ARCHIVES OF ACOUSTICS
14, 3-4, 145-160 (1989)

A-WEIGHTED SOUND PRESSURE LEVEL CALCULATION IN THE DISSIPATIVE ATMOSPHERE

KRYSTYNA BEREZOWSKA-APOLINARSKA, RUFIN MAKAREWICZ

Institute of Acoustics, A. Mickiewicz University
(60-769 Poznań, Matejki 48)

During propagation of sound in the atmosphere a certain fraction of acoustical energy is absorbed by air. Thus, acoustical pressure decreases exponentially with distance. The rate of this drop is expressed by coefficient α which depends on frequency and meteorological conditions.

Absorption coefficient $\alpha(f)$ is approximated in this study by binominal function of frequency with two coefficients depending on temperature and relative humidity. The A-weighted sound pressure level is expressed by the function depending on parameters which describe absorption and the distance between the source and the point of observation. Thus the integration is substituted for summation over frequency bands. Calculations are made and charts are drawn to show differences in A-weighted pressure level calculation which result from $\alpha(f)$ approximation by binominal function.

Podczas propagacji dźwięku w atmosferze, część energii akustycznej jest pochłaniana przez powietrze. W wyniku tego pochłaniania ciśnienie akustyczne spada wykładniczo z odległością, a prędkość tego spadku wyraża współczynnik α, który zależy od częstotliwości i warunków meteorologicznych.

W pracy przedstawiono współczynnik pochłaniania dźwięku przez powietrze $\alpha(f)$ jako jawną funkcję częstotliwości w postaci dwumianu ze współczynnikami β i γ zależnymi od temperatury i wilgotności względnej. Wyrażono poziom dźwięku mierzony w dB(A) w postaci wzoru zależnego m. in. od parametrów opisujących pochłanianie (β, γ) i odległości od źródła do punktu obserwacji. W ten sposób zastąpiono sumowanie względem pasm częstotliwości całkowaniem. Przeprowadzono rachunki i sporządzono wykresy ilustrujące różnice w obliczeniach poziomu dźwięku, które wynikają m. in. z zastosowania uproszczonej postaci funkcji $\alpha(f)$.

1. Introduction

Environmental noise depends on several basic phenomena such as reflection, diffraction, refraction, interaction with ground and air absorption.

A simplified description of the latter is the aim of the study. Dependence of the absorption coefficient on frequency, temperature and relative humidity is very complex. The accuracy of A-weighted sound pressure level calculations required by planners, traffic engineers, road designers, etc. makes possible to simplify some relations.

First attempt was taken up in Ref. [4]. Critical remarks on it are given in part 4. There we have (Eq. (18)) absorption coefficient α in a form of a binominal containing coefficients which depend on temperature and relative humidity. Numerical values of these coefficients are determined for temperature range from $-10°C$ till $40°C$ and relative humidities $10\%-100\%$.

The noise control requires explicit relations among quantities describing effect of noise on people i.e. noise indices and quantities describing the process of sound generation and propagation. Noise annoyance is generally evaluated in terms of *A*-weighted sound pressure level expressed in dB(A). We have used continuous form of *A*-weighting function (Eq. (9)). A continuous form of the function describing spectral density of source power which includes a fairly large class of real noise sources was introduced as well (Eq. (2)).

Introduction of continuous forms of functions describing absorption coefficient and spectral density of source power made possible to derive an explicit form of dependence between *A*-weigthed sound pressure level, source – point of observation distance, parameters describing absorption, and parameters characterizing a source (Eqs. (7), (19)).

2. Noise source

Spectral density of mean square sound pressure in far field of nondirectional source equals:

$$p^2(f) = \frac{P(f)\varrho c}{4\pi r^2}, \tag{1}$$

where $P(f)$ represents spectral density of source's power, $\varrho c = 414$ kg m^{-2} s^{-1} temperature 20°C for speed of sound $c = 342$ m s^{-1} and air density $\varrho = 1.21$ kg m^{-3}. Formula (1) doesn't allow for absorption.

We assume that spectral density of source power is an exponential frequency function:

$$P(f) = P^{(0)} \exp(-\mu f), \tag{2}$$

where μ is expressed in seconds. For example taking into account a passanger car one may assume $P^{(0)} = 4 \cdot 10^{-5}$, $\mu = 2 \cdot 10^{-3}$. There are many other real noise sources where formula (2) is valid. Source power in n^{th} frequency band determined by $f_n^{(1)}$ and $f_n^{(2)}$ frequencies equals:

$$P_n = \int_{f_n^{(1)}}^{f_n^{(2)}} P(f)\mathrm{d}f = \frac{P^{(0)}}{\mu}\{\exp[-\mu f_n^{(1)}] - \exp[-\mu f_n^{(2)}]\}. \tag{3}$$

Now, to show source power spectrum as a function of parameter *u* we introduce band sound power level, $L_{Wn} = 10 \lg(P_n/P_0)$, where $P_0 = 10^{-12}$ *W*. Difference between two successive band sound power levels $\Delta L_n = L_{Wn} - L_{Wn+1}$, equals:

$$\Delta L_n = 10\lg\left\{\frac{\exp[\mu f_n^{(2)}]-\exp[\mu f_n^{(1)}]}{\exp[\mu f_{n+1}^{(2)}]-\exp[\mu f_{n+1}^{(2)}]}\exp[\mu(f_{n+1}^{(2)}-f_n^{(1)})]\right\}. \tag{4}$$

If $L_{W1} = N$ dB, then $L_{W2} = N+\Delta L_1$, $L_{W3} = N+\Delta L_1+\Delta L_2$, etc. The results of calculations are shown in Fig. 1. Despite the fact that the spectral density of source's power is a decreasing function of frequency (Eq. 2), power spectrum expressed in

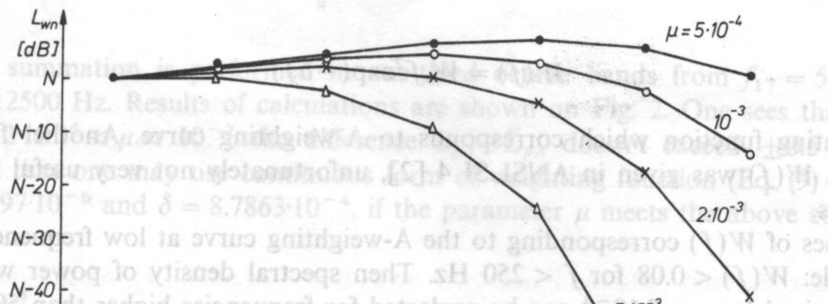

Fig. 1. Source power spectrum depending on parameter μ (Eqs (3), (4))

terms of band sound power level has a maximum. It is seen, that for increasing values of μ, L_{Wn} decreases with frequency. Then small values of μ characterize the power spectra of maximum above 5 kHz. Let f^* be the frequency which separates "low" ($f < f^*$) and "high" ($f > f^*$) parts of the spectrum. The powers P_l and P_h for the high and low sections are:

$$P_l = \int_0^{f^*} P(f)\,df \quad \text{and} \quad P_h = \int_{f^*}^{\infty} P(f)\,df. \tag{5}$$

