CONSTANT BENDING METHOD FOR DETERMINING MODULUS OF ELASTICITY OF LUMBER IN STRUCTURAL SIZE

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ABSTRACT

The objective of this study was to develop a method to determine modulus of elasticity of full-length lumber members loaded on edge in pure bending. This was achieved by loading the members by moments applied at each end of span using specially designed levers and a cable. To validate this approach, 38 × 89-mm (2 × 4-in.) spruce members of varying grades were instrumented with strain gauges and tested over a span/depth ratio of 17. Strain data revealed that the method developed was capable of subjecting members to pure bending except when a member twisted during loading due to the presence of large knots or spiral grain. A procedure is also presented to obtain shear modulus of elasticity of the members from a complementary test in center-point loading. Advantages of constant bending over standard test methods are discussed.

Keywords: Structural lumber, bending modulus of elasticity, shear modulus of elasticity.

INTRODUCTION

Wood is a material whose shear modulus of elasticity (G) is relatively small with respect to its bending modulus of elasticity (E). Typically, E/G ratios range between 2 and 4 for most metals, whereas for wood this parameter, extracted from different sources, ranges between 5 and 50 (Biblis 1965; Kollmann and Côté 1968; Curry 1976). Because of these high E/G ratios, deflection of a lumber member due to shear may not always be neglected, even for ordinary slender beams.

The elastic behavior of a lumber member in bending is influenced by a number of factors including density, temperature, and moisture content of the wood, testing geometry, and rate of loading. These effects have been the object of many experimental investigations well documented in the literature. Most investigators followed the approach of monitoring total deflection of test beams under varying conditions or treatments. This deflection is then used as a basis to infer the treatment effect on the elastic constants. As testing is normally carried out under conditions that include both axial and shearing stresses (such as in a center-point loading test), it is not possible from the data developed to determine how the true bending behavior has been affected by the treatment since total deflection includes both bending and shearing components. For a better understanding of the mechanical behavior of lumber, it would be desirable to evaluate these effects under the condition of pure bending or pure shear. These conditions are particularly difficult to achieve for lumber members in structural size.

A few attempts have been made to devise methods for determining true flexural properties. In an attempt to evaluate local modulus of elasticity of lumber members, Corder (1965) tried to subject members to a constant bending moment from attaching levers at the end of the member and pulling the levers inward. This method, however, cannot create a pure bending situation since under the action of the pulling force, the member is loaded in both bending and compression and behaves as an eccentrically loaded column. Pursuing Corder's objective, Kass (1975) obtained constant bending by hanging weights from the ends of a member while supporting the member equivalent distances inward from each end. Although this method is valid from the statics point of view, it has the disadvantage of evaluating only the central portion of the member. Furthermore, this method is hardly applicable to edge bending situations because weights and lever arms required would become too large.

ASTM D 198 (1988) proposes a method for determination of pure bending E. The member is subjected to a four-point loading arrangement, and deflection is measured within the shear free-span located between loading points. This method is prone to errors because deflections are very small and the deflected shape within the shear-free span is affected by the material outside loading points for usual lumber containing localized weak zones. Furthermore, this method evaluates only the central portion of the member.

SCOPE

The present study was carried out as part of a larger research project conducted at Université Laval aimed at assessing mechanical properties of lumber members. In the larger study, it was necessary to evaluate stiffness of full length lumber members loaded on edge in pure bending. A device based on a principle called constant bending was developed and evaluated for this purpose. This paper describes the device and assesses its performance.

PRINCIPLE

The principle of constant bending is illustrated in Fig. 1, showing a beam and its load application system in their original undeformed and deformed positions. At support points, A and A′ are connected two rigid levers AB and A′B′ normal to beam axis. Points B and B′ are connected by a cable. Under the influence of vertical load F applied at cable midlength, the cable, loaded in tension, pulls on the ends of the levers. This pulling force, through the action of the lever arm, loads the beam with two concentrated moments applied at support points and acting opposite to each other. As schematically shown in the figure, the supports are such that they resist both the vertical (R_v) and horizontal (R_h) force components developed at the base of the levers. Furthermore, the supports are designed to allow free translational motion of the beam along its axis at support points. As a result, the beam is free from any axial force and feels only the rotation applied at supports. Consequently, this arrangement should subject a uniform beam to pure bending.

