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# A PRACTICAL TRIAL OF STOCHASTIC SYSTEM IDENTIFICATION UNDER EXISTENCE OF BACKGROUND NOISE BASED ON EQUIVALENCE OF STATISTICS AND ITS APPLICATION TO RESPONSE PROBABILITY EVALUATION OF ACOUSTIC SYSTEM WITH MUSIC INPUT

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A practical identification method of functional type for sound insulation systems is derived especially in the form matched to the prediction problem of output response probability distribution once after introducing some new evaluation criteria on output statistics. The probability distribution of output response with an arbitrary input sound under the contamination of background noise is principally able to predict theoretically by positively using the statistics of input and background noises based on the physical law of intensity linearity. More specifically, the system parameters of the objective sound insulation system are systematically identified by employing an equivalence property between the experimentally observed output probability distribution and the theoretically predicted one as a criterion of the parameter identification. As an application to the actual noise environment, the proposed method has been concretely applied to a double walls sound insulation system and experimentally confirmed under a music sound input.

**Keywords:** *system identification, probabilistic evaluation, environmental noise, sound insulation system, prediction of output distribution*

## Notations

- $\eta$ intensity of output noise,
- $\xi_i$ intensity of input noise in the $i$-th frequency band,
- $\zeta$ intensity of external background noise,
- $a_i$ system parameter for $i$-th frequency band,
- $N$ number of frequency component band in overall frequency range,
- $x_i$ level observation of $\xi_i$,
- $y$ level observation of $\eta$,
- $v$ level observation of $\zeta$,
- $P(y)$ probability density function of $y$,
- $H_n()$ $n$-th order Hermite polynomial,
- $n$ order of approximation,
- $L$ upper limit of approximation,

| | |
|---|---|
| $\mu$ | mean value of $y$, |
| $\sigma^2$ | variance of $y$, |
| $A_n$ | $n$-th order expansion coefficient, |
| $\langle\ \rangle$ | time average, |
| $M_y()$ | moment generating function of $y$, |
| $M$ | $10/\log_e 10$, |
| $\varepsilon_0$ | $10^{-12}\ \mathrm{W/m^2}$, |
| $\mathbf{a}$ | vector of system parameter, |
| $\mu_T(\mathbf{a})$ | theoretically estimated mean value under the estimation $\mathbf{a}$, |
| $\sigma_T^2(\mathbf{a})$ | theoretically estimated variance under the estimation $\mathbf{a}$, |
| $A_{Tn}(\mathbf{a})$ | theoretically estimated coefficient under the estimation $\mathbf{a}$, |
| $\widehat{\mathbf{a}}^{(k)}$ | estimation of $\mathbf{a}$ at $k$-th stage, |
| $\Gamma$ | Robbins Monro's gain diagonal matrix, |
| $P_T(y:\mathbf{a})$ | theoretically estimated p.d.f. under the estimation $\mathbf{a}$, |
| $Q(y)$ | cumulative distribution of $P(y)$, |
| $z$ | integral variable, |
| $L_x$ | noise evaluation index. |

## 1. Introduction

As is well-known, a sound insulation system is usually evaluated by its transmission loss [1, 2, 3] given by the difference of sound pressure levels between input and output in each frequency band. On the other hand, it is usual that not only the input sound showing several types of probability distribution form but also the output response is observed under an inevitable contamination by a background noise of arbitrary distribution type. Furthermore, in addition to the lower order moment statistics like mean and variance of sound levels, $L_{\mathrm{eq}}$ and noise indexes like $(100-x)$ percentile $L_x$ closely connected with a whole cumulative probability distribution form of sound level are actually employed to evaluate the environmental noises [4].

In this study, first, we pay our attention to a probability prediction problem on the output response of a sound insulation system in a room acoustics. So, not from a bottom up way of thinking like the standard type of analytical method along to the physical law, our main concern is paid to the prediction on a whole probability distribution form of the system output response on a dB scale based on the functional introduction of some object-oriented type evaluation criteria. That is, the probability distribution of the output response with an arbitrary stochastic input (e.g. music sound) under an avoidable contamination of background noise is able to be predicted by using the statistics of input and background noises based on the physical law of intensity linearity. More concretely, by employing the equivalence property between the experimentally observed output probability distribution and the theoretically predicted one as a criterion of the parameter identification, the system parameters of the objective sound insulation system are systematically identified. Then, the probability distribution of output response can be estimated with use of the statistical Hermite type expansion expression [5] reflecting the system parameters, the statistics of input and background noises observed on dB scale.

