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THE ACOUSTIC NEARFIELD OF SOURCES OF HIGH DENSITY

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Introduction

In the present paper we compute the acoustic potential in the nearfield of a circular source placed in an infinite rigid baffle. Two cases of distribution of velocity amplitude of harmonic vibrations are considered for which high directivity of emitted acoustic field is obtained. As it has been proved in paper [6] such directivity is obtained if one of the following two distributions of velocity amplitude is applied: distribution according to the Gaussian curve and distribution defined by the Bessel function of first order divided by its argument. The first distribution gives, in the farfield, the directivity coefficient also according to the Gaussian curve, while the second produces radiation only in a defined cone. In view of practical applications it is important to investigate also the nearfield in these cases.

Theoretical basis of calculation of the nearfield of radially symmetric sources has been presented in paper [8] and here we limit ourselves only to presentation of general formulae.

We choose the cylindrical system of coordinates such that the polar axis r lies in the plane of the baffle (thus also in the plane of source being at rest) with the origin of the system at the centre of the source. We assume that distribution of velocity amplitude, and therefore the produced acoustic field, is independent on the polar angle φ. The z axis crosses the origin of the system perpendicularly to the baffle plan.

We denote by $u(r)$ the considered distribution of velocity amplitude on the source. The acoustic potential of resultant field can be expressed by the following integral formula [8]:

$$\Phi(r, z) = -\int_0^\infty \frac{J_0(\varrho r)}{\sqrt{\varrho^2-k^2}} H_0[u(r)]\, e^{-z\sqrt{\varrho^2-k^2}} \varrho d\varrho. \tag{1}$$

$J_0()$ denotes the Bessel function of zero order.

In Eq. (1) k denotes the wavenumber of the emitted harmonic wave, and $H_0[u(r)] = H_0(\varrho)$ is the Hankel transform of zero order of the considered distribution of velocity amplitude of vibrations. As the baffle is at rest by assumption, thus denoting by a the source radius we have [8]

$$H_0[u(r)] = \int_0^a ru(r)\, J_0(\varrho r)dr. \tag{2}$$

Considering $H_0[u(r)]$ in the above form we cut the $u(r)$ distribution curve at $r = a$. This is a realistic assumption, but it leads to serious mathematical difficulties. Calculations can be simplified by the following approximation: we choose the $u(r)$ distribution curve such that its ordinates for $r > a$ are negligible so that one can assume that $u(r)$ is stretched to infinity. Obviously the crucial criterion of validity of such approximation is the condition that with inreasing r the product $ru(r)$ under integral must tend to zero. This condition is obviously satisfied for both distributions of velocity amplitude considered in this paper. The form of expression (1) for the potential remains the same, but the transform (2) is now given by

$$H_0[u(r)] = \int_0^\infty ru(r)\, J_0(\varrho r)dr. \tag{3}$$

As we will see below, in both cases considered here integral (3) can be calculated analytically while the exact formula (2) can be integrated only numerically. In this paper we present results of both methods, the exact ("strict") and the approximate.

The references are quoted here in two different manners. Everywhere we refer to the whole item or to its greater part, we give only the number of its position on the reference list. On the other hand, quoting specific formulae from tables, we add also the appropriate page number.

1. Distribution of velocity amplitude of vibrations given by the Gaussian curve

We assume the following distribution of velocity amplitude of vibrations:

$$u(r) = \begin{cases} u_0 e^{-(nr/a)^2}, & 0 \leqslant r \leqslant a \\ 0, & r > a. \end{cases} \tag{4}$$

The coefficient n appearing in the exponent is called the coefficient of contraction. We choose the value of this coefficient such that the value of the velocity amplitude is sufficiently small on the edge of the source, what is specially important in approximate cases. We will perform numerical calculations for $n = 2$ and $n = 3$. In the first case we have for $r = a$

$$u(a)/u_0 = e^{-4} = 0.018316, \tag{5}$$

and in the second

$$u(a)/u_0 = e^{-9} = 0.000123. \tag{6}$$

On the other hand, considering the approximate case we have the amplitude distribution in the form

$$u(r) = u_0 e^{-(nr/a)^2}, \quad 0 \leqslant r \leqslant \infty. \tag{7}$$

In the first case (4) the Hankel transform reads

$$H_0[u(r)] = u_0 \int_0^a r e^{-(nr/a)^2} J_0(\varrho r) dr \tag{8}$$

and must be computed numerically. Instead, in the second case we obtain

$$H_0[u(r)] = u_0 \int_0^\infty r e^{-(nr/a)^2} J_0(\varrho r) dr. \tag{9}$$

We find this integral in tables of integrals [1, p. 731] and we get

$$H_0[u(r)] = \frac{u_0 a^2}{2n^2} e^{-(\varrho a/2n)^2}. \tag{10}$$

This result is consistent with the well known fact that the Hankel transform of the Gaussian function of argument r is also the Gaussian function of argument ϱ.