Inserting source power spectrum (Eq. 2), we get:

$$P_l/P_h = \exp(\mu f^*)-1. \tag{6}$$

For $\mu = 5{\cdot}10^{-4}$ and $P_l/P_h = 10$ from the above relation we get $f^* \sim 5$ kHz. It means, that for $\mu > 5{\cdot}10^{-4}$ (maximum of power spectra below 2 kHz, see Fig. 1), source's power and thus, spectral density of mean square sound pressure (Eq. (1)), may be neglected above 5 kHz. Minimal value of μ depends on $P_l\backslash P_h$ ratio. For example, $\mu \sim 3{\cdot}10^{-4}$ and $f^* \sim 5$ kHz correspond to the value of $P_l/P_h = 4$.

3. A-weighted sound pressure level

Sound level L_{pA} is commonly used index of noise assessment despite its weaknesses [9]. Its definition is:

$$L_{pA} = 10\lg(p_A^2/p_0^2), \tag{7}$$

where $p_0 = 2{\cdot}10^{-5}$ N m^{-2}. It was shown in Refs [4, 8], that A-weighted mean square sound pressure can be given as:

$$p_A^2 = \int_0^\infty W(f)p^2(f)df, \tag{8}$$

where

$$W(f) = W_0 f^2 \exp(-\delta f) \tag{9}$$

is weighting function which corresponds to A-weighting curve. Another form of function $W(f)$ was given in ANSI SI 4 [2], unfortunately not very useful for our purposes.

Values of $W(f)$ corresponding to the A-weighting curve at low frequencies are negligible: $W(f) < 0.08$ for $f < 250$ Hz. Then spectral density of power which is characterized by $\mu > 5{\cdot}10^{-4}$ can be neglected for frequencies higher than 5000 Hz. Thus, for $f > 5000$ Hz we can neglect the spectral density of mean square sound pressure. Within the frequency range 250 Hz < f < 5000 Hz, the application of the linear regression analysis yields: $W_0 = 2.5197{\cdot}10^{-6}$ and $\delta = 8.7863{\cdot}10^{-4}$ [4, 8].

From Eqs. (1), (2), (8), (9) we get [5]:

$$p_A^2 = \frac{P^{(0)} W_0 \varrho c}{4\pi r^2} \int_0^\infty f^2 \exp[-(\delta+\mu)f]df = \frac{P^{(0)} W_0 \varrho c}{2\pi r^2 (\delta+\mu)^3} \tag{10}$$

Classical method of calculation of the A-weighted mean square sound pressure requires summation over frequency bands:

$$\widetilde{p_A^2} = \sum_n 10^{0.1\Delta L_A(f_n)} p_n^2, \tag{11}$$

where values of ΔL_A relate to the A-weighted curve [2, 6], whereas p_n^2 is a mean-square sound pressure of n^{th} frequency band. Taking into account the same value of μ as in expression (10), the definition:

$$p_n^2 = \int_{f_n^{(1)}}^{f_n^{(2)}} p^2(f)df \tag{12}$$

yields (Eqs. (1), (3), (11)):

$$p_n^2 = \frac{P^{(0)} \varrho c}{4\pi r^2} \frac{\exp[\mu f_n^{(1)}] - \exp[-\mu f_n^{(2)}]}{\mu}. \tag{13}$$

We see that (Eg. 11):

$$\tilde{p}_A^2 = \frac{P^{(0)}\varrho c}{4\pi r^2}\sum_n 10^{0.1\Delta L_A(f_n)}\{\exp[-\mu f_n^{(1)}]-\exp[-\mu f_n^{(2)}]\}. \quad (14)$$

To compare $\tilde{p}_A^2$ with the value of p_A^2 we use the definition of A-weighted sound pressure level (Eq. (7)). The difference between $L_{pA} = 10\lg(p_A^2/p_0^2)$ and $\tilde{L}_{pA} = 10\lg(\tilde{p}_A^2/\tilde{p}_0^2)$ equals:

$$L_{pA}-\tilde{L}_{pA} = 10\lg\left[\frac{2W_0\mu}{(\delta+\mu)^3\sum_n 10^{0.1\Delta L_A(f_n)}\{\exp[-\mu f_n^{(1)}]-\exp[-\mu f_n^{(2)}]\}}\right], \quad (15)$$

where summation is performed in one-third octave bands from $f_{17} = 50$ Hz to $f_{41} = 12500$ Hz. Results of calculations are shown on Fig. 2. One sees that in the range $2\cdot10^{-4} < \mu < \cdot10^{-2}$ the difference $L_{pA}-\tilde{L}_{pA}$ doesn't exceed ± 0.5 dBA. It means, that one may use continuous form of weighting function (Eq. (9) with $W_0 = 2.5197\cdot10^{-6}$ and $\delta = 8.7863\cdot10^{-4}$, if the parameter μ meets the above condition.

