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STATISTICAL SOUND EVALUATION FOR THE IMPROVEMENT OF SOUND INSULATION SYSTEMS

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The random noise fluctuation encountered in our living environment, such as street noise, road traffic noise, etc. exhibits various kinds of probability distribution forms apart from the usual Gaussian distribution due to the diversified causes of the fluctuations. From the practical viewpoint of control and regulation of such environmental noise, several statistics, such as median, L_5 and L_{10} (in general, so called L_α sound level), directly connected with the probability distribution form of random noise fluctuation are very often used for evaluation of the human response. Thus, it is essential to establish a systematic method for evaluating the effect of the system change of noise control on the widely-used standard noise index such as L_α. In this paper, general and fundamental considerations for statistical evaluation of transmitted sound waves have been theoretically proposed, when the system characteristic of the sound insulation is changed by any improvement work. The theoretical result was experimentally confirmed not only by the result of the digital simulation technique, but also by actually observed data obtained using reverberation room method. The results of the experiment are in good agreement with our theory.

Praca poświęcona jest badaniu przechodzenia losowej fali akustycznej przez ekran dźwiękochłonny pojedynczy i podwójny. Oryginalnym wynikiem jest porównanie zjawiska przechodzenia energii w modelu z pojedynczą ścianką z modelem z podwójną ścianką. Przedstawiono porównania wyników analitycznych z rezultatami doświadczeń.

Introduction

The random fluctuation of noise and vibration encountered in our living environment, such as street noise, road traffic noise, machine or structure vibration, etc., exhibits various kinds of probability distribution forms apart from a usual

Gaussian distribution due to the diversie causes of the fluctuations. From the practical viewpoint of control and regulation for such environmental noise and vibration pollutions, several of statistics, such as median, L_5 and L_{10} (in general, L_α((100 − α) percentile) sound or vibration levels), directly connected with a whole shape of the probability distribution of random noise and vibration fluctuations are very important for an evaluation of the human response [1]. Thus, it must be an essential problem to establish a systematic method for the purpose of evaluating the effect of system change of noise or vibration controls on the standard noise index such as L_α. In this paper, general and fundamental considerations for the statistical evaluation of noise or vibration have been theoretically proposed, when the characteristic of the control system is changed by any improvement work. That is, when an arbitrarily distributed random signal is passed through noise or vibration control systems, a unified statistical treatment for the probability density function of its output energy fluctuation has been proposed in the universal form of an expansion series expression. Hereupon, an input random noise may have arbitrary types of the first and higher order correlations among arbitrarily chosen samples, and furthermore noise or vibration control systems have arbitrary linear characteristics of the finite memory type. For the purpose of finding systematically and universally the effect of the system change on the statistical evaluation quantities L_α of the output signal, the distribution function for the output energy fluctuation observed before the system characteristic is changed, has been taken into consideration as the first term of the unified expansion expression.

Furthermore, in view of the arbitrariness of possible input characteristics, the possible variety of noise or vibration control systems, and the complexity of the mathematical expressions involved and its statistical treatment, a digital simulation technique appears to be a powerful way of experimentally confirming the theoretical expressions. That is, in the simple and basic case when a homogeneous single wall is changed to a double wall by the improvement work as an example of a noise control system, the probability distribution function of the output sound energy fluctuation simulated on a digital computer has been drawn graphically for the comparison between theory and experiment. Finally, the validity and usefulness of our theoretical consideration have been confirmed experimentally by applying it to the actual observed noise data obtained using the reverberation room method. The experiment has been carried out with the single wall and double wall composed of an aluminum panel, using road traffic noise as an arbitrary random incident wave. We have been able to observe a good agreement between theory and experiment in the above two cases.

1. General theory

When a general random sound pressure wave of an arbitrary non-Gaussian distribution type $X(t)$ (let X_j be the sampled value at time point t_j of $X(t)$; $j = 1, 2, \ldots, K$) passes through the time-invariant linear system with an impulse response function $h(t)$ (let b_{ij} be the sampled weighting value of $h(t)$), the transmitted sound pressure wave $Y(t)$ (let Y_i be the sampled value at time point t_i of $Y(t)$; $i = 1, 2, \ldots, N$) can be easily given as the following equation in the discrete form [2, 3]:

$$Y_i = \sum_{j=1}^{K} b_{ij} X_j, \tag{1}$$

where K is the order of linear system.

On the other hand, an N-dimensional probability density function $P_z(\mathbf{z})$ for the transmitted sound pressure wave $Z(t)$ (let Z_i be the sampled value of $Z(t)$; $i = 1, 2, \ldots, N$) after changing the system characteristic from $h(t)$ to $W(t)$ (with sampled weighting value a_{ij}) can be expressed on the basis of the N-variate joint probability density function $P_{\mathbf{Y}}(\mathbf{Y})$ for the above transmitted sound pressure wave $Y(t)$ of the original system $h(t)$ as follows

$$P_z(\mathbf{Z}) = \sum_{r=0}^{\infty} \sum_{r_1+\ldots+r_N=r} \frac{B_r(r_1, r_2, \ldots, r_N)}{r_1!\, r_2! \ldots r_N!} (-1)^{r_1+r_2+\ldots+r_N} \frac{\partial^{r_1+r_2+\ldots+r_N}}{\partial Z_1^{r_1} \partial Z_2^{r_2} \ldots \partial Z_N^{r_N}} P_y(\mathbf{Z}), \tag{2}$$

where the above expansion coefficients are given as:

$$B_0(0, 0, \ldots, 0) = 1,$$

$$B_1(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0) = \sum_{p_1=1}^{K} (a_{ip_1} - b_{ip_1}) \varkappa_{x1}(0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0),$$

$$B_2(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0) = \sum_{p_1, p_2=1}^{K} (a_{ip_1} a_{jp_2} - b_{ip_1} b_{jp_2})$$

$$\times \varkappa_{x2}(0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0, \overset{p_2}{1}, 0, \ldots, 0) + B_1(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0)$$

$$\times B_1(0, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0),$$

$$B_3(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0, \overset{k}{1}, 0, \ldots, 0) = \sum_{p_1, p_2, p_3=1}^{K} \tag{3}$$

$$\times (a_{ip_1} a_{jp_2} a_{kp_3} - b_{ip_1} b_{jp_2} b_{kp_3}) \varkappa_{x3}(0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0, \overset{p_2}{1}, 0, \ldots, 0, \overset{p_3}{1}, 0, \ldots, 0)$$

$$+ B_2(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0) B_1(0, 0, \ldots, 0, \overset{k}{1}, 0, \ldots, 0)$$

$$+ B_2(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0, \overset{k}{1}, 0, \ldots, 0) B_1(0, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0)$$

$$+ B_2(0, 0, \ldots, 0, \overset{j_1}{1}, 0, \ldots, 0, \overset{k}{1}, \ldots, 0) B_1(0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0) - 2B_1$$

$$\times (0, 0, \ldots, 0, \overset{i}{1}, 0, \ldots, 0) B_1(0, 0, \ldots, 0, \overset{j}{1}, 0, \ldots, 0) B_1(0, 0, \ldots, 0, \overset{k}{1}, 0, \ldots, 0).$$

Hereupon, $\varkappa_{xl}(0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0, \overset{p_2}{1}, 0, \ldots, 0, \overset{p_l}{1}, 0, \ldots, 0)$ denotes the l-dimensional correlation function of $X(t)$.

