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ANALYSIS OF ACOUSTICS SURFACE WAVE REFLECTION ON THE INTERDIGITAL TRANSDUCER

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By applying the equivalent scheme, the analysis of the acoustics surface wave reflection from the interdigital transducer has been carried out. The explicit form of the coefficient of reflection was found, from which the following terms have been separated — the magnitude of the mechanical wave reflection due to variation of the acoustics impedance in the area of transducer, and magnitude of the wave regeneration by the electrically loaded transducer. The results of calculations has been presented for the coefficient of surface wave reflection from the simple transducer with single and double electrodes as a function of its electrical load. The experimental verification has been undertaken for transducers on $LiNbO_3$.

1. Introduction

The surface acoustic wave (SAW) incident upon the interdigital transducer is submitted to the reflection as result of:

acoustics impendance variation in the transducer domain (the mechanical type reflection),

regeneration by the electrically loaded transducer (the electrical type reflection).

In the case of a pair of two transducers in SAW filters the wave reflection generates multiple echoes, respectively high level of which can caused considerable distortions in amplitude and phase characteristics of filters. The phenomenon of wave reflection from the structure of interdigital transducer can be applied for constructing some SAW devices, for example resonators. On the other hand, by using the exact description of this phenomenon, the effect can be precisely eliminated in the undesirable cases. In this paper the above mentioned aspects of transducers are considered on the strength of its equivalent scheme.

2. A concept of equivalent scheme

The surface wave incident upon the interdigital transducer is submitted to reflection, transmission and convertion into electrical sygnal, Fig. 1a. The analysis of these phenomena can be carried out by using the method of equivalent scheme, [1]. The magnitude of incident wave is identified with a voltage source described by both 2V electromotive force and inner impedance z_N which denotes impedance of free surface of piezoelectric, Fig. 1b, [2]. The simple transducer is a combination of many

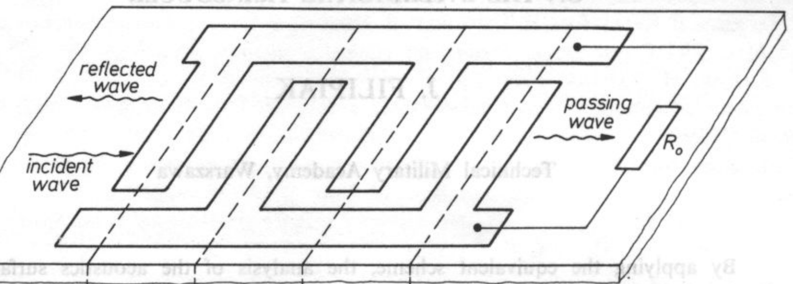

FIG. 1. a) Interdigital transducer, b) Equivalent scheme of transducer

three-port networks each being on equivalent scheme of its single section [3]. Single section corresponds to the area located between centres of two following electrodes. The scheme and notations are presented on Fig. 1b. The acoustics and electrical clamps of separate sections are connected in cascade and in parallel respectively considering polarization of electrodes. On the side of incident wave the three-port network is loaded by the voltage source but on its other side the acoustics clamps are loaded by impedance related to the free surface of piezoelectric. The load of electrical clamps is determined by impedance R_0.

By applying relations introduced in [3], the three-port network can be described in the following way:

$$i_k = i_1 \cos\beta + \frac{e_1}{jz_0}\sin\beta - \frac{2r\sin\frac{\alpha}{2}}{jz_0}\sum_{n=1}^{N} E_n \cos\left(N-n+\frac{1}{2}\right)\alpha$$

$$e_k = -ji_1 z_0 \sin\beta + e_1\cos\beta + 2r\sin\frac{\alpha}{2}\sum_{n=1}^{N} E_n \sin\left(N-n-\frac{1}{2}\right)\alpha, \tag{2.1a}$$

where N denotes the number of sections on which transducer is divided,

$$\beta = N\alpha \qquad \alpha = \pi\left(1 - j\frac{\sqrt{(\omega-\omega_3)(\omega-\omega_1)}}{\omega_1}\right)$$

$$\omega_3 = \pi v'_R p^{-1} \qquad \omega_1 = \pi v'_R p^{-1}\left(1+\frac{1}{4}\cos\Delta\right)^{-1} \qquad \Delta = \pi w p^{-1}$$

the notations of V, w and p denote respectively the velocity of surface wave, the width and the period of electrode.