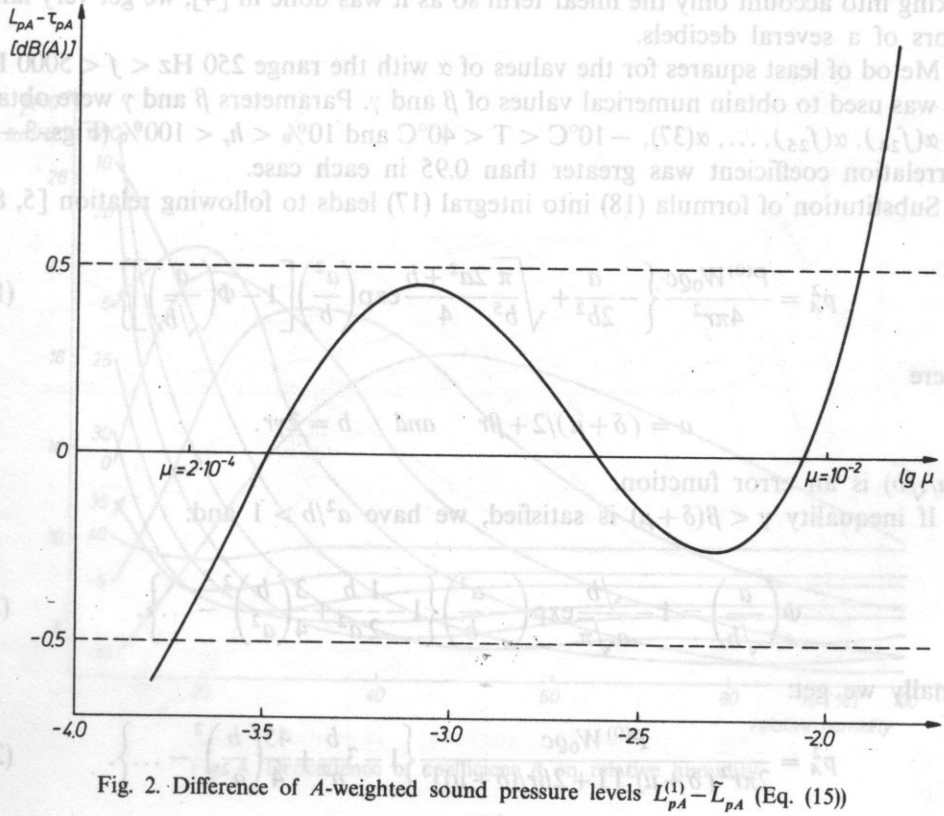

Fig. 2. Difference of A-weighted sound pressure levels $L_{pA}^{(1)} - \tilde{L}_{pA}$ (Eq. (15))

4. Air absorption of sound

The air absorption of sound is not negligible if the point of observation is in the long distance away from the source (Eq. (1)):

$$p^2(f) = \frac{P(f)\varrho c}{4\pi r^2}\exp[-2\alpha(f)r]. \tag{16}$$

Absorption coefficient α depends not only on frequency but also on temperature (T) and relative humidity (h_r).

For the spectral density of source's power determined by Eq. (2) and the weighting function (Eq. 9), we get from (Eq. (8)):

$$p_A^2 = \frac{P^{(0)}W_0\varrho c}{4\pi r^2}\int_0^\infty f^2\exp\{-[\delta+\mu+2\alpha(f)r]\}\,df. \tag{17}$$

Explicit form of the relation $\alpha(f)$ is very complex [1, 3] and the integration can not be done in closed form. The situation changes if one approximates $\alpha(f)$ by the function of a form:

$$\alpha(f) = \beta(T,h_r)f+\gamma(T,h_r)f^2. \tag{18}$$

Taking into account only the linear term so as it was done in [4], we get very large errors of a several decibels.

Metod of least squares for the values of α with the range 250 Hz $< f <$ 5000 Hz [7] was used to obtain numerical values of β and γ. Parameters β and γ were obtain for $\alpha(f_{24}), \alpha(f_{25}), \ldots, \alpha(37)$, $-10°\mathrm{C} < \mathrm{T} < 40°\mathrm{C}$ and $10\% < h_r < 100\%$ (Figs. 3–6). Correlation coefficient was greater than 0.95 in each case.

Substitution of formula (18) into integral (17) leads to following relation [5, 8]:

$$p_A^2 = \frac{P^{(0)}W_0\varrho c}{4\pi r^2}\left\{-\frac{a}{2b^2}+\sqrt{\frac{\pi}{b^5}}\frac{2a^2+b}{4}\exp\left(\frac{a^2}{b}\right)\left[1-\Phi\left(\frac{a}{\sqrt{b}}\right)\right]\right\}, \tag{19}$$

where

$$a = (\delta+\mu)/2+\beta r \quad \text{and} \quad b = 2\gamma r.$$

$\Phi(a/\sqrt{b})$ is an error function.

If inequality $\gamma < \beta(\delta+\mu)$ is satisfied, we have $a^2/b > 1$ and:

$$\Phi\left(\frac{a}{\sqrt{b}}\right) = 1-\frac{\sqrt{b}}{a\sqrt{\pi}}\exp\left(-\frac{a^2}{b}\right)\left\{1-\frac{1}{2}\frac{b}{a^2}+\frac{3}{4}\left(\frac{b}{a^2}\right)^2-\ldots\right\}. \tag{20}$$

Finally we get:

$$p_A^2 = \frac{P^{(0)}W_0\varrho c}{2\pi r^2(\delta+\mu)^3[1+2\beta r/(\delta+\mu)]^3}\left\{1-3\frac{b}{a^2}+\frac{45}{4}\left(\frac{b}{a^2}\right)^2-\ldots\right\}. \tag{21}$$