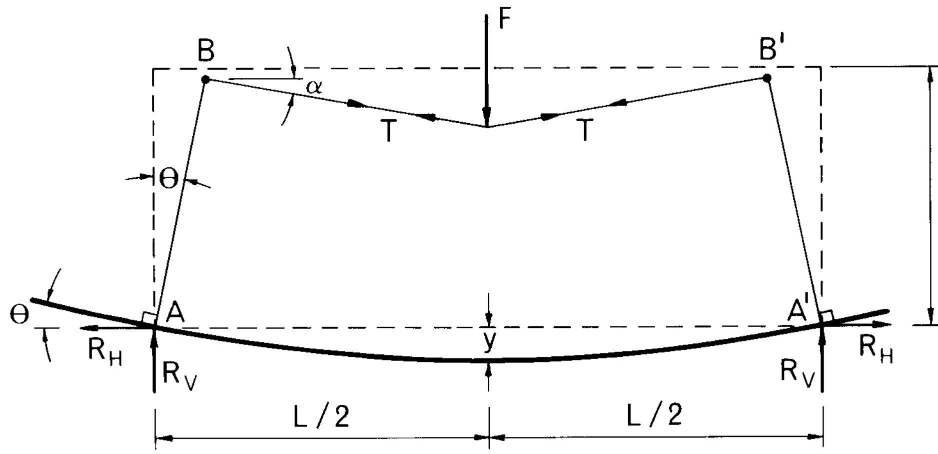

FIG. 1. Diagram illustrating constant bending method. Dashed lines show the beam and load application system in their undeformed position. Deformations are exaggerated.

A kinematic analysis of the loading system, carried out under the assumptions that the cable does not stretch and the levers remain normal to the beam axis during loading, leads to the following geometric relation

$$\text{Cos}\ \alpha = 1 - (2\text{h/L})\sin\theta \tag{1}$$

where

α = cable angle with respect to horizontal,
θ = beam rotation at supports,
h = lever arm,
L = span.

Relationships between the bending moment and the applied forces depend on this geometry. One of these relationships, obtained from applying static equilibrium equations to a single lever, is

$$\text{M} = \text{Th}\cos(\alpha - \theta) \tag{2}$$

where

M = bending moment applied at each support,
T = cable tension.

The deflection resulting from applying a bending moment at each end of span is given by simple beam theory as

$$y = \frac{3}{2}\frac{\text{ML}^2}{\text{Ebd}^3} \tag{3}$$

where

y = midspan deflection,
E = shear-free modulus of elasticity,
b = width of the beam,
d = depth of the beam.

Simple beam theory also provides the following relationship between midspan deflection and rotation at supports:

$$\theta = \frac{4y}{L}. \tag{4}$$

This relationship is particularly useful in the following analysis of system geometry under load.

Substituting Eq. (2) into Eq. (3) suggests that beam deflection and applied tension force are related in a nonlinear manner. However, using Eqs. (1) and (4), it is easy to demonstrate that for small values of the y/L ratio, the nonlinear term $\cos(\alpha - \theta)$ in Eq. (2) approaches unity when h is reasonably larger than y. For example, for y/L smaller than 1/150 and h greater than 50 times y, this term will deviate from unity by less than 1.5%, an error that is compatible with usual measurement errors in mechanical testing of lumber members. Under these conditions, Eq. (2) simplifies to

$$M = Th \tag{2'}$$

which, upon substituting into Eq. (3) and solving for E yields

$$E = \frac{3hL^2}{2bd^3}\frac{T}{y} \tag{5}$$

a relation which suggests that the modulus of elasticity could be derived from the slope of the T versus y diagram.

From an analysis similar to that carried out above, it is possible also to express E as a function of point load F. However, the resulting relationship is highly nonlinear and cannot be simplified for small beam deflections. Therefore, it is preferable to monitor cable tension and beam deflection to determine pure bending E using Eq. 5.

APPARATUS

An apparatus was built to evaluate the method described. This apparatus, shown in Fig. 2, was designed to test 38 × 89-mm (2 × 4-inch) members loaded on edge. The setup consists of two lever assemblies bolted to the bed of a universal testing machine and a loading head attached to the testing machine crosshead. The loading head is made of two 127-mm ball bearings mounted side by side. The cable connecting the two levers sits in the groove formed by bearing chamfers. These bearings may rotate slightly to allow for any axial motion of the cable resulting from unsymmetrical bending.