In the electrical side it is

$$I = \sum_{l=1}^{N} \mu(l)\{j\omega C_0 E_l + r(i_{l+1} - i_l)\} \tag{2.1b}$$

where

$$i_l = i_1 \cos l\alpha + \frac{e_1}{jz_0}\sin l\alpha - \frac{2r\sin\frac{\alpha}{2}}{jz_0}\sum_{n=1}^{l} E_l \mu(l)\cos\left(l-n+\frac{1}{2}\right)\alpha.$$

Rel. (2.1) and boundary conditions, derived for the three-port network are presented in the following forms

on the acoustic side

$$i_l z_n = 2V - e_1 \qquad i_k z_n = e_k \tag{2.2a}$$

on the electrical side

$$R_0 I = -E \tag{2.2b}$$

The set of equations (2.2a) and (2.2b) allows to determine all quantities for the three-port network. The reflection and transmission coefficients, derived in [4] can be presented in the following forms

$$\Gamma = \frac{z_n - \frac{e_1}{i_1}}{z_n + \frac{e_1}{i_1}} \tag{2.3}$$

$$T_z = \frac{e_k}{V} \tag{2.4}$$

respectively.

Hence, the identification of Eq. (2.3) and Eq. (2.4) requires finding of both magnitudes – the acoustics impedance of transducer defined by e_1/i_1 to be calculated on the side of incident wave and the value of voltage e_k defined by the loaded acoustic clamps of transducer to be calculated on its other side. A determination of these both magnitudes can be possible by solving the set of equations (2.1) and (2.2) which has been defined for the single three-port network.

3. Reflection and transmission coefficients

By applying relation (2.1a), the coefficient of reflection Γ, Eq. (2.3) can be presented in the following form

$$\Gamma = 1 - \frac{e_1}{V} \tag{3.1}$$

therefore, a determination of above mentioned coefficients requires finding dependence of voltages existed on acoustics clamps of the three-port network upon the value of V. Hence, from relations (2.1a) and (2.2a) a dependence of these voltages on the magnitude of source V and the voltage determined by the loaded transducer can be found in the form

$$e_1 = \frac{(1-R)(1+Re^{-j2\beta})}{1-R^2e^{-j2\beta}}V + \frac{jr(1+R)\sin\frac{\alpha}{2}e^{-j\frac{\beta}{2}}(H^*+Re^{-j\beta}H)}{1-R^2\mathrm{e}^{-j2\beta}}E$$

$$e_k = \frac{(1-R^2)e^{-j\beta}}{1-R^2e^{-j2\beta}}V + \frac{jr(1+R)\sin\frac{\alpha}{2}e^{-j\frac{\beta}{2}}(H+Re^{-j\beta}H^*)}{1-R^2\mathrm{e}^{-j2\beta}}E \tag{3.2}$$

where

$$R = \frac{z_N - z_0}{z_N + z_0}$$

$$H = \sum_{n=1}^{N}\mu(n)e^{-j(\frac{N-1}{2}+\imath)\alpha} \qquad \mu(n) = \frac{A_n - A_{n+1}}{|A_n - A_{n+1}|}$$

Hence, the value of $\mu(n) = 0, 1$ depends on the way in which the bus of transducer is connected to the electrode located in the area of n-th section of transducer. Moreover, the value of $A(n)$ determines position of the end of electrode. By assuming relations (2.1b), (2.2b) and (3.2), a dependence between voltage E of the loaded transducer and the magnitude of incident wave V can be determined in the form

$$E = \frac{2jr(1-R)\sin\frac{\alpha}{2}e^{-j\frac{\beta}{2}}}{z_0(1-R^2e^{-j2\beta})}\frac{(H^*+Re^{-j\beta}H)R_0}{1+Y_NR_0}V \tag{3.3}$$

where Y_N denotes the transducer admittance. By substituting Eq. (3.3) into (3.1), the value of acoustics voltages of the three-port network can be determined as the function of the magnitude of incident wave

$$e_1 = \left\{ \frac{(1+R)(1-Re^{-j2\beta})}{1-R^2e^{-j2\beta}} - \frac{2r^2\sin^2\frac{\alpha}{2}(1-R^2)e^{-j\beta}(H^*+Re^{-j\beta}H)^2R_0}{z_0(1-R^2e^{-j2\beta})^2(1+Y_NR_0)} \right\} V$$

$$e_k = \frac{(1+R^2)e^{-j\beta}}{1-R^2e^{-j2\beta}} \left\{ 1 - \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)R_0}{z_0(1-R^2e^{-j2\beta})^2(1+Y_NR_0)} \right\} V \tag{3.4}$$

By applying relation (3.4), the reflection and transmission coefficients can be found as

$$\Gamma = \frac{R(1-e^{-j2\beta})}{1-R^2e^{-j2\beta}} - \frac{(1-R^2)e^{-j2\beta}}{1-R^2e^{-j2\beta}} \cdot \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)^2R_0}{z_0(1-R^2e^{-j2\beta})(1+Y_NR_0)}$$

$$T = \frac{(1-R^2)e^{-j2\beta}}{1-R^2e^{-j2\beta}} \left\{ 1 - \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)R_0}{z_0(1-R^2e^{-j2\beta})(1+Y_NR_0} \right\} \tag{3.5}$$