In Eq. (2), it is noteworthy that the N-dimensional probability density function $P_{\mathbf{Y}}(\mathbf{z})$ is not directly related to Z_i itself, but is given by merely substituting $Z_i's$ for random variables $Y_i's$ in the joint probability density $P_{\mathbf{Y}}(\mathbf{Y})$ of a transmitted sound pressure wave $Y_i (i = 1, 2, \ldots, N)$. In the above expression $P_{\mathbf{Z}}(\mathbf{z})$, in a specific case when $N = 1$, $P_z(z)$ can be directly expressed as follows (The usefulness of this probability expression is briefly discussed in **Appendix 1**):

$$P_z(Z) = \sum_{r=0}^{\infty} (-1)^r \frac{B_r(r)}{r!} \frac{\partial^r}{\partial Z^r} P_y(Z), \tag{4}$$

where the expansion coefficients are simplified as follows:

$$B_0(0) = 1,$$

$$B_1(1) = \sum_{p_1=1}^{K} (a_{p_1} - b_{p_1}) \varkappa_{x1}(0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0), \tag{5}$$

$$B_2(2) = \sum_{p_1, p_2 = 1}^{K} (a_{p_1} a_{p_2} - b_{p_1} b_{p_2}) \varkappa_{x2}(0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0, \overset{p_2}{1}, 0, \ldots, 0) + B_1(1)^2,$$

$$B_3(3) = \sum_{p_1, p_2, p_3 = 1}^{K} (a_{p_1} a_{p_2} a_{p_3} - b_{p_1} b_{p_2} b_{p_3}) \varkappa_{x3}$$

$$\times (0, 0, \ldots, 0, \overset{p_1}{1}, 0, \ldots, 0, \overset{p_2}{1}, 0, \ldots, 0, \overset{p_3}{1}, 0, \ldots, 0) + 3B_2(2) B_1(1) - 2B_1(1)^3.$$

From the above theoretical results, Eqs. (2) or (4), we can find that the joint probability density expression of Z_i can be expressed in a universal expansion form in which the joint probability density $P_{\mathbf{Y}}(\mathbf{Y})$ of random process Y_i is taken into the first term (so that it may be convenient for our purpose of research) and its successive derivatives are taken in the second and higher expansion terms. Furthermore, the effect of system change on the resultant distribution form of the transmitted sound pressure fluctuation is explicitly reflected in each expansion coefficient (cf. Eqs. (3) or (5)).

Now, as was reported in the previous papers [9], we can employ the following probability density function of a statistical Hermite series-type expression as $P_y(z)$ for a transmitted sound pressure wave $Z(t)$ of an arbitrary distribution type for the non-changed system $h(t)$:

$$P_y(z) = \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-z^2/2\sigma_y^2} \sum_{n=0}^{\infty} A_n H_n\left(\frac{z}{\sigma_y}\right), \tag{6}$$

where:

$$A_n \overset{\Delta}{=} \frac{1}{n!}\left\langle H_n\left(\frac{Y}{\sigma_y}\right)\right\rangle,$$

($\langle * \rangle$ denotes a statistical mean operation with respect to random variable $*$). Substituting Eq. (6) into Eq. (4) and applying the well-known relation between the normal Gaussian distribution function and the Hermite polynomial:

$$\frac{1}{\sqrt{2\pi}} e^{-\eta^2/2} H_n(\eta) = (-1)^n \frac{d^n}{d\eta^n} \frac{1}{\sqrt{2\pi}} e^{-\eta^2/2}, \tag{7}$$

we can obtain the probability density function $P_z(z)$ of a transmitted sound wave after changing the system from $h(t)$ to $W(t)$ concerning its impulse response function as:

$$P_z(z) = \sum_{r=0}^{\infty} \sum_{n=0}^{\infty} \frac{B_r(r)}{r!} \frac{A_n}{\sigma_y^r} \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-z^2/2\sigma_y^2} H_{n+r}\left(\frac{z}{\sigma_y}\right). \tag{8}$$

The probability density function of the sound energy fluctuation is more important than that of the sound presssure wave fluctuation for the purpose of evaluation of noise pollution. By using an integral formula:

$$\int_{-\infty}^{\infty} e^{-B^2x^2} H_n(X)dx = \begin{cases} 0 & (n\text{: odd}) \\ (1/B)^{n+1}(1-2B^2)^{n/2}\,\Gamma((n+1)/2) & (n\text{: even})\end{cases}, \tag{9}$$

the moment generating function of Laplace transformation type for transmitted sound energy $E(\overset{\Delta}{=} z^2)$ is derived as follows:

$$M_E(\theta) \overset{\Delta}{=} \langle e^{\theta E}\rangle = \sum_{r=0}^{\infty}\sum_{n=0}^{\infty} \frac{B_r(r)}{r!} \frac{A_n}{\sigma_y^r} \frac{(\sqrt{2})^{n+r}}{\sqrt{\pi}}\, \Gamma\left(\frac{n+r+1}{2}\right)$$
$$\times \frac{1}{(1-2\sigma_y^2\theta)^{1/2}} \left(\frac{2\sigma_y^2\theta}{1-2\sigma_y^2\theta}\right)^{\frac{n+r}{2}} \quad (n+r\text{: even}). \tag{10}$$

By applying an integral formula on an associated Laguerre polynomial $L_n^{(\alpha)}(*)$:

$$\int_0^{\infty} e^{-pt} t^{\alpha} e^{\beta t} L_n^{(\alpha)}(at)dt = \frac{\Gamma(n+\alpha+1)}{n!} \frac{(p-a-\beta)^n}{(p-\beta)^{n+\alpha+1}} \tag{11}$$

to the inverse Laplace transformation of Eq. (10), the probability density function $P_E(E)$ of the transmitted sound energy through the changed system $W(t)$ is consequently given in the form of an expansion expression as follows (see Appendix 2):

$$P_E(E)=\sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\frac{B_r(r)}{r!}\frac{A_n}{\sigma_y^r}\frac{(\sqrt{2})^{n+r}}{\sqrt{\pi S}}(-1)^{\frac{n+r}{2}}\cdot\left(\frac{n+r}{2}\right)!\,E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n+r}{2}}^{(-\frac{1}{2})}\left(\frac{E}{s}\right)$$

$$=\sum_{m=0}^{\infty}\left\langle H_{2m}\left(\frac{Y}{\sigma_y}\right)\right\rangle\frac{(-1)^m}{\Gamma\left(m+\frac{1}{2}\right)2^m\sqrt{s}}E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_m^{(-\frac{1}{2})}\left(\frac{E}{s}\right) \quad (12)$$

$$+\sum_{r=1}^{\infty}\sum_{n=0}^{\infty}\frac{B_r(r)A_n(\sqrt{2})^{n+r}}{r!\,\sigma_y^r\sqrt{\pi s}}(-1)^{\frac{n+r}{2}}\left(\frac{n+r}{2}\right)!\cdot E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n+r}{2}}^{(-\frac{1}{2})}\left(\frac{E}{s}\right)$$

with $s\stackrel{\Delta}{=}2\sigma_y^2$.

If we introduce a dimensionless variable $\eta\,(\stackrel{\Delta}{=}E/s)$ for the purpose of obtaining the universal expression, we can easily find the following unified expression:

$$P_\eta(\eta)=\sum_{m=0}^{\infty}\left\langle H_{2m}\left(\frac{Y}{\sigma_y}\right)\right\rangle\frac{(-1)^m}{\Gamma\left(m+\frac{1}{2}\right)2^m}\eta^{-\frac{1}{2}}e^{-\eta}L_m^{(-\frac{1}{2})}(\eta)$$

$$+\sum_{r=1}^{\infty}\sum_{m=0}^{\infty}\frac{B_r(r)A_m(\sqrt{2})^{m+r}}{r!\,\sigma_y^r\sqrt{\pi}}(-1)^{\frac{m+r}{2}}\left(\frac{m+r}{2}\right)!\,\eta^{-\frac{1}{2}}e^{-\eta}L_{\frac{m+r}{2}}^{(-\frac{1}{2})}(\eta). \quad (13)$$

Hereupon, the relation

$$L_n^{(-1/2)}(\eta)=(-1)^n\frac{1}{2^n n!}H_{2n}(\sqrt{2\eta})$$

should be observed.