Details of one lever assembly are provided in Fig. 3. The lever is made of two triangular-shaped vertical plates connected at their top by a shaft, mounted on ball bearings, on which the cable is attached. In series with the cable, a load cell is mounted to monitor cable tension. At their base, the lever plates are connected by two horizontal plates which, together with the side plates, form a cage to receive the specimen. The specimen is held in the cage by clamping bolts pressing on thin plates, the details of which can be seen in Fig. 4. Flat cage needle bearings inserted between these plates allow free translational motion of the specimen along beam axis at supports while translation normal to beam axis is fully prevented. On the lever side plates are mounted ball bearings whose axis coincide

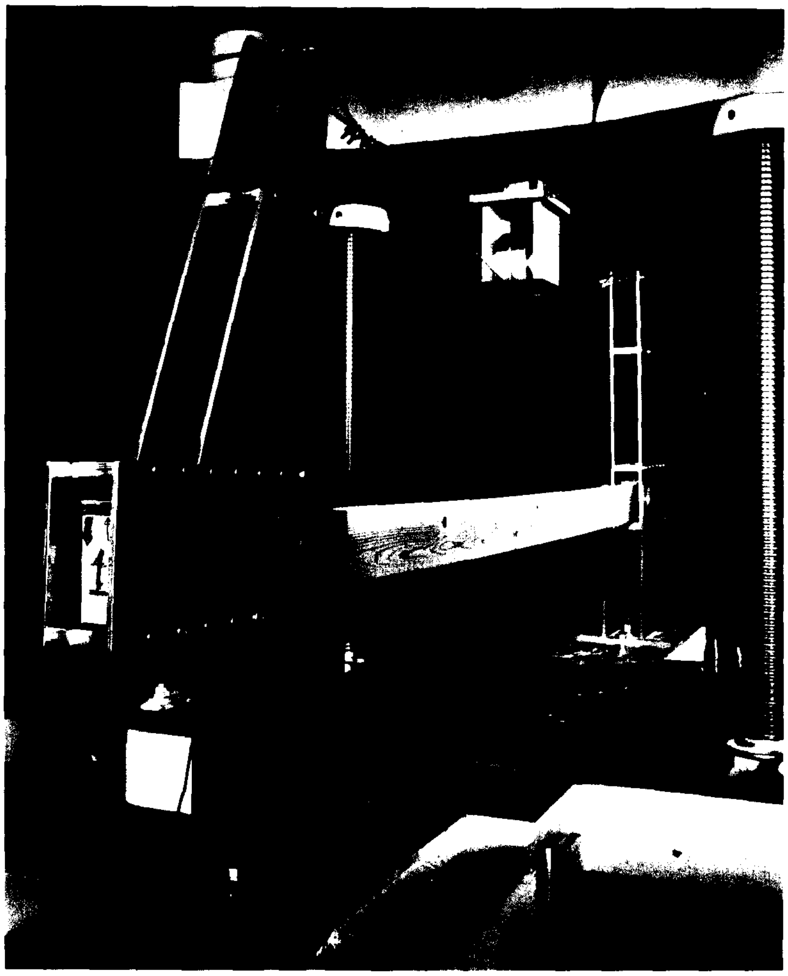

FIG. 2. Constant bending test apparatus.

with the specimen's neutral axis allowing free specimen rotation at support points. The pulling force exerted by the cable on the levers is transferred to the foundations through these bearings without being felt by the specimen.

Specimen deflection is measured using a yoke on which is attached the body of a displacement transducer. The yoke rests on two nails, delimiting the test span, driven at neutral axis on the specimen prior to its mounting in the frame. In the final positioning of the specimen in the frame, each nail coincides with a lever's pivot point, as shown in Fig. 3. A third nail is driven at midspan, also at the neutral axis, to receive the transducer's rod.