In the case of short-circuited transducer ($R_0 = 0$) the magnitudes of wave reflection and transmission depend only on the variation of acoustics impedance of the transducer domain

$$\Gamma_z = \frac{R(1-e^{-j2\beta})}{1-R^2e^{-j2\beta}}$$

$$T_z = \frac{(1-R^2)e^{-j\beta}}{1-R^2e^{-j2\beta}} \tag{3.6}$$

Relations (3.6) are identical with those determined in [5]. The final relations define the reflection and transmission coefficients are presented in the following form

$$\Gamma = \Gamma_z + T_z \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)^2R_0}{z_0(1-R^2e^{-j2\beta})(1+Y_NR_0)}$$

$$T = T_z - T_z \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)R_0}{z_0(1-R^2e^{-j2\beta})(1+Y_NR_0)} \tag{3.7}$$

where Γ_z denotes the magnitude of mechanical wave reflection but the term proportional to R_0 denotes electrical wave regeneration by the loaded transducer. The coefficient of wave reflection consists of the "mechanical" and "electrical" terms, Γ_z and Γ_E respectively, where Γ_E is presented in the following form

$$\Gamma_E = \frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)^2R_0}{z_0(1-R^2e^{-j2\beta})(1+Y_NR_0)} T_z \tag{3.8}$$

It can be proved that

$$\frac{2r^2 \sin^2 \frac{\alpha}{2} (H^* + Re^{-j\beta} H)(H + Re^{-j\beta} H^*)}{z_0 (1 - R^2 e^{-j2\beta})} = Re(Y_N) = G \tag{3.9}$$

where G denotes the transducer conductance. Because the transducer admittance has a form

$$Y_N = G + jB \tag{3.10}$$

the coefficient of wave reflection and transmission can be presented in the form

$$\begin{aligned} \Gamma &= \Gamma_z - T_z e^{j\Phi} \frac{GR_0}{1 + (G + jB) R_O} \\ T &= T_z - T_z \frac{GR_O}{1 + (G + jB) R_O} \end{aligned} \tag{3.11}$$

where

$$e^{j\Phi} = \frac{H^* + Re^{-j\beta} H}{H + Re^{j\beta} H^*}$$

4. Experimental results

The comparison of the theory with experiment has been carried out for both- electrically shorted and opened transducers with single and double electrodes. For these cases the value of R_0 is precisely determined and varied from 1 to ∞ respectively. For the baseband of shorted transducer with double electrodes the value of coefficient of reflection is equal to 0, because $R = 0$, but for the opened one the reflection and transmission coefficients are defined in the form

$$|\Gamma| = \frac{G}{\sqrt{G^2 + B^2}} \quad |T| = \frac{B}{\sqrt{G^2 + B^2}} \tag{4.1}$$

By finding the transducer admittance, the value of coefficient of wave reflection can be determined in very simple way. The values of measured admittance of $LiNbO_3$ base transducer with 10 pairs of double electrodes are presented on Fig. 2. Based on the relation (4.1), the coefficient of reflection has been calculated by applying the experimental data of admittance and then the values of coefficient have been compared with the experimental data obtained from [6]. In the case of short-circuited transducer with single electrodes, the coefficient of reflection is identified with Γ_z but for the opened transducer the coefficient has the following form

$$\Gamma = \Gamma_z + T_z \frac{Ge^{j\Phi}}{G+jB} \tag{4.2}$$

The theoretical relations of Γ_z, Γ_E and Γ for simple $LiNbO_3$ base transducer with 10 single electrodes are compared on Fig. 3 with the experimental results obtained from [6]. The worse coincidence between theory and experiment can be explained by the following reasons – the transducer admittance, mainly its static capacitance was defined with the less accuracy, the considery analitical model doesn't contain the phenomenon of the surface wave diffraction.