In the first case considered (Eq. (8)), we will not write *explicite* the formula for the potential (1) as it can be computed only numerically. On the other hand, in the approximate case (10) we get the acoustic potential in the nearfield in the form

$$\Phi(r, z) = -\frac{u_0 a^2}{2n^2} \int_0^\infty \frac{J_0(\varrho r)}{\sqrt{\varrho^2 - k^2}} e^{-\frac{\varrho^2 a^2}{4n^2}} e^{-z\sqrt{\varrho^2 - k^2}} \varrho d\varrho. \tag{11}$$

When integrating over variable ϱ we have to distinguish between two cases. For $0 \leqslant \varrho \leqslant k$ the square root has imaginary value, and for $\varrho > k$ it is real. Taking this into account and separating real and imaginary parts we write Eq. (11) in the form

$$\begin{aligned}\Phi(r, z) = \frac{u_0 a^2}{2n^2} \Bigg[\int_0^k \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} e^{-\frac{\varrho^2 a^2}{4n^2}} \sin\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho + \\ - \int_k^\infty \frac{J_0(\varrho r)}{\sqrt{\varrho^2 - k^2}} e^{-\frac{\varrho^2 a^2}{4n^2}} \exp\left(-z\sqrt{\varrho^2 - k^2}\right) \varrho d\varrho \Bigg] + \\ + i \int_0^k \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} e^{-\frac{\varrho^2 a^2}{4n^2}} \cos\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho.\end{aligned} \tag{12}$$

The integrals in the above formula must also be calculated numerically.

On the axis, i.e. for $r = 0$, we obtain the potential in the nearfield as

$$\Phi(0, z) = \frac{u_0 a^2}{2n^2}\Bigg[\int_0^k \frac{e^{-\frac{\varrho^2 a^2}{4n^2}}}{\sqrt{k^2-\varrho^2}} \sin\left(z\sqrt{k^2-\varrho^2}\right)\varrho d\varrho +$$

$$-\int_k^\infty \frac{e^{-\frac{\varrho^2 a^2}{4n^2}}}{\sqrt{\varrho^2-k^2}} \exp\left(-z\sqrt{\varrho^2-k^2}\right)\varrho d\varrho\Bigg] + \tag{13}$$

$$+i\int_0^k \frac{e^{-\frac{\varrho^2 a^2}{4n^2}}}{\sqrt{k^2-\varrho^2}} \cos\left(z\sqrt{k^2-\varrho^2}\right)\varrho d\varrho.$$

One can simplify the integrals in the above formula removing at the same time singularities of the integrand, by means of substitution in the first and in the third integral

$$k^2-\varrho^2 = t^2; \quad -\varrho d\varrho = tdt, \tag{14}$$

and in the second integral,

$$\varrho^2-k^2 = t^2; \quad \varrho d\varrho = tdt, \tag{15}$$

Fixing limits of integration and arranging terms we get finally

$$\Phi(0, z) = \frac{u_0 a^2}{2n^2} e^{-\frac{k^2 a^2}{4n^2}}\Bigg[\int_0^k e^{\frac{t^2 a^2}{4n^2}} \sin(tz)\, dt + \int_0^\infty e^{-\frac{t^2 a^2}{4n^2}} e^{-tz} dt +$$

$$+i\int_0^k e^{\frac{t^2 a^2}{4n^2}} \cos(tz)\, dt. \tag{16}$$

These integrals are much simpler and more convenient for numerical calculations. Let us note that the second of them can be expressed by means of the probability function. It should be mentioned here that the formula for the nearfield on axis of the source with Gaussian velocity amplitude distribution have been given in paper [7] in the form of a series.

2. The distribution of velocity amplitude of vibrations given by the first order Bessel function divided by the argument

As we have mentioned in Introduction and as it was proved in paper [6], distribution of velocity amplitude of vibrations in the form of the first order Bessel function divided by the argument gives so called absolute directivity in the farfield which means radiation exclusively in a definite cone without sidelobes. Presently we will consider the nearfield of such a source. Similarily as in the case of Gaussian distribution we will consider here the "strict" case with velocity distribution limited to the source surface, and the approximate one when the distribution curve is stretched to infinity under obvious assumption that for r greater than the source radius, ordinates of the distribution curve are negligible.