In this expression, the transmitted sound energy distribution through the original system $h(t)$ is taken as the first expansion term and the effect of characteristic change for the sound insulation due to changing the system. The statistical characteristic of an incident sound wave is reflected in the expansion coefficients of the remainder terms.

2. Step response function for single wall and double wall

2.1. Step response function for single wall

By use of the mass law, the frequency transfer function G_I for a single wall is directly given as [12]:

$$G_\mathrm{I}(j\omega)=\frac{1}{1+j\omega T},\qquad T\stackrel{\Delta}{=}\frac{m}{2\varrho c}\cos\theta, \quad (14)$$

where m, ϱc and θ denote respectively the surface mass of single wall, the characteristic impedance of air and the incident angle. From the above equation, the step response function $S_I(t)$ is easily derived as follows:

$$S_I(t) = 1 - e^{-t/T}. \tag{15}$$

2.2. *Step response function for double wall*

As reported in our previous paper from the viewpoint of the distributed constant circuit theory, the frequency transfer function G_{II} for a double wall is given as follows [13]:

$$G_{\text{II}}(j\omega) = \frac{1}{1 + j\omega(T_1 + T_2) + (j\omega)^2 T_1 T_2 (1 - e^{-j\omega\tau})},$$
$$T_1 \triangleq \frac{m_1}{2\varrho c}\cos\theta,\ T_2 \triangleq \frac{m_2}{2\varrho c}\cos\theta,\quad \tau_\theta \triangleq \frac{2d}{c}\cos\theta, \tag{16}$$

where m_1, m_2 denote the surface mass of each panel, and d denotes the width of the air gap. Hereupon, the above expression of $G_{\text{II}}(s)$ can be rewritten as the following expansion form (see Appendix 3):

$$G_{\text{II}}(s) = \frac{1}{(1+T_1 s)(1+T_2 s)} \sum_{n=0}^{\infty} \left[\frac{T_1 T_2 s^2}{(1+T_1 s)(1+T_2 s)} e^{-\tau_\theta s} \right]^n = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{1}{r!} \frac{T_2^n}{T_1^{n+2}}$$
$$\times \left(1 - \frac{T_2}{T_1}\right)^r (n+1)(n+2)\ldots(n+r) \cdot \frac{s^{2n+r}}{\left(s + \frac{1}{T_1}\right)^{2n+r+2}} e^{-n\tau_\theta s}. \tag{17}$$

Using a formula on the Laplace transform of a Laguerre polynomial (see Eq. (11)), the step response function for a double wall can be explicitly derived in the following expansion form as one of a Laguerre filter in some sense:

$$S_{\text{II}}(t) = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{1}{r!} \frac{T_2^n}{T_1^{n+2}} \left(1 - \frac{T_2}{T_1}\right)^r (n+1)(n+2)\ldots(n+r)$$
$$\times \frac{2n+r-1)!}{\Gamma(2n+r+2)} (t - n\tau_\theta)^2 e^{-\frac{t-n\tau_\theta}{T_1}} L_{2n+r-1}^{(2)}\left(\frac{t - n\tau_\theta}{T_1}\right). \tag{18}$$

Especially, if we focus our attention on the special case when every surface mass is equal (i.e., $m_1 = m_2 = m$ $(T_1 = T_2 = T)$), the above expression of $S_{\text{II}}(t)$ can be reduced to the expansion expression given by:

$$S_{II}(t)=\frac{1}{T^2}\sum_{n=0}^{\infty}\frac{(2n-1)!}{\Gamma(2n+2)}(t-n\tau_\theta)^2 e^{-\frac{t-n\tau_\theta}{T}} L_{2n-1}^{(2)}\left(\frac{t-n\tau_\theta}{T}\right). \tag{19}$$

Fig. 1 shows the step response curves for a single wall with various values of the incident angle θ obtained by use of Eq. (15). On the other hand, Eq. (19) has an infinite expansion form in the form of a Laguerre filter, but in practice we must use only the first finite term. Fig. 2 is introduced to find what number of expansion terms in the expression of $S_{II}(t)$ (see Eq. (19)) is needed to evaluate $S_{II}(t)$ as exactly as possible. In this figure a comparison between the step response curve of a single wall $S_I(t)$ in the specific case of $m=m_1+m_2$ and the step response curves of a double wall $S_{II}(t)$ with m_1 and m_2 is shown, where a single surface mass is taken as $m=5.5$ Kg/m^2 $(=m_1+m_2)$ for a single wall and two surface masses $m_i=2.75$ Kg/m^2 $(i=1, 2)$ are taken by letting $d=0$ and $\theta=0°$ for a double wall. After confirming an agreement between the two kinds of step response curves ($S_I(t)$ and $S_{II}(t)$), we can find that it is sufficient to take the first 8 expansion terms of Eq. (19) to evaluate $S_{II}(t)$. Fig. 3 shows the step response curves of a double wall with various values of the incident angle θ, where the two surface masses are equally taken as $m_1=m_2=3.22$ Kg/m^2 and the width of the air gap is set as $d=0.05$ m.

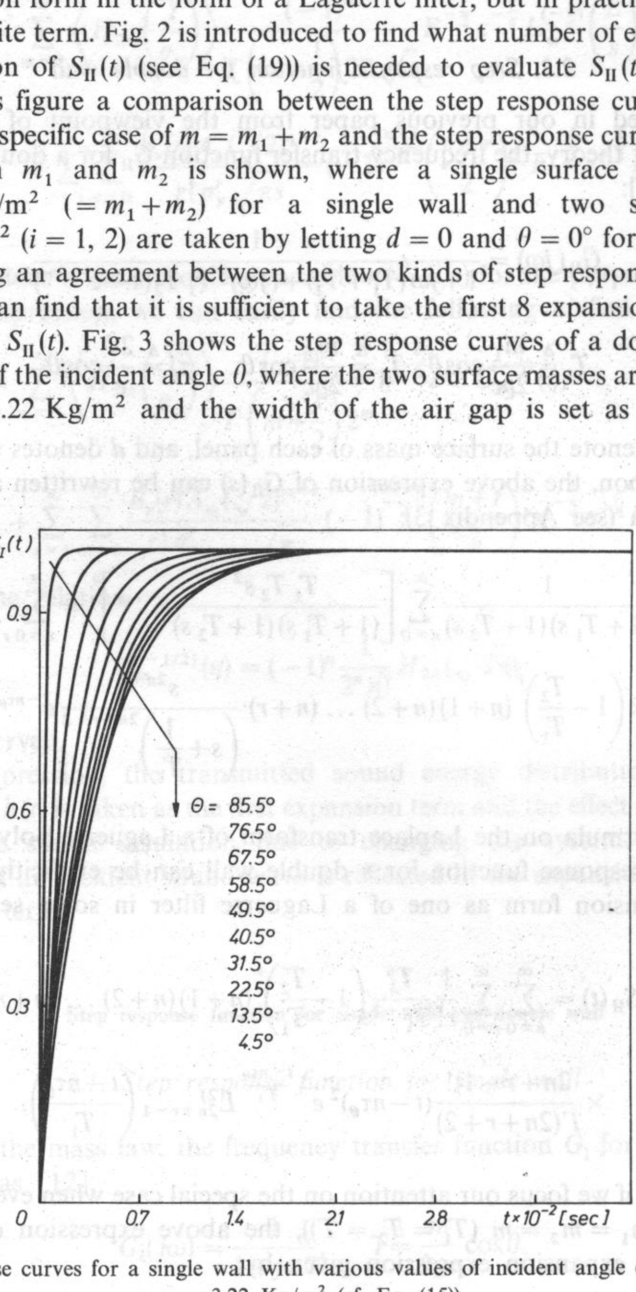

Fig. 1. Step response curves for a single wall with various values of incident angle θ, in the case with $m=3.22$ Kg/m^2 (cf. Eq. (15))

