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AN ANALYSIS OF ACOUSTIC WAVE RADIATION CONDITIONS WITHIN THE STRUCTURAL SYSTEM OF AN AXIAL DYNAMIC GENERATOR

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This paper analysis radiation conditions of an acoustic wave in an axial generator with outlets of the stator entering a common horn with a ring-shaped cross section. Acoustic properties of the generator horn, including the wave reflected from the outlet, were considered. The influence of the impedance misfit in the outlets of stater openings-horn inlet system, on the acoustic power emission was estimated. A theoretical and experimental analysis was performed in order to determine the influence of a finite horn length on the power emission and generator efficiency.

1. Introduction

Acoustic axial dynamic generators belong to a group of flow generators which convert energy of compressed air into acoustic energy. The vertical section in Fig. 1

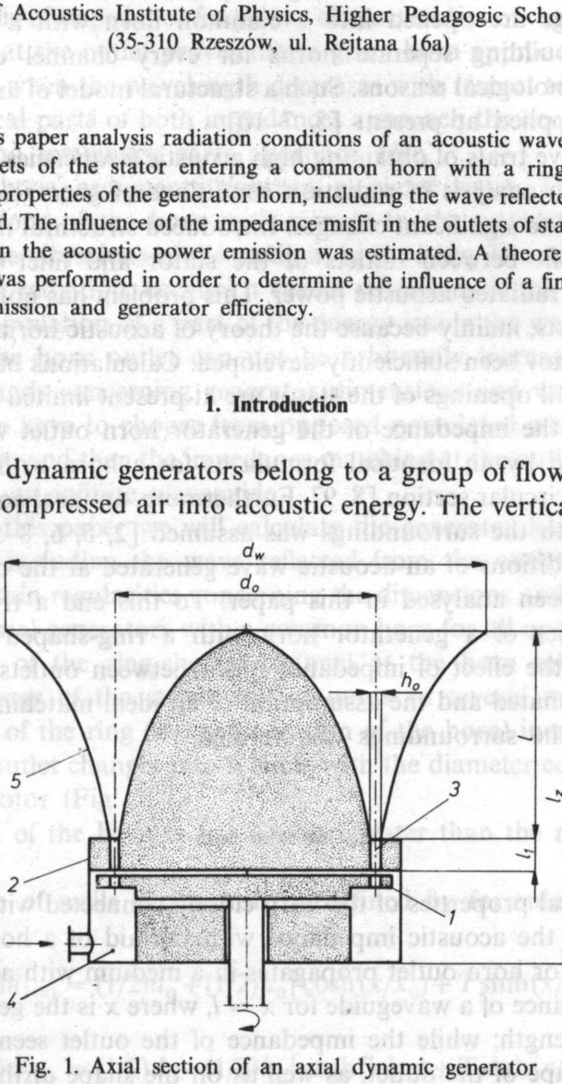

Fig. 1. Axial section of an axial dynamic generator

shown main elements of the structural system of an axial dynamic generator. An air flux which escapes the pressure chamber (4) through channels (3) in the plate of the stator (2) is periodically cut-off by a rotating rotor (1) with openings spaced in a regular manner on its perimeter. Thus, a sound wave is generated at the outlet of the stator channel and radiated outside through the horn (5).

In first constructions of generators [2] the stator was equipped with few holes only. Every one of them had a separate acoustic horn. Such a structure is still applied in audio frequency, high power generators [11]. Trials of constructing generators of high frequency waves have led to structural changes in the stator-horn system, such as increasing the number of openings (from tens to several hundred) with small diameter (several milimeters), spaced regularly on the perimeter of the rotor and stator. All openings are opened into a common horn with a ring-shaped cross section, because building seperate horns for every channel of the stator was impossible for technological reasons. Such a structural model of an axial generator is most frequently applied at present [5, 7–10].