As an application to the actual noise environment, the proposed method is concretely applied to an evaluation problem of double walls sound insulation system excited by a music sound input under the contamination of background noise at an observation point. The system parameters and the probability distribution form of output response are theoretically predicted and compared with the experimentally observed data. It is noteworthy that the system parameters are identified without employing the standard type frequency analysis like the 1/1 or 1/3 octave band analysis for the system output.

## 2. Model of sound environmental system

In the standard frequency analysis directly connected to the physical countermeasure, it is usual that a sound insulation system can be usually described by a linear system on the intensity scale in a frequency domain and is contaminated by an additional background noise of arbitrary distribution type at an observation point as follows:

$$\eta = \sum_{i=1}^{N} a_i \xi_i + \zeta. \tag{1}$$

## 3. The output response probability distribution of sound insulation system

As is well-known, an arbitrary type probability density function (abbr. p.d.f.) on the continuous level can be universally expressed in the form of statistical Hermite type expansion expression [5].

$$P(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \left\{ 1 + \sum_{n=3}^{L} \frac{1}{n!} A_n H_n\left(\frac{y-\mu}{\sigma}\right) \right\} \qquad (L \to \infty) \tag{2}$$

with distribution parameters

$$\mu \triangleq \langle y \rangle, \qquad \sigma^2 \triangleq \langle (y-\mu)^2 \rangle \qquad \text{and} \qquad A_n \triangleq \left\langle H_n\left(\frac{y-\mu}{\sigma}\right) \right\rangle. \tag{3}$$

On the other hand, by considering Eq. (1), the moment generating function of the above p.d.f. $P(y)$ can be easily derived as follows:

$$M_y\left(\frac{\theta}{M}\right) \triangleq \left\langle e^{\frac{\theta}{M} y} \right\rangle = \frac{\langle \eta^\theta \rangle}{\varepsilon_0^\theta} = \sum_{r=0}^{\theta} \binom{\theta}{r} X_r \left\langle e^{\frac{\theta - r}{M} v} \right\rangle, \tag{4}$$

where

$$X_r \triangleq \sum_{r_1 + r_2 + \cdots + r_N = r} \frac{a_1^{r_1} a_2^{r_2} \cdots a_N^{r_N}}{r_1! r_2! \cdots r_N!} \left\langle e^{\frac{r_1}{M} x_1 + \frac{r_2}{M} x_2 + \cdots + \frac{r_N}{M} x_N} \right\rangle,$$

$$M \triangleq 10/\log_e 10 \qquad \text{and} \qquad \varepsilon_0 = 10^{-12}\ \mathrm{W/m^2}.$$

By taking the well-known relationship between the moment generating function, Eq. (4) and the cumulant statistics of $y$ into consideration, the distribution parameters defined in Eq. (3) can be estimated by employing the statistics of input and background noises as follows:

$$\begin{bmatrix} \dfrac{\mu}{M} \\ \dfrac{\sigma^2}{M^2} \\ \dfrac{\sigma^3}{M^3}A_3 \\ \dfrac{\sigma^4}{M^4}A_4 \end{bmatrix} = \begin{bmatrix} 4 & -3 & \dfrac{4}{3} & -\dfrac{1}{4} \\ -\dfrac{26}{3} & \dfrac{19}{2} & -\dfrac{14}{3} & \dfrac{11}{12} \\ 9 & -12 & 7 & -\dfrac{3}{2} \\ -4 & 6 & -1 & 1 \end{bmatrix} \begin{bmatrix} \ln\left\{\sum_{r=0}^{1} X_r \left\langle e^{\frac{1-r}{M}v}\right\rangle\right\} \\ \ln\left\{\sum_{r=0}^{2} \binom{2}{r} X_r \left\langle e^{\frac{2-r}{M}v}\right\rangle\right\} \\ \ln\left\{\sum_{r=0}^{3} \binom{3}{r} X_r \left\langle e^{\frac{3-r}{M}v}\right\rangle\right\} \\ \ln\left\{\sum_{r=0}^{4} \binom{4}{r} X_r \left\langle e^{\frac{4-r}{M}v}\right\rangle\right\} \end{bmatrix}. \tag{5}$$

Hereupon, the expression of p.d.f. in Eq. (2) is approximately employed by letting $L = 4$. Therefore, by substituting the solution of Eq. (5) into Eq. (2), the output probability distribution of a sound insulation system can be explicitly estimated.