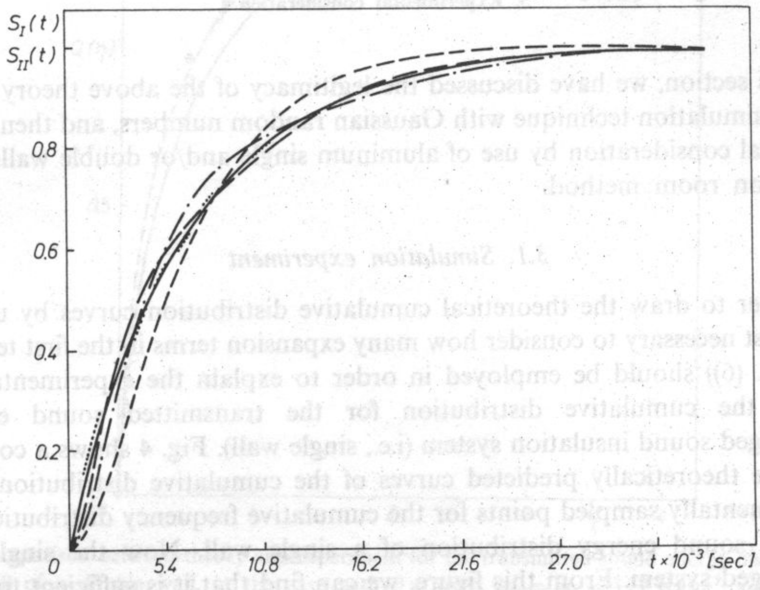

Fig. 2. A comparison between $S_{\mathrm{I}}(t)$ for a single wall and $S_{\mathrm{II}}(t)$ for a double wall, in the case with $m_1 = m_2 = 2.75$ Kg/m^2, $d = 0$ m and $\theta = 0°$. Theoretical curve of $S_{\mathrm{I}}(t)$ (cf. Eq. (15)) is shown by (———). Theoretical curves of $S_{\mathrm{II}}(t)$ (cf. Eq. (19)) are shown with the degree of approximation n [$n = 0$ (– – – –), $n = 1$ (—·—), $n = 2$ (— —) and $n = 7$ (..........)]

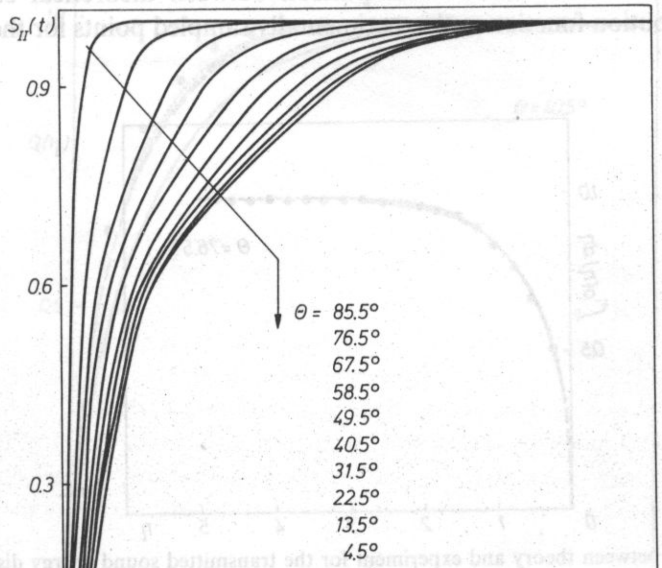

Fig. 3. Step response curves for a double wall with various values of incident angle θ, in the case with $m_1 = m_2 = 3.22$ Kg/m^2, $d = 0.05$ m (cf. Eq. (19))

3. Experimental consideration

In this section, we have discussed the legitimacy of the above theory by use of the digital simulation technique with Gaussian random numbers, and then by actual experimental consideration by use of aluminum single and/or double walls with the reverberation room method.

3.1. *Simulation experiment*

In order to draw the theoretical cumulative distribution curves by use of Eq. (13), it is first necessary to consider how many expansion terms in the first term of Eq. (13) (cf. Eq. (6)) should be employed in order to explain the experimental sample points of the cumulative distribution for the transmitted sound energy of a non-changed sound insulation system (i.e., single wall). Fig. 4 shows a comparison between the theoretically predicted curves of the cumulative distribution function and experimentally sampled points for the cumulative frequency distribution on the transmitted sound energy distribution of a single wall. Now the single wall is a non-changed system. From this figure, we can find that it is sufficient to take the first 3 expansion terms to calculate the distribution curve in the case with $\theta = 76.5°$ (we have confirmed that it is sufficient to take the first three expansion terms to fit the experimental sample points in other cases with various values of θ).

Figures 5, 6, 7 and 8 show a comparison between theoretical curves of the cumulative distribution function and experimentally sampled points for the cumulative

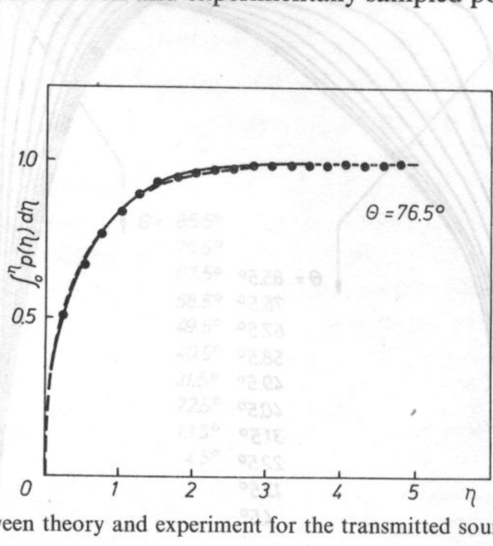

Fig. 4. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ ($\stackrel{\Delta}{=} \int_0^\eta P(\eta)d\eta$; $P(\eta)$ is evaluated by use of the first terms ($r = 0$) in Eq. (13) corresponding to a single wall) of a single wall, in the case with $\theta = 76.5$ and $m = 3.22$ Kg/m^2. Experimentally sampled points are marked by (•) and theoretical curves are shown with the degree of approximation m [$m = 0$ and 1 (----), $m = 2$ (——)]

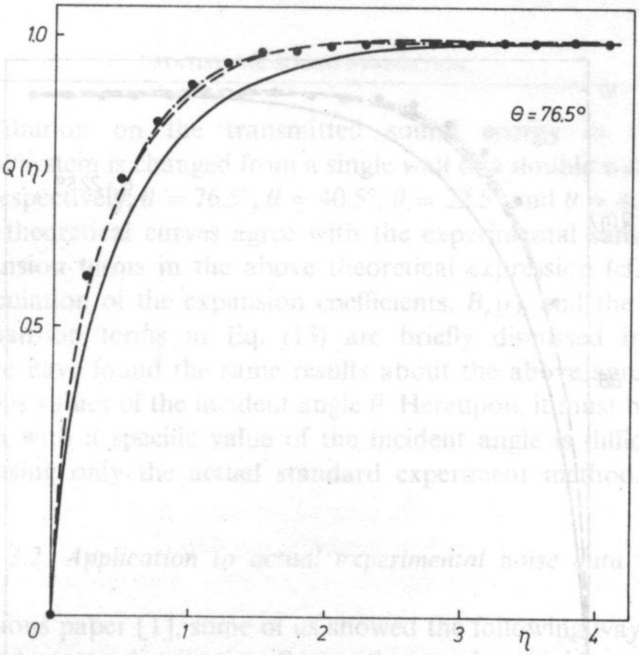

Fig. 5. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ (cf. Eq. (13)) of a double wall, in the case with $\theta = 76.5°$, $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1, 2, 3, 4$ and 5 (– – –), $r = 6$ (—·—)]

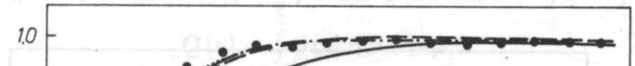

Fig. 6. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ (cf. Eq. (13)) of a double wall, in the case with $\theta = 40.5°$, $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1, 2$ and 3 (– – –), $r = 4$ and 5 (—·—), $r = 6$ (.........)]