EXPERIMENTS

The adequacy of the proposed method for conducting pure bending tests was assessed on the basis of an analysis of strain data measured on selected lumber

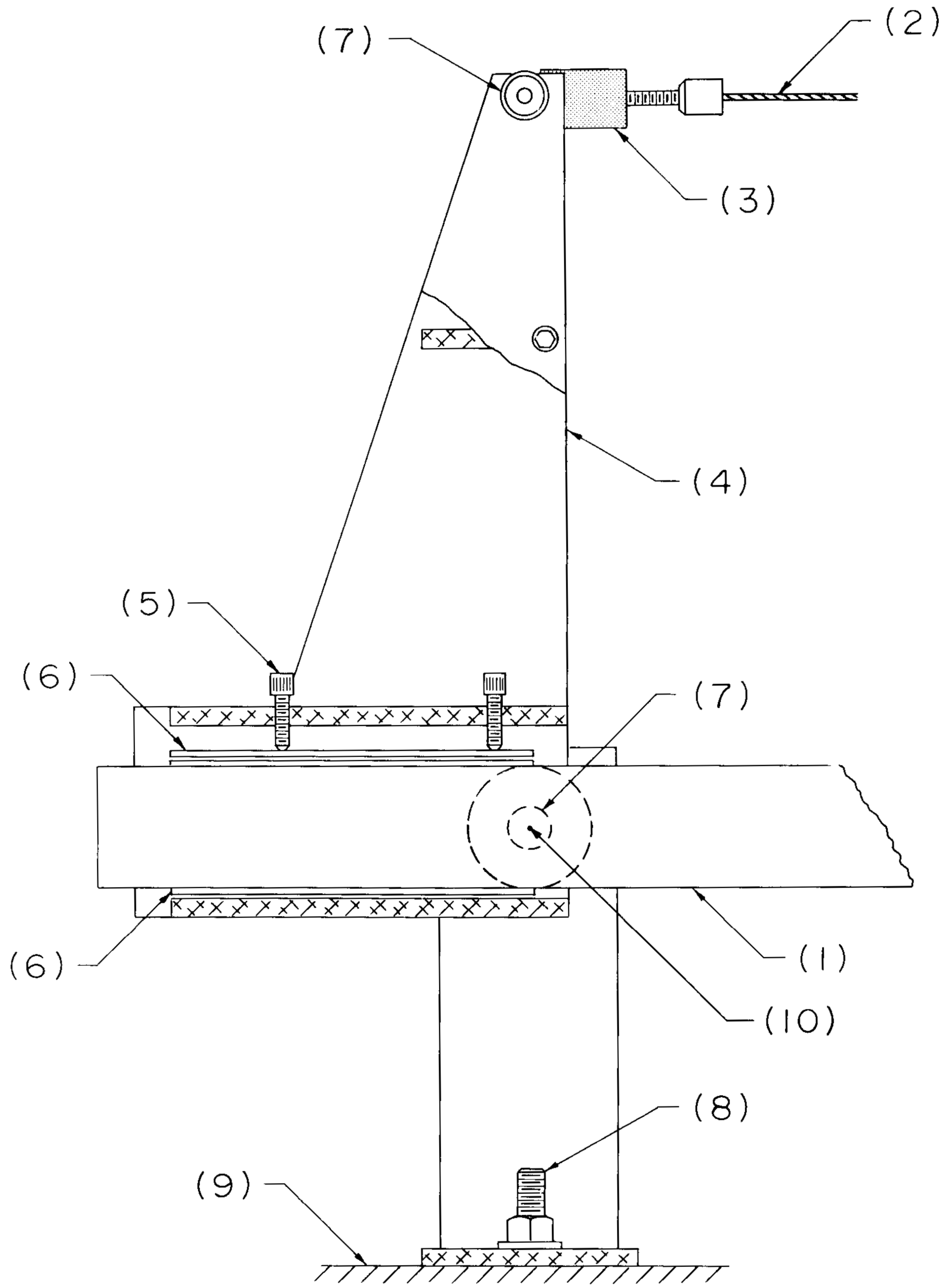

FIG. 3. Details of a lever assembly: (1) test specimen, (2) cable, (3) tension load cell, (4) lever, (5) clamping bolt, (6) clamping plates and thrust bearings, (7) ball bearing, (8) anchoring bolt, (9) testing machine bed, (10) nail driven in the specimen.