## 4. Identification of system parameters

The Eq. (5) can be regarded as a simultaneous nonlinear equation with respect to system paremeters $a_i$'s through the variables $X_r$'s. So, the system parameters can be estimated by solving the some complicated nonlinear equation Eq. (5) with respect to $a_i$'s. Here, let us employ the well-known stochastic approximation method to solve it under two kinds of criterions on the equivalency of theoretically estimated p.d.f. and experimentally observed one of the output response.

*Agreement of distribution parameters* [Method I]

Let us introduce the following criterions with respect to each distribution parameter:

$$\begin{aligned} \mu_T(\mathbf{a}) - \mu &= 0, \\ \sigma_T^2(\mathbf{a}) - \sigma^2 &= 0, \\ A_{Tn}(\mathbf{a}) - A_n &= 0, \end{aligned} \tag{6}$$

where $\mu_T(\mathbf{a})$, $\sigma_T^2(\mathbf{a})$ and $A_{Tn}(\mathbf{a})$ are obtained by Eq. (5) with use of $\mathbf{a} \triangleq (a_1, a_2, \ldots, a_N)^T$. Parameters $\mu$, $\sigma^2$ and $A_n$ are directly obtained ones from the actual output response observation. Then, the system parameters can be estimated successively by using the following algorithm of the stochastic approximation method [6]:

$$\widehat{\mathbf{a}}^{(k)} = \widehat{\mathbf{a}}^{(k-1)} - \Gamma_k \begin{bmatrix} A_{T1}\left(\widehat{\mathbf{a}}^{(k-1)}\right) - H_1\left(\dfrac{y^{(k-1)} - \mu}{\sigma}\right) \\ \vdots \\ A_{TN}\left(\widehat{\mathbf{a}}^{(k-1)}\right) - H_N\left(\dfrac{y^{(k-1)} - \mu}{\sigma}\right) \end{bmatrix}, \tag{7}$$

with

$$\widehat{\mathbf{a}}^{(k)} \triangleq \left(\widehat{a}_1^{(k)}, \widehat{a}_2^{(k)}, \ldots, \widehat{a}_N^{(k)}\right)^T,$$

$$A_{T1}(\mathbf{a}) \triangleq \frac{\mu_T(\mathbf{a}) - \mu}{\sigma} \quad \text{and} \quad A_{T2}(\mathbf{a}) \triangleq \frac{\sigma_T^2(\mathbf{a}) - \sigma^2}{\sigma^2}.$$

*Agreement of whole distribution shape* [Method II]

Let us introduce the well-known least-squares error criterion with respect to the estimation error between $P_T(y : \mathbf{a})$ calculated by Eq. (2) and the corresponding experimental one, as follows:

$$\left\langle \{P_T(y : \mathbf{a}) - P(y)\}^2 \right\rangle \longrightarrow \min. \tag{8}$$

Then, the system parameters can be estimated successively by using the following algorithm on the stochastic approximation method:

$$\widehat{\mathbf{a}}^{(k)} = \widehat{\mathbf{a}}^{(k-1)} - \Gamma_k \begin{bmatrix} \dfrac{\partial}{\partial \widehat{a}_1^{(k-1)}} P_T(y_k; \widehat{\mathbf{a}}^{(k-1)}) \\ \vdots \\ \dfrac{\partial}{\partial \widehat{a}_N^{(k-1)}} P_T(y_k; \widehat{\mathbf{a}}^{(k-1)}) \end{bmatrix} \left\{ P_T(y_k; \widehat{\mathbf{a}}^{(k-1)}) - P(y_k) \right\}. \tag{9}$$

## 5. Experimental consideration

### 5.1. Experimental arrangement

Figure 1 shows an experimental arrangement in two reverberation rooms. Here, the speaker 1 excites the transmission room, two microphones can receive the input and output sound level fluctuations of an insulation system and the speaker 2 radiates the background noise into the reception room. For simulating an actual living environment, a music sound (rock music) has been positively employed as the input noise. A white noise has been employed as the background noise. Double walls consisting of aluminum panel with 1.2 mm thickness and surface mass 3.22 kg/m$^2$ have been employed as a partition wall. The overall response fluctuation has been observed over about 10 minutes with a sampling rate of 0.5 sec under the contamination of a background noise. Each frequency component of the input has been obtained simultaneously by employing the octave band analysis.