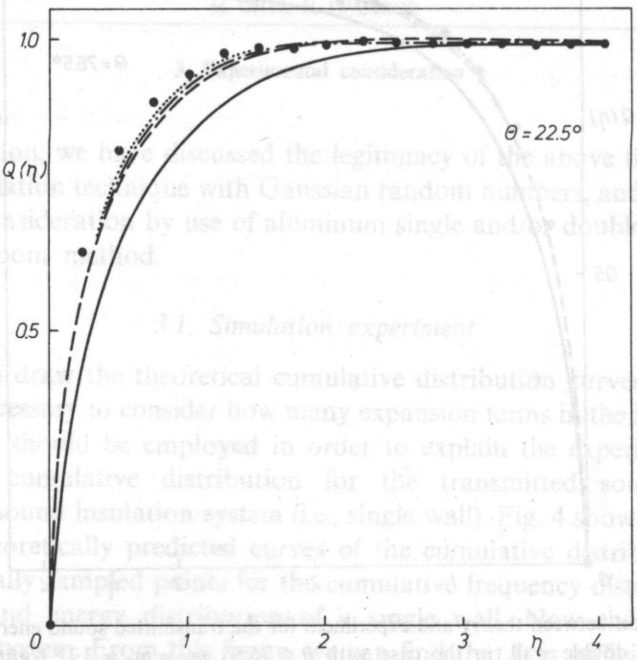

Fig. 7. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ (cf. Eq. (13)) of a double wall, in the case with $\theta = 22.5°$, $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1$, 2 and 3 (– – – –), $r = 4$ and 5 (—·—), $r = 6$ (........)]

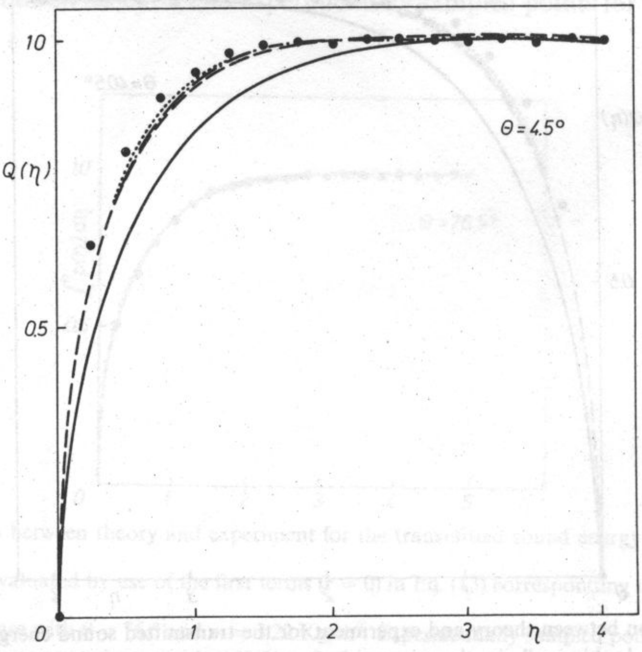

Fig. 8. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ (cf. Eq. (13)) of a double wall, in the case with $\theta = 4.5°$, $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1$, 2 and 3 (– – – –), $r = 4$ and 5 (—·—), $r = 6$ (........)]

frequency distribution on the transmitted sound energy in the case when the noise control system is changed from a single wall to a double wall, by setting the incident angle respectively; $\theta = 76.5°$, $\theta = 40.5°$, $\theta = 22.5°$ and $\theta = 4.5°$. It is obvious that the above theoretical curves agree with the experimental sample points with increasing expansion terms in the above theoretical expression (cf. Eq. (13)) (The methods of calculation of the expansion coefficients, $B_r(r)$, and the decision of the number of expansion terms in Eq. (13) are briefly discussed in Appendix 4). Furthermore, we have found the same results about the above agreement in other cases with various values of the incident angle θ. Hereupon, it must be observed that such a situation with a specific value of the incident angle is difficult to measure practically by using only the actual standard experiment method.

3.2. *Application to actual experimental noise data*

In the previous paper [1], some of us showed the following way of treating the cumulative sound energy distribution $Q(\eta)$ with a random incidence. By letting the transmitted sound energy distribution with a specific incident angle θ be the conditional distribution $P(\eta/\theta)$ with a fixed value of θ, the transmitted cumulative sound energy distribution with a random incidence can be directly expressed as follows (see Appendix 5):

$$Q(\eta) = \int_0^{\frac{\pi}{2}} Q(\eta|\theta)P(\theta)d\theta, \tag{20}$$

where:

$$Q(\eta/\theta) = \int_0^{\eta} P(\eta'/\theta)d\eta' \;\; (\eta' = E'/S) \quad \text{and:} \; P(\theta) = \sin 2\theta.$$

Our experiments corresponding to this case with a random incidence have been done in the laboratory of Hiroshima University by use of an aluminum single wall and/or double wall. Then, we have first recorded both the sound pressure waves in a source room and in a reception room (which are reverberant) by use of a data recorder. Next, we have converted these analogue waves to digital quantities by an A–D converter and then evaluated them with use of a digital computer. We have used road traffic noise actually recorded in Hiroshima City as one example of an arbitrary sound input source. Fig. 9 shows a comparison between the theoretically predicted curves and experimentally sampled points for the cumulative distribution function on the transmitted sound energy for a single wall ($m = 3.22$ Kg/m^2) before our system change. As seen too evidently in this figure, we find that it is sufficient to take the first 3 expansion terms in Eq. (13).

Figure 10 shows a comparison between the theoretically predicted curves and experimentally sampled points for the cumulative distribution on a transmitted sound energy for a double wall ($m_1 = m_2 = 3.22$ Kg/m^2, $d = 0.05$ m) after changing the system from a single wall to a double wall. It is obvious that the present theoretically predicted curves agree with the experimentally sampled points with increasing expansion terms in our theoretical expression.

Hereupon, from the fundamental viewpoint, we have applied a new consideration for a random incidence (cf. Eq. (20)) to the present case. However, in the architectural acoustics field, the coefficient of transmission of the sound intensity with a random incident property is usually given by use of the frequency transfer function of α as follows:

$$\tau = \int_0^{\frac{\pi}{2}} \frac{1}{|\alpha|^2} P(\theta) d\theta, \quad \text{with } P(\theta) = \sin 2\theta, \tag{21}$$

after first averaging $1/|\alpha|^2$ by $P(\theta)$ instead of averaging $Q(\eta|\theta)$ by $P(\theta)$.

Figure 11 shows a comparison between the theoretically predicted curves and experimentally sampled points for the cumulative distribution on a transmitted sound energy by use of the latter simplified method. In the above case, after finding the system characteristics due to averaging Eqs. (15) and (19) by $P(\theta)$, Eq. (13) itself has been used instead of using Eqs. (13) and (20) to draw the theoretically predicted distribution curves. By comparing Fig. 11 with Fig. 10 evaluated by the former exact method, though there is some difference between theory and experiment in this figure, in practice this simplified method is still useful for the case with a rough evaluation. Finally, a practical example of applying the proposed evaluation method to actual design problems has been illustrated in Appendix 6.