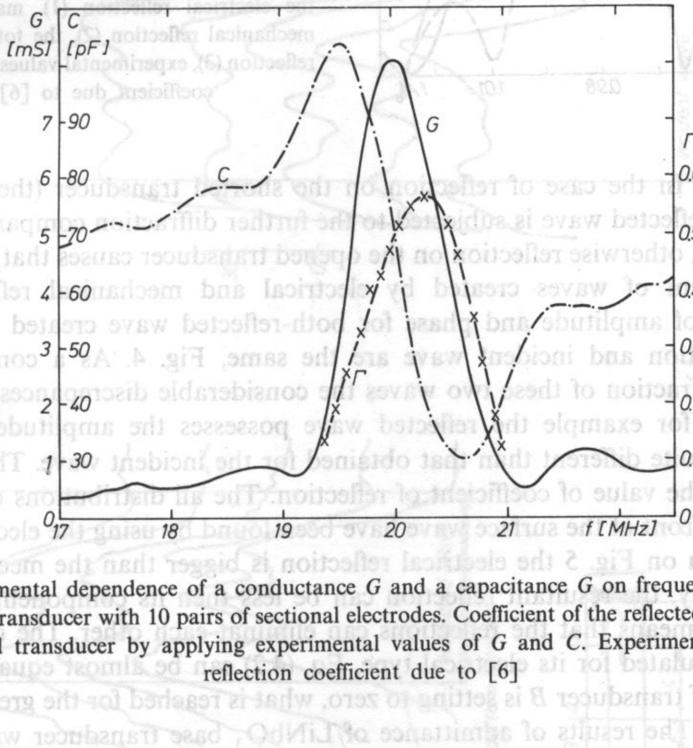

FIG. 2. Experimental dependence of a conductance G and a capacitance G on frequency of the simple $LiNbO_3$ base transducer with 10 pairs of sectional electrodes. Coefficient of the reflected wave, calculated for the opened transducer by applying experimental values of G and C. Experimental values of the reflection coefficient due to [6]

The reflected wave is submitted to the diffraction, evaluation of which is related to its point of generation or its point of regeneration taking into consideration respectively the mechanical or electrical type of reflection. Amplitude and phase distributions of front of the surface wave propagating between two $LiNbO_3$ base transducers with double electrodes are presented on Fig. 4. The distributions of both induced and regenerated wave on the opened transducer are the same in this case. On the contrary, Fig. 5 shows distribution of amplitude and phase of front of the surface wave incident and reflected from shorted and opened simple transducer composed of 10 pairs of single electrodes. These distributions have been obtained at the half distance between transducers in which the surface wave has been induced

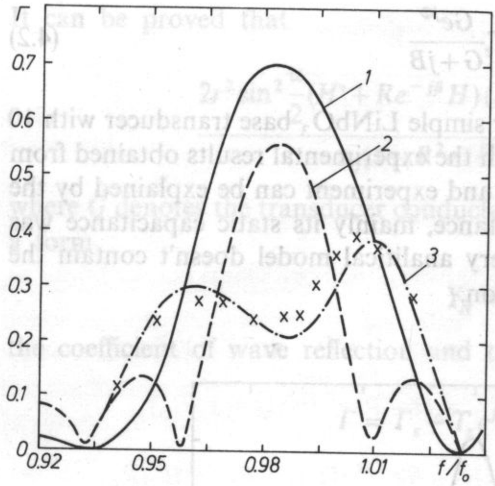

FIG. 3. Theoretical values of the reflection coefficient for wave reflected on the simple transducer with 10 pairs of single electrodes: magnitude of the electrical reflection (1), magnitude of the mechanical reflection (2), the total coefficient of reflection (3), experimental values of the reflection coefficient due to [6] (×).

and reflected. In the case of reflection on the shorted transducer (the mechanical reflection) a reflected wave is subjected to the further diffraction comparing with the incident wave, otherwise reflection on the opened transducer causes that the reflected wave is a sum of waves created by electrical and mechanical reflection. The distributions of amplitude and phase for both-reflected wave created by electrical type of reflection and incident wave are the same, Fig. 4. As a consequence of a different diffraction of these two waves the considerable discrepances occur in its distributions, for example the reflected wave possesses the amplitude and phase distribution quite different than that obtained for the incident wave. This effect can influence on the value of coefficient of reflection. The all distributions of amplitude and phase of front of the surface wave have been found by using the electrical probe, [7]. As shown on Fig. 5 the electrical reflection is bigger than the mechanical one. Simultaneously, the resultant reflection can be less then its components calculated separately. It means that the reflections can eliminat each other. The coefficient of reflection calculated for its electrical type, Eq. (4.2) can be almost equal to 1, if the susceptance of transducer B is setting to zero, what is reached for the greater number of electrodes. The results of admittance of $LiNbO_3$ base transducer with 78 single simple electrodes, presented on Fig. 6 are complianced to the experimental data obtained from [8]. The susceptance is equal to zero for two values of frequency. The coefficient of reflection, the values of which have been calculated based on the theoretical model, are presented on Fig. 7. The coefficient of reflection calculated for its electrical type is equal to 1, $\Gamma_E = 1$ at points for which the susceptance is setting to zero. Also this coefficient calculated for mechanical type of reflection is almost equal to 1. After all the value of total coefficients of reflection is less than 1 and it reaches the value of 0.34 for 0.99 f_{00}. Hence, the value of total coefficient varies with the phase changing of the coefficient of reflection calculated for its electrical type, what can be reached by changing the load impedance R_0. It means that the value