Assuming distribution limited to the disk surface we write

$$u(r) = \begin{cases} u_0 \dfrac{J_1(nr/a)}{nr/a}, & 0 \leqslant r \leqslant a, \\ 0, & r > a, \end{cases} \tag{17}$$

where a, as previously, is the source radius. Coefficient n plays here analogous role as for the Gaussian distribution but now we apply an additional condition that the velocity amplitude of vibrations on the source edge $u(a)$ should be equal to zero. This reasonable assumption implies

$$J_1(n) = 0, \tag{18}$$

which gives a discrete series of admissible values of n.

We will perform numerical calculations for n equal to the three first zeros of the first order Bessel function, namely [2]

$$\begin{aligned} n_1 &= 3.8317, \\ n_2 &= 7.0156, \\ n_3 &= 10.1735. \end{aligned} \tag{19}$$

Of course, in the approximate case we apply the same values of n and we will consider the distribution of the form

$$u(r) = u_0 \frac{J_1(nr/a)}{(nr/a)}, \quad 0 \leqslant r \leqslant \infty. \tag{20}$$

We pass on to the calculation of the Hankel transform of zero order for both exact and approximate cases. In the first one we have

$$H_0[u(r)] = \frac{u_0 a}{n} \int_0^a J_1(nr/a) J_0(\varrho r)\, dr. \tag{21}$$

Formally the above integral can be expressed in analytical form [3, p. 259]:

$$H_0[u(r)] = \frac{u_0 a^2}{\Gamma(1)\Gamma(2)} \sum_{p=1}^{\infty} \frac{(-1)^p (n/2)^2 p\, {}_2F_1(-p, -1-p, 1, \varrho^2 a^2/n^2)}{2p(p+1)!}. \tag{22}$$

Anyhow, in either case the Hankel transform must be computed numerically and there is no need to present here the formula (1) for the potential in explicit form.

In the approximate case (Eq. (20)) the appropriate Hankel transform is

$$H_0[u(r)] = \frac{u_0 a}{n} \int_0^{\infty} J_1(nr/a) J_0(\varrho r)\, dr. \tag{23}$$

This integral can be found in literature [4, p. 100]:

$$\int_0^\infty J_1(nr/a)\,J_0(\varrho r)\,dr = \begin{cases} a/n, & 0 < \varrho < n/a, \\ 1/2\varrho, & \varrho = n/a, \\ 0, & \varrho > n/a. \end{cases} \tag{24}$$

Substituting this into Eq. (23) we obtain the Hankel transform of interest in the form

$$H_0[u(r)] = \begin{cases} \dfrac{u_0 a^2}{n^2}, & 0 < \varrho < n/a, \\ \dfrac{u_0 a}{2n\varrho}, & \varrho = n/a, \\ 0 & \varrho > n/a. \end{cases} \tag{25}$$

Substituting this into Eq. (1) representing the acoustic potential note that integration is now extended only to $\varrho = n/a$. For greater values of ϱ the integrand becomes zero. Therefore we get

$$\Phi(r, z) = -\frac{u_0 a^2}{n^2} \int_0^{n/a} \frac{J_0(\varrho r)}{\sqrt{\varrho^2 - k^2}} e^{-z\sqrt{\varrho^2 - k^2}}\,\varrho d\varrho. \tag{26}$$

This integral will have two different forms depending on value of k compared to the limiting value

$$k_{\lim} = n/a. \tag{27}$$

If $k > k_{\lim}$ than integrating over ϱ from 0 to n/a we have everywhere $\varrho^2 < k^2$ and formula for the acoustic potential takes form

$$\Phi(r, z) = \frac{u_0 a^2}{n^2} \left[\int_0^{n/a} \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} \sin\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho + \right. \tag{28}$$

$$\left. + i \int_0^{n/a} \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} \cos\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho \right].$$

Instead, when $k < k_{\lim}$, the interval of integration must be divided into two parts: for $0 < \varrho < k$ the integrands will be the same as in Eq. (28), while for $k < \varrho < n/a$ we have $\varrho^2 > k^2$. Thus in this case we have

$$\Phi(r, z) = \frac{u_0 a^2}{n^2} \left[\int_0^{k} \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} \sin\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho + \right. \tag{29}$$

$$- \int_k^{n/a} \frac{J_0(\varrho r)}{\sqrt{\varrho^2 - k^2}} \exp\left(-z\sqrt{\varrho^2 - k^2}\right) \varrho d\varrho +$$

$$\left. + i \int_0^{k} \frac{J_0(\varrho r)}{\sqrt{k^2 - \varrho^2}} \cos\left(z\sqrt{k^2 - \varrho^2}\right) \varrho d\varrho \right].$$