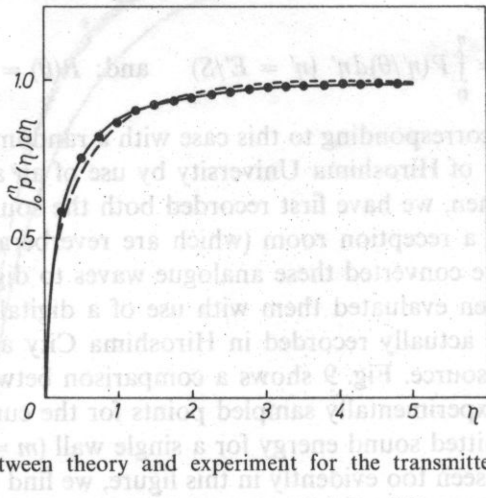

Fig. 9. A comparison between theory and experiment for the transmitted sound distribution $Q(\eta)$ $(\overset{\Delta}{=} \int_0^\eta P(\eta) d\eta$; $P(\eta)$ is evaluated by use of the first terms $(r = 0)$ in Eq. (13) corresponding to a single wall and the random incident property Eq. (20)) of single wall, in a case with $m = 3.22$ Kg/m^2. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation $m[m = 0$ (– – –), $m = 1, 2, 3$ and 4 (——)]

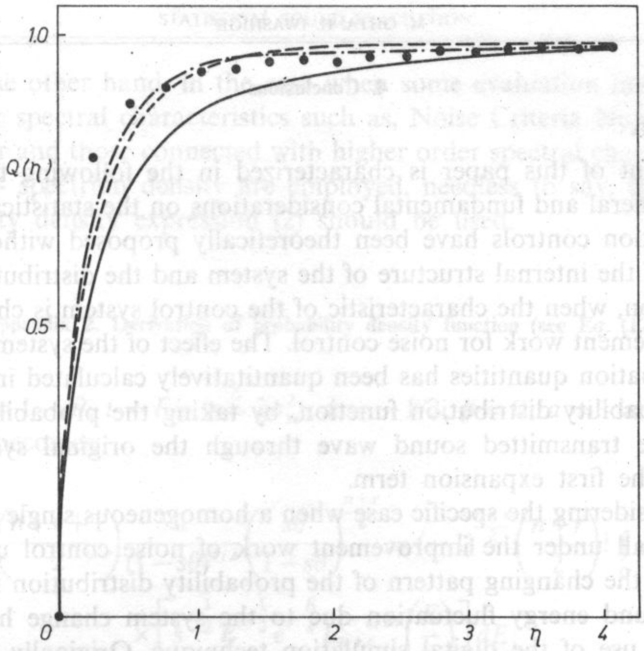

Fig. 10. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ (cf. Eqs. (13) and (20)) of a double wall, in the case with $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1, 2, 3$ and 4 (– – – –), $r = 5$ and 6 (—·—)]

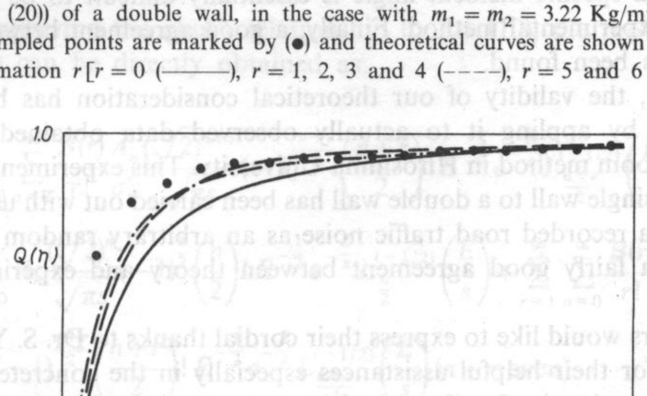

Fig. 11. A comparison between theory and experiment for the transmitted sound energy distribution $Q(\eta)$ of a double wall by use of a simplified method (cf. Eqs. (21) and (13)), in the case with $m_1 = m_2 = 3.22$ Kg/m^2 and $d = 0.05$ m. Experimentally sampled points are marked by (●) and theoretical curves are shown with the degree of approximation r [$r = 0$ (———), $r = 1, 2, 3$ and 4 (– – – –), $r = 5$ and 6 (—·—)]

4. Conclusions

The content of this paper is characterized in the following three points:

(1) The general and fundamental considerations on the statistical evaluation of noise or vibration controls have been theoretically proposed without any special assumptions of the internal structure of the system and the distribution type of the input fluctuation, when the characteristic of the control system is changed by some kind of improvement work for noise control. The effect of the system change on the statistical evaluation quantities has been quantitatively calculated in the expansion form of a probability distribution function, by taking the probability distribution function of the transmitted sound wave through the original system into consideration as the first expansion term.

(2) By considering the specific case when a homogeneous single wall is changed to a double wall under the improvement work of noise control under a specific incident angle, the changing pattern of the probability distribution function on the transmitted sound energy fluctuation due to the system change has been drawn graphically by use of the digital simulation technique. Originally, such a typical situation with a specific incident angle is essentially difficult to be examined only from a usual experimental method. Finally, a good agreement between theory and experiment has been found.

(3) Finally, the validity of our theoretical consideration has been confirmed experimentally by appling it to actually observed data obtained by using the reverberation room method in Hiroshima University. This experiment on the system change from a single wall to a double wall has been carried out with use of aluminum panels, taking a recorded road traffic noise as an arbitrary random incident wave. Consequently a fairly good agreement between theory and experiment has been found.

The authors would like to express their cordial thanks to Dr. S. Yamaguchi and Dr. K. Nagai for their helpful assistances especially in the concrete calculation of probability expression in Eq. (3) and of frequency transfer function in Appendices **2** and **3**.

Appendix 1. Usefulness of probability expression (see Eq. (4))

A typical case with $N = 1$ presented by Eq. (4) is obviously valid, if the statistics such as L_{eq} (equivalent sound pressure level), median, L_5 and L_{10} directly connected with a single-variate probability distribution form are employed as standard noise indexes. But, in order to estimate the expansion coefficients $B_r(r)$ ($r = 1, 2, 3, \ldots$) (cf. Eq. (5)), it should be observed that the statistical information on the bivariate correlation function so-called usual auto-correlation function and higher order multi-variate correlation functions for the input sound pressure fluctuation X_i must be used. On the other hand, in the case when some evaluation indexes connected with the power spectral characteristics such as, Noise Criteria Number and Noise Rating Number and those connected with higher order spectral characteristics (such as, bi- or poly- spectrum density are employed, needless to say, the multi-variate joint probability density expression (2) should be used.