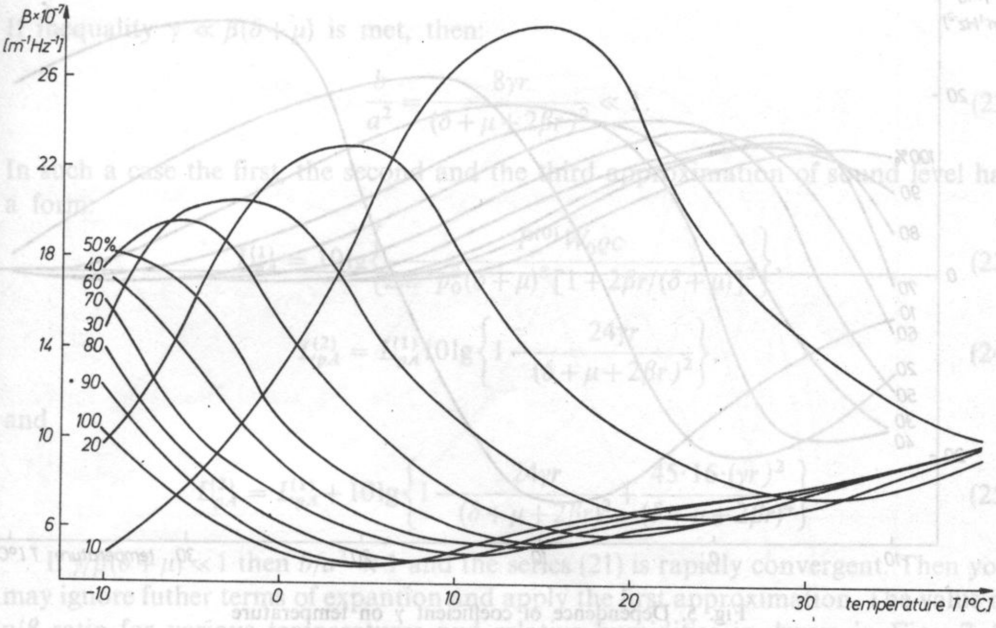

Fig. 3. Dependence of coefficient β on temperature

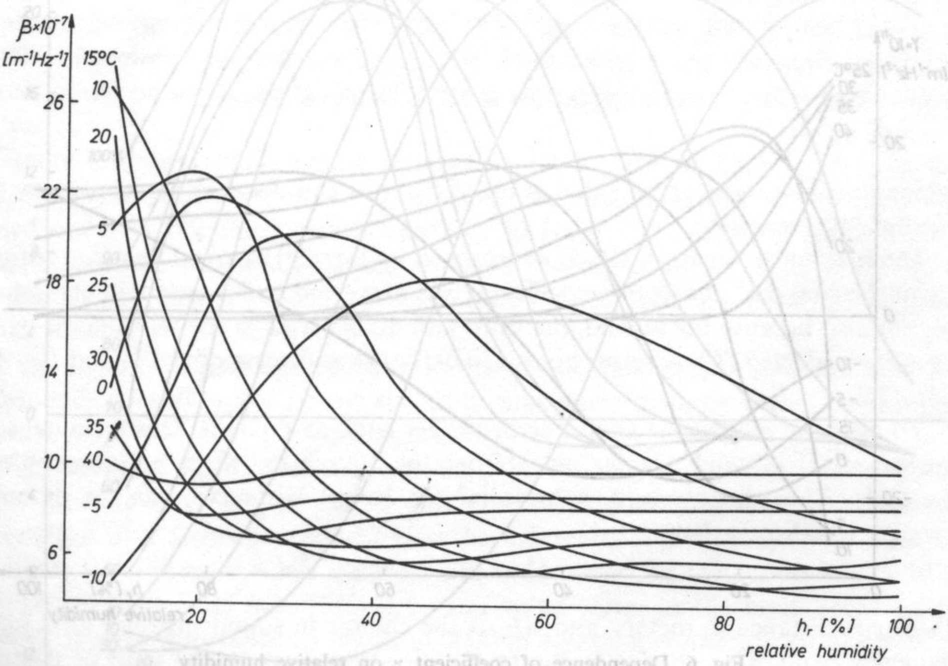

Fig. 4. Dependence of coefficient β on relative humidity

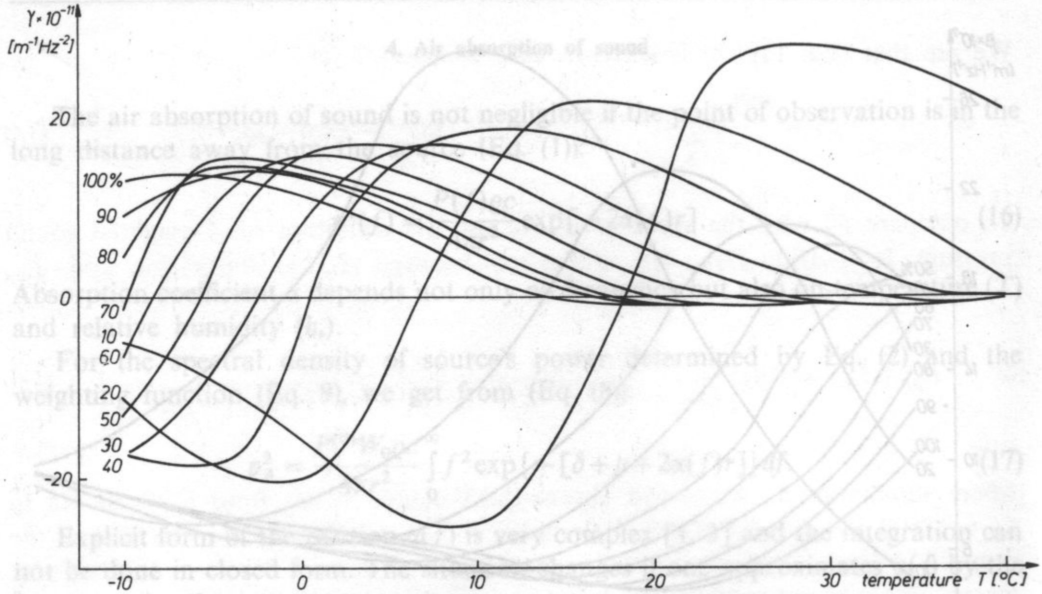

Fig. 5. Dependence of coefficient γ on temperature

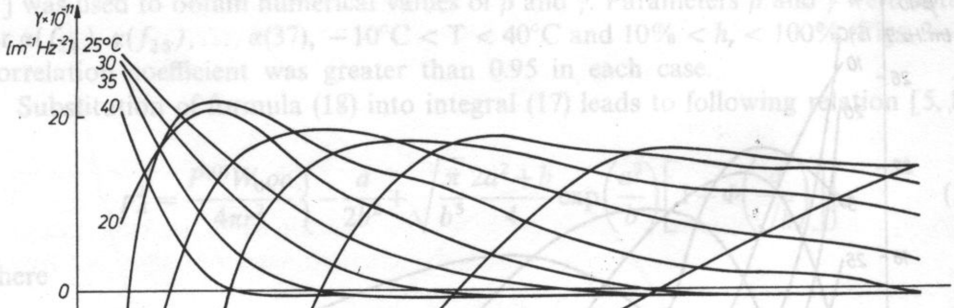

Fig. 6. Dependence of coefficient γ on relative humidity

If inequality $\gamma \ll \beta(\delta+\mu)$ is met, then:

$$\frac{b}{a^2} = \frac{8\gamma r}{(\delta+\mu+2\beta r)^2} \ll 1. \tag{22}$$

In such a case the first, the second and the third approximation of sound level has a form:

$$L_{pA}^{(1)} = 10\lg\left\{\frac{P^{(0)} W_0 \varrho c}{2\pi r^2 p_0^2 (\delta+\mu)^3 [1+2\beta r/(\delta+\mu)]^3}\right\}, \tag{23}$$

$$L_{pA}^{(2)} = L_{pA}^{(1)} 10\lg\left\{1 - \frac{24\gamma r}{(\delta+\mu+2\beta r)^2}\right\}, \tag{24}$$

and

$$L_{pA}^{(3)} = L_{pA}^{(1)} + 10\lg\left\{1 - \frac{24\gamma r}{(\delta+\mu+2\beta r)^2} + \frac{45\cdot 16\cdot(\gamma r)^2}{(\delta+\mu+2\beta r)^4}\right\}. \tag{25}$$

If $\gamma/\beta(\delta+\mu) \ll 1$ then $b/a^2 \ll 1$ and the series (21) is rapidly convergent. Then you may ignore futher terms of expantion and apply the first approximation. The value of γ/β ratio for various temperatures and relative humidities is shown in Figs. 7, 8.