Integrals in the above equations will be computed numerically. However, the field on z axis can be given analitycally. Namely, when $k > k_{\lim}$ we have from Eq. (28) for $r = 0$

$$\Phi(0, z) = \frac{u_0 a^2}{n^2}\left[\int_0^{n/a} \frac{\sin(z\sqrt{k^2-\varrho^2})}{\sqrt{k^2-\varrho^2}}\,\varrho d\varrho + i\int_0^{n/a} \frac{\cos(z\sqrt{k^2-\varrho^2})}{\sqrt{k^2-\varrho^2}}\,\varrho d\varrho\right], \tag{30}$$

and when $k < k_{\lim}$ from Eq. (29) we get

$$\Phi(0, z) = \frac{u_0 a^2}{n^2}\left[\int_0^{k} \frac{\sin(z\sqrt{k^2-\varrho^2})}{\sqrt{k^2-\varrho^2}}\,\varrho d\varrho - \int_k^{n/a} \frac{e^{-(z\sqrt{\varrho^2-k^2})}}{\sqrt{\varrho^2-k^2}}\,\varrho d\varrho + i\int_0^{k} \frac{\cos(z\sqrt{k^2-\varrho^2})}{\sqrt{k^2-\varrho^2}}\,\varrho d\varrho\right]. \tag{31}$$

The integrals in Eqs. (30) and (31) can be reduced easily to elementary ones. For this purpose one has to apply substitution (14) in Eq. (30) and in the first and the third integral of Eq. (31), and substitution (15) in the second one. Thus for $k > k_{\lim}$ we have from Eq. (30)

$$\Phi(0, z) = \frac{u_0 a^2}{n^2}\Big[\int_{\sqrt{k^2-(n/a)^2}}^{k} \sin(tz)dt + i\int_{\sqrt{k^2-(n/a)^2}}^{k} \cos(tz)dt\Big]. \tag{32}$$

Performing integration we can rewrite this in more convenient exponential form:

$$\Phi(0, z) = \frac{u_0 a^2}{zn^2}\left(e^{-iz\sqrt{k^2-(n/a)^2}} - e^{-ikz}\right). \tag{33}$$

Applying formula given in [5, p. 291] to the above expression one can separate the amplitude and the phase:

$$\Phi(0, z) = \frac{2u_0 a^2}{zn^2}\sin\left(\frac{z}{2}\left(\sqrt{k^2-(n/a)^2}-k\right)\right)e^{-i\left(\frac{\pi}{2}+\frac{z}{2}\left(\sqrt{k^2-(n/a)^2}+k\right)\right)}. \tag{34}$$

As we can see, the amplitude of the acoustic potential is greatest for $z = 0$,

$$\Phi(0, 0) = \frac{u_0 a^2}{n^2}\left(\sqrt{k^2-(n/a)^2}-k\right), \tag{35}$$

and then oscillates with increasing z with local extrema decreasing as $1/z$.

Let us now consider the case of Eq. (31), $k > k_{\lim}$. Using substitutions given in Eqs. (14) and (15) we get

$$\Phi(0, z) = \frac{u_0 a^2}{n^2}\Big[\int_0^{k} \sin(tz)dt - \int_0^{\sqrt{(n/a)^2-k^2}} e^{-zt}dt + i\int_0^{k} \cos(tz)dt\Big]. \tag{36}$$

After performing elementary integration we turn back to convenient exponential notation and get

$$\Phi(0, z) = \frac{u_0 a^2}{n^2 z}\left[2 - e^{-ikz} - e^{-z\sqrt{(n/a)^2 - k^2}}\right]. \tag{37}$$

Calculating $\Phi(0, 0)$ we have to find the limiting value of Eq. (37) for $z = 0$, similarly as in case of Eq. (34). Applying the l'Hopital's rule we obtain

$$\Phi(0, 0) = \frac{u_0 a^2}{n^2}\left(\sqrt{(n/a)^2 - k^2} + ik\right). \tag{38}$$

It is easy to prove that in the limit $k \Rightarrow k_{\lim}$ the absolute values of $\Phi(0, 0)$ from Eqs. (35) and (38) are equl to each other.