Appendix 2. Derivation of probability density function (see Eq. (12))

Setting $p = 1 - s\theta$, $t = E/s$ $(s \overset{\Delta}{=} 2\sigma_y^2)$, $\alpha = -1/2$, $\beta = 0$, $a = 1$ and $n = (n+r)/2$, equation (11) becomes:

$$\Gamma\left(\frac{n+r+1}{2}\right)\frac{1}{(1-s\theta)^{1/2}}\left(\frac{s\theta}{1-s\theta}\right)^{\frac{n+r}{2}} = (-1)^{\frac{n+r}{2}}\left(\frac{n+r}{2}\right)!\int_0^\infty e^{\theta E} \times\left[s^{-\frac{1}{2}}E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n+r}{2}}^{(-1/2)}\left(\frac{E}{s}\right)\right]dE. \tag{A2.1}$$

Thus, by applying the above relationship to Eq. (10), the probability density function $P_E(E)$ can be directly obtained as:

$$\begin{aligned} P_E(E) &= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\frac{B(r)}{r!}\frac{A_n}{\sigma_y^r}\frac{(\sqrt{2})^{n+r}}{\sqrt{\pi s}}(-1)^{\frac{n+r}{2}}\left(\frac{n+r}{2}\right)!E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n+r}{2}}^{(-1/2)}\left(\frac{E}{s}\right) \\ &= \sum_{n=0}^{\infty}A_n\frac{(\sqrt{2})^n}{\sqrt{\pi s}}(-1)^{\frac{n}{2}}\left(\frac{n}{2}\right)!E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n}{2}}^{(-1/2)}\left(\frac{E}{s}\right) + \sum_{r=1}^{\infty}\sum_{n=0}^{\infty}\frac{B(r)}{r!}\frac{A_n}{\sigma_y^r}\frac{(\sqrt{2})^{n+r}}{\sqrt{\pi s}} \\ &\quad \times(-1)^{\frac{n+r}{2}}\left(\frac{n+r}{2}\right)!E^{-\frac{1}{2}}e^{-\frac{E}{s}}L_{\frac{n+r}{2}}^{(-1/2)}\left(\frac{E}{s}\right)(n+r\text{: even}). \end{aligned} \tag{A2.2}$$

In the first expansion term of Eq. (A2.2), setting $n = 2m$ and using the relationship $\Gamma(n+1/2) = (2n)!\sqrt{\pi}/2^{2n}n!$, we can obtain the probability density expression (12).

Appendix 3. Derivation of Expansion Expression Eq. (17)

Using the relationship:

$$\frac{1}{a-x} = \sum_{n=0}^{\infty}\frac{x^n}{a^{n+1}}, \tag{A3.1}$$

equation (16) can be rewritten as:

$$G_{II}(s)=\frac{1}{(1+T_1 s)(1+T_2 s)-T_1 T_2 s^2 e^{-\tau s}}$$

$$=\frac{1}{(1+T_1 s)(1+T_2 s)\left[1-\dfrac{T_1 T_2 s^2}{(1+Tsx)(1+T_2 s)}e^{-\tau s}\right]}$$

$$=\frac{1}{(1+T_1 s)(1+T_2 s)}\sum_{n=0}^{\infty}\left[\frac{T_1 T_2 s^2}{(1+T_1 s)(1+T_2 s)}e^{-\tau s}\right]^n. \tag{A3.2}$$

After setting $T_2 = T_1+\Delta$ in the above equation, and making use of the formula:

$$(a+x)^k=\sum_{r=0}^{\infty}\frac{k(k-1)\ldots(k-r+1)}{r!}a^{k-r}x^r, \tag{A3.3}$$

we have:

$$G_{II}(s)=\sum_{n=0}^{\infty}\frac{T_1^n.(T_1+\Delta)^n s^{2n}}{(1+T_1 s)^{n+1}(1+T_1 s+\Delta s)^{n+1}}e^{-n\tau s}$$

$$=\sum_{n=0}^{\infty}\frac{T_1^n(T_1+\Delta)^n s^{2n}}{(1+T_1 s)^{2n+2}}\left[1+\frac{\Delta s}{1+T_1 s}\right]^{-(n+1)}e^{-n\tau s}$$

$$=\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}T_1^n(T_1+\Delta)^n\frac{(-1)^r(n+1)(n+2)\ldots(n+r)}{r!}\frac{\Delta^r s^{2n+r}}{(1+T_1 s)^{2n+r+2}}e^{-n\tau s}$$

$$=\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{1}{r!}T_1^{n+r}T_2^n\left(1-\frac{T_2}{T_1}\right)^r(n+1)(n+2)\ldots(n+r)\frac{s^{2n+r}}{(1+T_1 s)^{2n+r+2}}e^{-n\tau s}$$

$$=\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{1}{r!}\frac{T_2^n}{T_1^{n+2}}\left(1-\frac{T_2}{T_1}\right)^r(n+1)(n+2)\ldots(n+r)\frac{s^{2n+r}}{\left(s+\dfrac{1}{T_1}\right)^{2n+r+2}}e^{-n\tau s}. \tag{A3.4}$$

Appendix 4. Evaluation of expansion coefficient and the number of expansion terms in Eq. (13)

First, the value of finite order K in Eq. (1) should be chosen (In our experiment related to Figs. 10 and 11, we have set $K = 10$). The sampled weighting values b_i and $a_i (i = 1, 2, \ldots, K)$ are concretely calculated by using step response curves of the original and the altered systems as:

$$b_i = S_I(t_i)-S_I(t_{i-1}),\quad a_i = S_{II}(t_i)-S_{II}(t_{i-1}),\ t_i \overset{\Delta}{=} i\times T, \tag{A4.1}$$

where T is the sampling period (in the present experiment, T is set to 0.003 s.). On the other hand, the statistical properties related to the lower order cumulant functions on the input sound pressure fluctuation can be experimentally evaluated as follows:

$$
\begin{aligned}
&\varkappa_{X1}(0, 0, \ldots, 0, \overset{P_1}{1}, 0, \ldots, 0) = \langle X_{P_1} \rangle, \\
&\varkappa_{X2}(0, 0, \ldots, 0, \overset{P_1}{1}, 0, \ldots, 0, \overset{P_2}{1}, 0, \ldots, 0) = \langle (X_{P_1} - \langle X_{P_1} \rangle)(X_{P_2} - \langle X_{P_2} \rangle) \rangle, \\
&\varkappa_{X3}(0, 0, \ldots, 0, \overset{P_1}{1}, 0, \ldots, 0, \overset{P_2}{1}, 0, \ldots, 0, \overset{P_3}{1}, 0, \ldots, 0) \\
&\qquad = \langle (X_{P_1} - \langle X_{P_1} \rangle)(X_{P_2} - \langle X_{P_2} \rangle)(X_{P_3} - \langle X_{P_3} \rangle) \rangle, \\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
\tag{A4.2}
$$

by using experimentally sampled data X_i $(i = 1, 2, \ldots)$ on the input sound pressure waves $X(t)$. Thus, substituting the concrete values of b_i, a_i $(i = 1, 2, \ldots, K)$ and Eq. (A4.2) into Eq. (5), the expansion coefficients, $B_r(r)$ $(r = 1, 2, \ldots)$, can be concretely calculated.

Next, let us discuss the decision problem on the number of expansion terms for the probability density expression (13). From the arbitrariness of the weighting functions, $h(t)$ and $W(t)$, for the original and the altered noise control systems, the probability density distribution and correlation functions of input signal $X(t)$, and the functional form of $P_y(y)$ in Eq. (4), we can not find generally and systematically any conclusion for the above decision problem only from the mathematical viewpoint. Principally, this problem must be considered by taking both sides on the theoretical convergency property of the expansion expression and the experimental reliability in an estimation of the expansion coefficients $B_r(r)$ (especially related to the higher order cumulants). In this paper, we have only pointed out in the conclusion section that this problem will have to be considered as future study. However, for the purpose of examining partly the legitimacy of the proposed expression Eq. (4), it can be shown in the following that the well-known Gram-Charlier A type series expansion [8] on the arbitrary type probability expression is contained as a special case in Eq. (4).