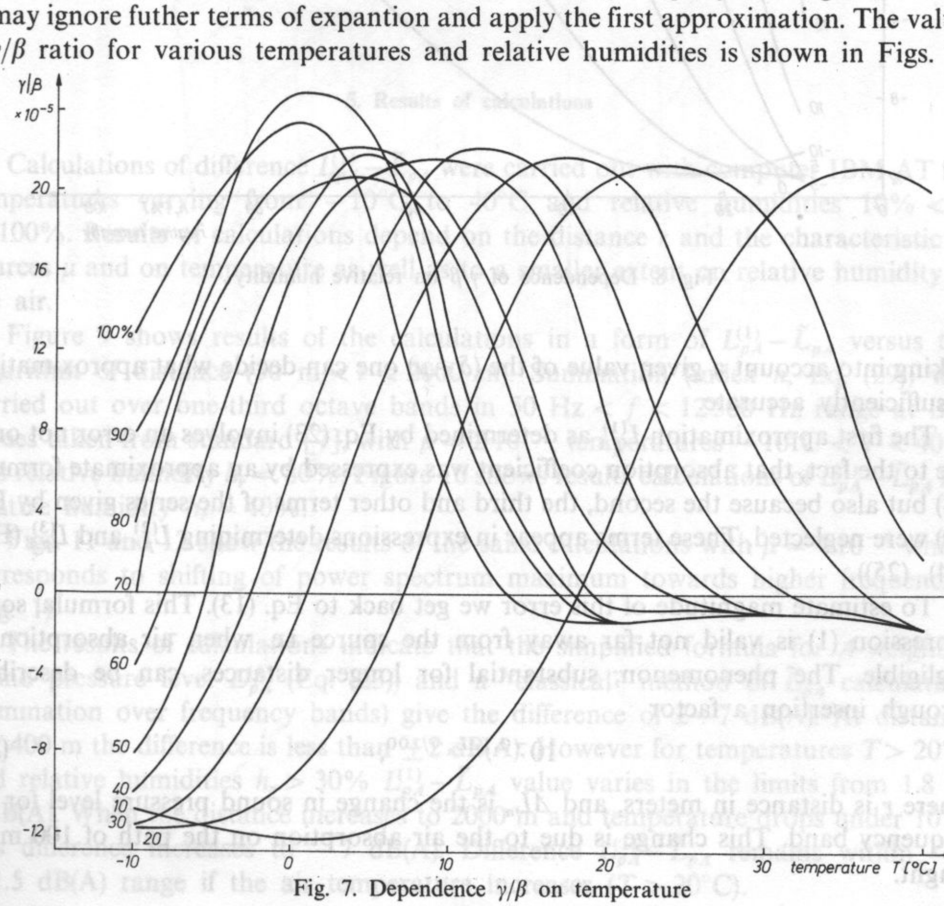

Fig. 7. Dependence γ/β on temperature

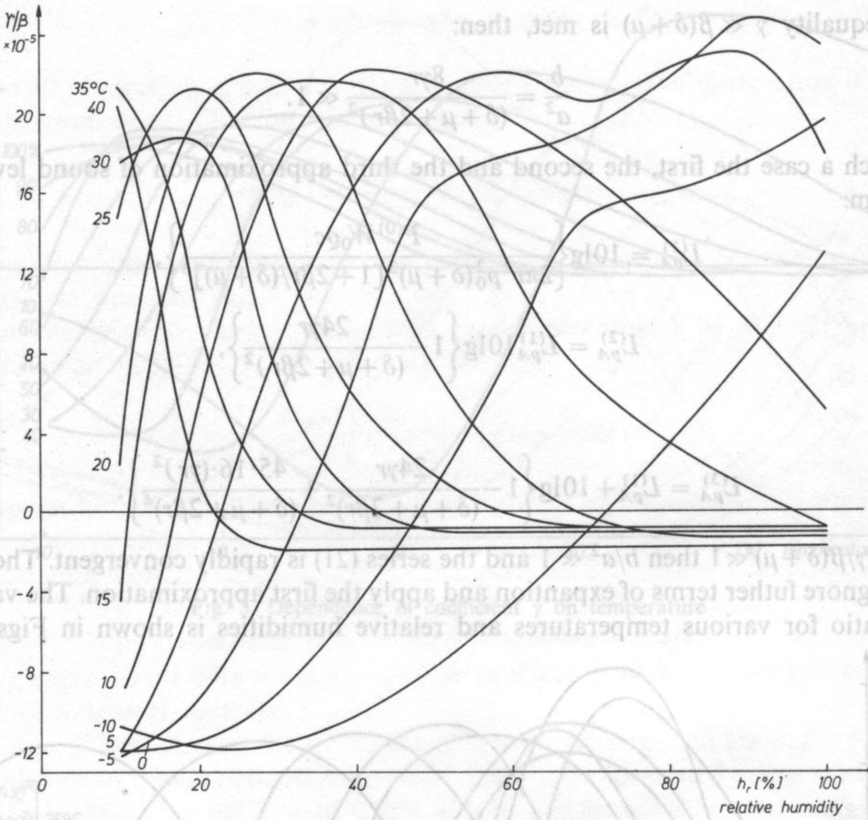

Fig. 8. Dependence of γ/β on relative humidity

Taking into account a given value of the $(\delta+\mu)$ one can decide what approximation is sufficiently accurate.

The first approximation $L_{pA}^{(1)}$ as determined by Eq. (23) involves an error not only due to the fact, that absorption coefficient was expressed by an approximate formula (18) but also because the second, the third and other terms of the series given by Eq. (20) were neglected. These terms appear in expressions determining $L_{pA}^{(2)}$ and $L_{pA}^{(3)}$ (Eqs 124), (25)).

To estimate magnitude of this error we get back to Eq. (13). This formula, so as expression (1) is valid not far away from the source i.e. when air absorption is negligible. The phenomenon, substantial for longer distances, can be described through insertion a factor

$$10^{-0,1\Delta L_n \cdot r/100}, \tag{26}$$

where r is distance in meters, and ΔL_n is the change in sound pressure level for n^{th} frequency band. This change is due to the air absorption on the path of 100 m of lenght.