3. Normalization of the output

Performing numerical calculations we assume that the two considered types of sources have the same volume output. In general, if the distribution of velocity amplitude of vibrations on the source of radius a is given by function $u(r)$, the output Q is expressed by

$$Q = 2\pi \int_0^a u(r)\, r dr. \tag{39}$$

In order to distinguish between the two distributions considered in this paper, we will add indices G and B to respective quantities representing the Gaussian distribution and the Bessel distribution. The source radius will be taken the same in both cases. For "rigorous" Gaussian distribution Eq. (4) we have from Eq. (39)

$$Q_G = 2\pi u_{OG} \int_0^a e^{-(n_G r/a)^2}\, r dr. \tag{40}$$

This integral is elementary and we get

$$Q_G = \pi u_{OG} (a/n_G)^2 (1 - e^{-n_G^2}). \tag{41}$$

For the Bessel distribution (Eq. (17)) we obtain from Eq. (39)

$$Q_B = 2\pi u_{OB} \int_0^a \frac{J_1(n_B r/a)}{(n_B r/a)}\, r dr. \tag{42}$$

This integral is also elementary [2] and we obtain

$$Q_B = 2\pi u_{OB} (a/n_B)^2 [1 - J_0(n_B)]. \tag{43}$$

Imposing the condition that both outputs are equal, i.e. $Q_G = Q_B$, we obtain a relation between respective maximal amplitudes of velocity which can be written as

$$u_{OB} = \frac{1}{2} u_{OG}(n_B/n_G)^2 \frac{1-e^{-n_G^2}}{1-J_0(n_B)}. \tag{44}$$

For distributions called above the "approximate" ones, i.e. stretched to infinity calculations are even easier. For the Gaussian distribution (Eq. (7)) we have

$$Q_G = 2\pi u_{OG} \int_0^\infty e^{-(n_G r/a)^2} r dr. \tag{45}$$

Elementary integration gives

$$Q_G = \pi u_{OG}(a/n_G)^2. \tag{46}$$

For the Bessel distribution (Eq.(20)) we get

$$Q_B = 2\pi u_{OB} \int_0^\infty \frac{J_1(n_B r/a)}{(n_B r/a)} r dr. \tag{47}$$

Using the well-known property of definite integral of the Bessel function of any order [1, p. 679],

$$\int_0^\infty J_r(x) dx = 1, \tag{48}$$

we obtain from Eq. (47)

$$Q_B = 2\pi u_{OB}(a/n_B)^2. \tag{49}$$

Comparing both outputs also in this case we obtain the relation between maximal amplitudes of velocity:

$$u_{OB} = \frac{1}{2} u_{OB}(n_B/n_G)^2. \tag{50}$$

As we have noted above, coefficient n_B can be chosen as a zero of the Bessel function of first order, while coefficient n_G can be arbitrary in principle. Then, if we put $u_G = u_B$ in Eq. (50), maximal amplitudes of the velocity will satisfy the relation

$$u_{OB} = \frac{1}{2} u_{OB}. \tag{51}$$

This means that with the same values of maximal amplitudes of the velocity, the Bessel distribution gives twice as great output as the Gaussian distribution.

4. Results and conclusions

Using the formulae derived in previous paragraphs we have performed numerical calculations of the acoustic nearfield for the two types of considered distributions of velocity on the source. In both cases we have compared the results obtained in the "strict" case, i.e. for the distribution function limited to the finite source surface, and in the "approximate" one, when the distribution is stretched to infinity. To represent the acoustic nearfield, we have plot diagrams of the normalized pressure modulus *versus* normalized coordinates in the neighbourhood of the source. Given the acoustic potential $\Phi(r, z)$, one can calculate the acoustic pressure p as

$$p = ik\varrho_D c\Phi(r, z), \tag{52}$$

where ϱ_D — rest density of the medium, c — speed of sound. To generalize results, we introduce the normalized pressure defined as

$$P = \frac{p}{\varrho_D c \langle v \rangle}, \tag{53}$$

where $\langle v \rangle$ is the average velocity amplitude over the source surface S and

$$\langle v \rangle = \frac{1}{\pi a^2} \int_S u(r)\, dS = \frac{Q}{\pi a^2}. \tag{54}$$

Preparing formulae for numerical calculations, it is convenient also to introduce everywhere: the normalized wavenumber $\varkappa = ka$, the normalized radial coordinate $\mathscr{R} = r/a$ and the normalized orthogonal coordinate $\mathscr{Z} = z/a$ (the procedure is explained in details in paper [8]).

Numerical calculations show that in the case of the Gaussian distribution, for both considered values of the contraction parameter, $n_G = 2$ and $n_G = 3$, the "approximate" and "strict" formulae give practically the same values of pressure amplitude in the nearfield. As the differences are of no importance and even difficult to represent on graphs, we present results for Gaussian source on Fig. 1 and Fig. 2 without specifying the type of approximation. It can be seen from the figures that the pressure amplitude decreases monotonically with respect to both coordinates $\mathscr{R}$ and $\mathscr{Z}$ and for both values of the contraction parameter n_G the pressure amplitude on axis decreases the same way, which is the effect of adequate normalization of outputs.