### 5.2. Identification of system parameters

The system identification has been carried out by solving the Eq. (7) with the first half of observed data. The estimation results of system parameters calculated under two methods ([Method I] and [Method II]) are shown in Table 1 as the transmission losses on dB. Experimentally observed transmission losses obtained actually by the usual octave band analysis with a usual white noise are also shown in Table 1 as "Experiment".

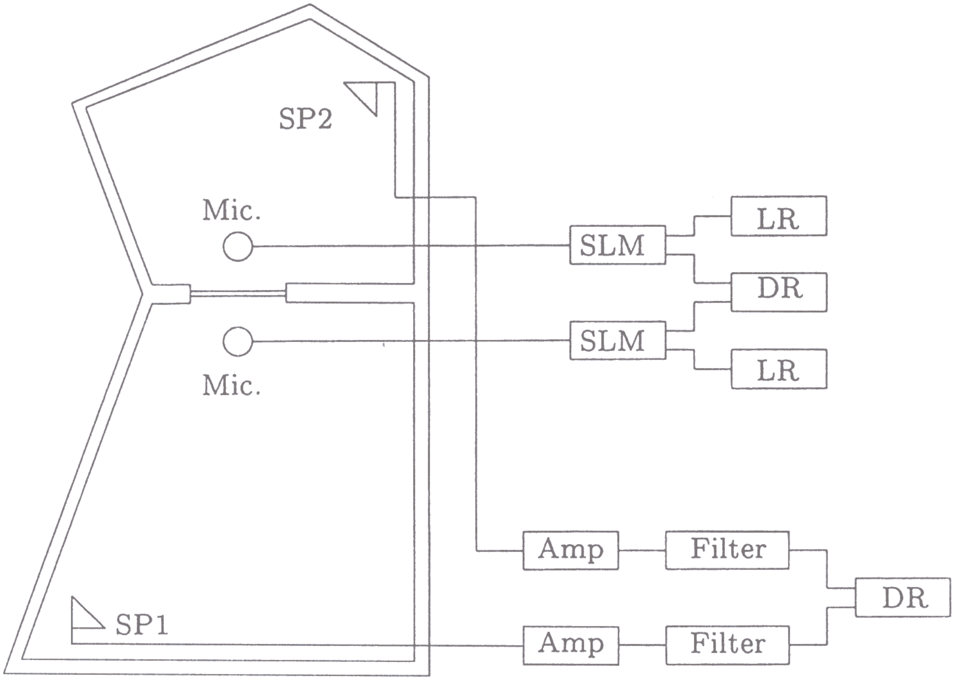


Fig. 1. Experimental arrangement in two reverberation rooms.

**Table 1.** Estimation results of the system parameters (Transmission loss).

| Method | $a_1$(250 Hz) | $a_2$(500 Hz) | $a_3$(1 kHz) |
|---|---|---|---|
| Method I | 9.72 | 20.46 | 25.63 |
| Method II | 9.71 | 20.67 | 26.75 |
| Experiment | 11.69 | 21.05 | 28.19 |

[dB]

### *5.3. Probability distribution for output response of sound insulation system*

The probability distribution form of output response can be estimated with use of Eq. (2) by substituting the distribution parameters calculated in Eq. (5) with previously estimated system parameters, the statistics of input and background noises. Here, in close relation to the usual noise evaluation index, $L_x$ noise level, Eq. (2) is evaluated especially in a cumulative distribution form as:

$$Q(y) = \int_{-\infty}^{y} P(z)\, dz. \tag{10}$$

The output cumulative distribution of a double walls sound insulation system is estimated for the first half of total observation data of non-stationary type used in the identification process of the system parameters. The estimations were carried out for two cases with and without background noise at the system output. The results are shown in Fig. 2 with comparisons to the experimentally observed cumulative probability distributions of the system output noise.