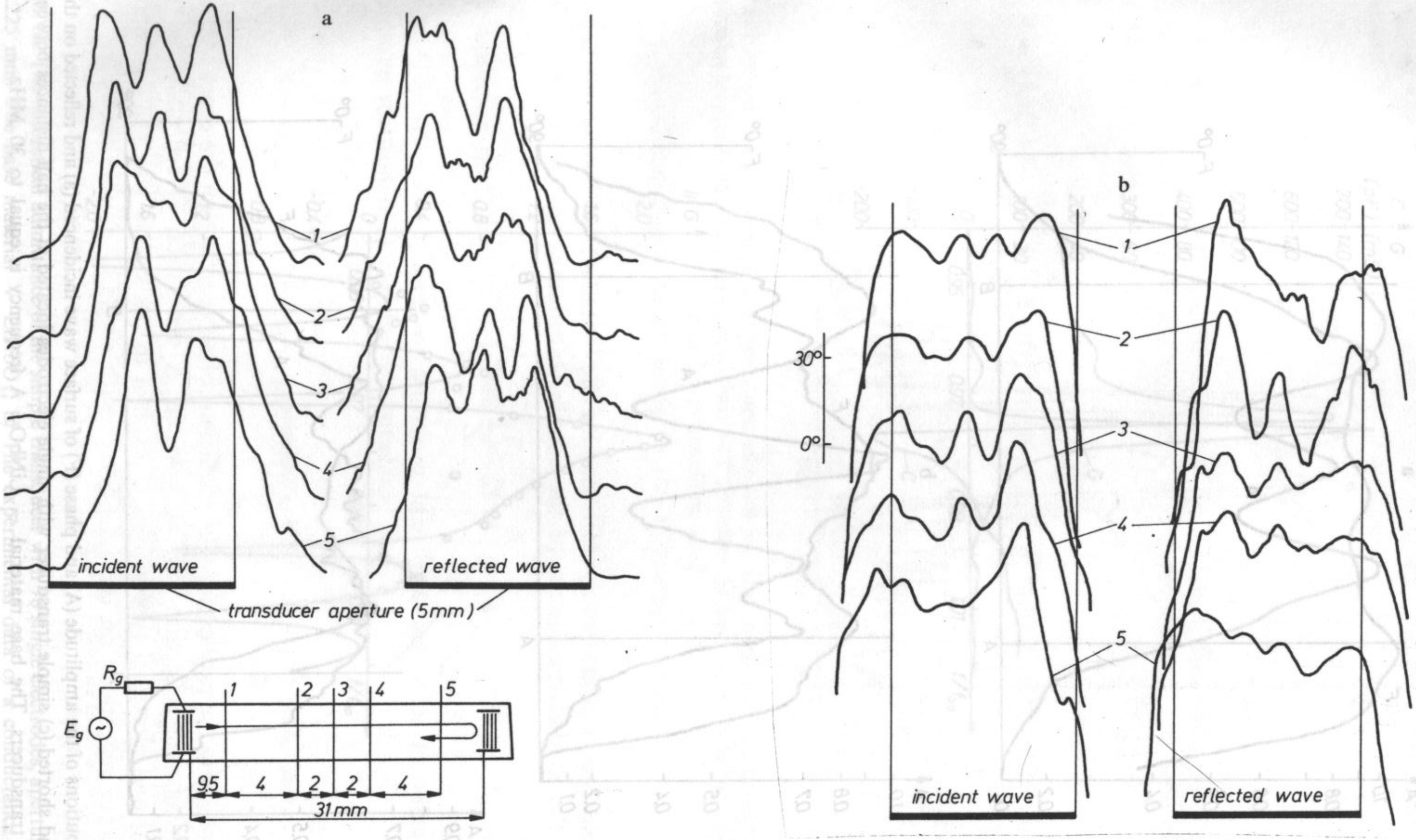

FIG. 4. a) Distributions of amplitude of the surface wave incidenced and reflected on the simple opened $LiNbO_3$ base transducer with double electrodes corresponded to the appropriate distances. Frequency is equal to 30 MHz; b) Distributions of phase of the surface wave corresponded to the distributions of amplitude shown on Fig. 4a