Instead, for the Bessel distribution the differences between the "strict" and "approximate" cases become important. They are less visible in vicinity of the source, presented on Fig. 3 ($\mathscr{R} < 1$ and $0 < \mathscr{Z} < 1$ for the "approximate" case). For greatest distances, in the "strict" case when the distribution of velocity is nonzero only on the limited circular source surface, the field on axis exhibits characteristic minima (Fig. 4) similar to these observed in the case of rigid piston [5]. The minima become less significant when the contraction parameter n_B becomes greater then the normalized wavenumber (Fig. 4, dotted line). In the "approximate" case, when the distribution is assumed to be stretched to infinity, no minima are observed for all values of n_B (Fig. 5).

Recapitulating approximation consisting in extending the velocity distribution function from finite circular source surface to infinite plane is plausible in the case of

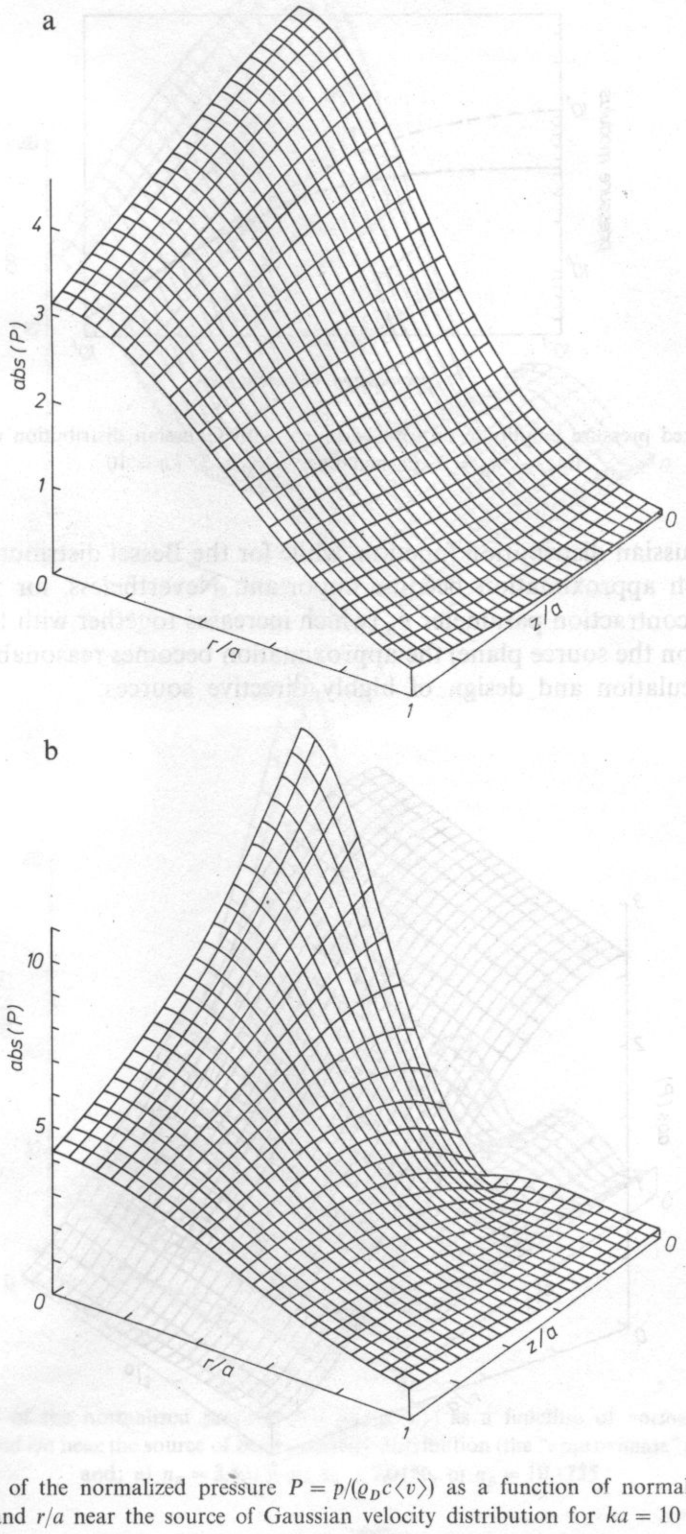


FIG. 1. Modulus of the normalized pressure $P = p/(\varrho_D c\langle v\rangle)$ as a function of normalized cylindrical coordinates z/a and r/a near the source of Gaussian velocity distribution for $ka = 10$ and: a) $n_G = 2$, b) $n_G = 3$