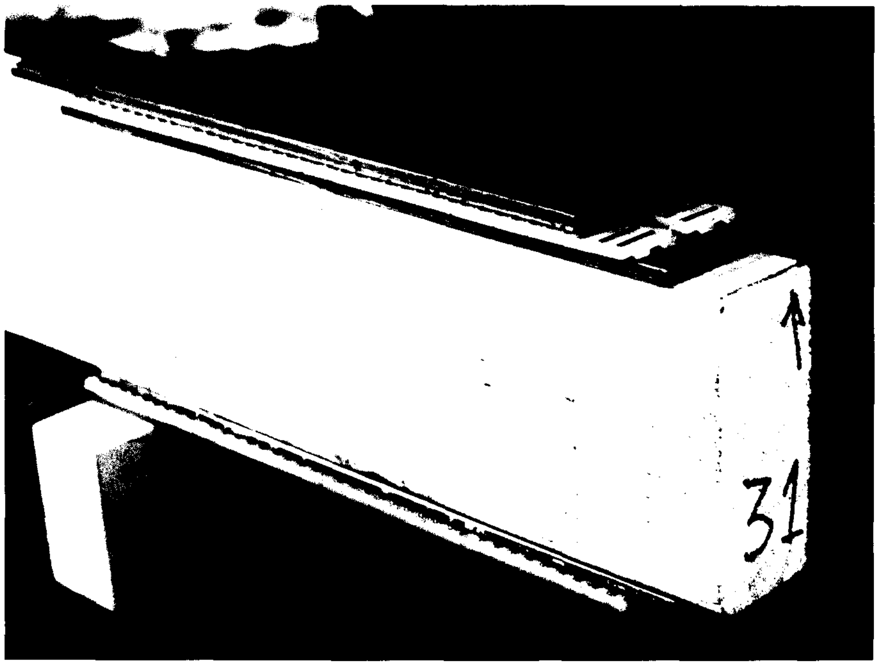

FIG. 4. A view of clamping plates and flat cage needle bearings positioned on a specimen.

members. The state of strain within span for a prismatic beam of uniform material and rectangular cross section loaded in pure bending could be described as follows:

a) At any section along beam axis, given a distance v on either side of the neutral axis, axial strain on the compression side of the beam is equal to the axial strain on the tension side of the beam;
b) Given a distance v from the neutral axis, axial strain at this position is constant along beam axis; and
c) At any section, shearing strain vanishes.

To verify if the method developed could produce this state of strain, 7 kiln-dried 38 by 89-mm white spruce lumber members of varying grades were selected and tested. One specimen was a glued-laminated member prepared by gluing flatwise four straight-grained 23 by 38-mm laminations free of knots and other structural defects. The other specimens were grade stamped material from the following grades of the spruce-pine-fir species group category: 1650f-1.5E (2 specimens), Select Structural (2 specimens) and No. 1 (2 specimens). Specimens selected were typical of commercial grades as found in Eastern Canada. All members were conditioned at 20 C and 65% relative humidity for a period of about two months prior to testing.

Specimens were tested over a span of 1,500 mm (corresponding approximately to 17 times the specimen's depth) with the clamping plates positioned near the ends. In the current test program, 250-mm-long clamping plates were used. Therefore, all specimens were cut to 2,000 mm, a total length corresponding to span plus twice the clamping length.

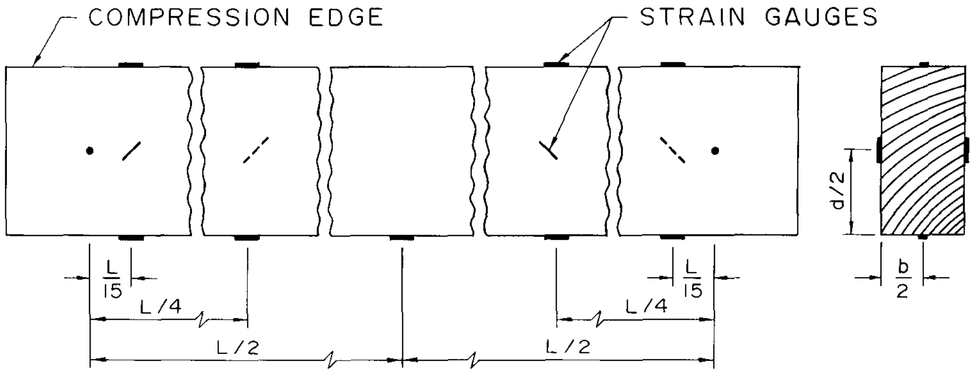

FIG. 5. Strain gauge position relative to span L. Dashed lines represent gauges positioned on face opposite to face shown.

To monitor axial strains, electrical resistance strain gauges were glued parallel to beam axis on both narrow faces of the specimens at locations shown in Fig. 5. Shearing strains were monitored with similar gauges glued at mid-depth of the specimen wide faces and oriented at 45 degrees from beam axis. These gauges were oriented such that all gauges were shortened when a vertical concentrated load was applied on the beam. In selecting the specimens, special care was taken to insure that no knots were located within 25 mm from any gauge.