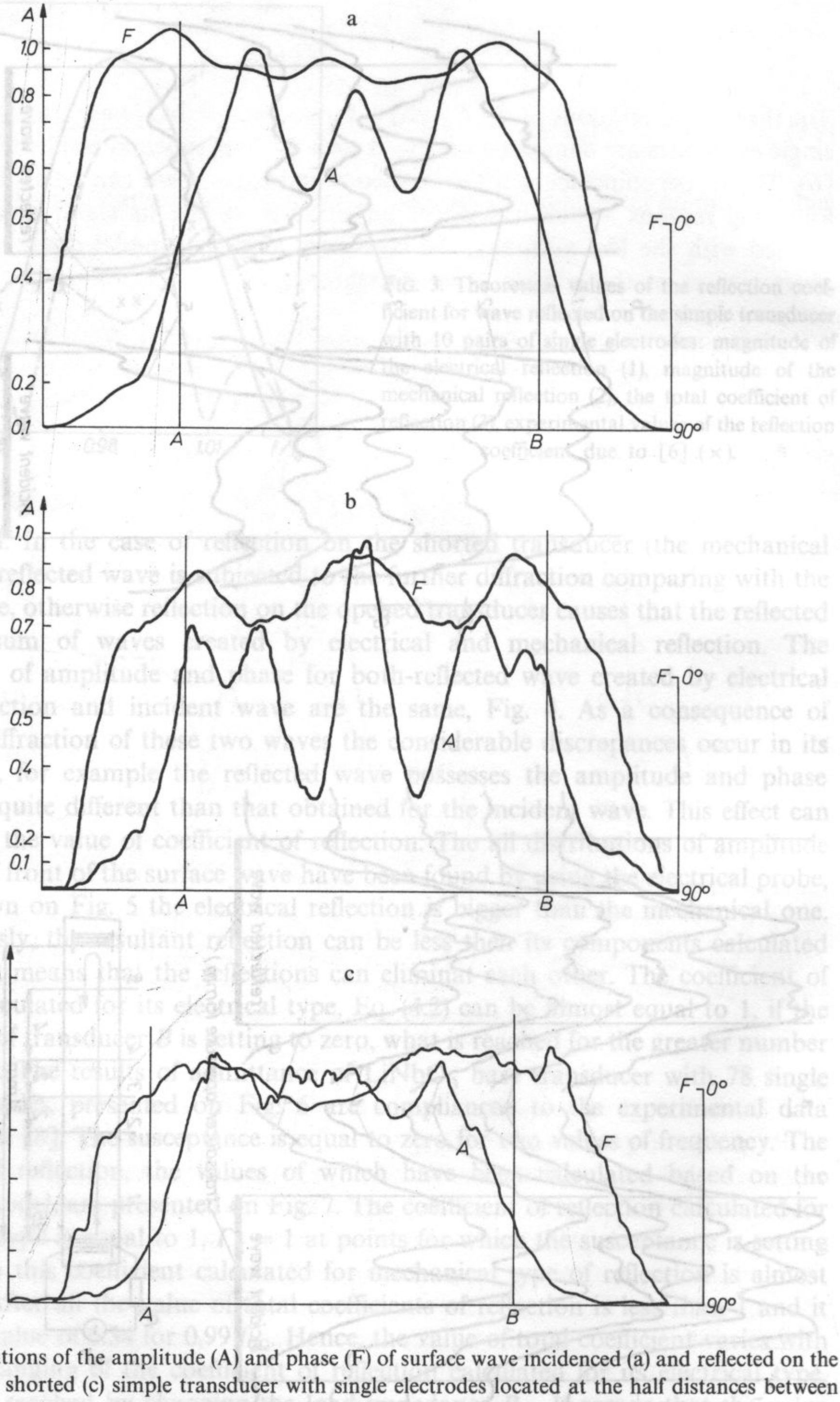

FIG. 5. Distributions of the amplitude (A) and phase (F) of surface wave incidenced (a) and reflected on the opened (b) and shorted (c) simple transducer with single electrodes located at the half distances between the transducers. The base material — $LiNbO_3$. A frequency is equal to 30 MHz

