical effect of gross anatomy upon conductivity of wood. Forest Prod J 14(1):25–32.

HUNT JF, GU HM (2006) Two dimensional finite element heat transfer model of wood—Part I. Effective thermal conductivity. Wood Fiber Sci 38(4):592–598.

IERARDI JA (1999) A computer model of fire spread from engine to passenger compartments in post-collision vehicles. M.S. thesis., Worcester Polytechnic Institute, Worcester, MA, Fire Protection Engineering: 205 pp.

INCROPERA FP, DEWITT DP (1981) Fundamentals of heat and mass transfer. 4th Ed. John Wiley & Sons, New York.

INSIGHTFUL CORPORATION (2001) S-PLUS 6 for Windows Programmer's Guide. Insightful Corporation, Seattle, WA.

MACLEAN JD (1941) Thermal conductivity of wood. Heat Piping Air Cond 13:380–391.

SIAU JF (1995) Wood: Influence of moisture on physical properties. Department of Wood Science and Forest Products, Virginia Polytechnic Institute and State University, Blacksburg, VA. 227 pp.

STAMM AJ, SMITH WE (1969) Laminar sorption and swelling theory for wood and cellulose. Wood Sci Technol 3(1): 301–323.

APPENDIX

The use of resistive circuits to determine thermal conductivity is well documented in elementary heat transfer textbooks (Incropera and DeWitt 1981). The arrangement of the resistive elements that describe the heat flow can vary depending on the description of the path and may result in slightly different end values. To evaluate the effective thermal conductivity in a wood cell using the resistive-circuit models, several assumptions were made in the geometry and for the moisture distribution within the cell. The assumptions are as follows:

STRUCTURAL ASSUMPTIONS

The cell and associated geometry are square as shown in Fig 4.

Unit dimensions are for cells at oven-dry conditions.

All moisture absorbed by the cell wall (bound water) is added as a dimension change to the outside of the unit cell (L-1), and the change to the outside of the cell is used for simple flow path defined in the resistive models.

The temperature for all the materials is 30°C.

Maximum expanded dimension to the unit cell is at the FSP of 30% MC.

Assumptions for MC conditions from 0% to FSP

At 0% MC, the lumen has 100% air in the lumen.

At MC conditions from 0% to FSP, the initial moisture goes to the lumen first to create a vapor-saturated lumen condition with the remaining moisture going into the cell wall.

With an increase in moisture, the lumen size remains the same and all dimensional change because of the increase in MC is added to the outside of cell-wall dimension.

Thermal conductivity of the cell wall is determined using the rule of mixtures.

Assumptions for MC conditions from FSP to FS

The lumen contains a combination of saturated vapor and free-water in the lumen.

For MCs above FSP, the vapor in the lumen is at saturated vapor conditions; the cell wall has a constant weight ratio for bound water to cell-wall substance (MC_f) at 0.3; and the remaining moisture goes to free water in the lumen.

Free water in the lumen is bound uniformly to

the inner surfaces of the lumen due to capillary forces, and gravity does not have an effect.

The lumen size remains the same and the maximum expanded dimension of the cell remains the same because of the constant MC in the cell wall after reaching FSP. The addition of free water in the lumen does not increase the size of the unit cell.

Thermal conductivity of the cell wall is constant after the FSP and is determined by the rule of mixtures with bound water and cell-wall substances.

PARALLEL RESISTIVE MODEL

The representative effective resistance (Eq (A1)), R_{effp}, for the parallel model (Fig 4a) is:

$$R_{effp} = \frac{C1C2C3R1R2R3}{C1C2R1R2 + C1C3R1R3 + C2C3R2R3} \quad \text{(A1)}$$

where C1, C2, and C3 are experimental constants.

Resistances R1, R2, and R3 through for the full length of the cell were calculated using the following equations:

$$R1 = Y1 = \frac{L}{K_f(L - a)} \quad \text{(A2)}$$

$$R2 = Y2 + Y3 = \frac{L - a}{K_f(a - b)} + \frac{a}{K_{fw}(a - b)} \quad \text{(A3)}$$

$$R3 = Y4 + Y5 + Y6 = \frac{L - a}{K_f(b)} + \frac{a - b}{K_{fw}(b)} + \frac{b}{K_v(b)} \quad \text{(A4)}$$

L is full width and height of the cell.

a is lumen dimension.

b is vapor dimension inside the lumen.

K_f is thermal conductivity for the fiber (or bound water saturated cell wall), determined by the rule of mixtures using volume percent of both bound water and cell-wall substance.