Equations (13) and (26) combine into:

$$p_n^2 = \frac{P^{(0)}\varrho c}{4\pi r^2} \frac{\exp[-\mu f_n^{(1)}] - \exp[-\mu f_n^{(2)}]}{\mu} \cdot 10^{-0.1\Delta L_n \cdot r/100}. \tag{27}$$

Summation over frequency bands yields expression for A-weighted mean square sound pressure:

$$\widetilde{p_A^2} = \frac{P^{(0)}\varrho c}{4\pi r^2 \mu} \sum_n 10^{0.1[\Delta L_A(f_n) - \Delta L_n \cdot r/100} \left\{\exp[-\mu f_n^{(1)}] - \exp[-\mu f_n^{(2)}]\right\}. \tag{28}$$

Difference of sound levels $L_{pA}^{(1)}$ and $\tilde{L}_{pA} = 10\lg(\tilde{p}_A^2/p_0^2)$ (Eqs. (23), (28)) gives:

$$L_{pA}^{(1)} - \tilde{L}_{pA} =$$

$$10\lg\left[\frac{2W_0\mu}{(\delta+\mu+2\beta r)^3 \sum_n 10^{0.1[\Delta L_A(f_n) - \Delta L_n \cdot r/100]}\{\exp[-\mu f_n^{(1)}] - \exp[-\mu f_n^{(2)}]\}}\right]. \tag{29}$$

5. Results of calculations

Calculations of difference $L_{pA}^{(1)} - \tilde{L}_{pA}$ were carried out with computer IBM AT for temperatures varying from $-10°C$ to $40°C$ and relative humidities $10\% < h_r < 100\%$. Results of calculations depend on the distance r and the characteristic of sources μ and on temperature as well as to a smaller extent on relative humidity of the air.

Figure 9 shows results of the calculations in a form of $L_{pA}^{(1)} - \tilde{L}_{pA}$ versus the logarithm of distance (50 m $< r <$ 2000 m). Summation (index n, Eq. (29)) was carried out over one-third octave bands in 50 Hz $< f <$ 12 500 Hz range at ΔL_n values taken from standard [7], with $\mu = 2 \cdot 10^{-3}$, temperatures $-10°C < T < 40°C$ and relative humidity $h_r = 80\%$. Figure 10 shows results calculations of $L_{pA}^{(1)} - \tilde{L}_{pA}$ for relative humidity $h_r = 40\%$.

Figs. 11 and 12 show the results of the same calculations with $\mu = 4 \cdot 10^{-4}$ which corresponds to shifting of power spectrum maximum towards higher frequencies (Fig. 1).

The results of calculations indicate that the simplified formula for A-weighted sound pressure level $L_{pA}^{(1)}$ (Eq. (23)) and a "classical" method of $\tilde{L}_{pA}$ calculation (summation over frequency bands) give the difference of $2 \div 7$ dB(A). At distance $r < 400$ m the difference is less than ± 2 dB(A). However for temperatures $T > 20°C$ and relative humidities $h_r > 30\%$ $L_{pA}^{(1)} - \tilde{L}_{pA}$ value varies in the limits from 1.8 to 0 dB(A). When the distance increases to 2000 m and temperature drops under 10°C this difference increases to -7 dB(A). Difference $L_{pA}^{(1)} - \tilde{L}_{pA}$ remains within $-1 \div 1.5$ dB(A) range if the air temperature increases ($T > 20°C$).

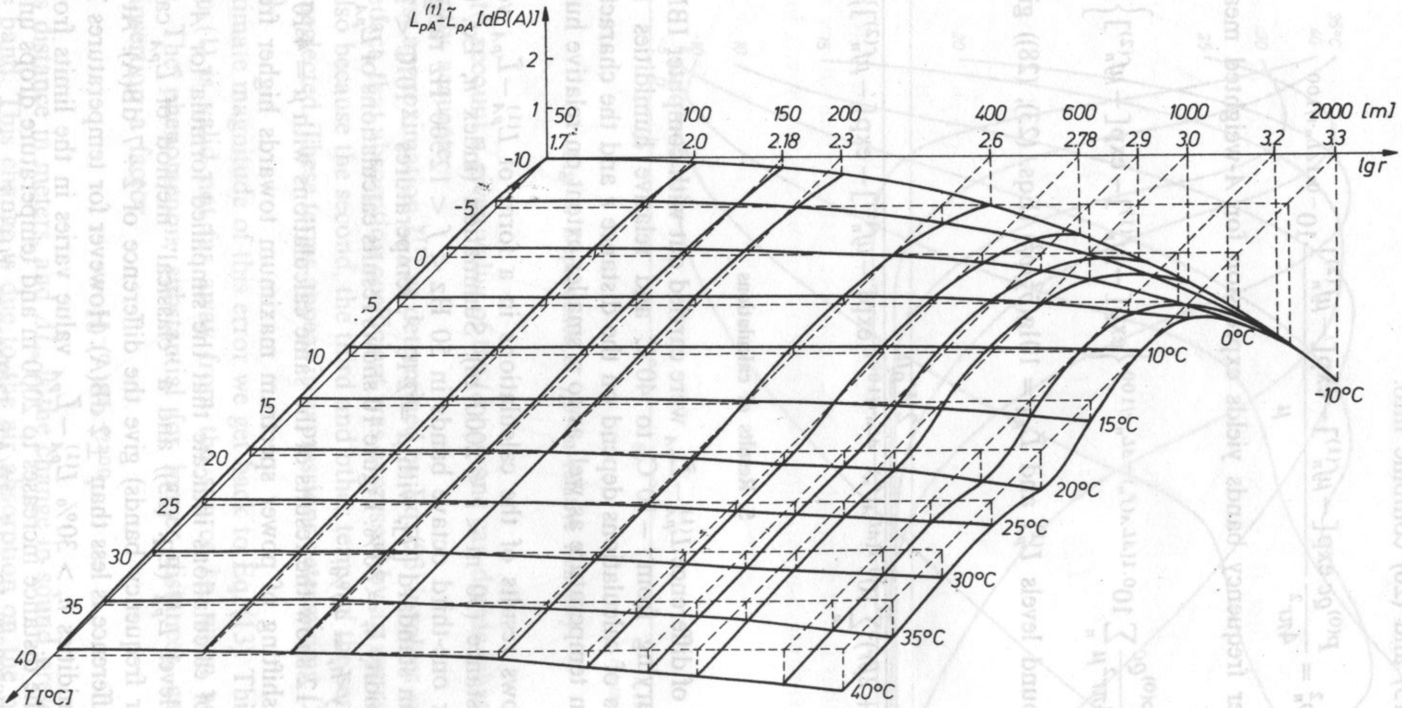

Fig. 9. Difference $L_{pA}^{(1)}-\tilde{L}_{pA}$ (Eq. (29)) versus logarithm of distance for relative humidity $h_r = 80\%$ and parameter $\mu = 2\cdot10^{-3}$