Presented above trials of obtaining high acoustic frequencies generally were not accompanied by an analysis of radiation conditions of generated waves, although they were subjected to significant changes. Introduced structural modification causes an impedance misfit between outlets of the stator and inlet of the horn, thus inevitable losses of radiated acoustic power. This problem has not hitherto attracted attention of scientists, mainly because the theory of acoustic horns with ring-shaped cross sections has not been sufficiently developed. Calculations of generators with a common horn for all openings of the stator are at present limited to approximations which accept that the impedance of the generator horn outlet with a ring-shaped section is expressed by an identical formula as for a horn with an identical wall profile, but with a circular section [8, 9]. Furthermore, an ideal acoustic matching of the horn outlet with the surroundings was assumed [2, 5, 6, 8–10, 12–15, 18, 19].

Radiation conditions of an acoustic wave generated at the outlet of the stator channel (3) have been analysed in this paper. To this end a trial of determining acoustical properties of a generator horn with a ring-shaped section has been undertaken. Then the effect of impedance misfit between outlets of the stator and horn inlet was estimated and the assumption of an ideal matching of the generator horn outlet with the surroundings was verified.

2. Generator horn

Certain physical properties of the horn closely connected with its geometry are required to match the acoustic impedance with the aid of a horn. A wave which reaches the generator horn outlet propagates in a medium with acoustic impedance equal to the impedance of a waveguide for $x = l$, where x is the generator axis of the horn and l is its length; while the impedance of the outlet seen from the outside depends on the shape of the outlet, as well as on the shape of the isophase surface and length of the wave leaving the horn. If real parts of both impedances are equal and their imaginary parts are equal to zero, then a wave which reaches the horn outlet will be radiated into the surrounding medium without obstacles and the generator will radiate outside maximum of the acoustic energy. The equality of real parts of the impedance prevents a reflection of the wave from the outlet, while the absence of their imaginary parts means that a phase shift between the pressure and vibration velocity of particles of the medium does not occur. Such a specific case of ideal impedance matching at the outlet is called a "horn with infinite length" [16], because only in a hypothetical, infinite horn a wave propagating towards the outlet is not accompanied by a reflected wave. In such a case the total acoustic power is released on the real part of the outlet impedance. This impedance can be considered focused. An analysis of impedance frequency characteristics of the outlet as well as of an arbitrary horn at the outlet proves that a given horn approaches the ideal of an infinite waveguide when the wavelength decreases with respect to the dimensions of the outlet. Then real parts of both impedances approach the acoustic self-resistance of the medium and imaginary parts approach zero.

Mentioned above arguments concern a plane wave. The enlargement of transverse dimensions of the horn with respect to the wavelength could lead to amplified transverse vibrations of the medium in the horn. It would have an adverse influence on power radiation, because it causes reflections of the wave from walls of the horn and accumulation of a part of the energy inside the waveguide. Therefore, the diameter of the horn outlet can not be arbitrarily increased, even when the construction demands concerning generator dimensions and weight are neglected.

In practice we have to choose from opposed postulated when determining the dimension of a horn and then the impedance matching at the outlet can greatly differ from the ideal of an infinite waveguide.

Therefore, in this paper we will calculate the generator horn as a "horn with finite length", i.e. including the wave reflected from the outlet.

There are certain regularities concerning the dimensions and shape of a horn in constructions of axial generators with a common horn for all openings of the stator:

– the width of the ring-shaped channel at the horn inlet is equal to the diameter of openings of the stator and amounts to several milimeters,

– the width of the ring (the cross-section of the horn) increases from inlet to outlet and at the outlet changes into a circle with the diameter equal to the doubled diameter of the rotor (Fig. 2),

– the length of the horn is 1–1.5 times greater than the radius of the outlet opening,

– the profile of walls of the horn is defined by formulae [17]

$$(1/2)d_{1(x)} = (1/2)d_0 + (1/2)h_0\left[\cosh(x/x_0) + T\sinh(x/x_0)\right], \tag{1}$$

$$(1/2)d_{2(x)} = (1/2)d_0 - (1/2)h_0\left[\cosh(x/x_0) + T\sinh(x/x_0)\right], \tag{2}$$