Let us especially choose the Gaussian distribution as an arbitrary probability density function, $P_y(z)$, in Eq. (4), as:

$$
P_y(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{(z-\mu_z)^2}{2\sigma_z^2}}, \quad (\mu_z = \langle z \rangle, \quad \sigma_z^2 = \langle (z - \langle z \rangle)^2 \rangle), \tag{A4.3}
$$

by setting $\varkappa_{y1}(1) = \varkappa_{z1}(1) \overset{\Delta}{=} \mu_z$, $\varkappa_{y2}(2) = \varkappa_{z2}(2) \overset{\Delta}{=} \sigma_z^2$ $(\varkappa_{yl}(l) = 0,\ l \geqslant 3)$. At this time, the probability density expression (4) can be easily rewritten as follows (see Eq. (7)):

$$
P_z(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{(z-\mu_z)^2}{2\sigma_z^2}} \sum_{r=0}^{\infty} C_r H_r\left(\frac{z-\mu_z}{\sigma_z}\right). \tag{A4.4}
$$

Hereupon, the expansion coefficient, C_r, is given as:

$$C_r \triangleq \frac{1}{r!\,\sigma_z^r} B_r(r) = \frac{1}{r!\,\sigma_z^r} \frac{\partial^r}{\partial\theta^r} \exp\left\{ \sum_{l=1}^{\infty} \frac{1}{l!} \left[\varkappa_{zl}(l) - \varkappa_{yl}(l) \right] \theta^l \right\}\bigg|_{\theta=0}$$

$$= \frac{1}{r!\,\sigma_z^r} \frac{\partial^r}{\partial\theta^r} \left[\langle \exp(z\theta) \rangle \exp\left(-\mu_z \theta - \frac{1}{2}\sigma_z^2 \theta^2\right) \right]\bigg|_{\theta=0}$$

$$= \frac{1}{r!} \left\langle \frac{\partial^r}{\partial(\sigma_z \theta)^r} \exp\left(z\theta - \mu_z \theta - \frac{1}{2} \sigma_z^2 \theta^2 \right)\bigg|_{\theta=0} \right\rangle \tag{A4.5}$$

$$= \frac{1}{r!} \left\langle \frac{\partial^r}{\partial t^r} \exp\left(\frac{z-\mu_z}{\sigma_z} t - \frac{1}{2} t^2 \right)\bigg|_{t=0} \right\rangle = \frac{1}{r!} \left\langle H_r\left(\frac{z-\mu_z}{\sigma_z} \right) \right\rangle.$$

Here, the relationship between the exponential function and the Hermite polynomials:

$$H_r(\xi) = \frac{\partial^r}{\partial t^r} \exp\left(\xi t - \frac{1}{2} t^2 \right)\bigg|_{t=0} \tag{A4.6}$$

has been used. Furthermore, we can easily find $C_0 = 1$, $C_1 = C_2 = 0$ from $\mu_z = \langle Z \rangle$ and $\sigma_z^2 = \langle (Z - \langle Z \rangle)^2 \rangle$. Thus, equation (A4.4) is reduced accurately to the well-known Gram-Charlier A type series expansion expression, as follows:

$$P_z(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{(z-\mu_z)^2}{2\sigma_z^2}} \left\{ 1 + \sum_{r=3}^{\infty} \frac{1}{r!} \left\langle H_r\left(\frac{z-\mu}{\sigma_z} \right) \right\rangle H_r\left(\frac{z-\mu_z}{\sigma_z} \right) \right\}. \tag{A4.7}$$

Finally, in all figures in this paper, it should be noticed that the tendency of converging to a certain proper cumulative probability distribution curve can be obviously found out and so can be examined too by finding such a saturation tendeny from an experimental viewpoint, as pointed out in section 3.

Appendix 5. Engineering background of Eq. (20)

As is well-known, by letting $E(\theta_i)$ be a transmitted output energy component from the input sound energy with a specific incident angle θ_i, the total energy E of the transmitted sound wave with random incident angles can be expressed from a deterministic viewpoint as follows:

$$E = \sum_i E(\theta_i) P(\theta_i) d\theta \rightarrow \int_0^{\pi/2} E(\theta) P(\theta) d\theta, \tag{A5.1}$$

$$P(\theta) = \frac{\cos\theta \sin\theta}{\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta} = \sin 2\theta \quad (0 \leqslant \theta \leqslant \pi/2),$$

where $p(\theta_i)\,d\theta$ denotes the rate of energy component between θ_i and $\theta_i + d\theta$. If the above viewpoint is generalized for the statistical evaluation of the insulation system by newly introducing any kind of probabilistic viewpoint, equation (A5.1) must be first satisfied in the averaged form of energy fluctuation:

$$\langle E \rangle = \sum_i \langle E|\theta_i \rangle P(\theta_i) d\theta \rightarrow \int_0^{\pi/2} \langle E|\theta \rangle P(\theta)\,d\theta, \tag{A5.2}$$

by use of the well-known additive property of mean energy. Hereupon, $\langle E|\theta \rangle$ is the conditional expectation of energy with a fixed incident angle θ. For the purpose of establishing some kind of probabilistic law for the total energy fluctuation of the transmitted sound wave with random incident angles, naturally we have to find it in a unified probabilistic expression form of generalizing the above equation (A5.2) Under the above background, equation (20) has been rationally introduced as a unified probabilistic law of transmission with random incident angles.

Appendix 6. An example of applying the present theory to an actual design problem

As an application example of the present statistical method, let us consider a statistical prediction method for the sound insulation effect of single and double walls. Let E_α and E'_α, respectively, be the $(100 - \alpha)$ percentage point of the output sound energy distribution of the original noise control system with $h(t)$ and that of the changed system with $W(t)$. At this time, an improvement of the evaluation index L_α can be quantitatively given as follows:

$$\Delta L_\alpha \overset{\Delta}{=} L_\alpha - L'_\alpha = 10 \log_{10} \frac{E_\alpha}{E'_\alpha}, \tag{A6.1}$$

where E'_α is calculated from the first expansion term of Eq. (12). Now, let us consider a prediction problem of improving the above L_α in a typical case when a sound insulation wall such as single or double walls is newly constructed. In this case, the present theory can be directly used by setting $h(t) = \delta(t)$ (Dirac's δ-function). We have employed the following two kinds of walls as examples of noise insulation systems:

Case A: A single wall with surface mass $m = 3.22$ (Kg/m^2).

Case B: A double wall with two panels of surface mass 3.22 Kg/m^2 (i.e., $m_1 = m_2 = 3.22$ Kg/m^2) and width of air gap $d = 0.05$ m.

Furthermore, the Gaussian random numbers have been used as the standard input sound pressure fluctuation to the noise control system, in the present simulation experiment.

Figure A6.1 shows a comparison between theoretically predicted values and

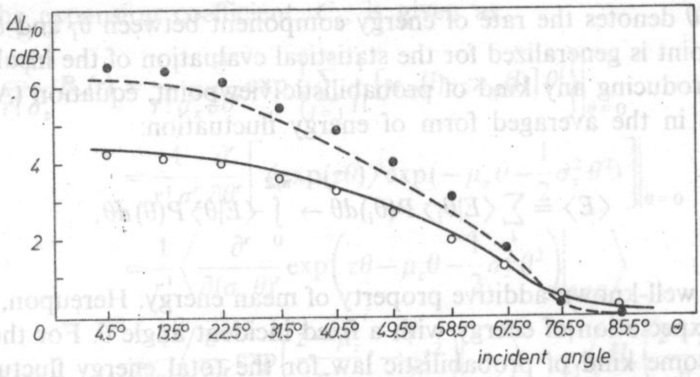

Fig. A6.1. A comparison between theoretically predicted values and experimentally sampled points for ΔL_{10}. Experimentally sampled points are marked by (○) and theoretical values are shown by (——) for Case A. Experimentally sampled points are marked by (●) and theoretical values are shown by (– – – –) for Case B

experimentally sampled points for ΔL_{10} with various values of the incident angle θ, where it is very difficult to measure by using only the actual standard experiment method. It must be observed that the prediction accuracy remains within the range less than a practically permissible error 1 dB.

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