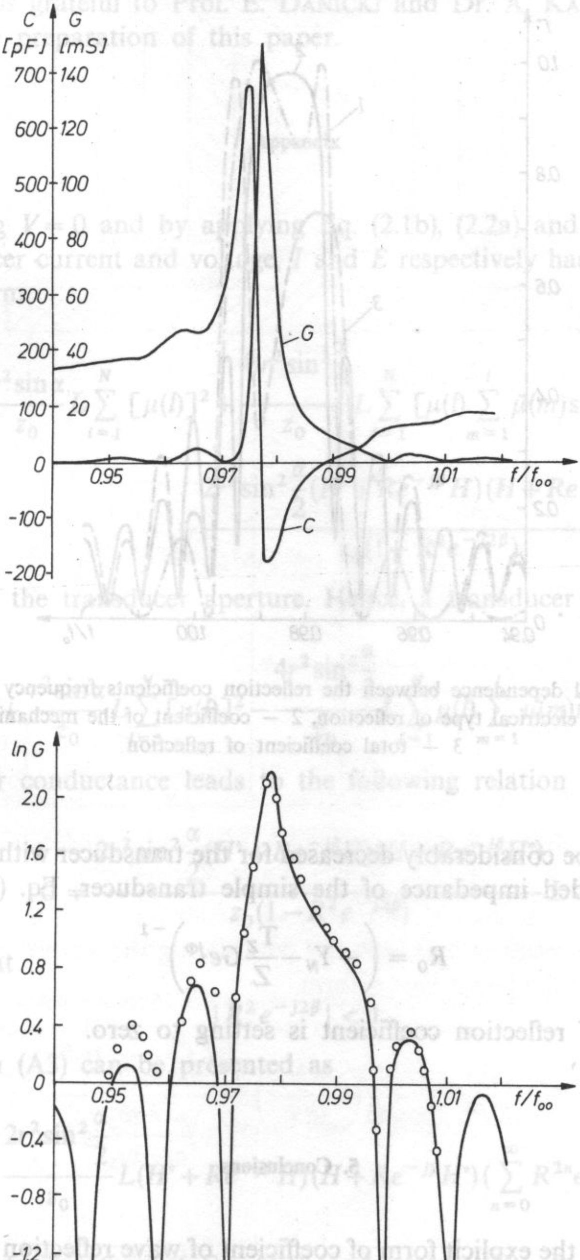

FIG. 6a. The theoretical admittance of the simple transducer with 78 single electrodes, aperture is equal to 0.225 mm. The base material — $LiNbO_3$; b) The experimental data of a conductance due to [8]

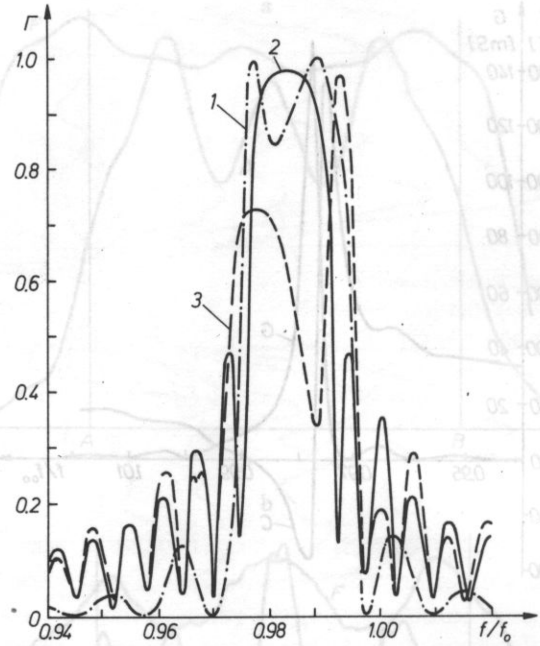

FIG. 7. The theoretical dependence between the reflection coefficients frequency presented on Fig. 5. 1 – coefficient of the electrical type of reflection, 2 – coefficient of the mechanical type of reflection, 3 – total coefficient of reflection

of coefficient can be considerably decreased for the transducer with single electrodes. Hence, if the loaded impedance of the simple transducer, Eq. (3.11) has a form

$$R_0 = \left(-Y_N - \frac{T_Z}{Z} G e^{j\Phi}\right)^{-1}$$

then the value of reflection coefficient is setting to zero.

5. Conclusions

By obtaining the explicit form of coefficient of wave reflection the exact analysis of phenomenon of reflection can be possible. A separation of reflection on the mechanical and electrical type allows to explain in the simple way the differences between wave reflections on the transducers with single or double electrodes. The considerably decrease of value of the reflection on the simple transducer can be reached by its load stimulation. It allows also to apply a transducer as the surface wave reflector.

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Appendix

By assuming $V=0$ and by applying Eq. (2.1b), (2.2a) and (3.2), the relation between transducer current and voltage, I and E respectively has been obtained in the following form

$$I=\Bigg\{j\omega C+j\frac{r^2\sin\alpha}{z_0}L\sum_{i=1}^{N}[\mu(l)]^2+\frac{4jr^2\sin^2\frac{\alpha}{2}}{z_0}L\sum_{l=1}^{N}[\mu(l)\sum_{m=1}^{l}\mu(m)\sin(l-m)\alpha]+$$
$$+\frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)}{z_0(1-R^2e^{-j2\beta})}L\Bigg\}E \quad \text{(A1)}$$

where L denotes the transducer aperture. Hence, a transducer susceptance is

$$B=\omega C+\frac{r^2\sin\alpha}{z_0}L\sum_{l=1}^{N}[\mu(l)]^2-\frac{4r^2\sin^2\frac{\alpha}{2}}{z_0}L\sum_{l=1}^{N}\mu(l)\sum_{m=1}^{l}\mu(m)\sin(l-m)\alpha \quad \text{(A2)}$$

But a transducer conductance leads to the following relation

$$G=\frac{2r^2\sin^2\frac{\alpha}{2}(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)}{z_0(1-R^2e^{-j2\beta})}L \quad \text{(A3)}$$

By assuming that

$$|R^2e^{-j2\beta}|<1 \quad \text{(A4)}$$

then the relation (A3) can be presented as

$$G=\frac{2r^2\sin^2\frac{\alpha}{2}}{z_0}L(H^*+Re^{-j\beta}H)(H+Re^{-j\beta}H^*)\Big(\sum_{n=0}^{\infty}R^{2n}e^{-j2n\beta}\Big) \quad \text{(A5)}$$