K_{fw} is thermal conductivity for free water.

K is thermal conductivity for water vapor.

Y1 is resistance of the top and bottom cell wall with the full length of the cell, L, and across the effective area (L-a) (Fig 4).

Y2 is resistance of the cell wall along the horizontal path of the cell wall, L-a, across the effective area (a-b) with b = a at the fiber saturation point (FSP) (Fig 4).

Y3 is resistance of the free water along horizontal path of the lumen, a, across the effective area (a-b) with b = a at the FSP (Fig 4).

Y4 is resistance of the cell wall along the horizontal path, L-a, across the effective area (b) with b = a at the FSP (Fig 4).

Y5 is resistance of the free water along the horizontal path, a-b, across the effective area (b) with b = a at the FSP (Fig 4).

Y6 is resistance of the water vapor along the horizontal path, b, across the effective area (b) with b = a at the FSP (Fig 4).

The experimental constants C1, C2, and C3 were not determined in this study because efforts were focused on determining the experimental constants for the series model because the series circuit's better representation of the FE model results, which is discussed in the next section.

SERIES RESISTIVE MODEL

For series flow model, the cell was divided into vertical sections, Fig 4b. The representative effective resistance, R_{effs} for the series model is

$$\begin{aligned} R_{effs} &= C4R4 + C5R5 + C6R6 \\ &= C4X1 + C5\frac{X2X3}{X2 + X3} \\ &\quad + C6\frac{X4X5X6}{X4X5 + X4X6 + X5X6} \end{aligned} \quad \text{(A5)}$$

The series resistance components were combined using the parameter weights C4, C5, and C6 for the respective resistances R4, R5, and R6.

where

R4 is the horizontal flow through the cell wall along the thickness of the side cell wall (L-a) (Fig 4b).

R5 is the parallel horizontal flow through the top and bottom cell wall and free water (depending on moisture content) along the thickness of the free water (a-b) (Fig 4b).

R6 is the parallel horizontal flow through the top and bottom cell wall, free water, and water vapor along for the thickness of the water vapor (b) (Fig 4b).

The individual resistances X1, X2, X3, X4, X5, and X6 were calculated using the following equations:

$$X1 = \frac{L - a}{K_f(L)} \quad \text{(A6)}$$

$$X2 = \frac{a - b}{K_f(L - a)} \quad \text{(A7)}$$

$$X3 = \frac{a - b}{K_{fw}(a)} \quad \text{(A8)}$$

$$X4 = \frac{b}{K_f(L - a)} \quad \text{(A9)}$$

$$X5 = \frac{b}{K_{fw}(a - b)} \quad \text{(A10)}$$

$$X6 = \frac{b}{K_v(b)} \quad \text{(A11)}$$

INPUT VARIABLES FOR THE RESISTIVE-CIRCUIT MODELS

The FE models that were developed (Hunt and Gu 2006; Gu and Hunt 2006) to determine K_{eff} values under geometrical conditions (% cell porosity) and known moisture content conditions (FSP, 50–50% water vapor/free water in the lumen, and FS) were used as input values for the resistive models. The goal of this paper was to describe thermal conductivity of the wood cell in terms of oven-dry density (ODD and MC). The following equations were used to describe the basic geometry of the wood cell in terms of ODD and MC. The initial description is based on the cell porosity P_d or lumen area a^2 to unit cell ratio 1^2 (Fig 4) at 0% MC. The ODD or ρ_{OD}, of the cell is described by density of the cell wall, ρ_{cw}, density of the air, ρ_{air}, and the oven-dry porosity, P_d, in Eq (A12).

$$\rho_{OD} = \rho_{cw}(1 - P_d) + \rho_{air}P_d \quad \text{(A12)}$$

P_d is determined by rearranging Eq (A12), Eq (A13).

$$P_d = \frac{\rho_{cw} - \rho_{od}}{\rho_{cw} - \rho_{air}} \quad \text{(A13)}$$

Dry porosity can also be described in geometrical terms, Eq (23).

$$P_d = \frac{a^2}{1^2} \quad \text{(A14)}$$

With the addition of moisture into the cell, the volume percent of bound water, $V\%_{bw}$, in the cell wall needs to be described. By definition, fiber moisture content MC_f is calculated by dividing bound water weight by the oven-dry cell weight, which is the cellulose material weight, when assuming UNIT (1 × 1 volume dimensions) volume for the wood cell. The MC in the fiber cell wall (MC_f) can be calculated using Eq (A15).