Micro-measurement gauges, 12.5 mm long, with flexible polymide film backing were used. Strain gauge mounting technique was identical to that developed by Cooper et al. (1981), who used strain gauges successfully on wood members. Strains were recorded on a Measurement Group indicator Model P-3500, with a 120 ohm resistor and a bridge voltage of 2 volts. Dummy gauges mounted on an identical specimen were used to balance out false strains resulting from temperature and moisture effects.

In all, each specimen carried 13 gauges. No gauges were glued on the compression edge at the midspan position because a concentrated load was applied at this point during the tests. Some gauges were purposely located in the vicinity of the supports (at L/15 and 14L/15) to assess the possible effect of stress concentration in these zones arising from the transverse clamping forces.

Each specimen was tested twice. Using the apparatus developed, the specimens were first tested in constant bending. Then, the cable connecting the two levers was removed, the loading head was lowered, and the specimen was loaded under a center-point loading condition. During each test, midspan deflection was measured on each wide face using two linear variable differential transducers mounted on individual yokes. Depending on the test carried out, cable tension T or center-point load F was recorded using an electronic load cell. Loading was interrupted upon reaching a load corresponding to an extreme fiber stress of 13.5 MPa, calculated from specimen dimensions and testing geometry. Testing machine crosshead motion was set so that maximum load was reached in approximately 1 minute. After testing, moisture content and specific gravity were measured on all specimens.

Signals from the displacement transducers were processed to obtain an average deflection. Each test yielded a load versus deflection plot, the slope of which was

TABLE 1. *Normalized strains for the constant bending test.*

Specimen		Position relative to span			
No.	Quality	L/15	L/4	3L/4	14L/15
		Compression edge			
1	Glulam	−0.995	−1.105	−1.040	−1.018
2	Select	−0.886	−0.929	−1.041	−1.034
3	Select	−0.933	−1.020	−0.984	−0.972
4	1650f-1.5E	−1.007	−0.995	−1.058	−0.882
5	1650f-1.5E	−0.742	−0.910	−0.876	−0.843
6	No. 1	−0.880	−1.008	−1.066	−1.095
7	No. 1	−0.938	−0.931	−0.912	−0.666
Mean strain		−0.912	−0.997	−0.997	−0.930
		Tension edge			
1	Glulam	0.939	0.875	1.058	1.109
2	Select	0.932	0.933	1.029	1.076
3	Select	1.025	0.993	1.012	1.003
4	1650f-1.5E	1.229	1.002	1.076	1.118
5	1650f-1.5E	1.055	0.881	0.988	0.935
6	No. 1	1.097	1.082	1.100	1.197
7	No. 1	0.984	1.102	0.885	0.993
Mean strain		1.037	0.981	1.021	1.062

calculated and retained for further analysis. Strain was recorded at each location before loading and upon reaching maximum load. Initial reading was subtracted from final reading to obtain an applied strain.

RESULTS AND DISCUSSION

Moisture content ranged from 9 to 12%. This variation was considered too small to require adjustment of test data for moisture content. Specific gravity ranged from 0.4 to 0.5, a variation usual for the species investigated.

Load versus deflection plots obtained for the constant bending tests were essentially linear up to maximum load. These plots did not vary appreciably in shape from the plots obtained for the center-point loading tests. This observation suggests that the simplification introduced in Eq. (2) was justified in the present experiments. More importantly, this conclusion suggests that E can be calculated with sufficient accuracy from Eq. (5). Actually, deflections never exceeded 10 mm or L/150, a value below which deviation from linearity was expected to be negligible. According to the load deflection diagrams, no specimen was loaded beyond proportional limit.