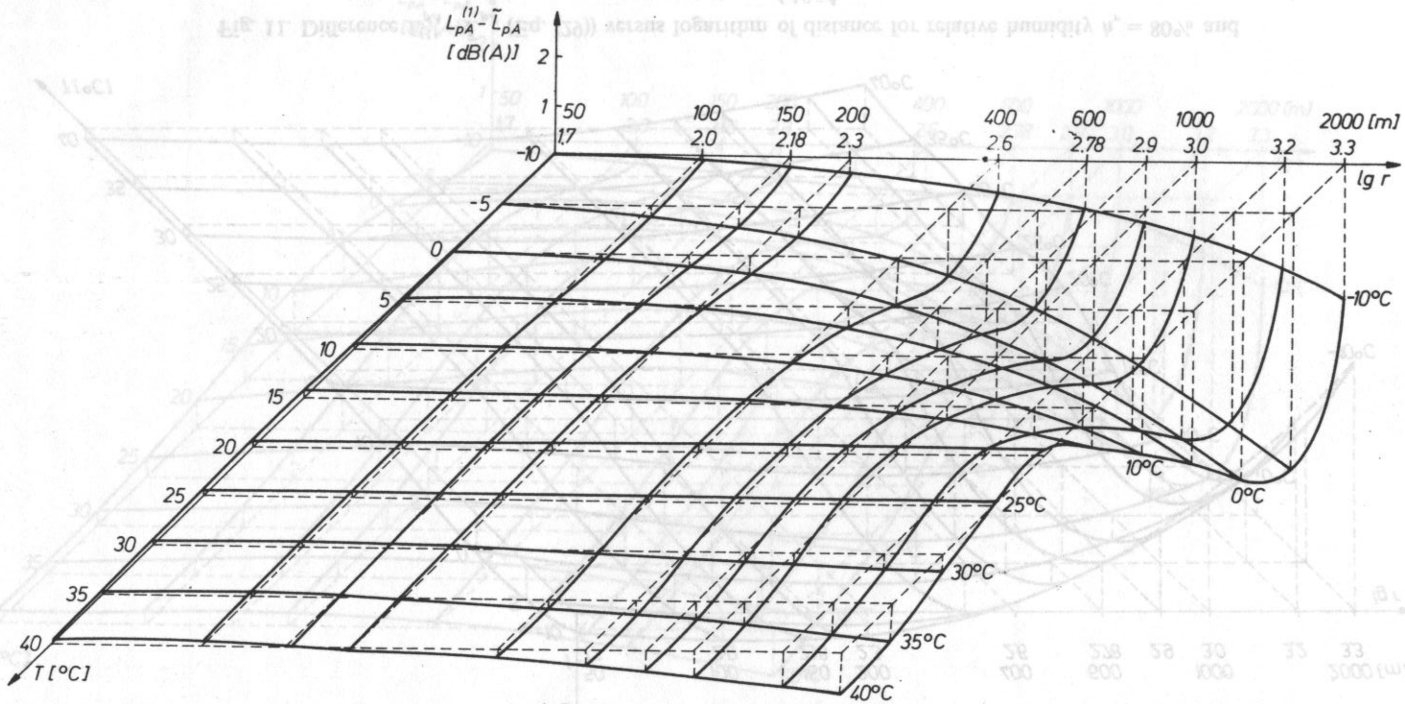

Fig. 10. Difference $L_{pA}^{(1)} - \tilde{L}_{pA}$ (Eq. (29)) versus logarithm of distance for relative humidity $h_r = 40\%$ and parameter $\mu = 2 \cdot 10^{-3}$

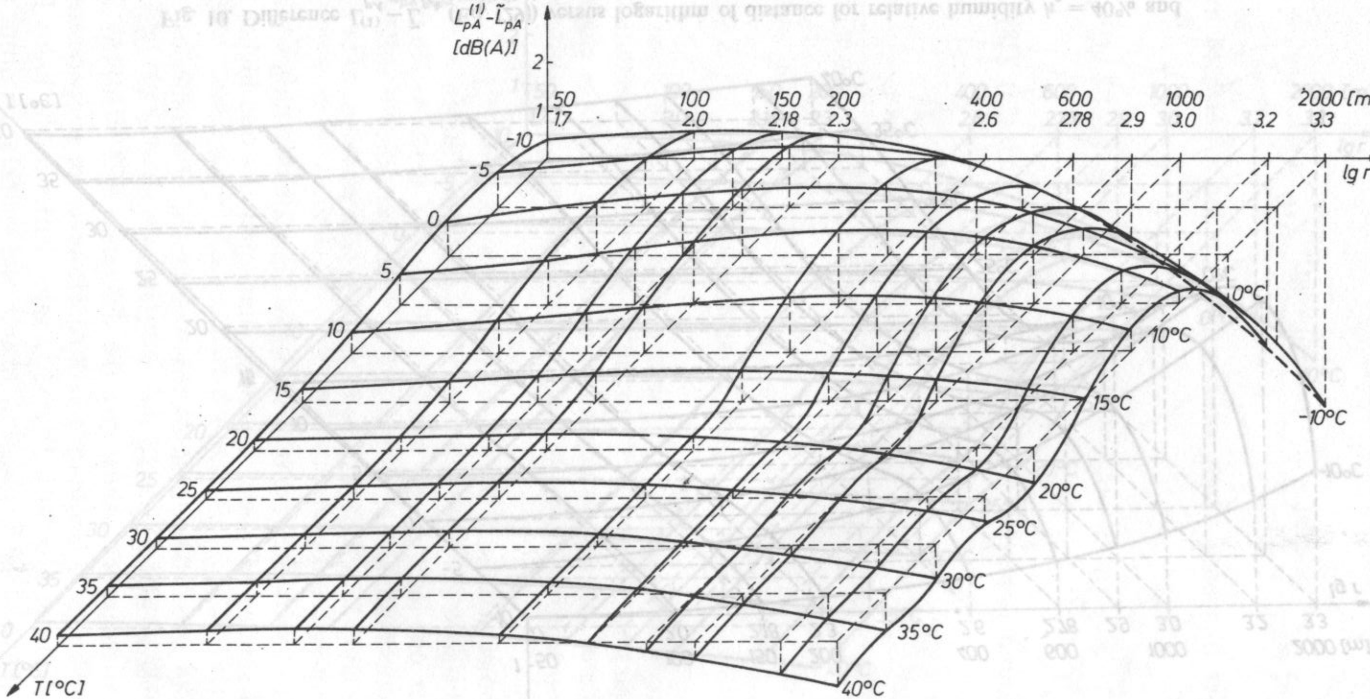

Fig. 11. Difference $L_{pA}^{(1)} - \tilde{L}_{pA}$ (Eq. (29)) versus logarithm of distance for relative humidity $h_r = 80\%$ and parameter $\mu = 4 \cdot 10^{-4}$