$$MC_f = \frac{V\%_{bw}\rho_{cw}}{\rho_{cw}(1 - V\%_{bw})} \quad \text{(A15)}$$

By rearranging Eq (A15), $V\%_{bw}$ can be determined for MC < 0.3. For MC > 0.3 $V\%_{bw}$ is a constant at 0.293. For MC from 0–0.3 then MC_f @ MC. For MC > 0.3 $MC_f = 0.3$

$$V\%_{bw} \cong \frac{MC_f\rho_{cw}}{MC_f\rho_{cw} + \rho_{bw}} \quad \text{(A16)}$$

With the addition of moisture into the cell, wet porosity, P_w, is introduced and can be described using Eq (A17).

$$P_w = \frac{(1 - V\%_{bw})P_d}{1 - V\%_{bw}P_d} \quad \text{(A17)}$$

Wet porosity can also be described in geometrical terms, Eq (A18).

$$P_w = \frac{a^2}{L^2} \tag{A18}$$

The outer dimension, L, and area of the wet cell, L^2, changes with increasing moisture up to the FSP and can be described using Eq (A19).

$$L^2 = \frac{\rho_{bw} + MC_f\rho_{cw}}{\rho_{bw} + MC_f P_w \rho_{cw}} \tag{A19}$$

The specific equation for MC at all moisture conditions is described in Eq (A20).

$$MC = \frac{(\rho_{cw}V\%_{bw})(1 - P_w) + \frac{\rho_v b^2 P_w}{a^2} + \rho_{fw}\left(1 - \frac{b^2}{a^2}\right)P_w}{(\rho_{cw}(1 - V\%_{bw}))(1 - P_w)} \tag{A20}$$

Free water is assumed to be evenly distributed around the inner lumen that extends into the middle of the lumen. For MC $\leq$ 0.3, there is assumed no free water, and b = a (Fig 4). For MC > 0.3, it is assumed the cell wall is saturated and the remaining water goes into the lumen as free water. The extent to which free water fills the lumen can be determined by rearranging Eq (A19) and solving for b, Eq (A21). The area defined by b, inside the free water, is assumed to be filled with saturated vapor.

$$b = \sqrt{\frac{a^2\{(1 - P_w)[V\%bw\rho_{bw} - (1 - V\%_{bw})MC\rho_{cw}] + P_w\rho_{fw}\}}{P_w(\rho_{fw} - \rho_v)}} \tag{A21}$$

Thermal conductivity for the fiber cell-wall material, K_f, needs to also be determined before calculating the effective thermal conductivity. Using rules of mixtures for MC < 0.3 then, K_f can be determined by Eq (A22).

$$K_f = K_{cw}(1 - V\%_{bw}) + K_{bw}V\%_{bw} \tag{A22}$$

For MC > 0.3 the fiber cell wall is fully saturated with water and is assumed not to change. Using the rule of mixtures for MC_f = 0.3, then the fiber cell-wall thermal conductivity is a constant at K_f = 0.4891 (Table 1).

The effective thermal conductivity, K_{eff}, for the entire range of moisture contents and densities can be determined by only knowing the ODD and MC conditions for the sample by substituting Eqs (A6–A11) into Eq (A5) and simplifying the equation as Eq (A23).

$$K_{eff} = \frac{(L - a)K_f}{(a - L)\left(\left(\frac{a}{L} - 1\right)C4 + K_f\left(\frac{(a - b)C5}{(a - L)K_f - aK_{fw}} + \frac{bC6}{(a - L)K_f + (b - a)K_{fw} - bK_v}\right)\right)} \tag{A23}$$

By solving for and substituting the appropriate values for variables a, b, L, K_f and substituting the appropriate C4, C5, and C6 parameters from Table 2, the entire range of thermal conductivities can be determined.

If FSP is other than the assumed 30%, the same set of equations can be used to determine estimated thermal conductivity values. By changing the volume percent of bound water, $V\%_{bw}$, (Eq (A16)) and recalculating the values in the subsequent equations, a K_{eff} (Eq (A23)) can be determined.

It is also possible to input new or improved values for the cell-wall material, bound water, free water, and water vapor and the equations used to estimate new thermal conductivity estimated values. Or a more rigorous effort could include equations that characterize the changes to density and thermal conductivity values for the cell-wall material, bound water, free water, and water vapor described as a function of temperature. These could then be inserted into the appropriate equations listed above.