Axial strain

To facilitate the analysis of axial strain data, all applied strains measured on the narrow faces were normalized according to the following procedure. Applied strains measured at individual locations during a test were divided by the applied strain measured on the tension edge at the midpoint location during the same test. This normalization provides a basis for comparing variations of strain with position, irrespective of the testing geometry employed. Axial strain data measured in the constant bending test for all specimens are listed in Table 1 in their normalized form. These strains are plotted in Fig. 6 as a function of position. Nor-

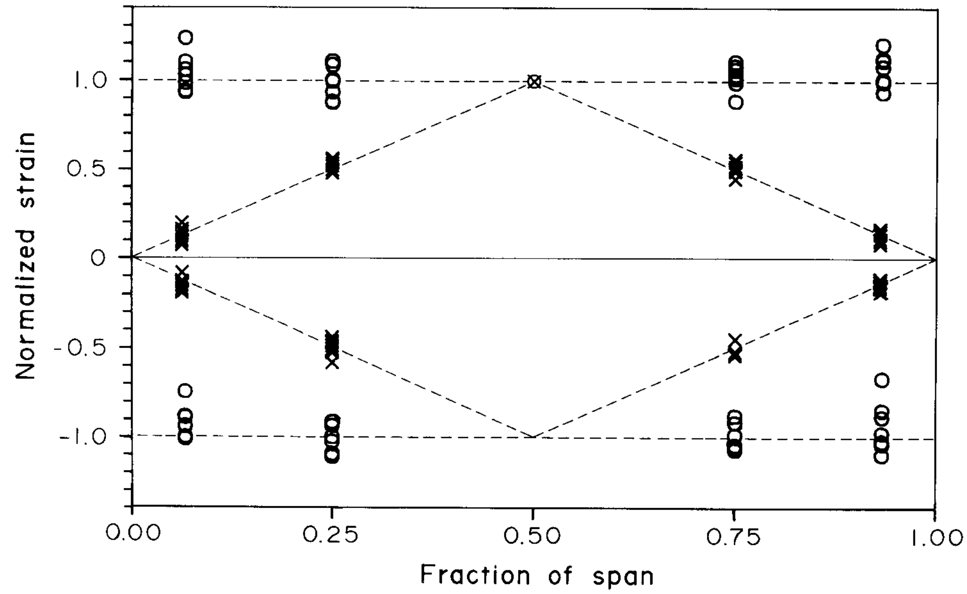

FIG. 6. Axial strains expressed in normalized form as a function of position expressed as a fraction of span (O = constant bending, X = center point loading).

malized strains measured for the center-point loading test are also shown on the plot for comparison purposes. The heavy dashed lines on the plot are the theoretical functions relating strain to position for both tests. These functions have the shape of the bending moment diagram corresponding to the test geometry employed.

Visual examination of Fig. 6 suggests that the mean trend of the data points follows closely theoretical strain functions for both testing geometries. Strain data collected in constant bending were statistically analyzed to verify if conditions a) and b) described earlier prevailed. This analysis was carried out considering the 8 groups of 7 data points appearing in Fig. 6 as individual samples of strain. A single classification analysis of variance carried out on the absolute value of these strain data revealed that the sample means were not statistically different at 5% significance level. At the same significance level, hypothesis testing involving the Student's *t* distribution indicated that these means were not significantly different from unity. Therefore, extreme fiber compression and tension strains would be equal regardless of position within span suggesting that conditions a) and b) are verified, at least in the portion of span included between L/15 and 14L/15. This result also indicates that no significant axial force was applied on the specimens during testing, a condition which could not be fulfilled by Corder (1965). No noticeable trend was detected in the axial strain data which would suggest that low grade specimens would deviate more from pure bending than higher grade specimens. This is why individual specimens were not identified in Fig. 6.

Shearing strain

Applicability of condition c) above was assessed by comparing the applied strains measured with 45-degree gauges in constant bending to those measured

TABLE 2. *Shearing strain ratio and moduli of elasticity.*

Specimen		Shearing strain ratio	Ea (MPa)	E (MPa)	G (MPa)	E/G
No.	Quality					
1	Glulam	0.02	12,300	12,900	1,100	11.7
2	Select	0.04	7,050	7,300	850	8.6
3	Select	0.06	10,550	11,600	500	23.2
4	1650f-1.5E	0.30	11,350	12,100	800	15.1
5	1650f-1.5E	0.03	11,900	12,200	2,050	6.0
6	No. 1	0.19	14,000	15,400	650	23.7
7	No. 1	0.19	9,800	10,400	700	14.9
Mean value		0.12	11,000	11,700	950	14.7

in third-point loading. Theoretically, the magnitude of the shearing force in third-point loading is constant at F/2 whereas in constant bending, this shearing force is expected to vanish. Therefore, a large difference in shearing strain is expected to exist between the two testing geometries. To evaluate the magnitude of shear in both geometries, strain data collected at the four locations on a specimen were averaged after each test. Average strain measured in constant bending was divided by the average strain measured in third-point loading to obtain for each specimen a shearing strain ratio. This ratio, given in Table 2 for all specimens, expresses the amount of shear present in one test relative to that in the other test when the same maximum bending stress (13.5 MPa) is applied in each test.