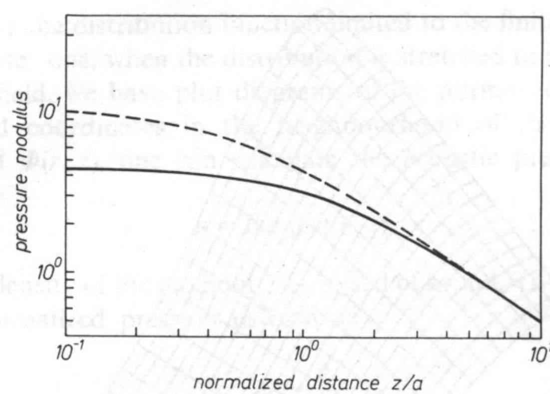
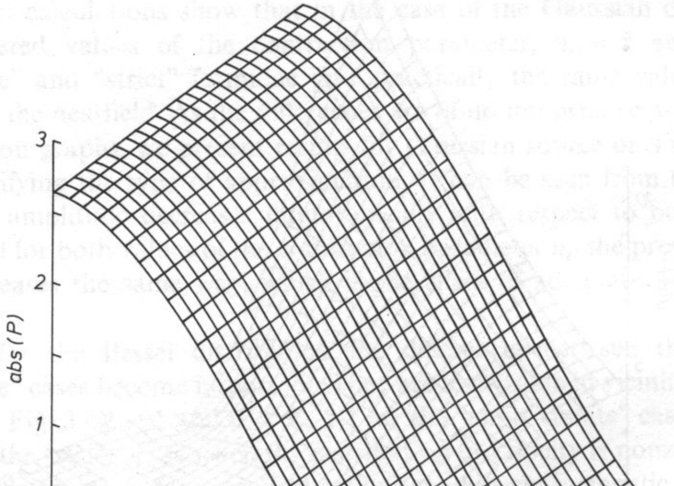


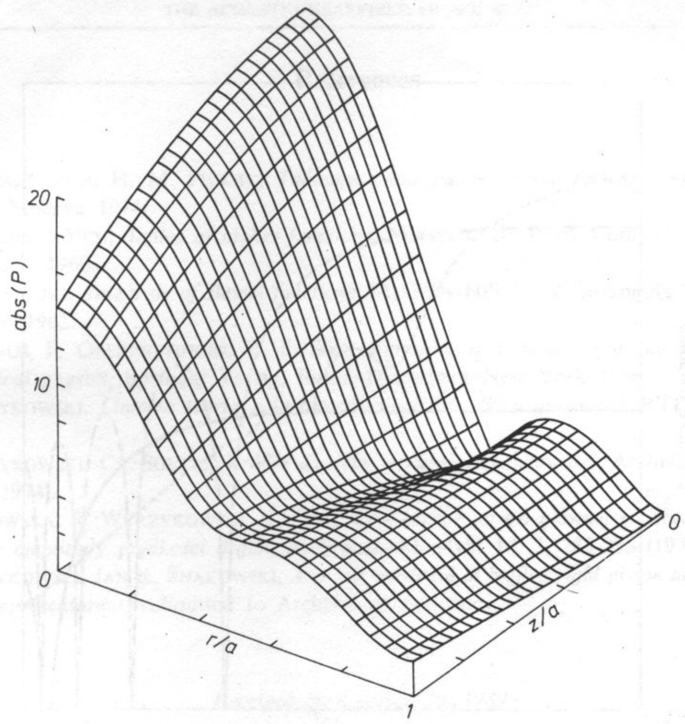
FIG. 2. Normalized pressure amplitude on axis of source with Gaussian distribution of velocity; solid line — $n_G = 2$, dashed line — $n_G = 3$; $ka = 10$

$n > 2$ the Gaussian distribution function, while for the Bessel distribution the errors following such approximation become important. Nevertheless, for wavenumbers less than the contraction parameter n_B (which increases together with the number of nodal circles on the source plane) the approximation becomes reasonable and can be useful in calculation and design of highly directive sources.

a)



b)



c)