The graphical interpretation of Eq. (3.1) is shown on Fig. 8. A conductance is proportional to the magnitudes of both-received signal and wave induced by the same considerated transducer, so for the lack of reflections ($R=0$) a conductance is proportional to HH^*. In the case of $R\neq 0$ the received signal is changed in terms of the waves to be reflected on the transducer boundary. The every n-th term of infinite series is related to the next twofold wave transmission in the area of whole transducer,

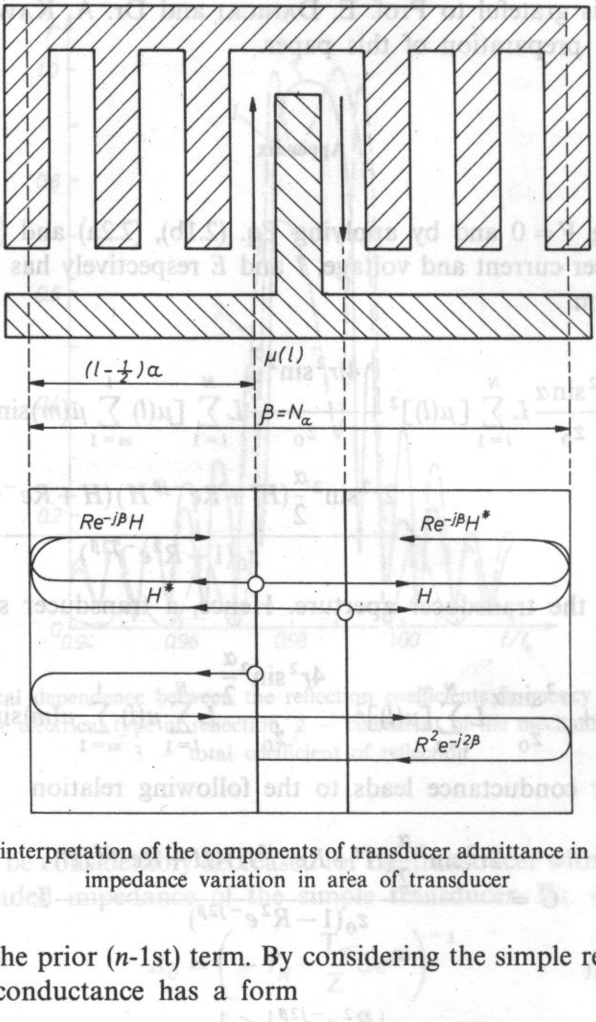

FIG. 8. The graphical interpretation of the components of transducer admittance in terms of the acoustics impedance variation in area of transducer

comparing with the prior (n-1st) term. By considering the simple regular transducer ($\mu_n = (-1)^n$) its conductance has a form

$$G = \frac{\omega\varepsilon_0\varepsilon_\infty k^2 L}{2P^2_{-s}(\cos\Delta)}\left(\frac{\sin Nx}{\operatorname{tg} x}\right)^2 \frac{1+j\dfrac{\omega-\omega_1}{\omega-\omega_3}\operatorname{tg} Nx}{1+j\dfrac{\omega-\omega_3}{\omega-\omega_1}\operatorname{tg} Nx} \tag{A6}$$

and a susceptance

$$B = \omega C_0 + \frac{\omega\varepsilon_0\varepsilon_\infty k^2 L}{2P^2_{-s}(\cos\Delta)}\sqrt{\frac{\omega-\omega_1}{\omega-\omega_3}}\,\frac{\sin 2Nx - 2N\operatorname{tg} x}{2\operatorname{tg}^2 x} \tag{A7}$$

because, due to [3]

$$\frac{r^2}{z_0} = \frac{\omega\varepsilon_0\varepsilon_\infty k^2}{2P^2_{-s}(\cos\Delta)}$$

where

$$X = \frac{\pi(\omega-\omega_1)(\omega-\omega_3)}{2\omega_1}$$

$$S = \frac{\omega}{2\omega_1}$$

and P_{-s} denotes the Legendre's polynomials. A conductance of refular transducer with double electrodes has a form

$$G = \frac{\omega\varepsilon_0\varepsilon_\infty k^2 L}{2P^2_{-s}(\cos\Delta)}\left(\frac{\sin Nx}{\operatorname{tg}x}\right)^2 \tag{A8}$$

and susceptance

$$B = \omega C_0 + \frac{\omega\varepsilon_0\varepsilon_\infty k^2 L}{2P^2_{-s}(\cos\Delta)}\left(\frac{\sin 2Nx - 2N\operatorname{tg}x}{2\operatorname{tg}^2 x}\right) \tag{A9}$$

therefore, the forms of both variables are well-known.

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