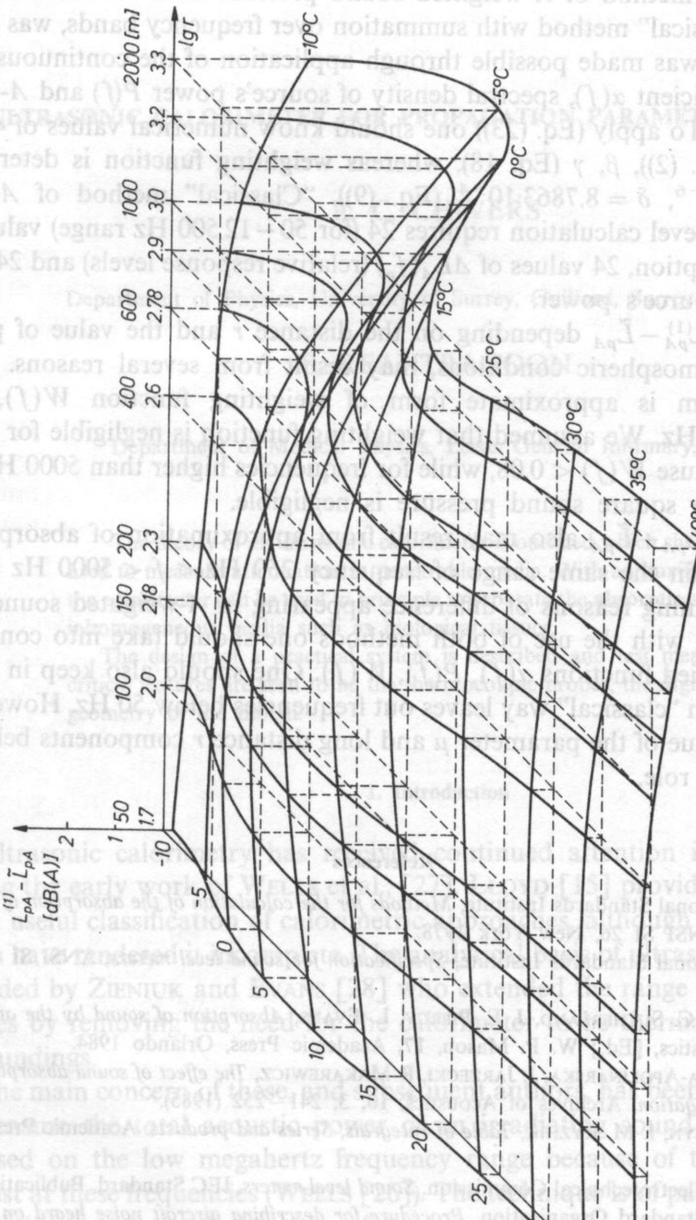

Fig. 12. Difference $L_{pA}^{(1)} - \bar{L}_{pA}$ (Eq. (29)) versus logarithm of distance for relative humidity $h_r = 40\%$ and parameter $\mu = 4 \cdot 10^{-4}$

6. Conclusions

A simplified method of A-weighted sound pressure level calculation (Eq. (23)) instead of "classsical" method with summation over frequency bands, was presented in the paper. It was made possible through application of the continuous forms of absorption coefficient $\alpha(f)$, spectral density of source's power $P(f)$ and A-weighting function $W(f)$. To apply (Eq. (23)) one should know numerical values of 4 parameters: $P^{(0)}$, μ (Eq. (2)), β, γ (Eq. 18), whereas weighting function is determined by $W_0 = 2.5197 \cdot 10^{-6}$, $\delta = 8.7863 \cdot 10^{-4}$ (Eq. (9)). "Classical" method of A-weighted sound pressure level calculation requires 24 (for 50 – 12 500 Hz range) values of ΔL_n describing absorption, 24 values of $\Delta L_A(f_n)$ (relative response levels) and 24 values of p_n describing source's power.

Differences $L_{pA}^{(1)} - \tilde{L}_{pA}$ depending on the distance r and the value of parameter μ as well as atmospheric conditions, may result from several reasons.

One of them is approximate form of weighting function $W(f)$, for 250 Hz $< f <$ 5000 Hz. We assumed that weighting function is negligible for frequency $f <$ 250 Hz because $W(f) < 0.08$, while for frequencies higher than 5000 Hz spectral density of mean square sound pressure is negligible.

Differences $L_{pA} - \tilde{L}_{pA}$ also may result from approximation of absorption coefficient $\alpha(f)$ within the same range of frequency 250 Hz $< f <$ 5000 Hz (Eq. (18)).

When explaining reasons of difference appearing in A-weighted sound pressure level calculation with the use of both methods one should take into consideration not only simplified functions $\alpha(f)$, $P(f)$, $W(f)$. One should also keep in mind that L_{pA} calculated in "classical" way leaves out frequencies below 50 Hz. However in the case of great value of the parameter μ and long distance r components below 50 Hz play important role.

References

[1] American National Standards Institute, *Methods for the calculation of the absorption of sound by the atmosphere*, ANSI SI 26, New York 1978.

[2] American National Standards Institute, *Specification for sound level meters*, ANSI SI 4, New York 1983.

[3] H. E. BASS, L. C. SUTHERLAND, J. E. PIERCY, L. EVANS, *Absorption of sound by the atmosphere*, in: Physical Acoustics, [Ed.] W. P. Mason, **17,** Academic Press, Orlando 1984.

[4] K. BEREZOWSKA-APOLINARSKA, J. JARZECKI, R MAKAREWICZ, *The effect of sound absorption by the air on noise propagation.* Archives of Acoustics, **10,** 3, 241 – 252 (1985).

[5] I. S. GRADSHTEYN, I. M. RYZHIK, *Table of Integrals, Series and products*, Academic Press, New York 1980.

[6] International Electrotechnical Commission, *Sound level meters*, IEC Standard, Publication 651, 1979.

[7] International Standard Organization, *Procedure for describing aircraft noise heard on ground*, ISO 3891, 1978.

[8] R. MAKAREWICZ, *Air absorption of motor vehicle*, J. Acoust. Soc. Am., **80,** 2, 561 – 568 (1986).

[9] E. ZWICKER, *What is a meaningful value for guantifying noise reduction*?, Proc. Inter – Noise 85, 47 – 56 (1985).

Received February 2, 1988.