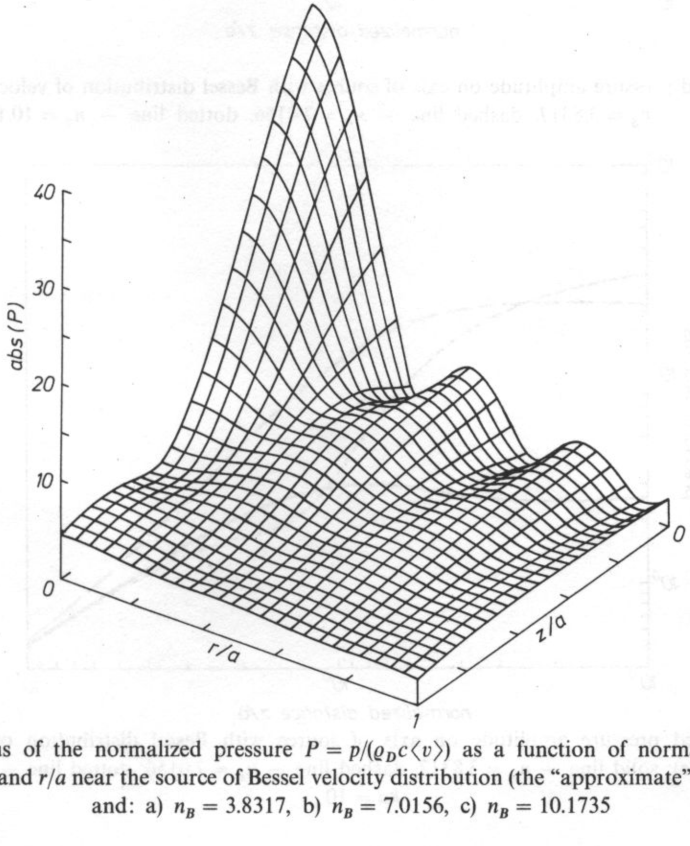


FIG. 3. Modulus of the normalized pressure $P = p/(\varrho_D c\langle v\rangle)$ as a function of normalized cylindrical coordinates z/a and r/a near the source of Bessel velocity distribution (the "approximate" case) for $ka = 10$ and: a) $n_B = 3.8317$, b) $n_B = 7.0156$, c) $n_B = 10.1735$

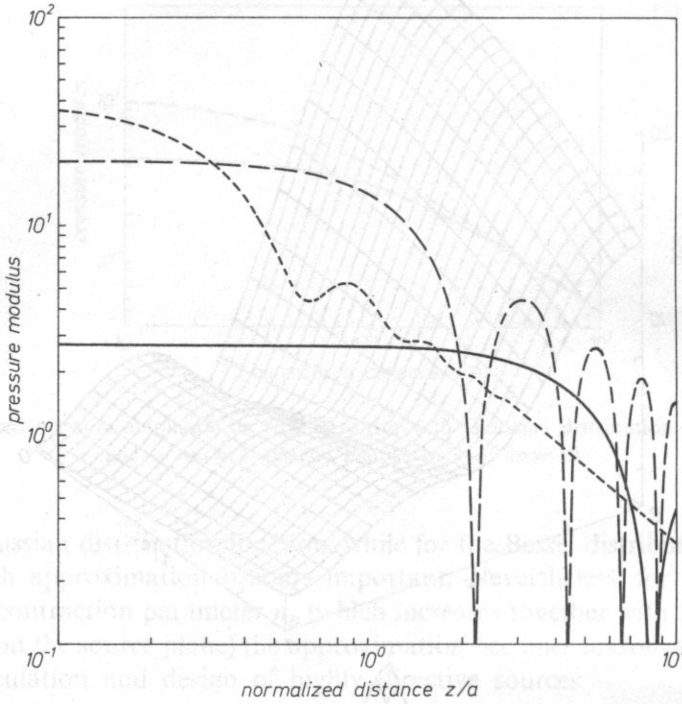


FIG. 4. Normalized pressure amplitude on axis of source with Bessel distribution of velocity (the "strict" case); solid line – $n_B = 3.8317$, dashed line – $n_B = 7.0156$, dotted line – $n_B = 10.1735$; $ka = 10$

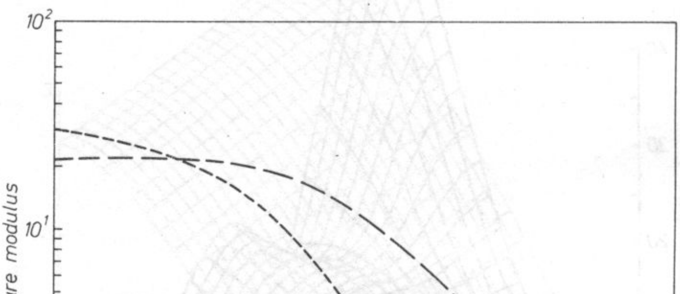


FIG. 5. Normalized pressure amplitude on axis of source with Bessel distribution of velocity (the "approximate" case); solid line – $n_B = 3.8317$, dashed line – $n_B = 7.0156$, dotted line – $n_B = 10.1735$; $ka = 10$

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