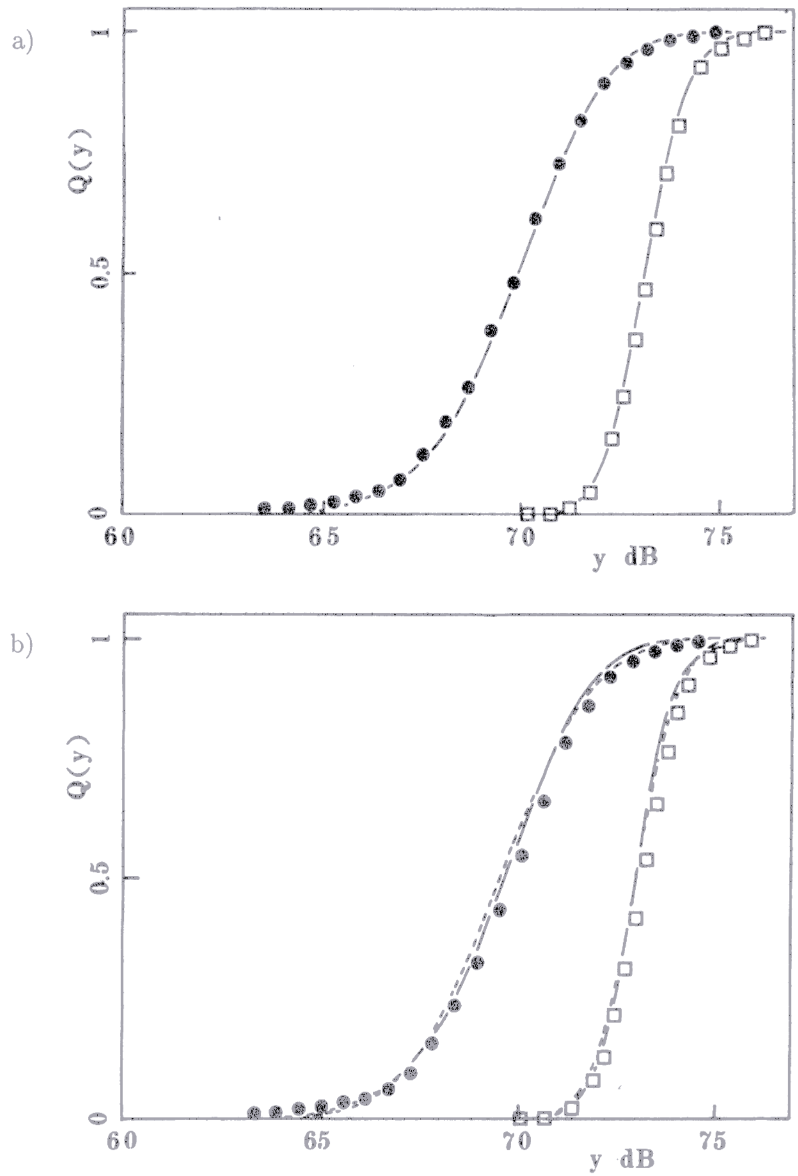


Fig. 2. Comparisons between experimentally sampled points and theoretically estimated curves for the output probability distribution of double walls sound insulation system with and without the effect of background noise under a music sound input: (a) Method I and (b) Method II. Experimentally observed points for output noise response are marked by • without background noise and □ with background noise. Theoretical curves are shown with the degree of approximation $n$ in Eq. (2) as - - - - -, $n$=0 (for case (b)); ——, $n$=4.

## 6. Conclusions

In this study, first, a systematical identification method of sound insulation system with arbitrary input fluctuation of non-Gaussian type has been theoretically proposed under a contamination of arbitrary background noise especially by introducing some new evaluation criterion on the agreement between theory and observation for input and output probability distribution forms or their distribution parameters. Then, as an application to the actual sound environment, the proposed evaluation method is employed for the evaluation of double walls sound insulation system excited by a music sound. Finally, by using the proposed method, it seems possible to identify the actual sound insulation system in a daily living environment without employing the standard type usual frequency analysis.

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