According to Table 2, shearing strain in constant bending would average 12% of the shearing strain in center-point loading for the 7 specimens selected. Three specimens, however, exhibit strain ratios well above average. One possible explanation for this deviation is twisting of the specimen when bending is taking place. In fact, twisting induces additional shearing stresses on the specimen surface. For rectangular beams, shearing stress induced by twisting is maximum at mid-depth, a position corresponding to strain gauge location. Consequently, test beams were very sensitive to twist. This sensitivity, however, was greater in constant bending since the unsupported length was twice as long as that of center-point loading. As a result, specimens prone to twist would exhibit a larger strain ratio. Specimen 4 exhibiting a strain ratio of 0.30 complies with this explanation since this specimen was found to contain spiral grain. As for specimens 6 and 7 from the No. 1 grade, twisting might have occurred due to skewed bending taking place in sections occupied by knots as these sections are often unsymmetrical. Disregarding specimens 4, 6, and 7, shearing strain in constant bending would average 4% of that of center-point loading, suggesting that for straight and relatively clear beams condition c) above is satisfied.

Moduli of elasticity

Load and deflection data developed were used to calculate bending and shear moduli of elasticity for comparison with published values. Bending modulus of elasticity E was directly obtained from the slopes of the T versus deflection diagrams using Eq. (5). Shear modulus G was calculated from the following equation relating the apparent modulus of elasticity E_a measured under center-point load to slenderness ratio

$$\frac{1}{E_a} = \frac{1}{E} + \frac{1.2}{G}\left(\frac{h}{L}\right)^2 \qquad (6)$$

where

$$E_a = \frac{L^3}{4bd^3}\frac{F}{y}.$$

These equations are provided in ASTM D 198 (1988) to calculate G for rectangular lumber members from a series of center-point load tests conducted over increasing span lengths. In the present investigation, Eq. (6) can be applied to the single center-point load test carried out on each specimen since E is known from the companion constant bending test. Therefore, after calculating E_a from the slope of the F versus deflection diagrams, G was obtained for each specimen from substituting E and E_a values into Eq. (6). These moduli are listed in Table 2 for all specimens together with the calculated E/G ratio.

From Table 2, it can be seen that in the present experiment E_a is consistently lower than E, as would be expected from shear effects. Differences of the magnitude observed here have been reported for other species of structural lumber members (Grant 1979; Lundberg and Thunell 1978; Fewell 1980). Shear modulus data are more difficult to analyze since very little information is available on this property in the literature. Although G values have been determined in various investigations of the elastic constants of wood, these generally relate to small clear specimens with careful orientation of growth rings and fiber direction. What is really needed are gross values of G, similar to those for E and directly associated with shear deflection of beams. The basic property involved in shear deflection is the ratio E/G and for Canadian hemlock members in structural sizes, Curry (1976) found E/G ratios ranging from 8 to 40, a range compatible with that obtained in the present investigation. Furthermore, the average E/G value measured in the present tests is very close to 16, the value generally assumed for testing (ASTM D 2915 1988) and design purposes (Hoyle and Woeste 1989). This analysis, therefore, suggests that the method developed in this study would also be suitable to measure G in structural lumber members.

The ASTM method that determines G from a series of tests where span is increased assumes that the E/G ratio is constant and is unaffected by change in span. The validity of this assumption is questionable for lumber members containing localized weak zones. One advantage of the present method over ASTM is that such an assumption is not necessary since all tests are carried out over the same span.

CONCLUSION

The data developed in this study show that shear-free modulus of elasticity of lumber members can be determined using constant bending. This method offers the advantage of testing the member uniformly over almost its full length, a condition that cannot be achieved with center-point or four-point loading arrangements usual in standard test methods. Constant bending requires special supports, but these can easily be adapted in test frames commonly used for performing standard bending tests. When a constant bending test is complemented by a center-point loading test carried out over the same material, the data developed from these tests can be used to determine the shear modulus of elasticity of the member. The merit of constant bending for determination of bending strength of lumber members remains to be assessed.

ACKNOWLEDGMENTS

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