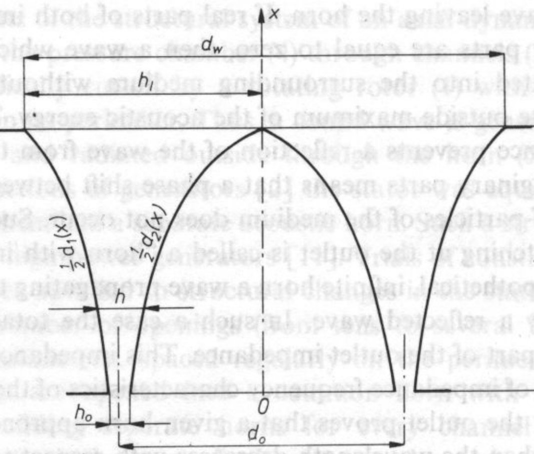

Fig. 2. Axial section of the horn of an axial generator

where x_0 is a constant determining the rate of flare. In practice parameter T most frequently equals one or zero, what corresponds to an exponential or catenoidal profile of walls, respectively. Hence, we will limit further consideratons to the case of $T \in [0, 1]$.

It results from formulae (1) and (2) that the crosssectional area S of the generator horn has the following form

$$S = S_0(\cosh\alpha + T\sinh\alpha), \tag{3}$$

where $\alpha = x/x_0$, $S_0 = \pi d_0 h_0$ is the area of the inlet. On the basis of [16] and [17] it can be stated that the equation of wave propagation in a horn (3) has the following form

$$d^2F/d\alpha^2 + [\mu^2 - V_{(\alpha)}]F = 0, \tag{4}$$

where F is the so-called wave function, μ is the dimensionless frequency and function $V_{(\alpha)}$, which characterizes the geometry of the horn, is expressed by

$$V_{(\alpha)} = 1/2 - (1/4)\mathrm{tgh}^2(\alpha + \Omega), \tag{5}$$

where Ω is an abbreviated notation of

$$\Omega = \mathrm{arctgh}\, T. \tag{6}$$

It should be noted that equation (4) describes the propagation of a plane harmonic wave, which propagates in the generator horn without energy loss [16]. This equation can be solved in an approximate manner for values $T \in [0, 1)$, while for $T = 1$ we achieve a linear differential equation with constant coefficients and with an accurate solution in the finite form [3].

Let us consider closer the case of $T = 1$. From (6) and (5) it results that wave equation (4) will accept the following form

$$d^2F/d\alpha^2 + K^2F = 0 \tag{7}$$

where K is independent of position and is expressed by formula

$$K^2 = \mu^2 - 1/4. \tag{8}$$

The solution of equation (7) has the following form [16]

$$F = Ae^{\pm iK\alpha}, \tag{9}$$

where A is independent of position. It results from (8) and (9) that the frequency which delimits periodic solutions from aperiodic ones, and is frequently called the cut-off frequency of a horn, is equal to

$$\mu_{gr} = 1/2. \tag{10}$$

From the theory of acoustical horns [16] we know that for a case of K independent of position, the admittance* of a horn with finite length is expressed by formula

$$\beta = \frac{K}{\mu}\text{ctgh}\left\{\pi\left[\varepsilon - i\left(\delta - \frac{K\alpha}{\pi}\right)\right]\right\} + \frac{i}{2\mu}\frac{dS}{Sd\alpha}. \tag{11}$$

The real and imaginary part of the expression under the hyperbolic cotangent characterize the amplitude decrease and phase shift of the wave reflected from the horn outlet, respectively. Within the framework of mathematic formalizm accepted in the theory of horns [16], quantity $2\pi\varepsilon$ is equal to the amplitude ratio of the reflected and incident wave, and expression $2\pi(\delta - K\alpha/\pi)$ characterizes the phase shift between the wave progressing towards the outlet and the reflected wave.

Including (3), (8), (11) in the case of the horn $T = 1$ under consideration, $\mu > \mu_{gr}$, we achieve the following formula for admittance

$$\beta = \sqrt{1 - \frac{1}{4\mu^2}}\text{ctgh}\left\{\pi\left[\varepsilon - i\left(\delta - \frac{\sqrt{\mu^2 - \frac{1}{4}}}{\pi}\alpha\right)\right]\right\} + \frac{i}{2\mu}. \tag{12}$$

Quantities ε and δ can be derived from the boundary condition

$$\beta_l = \beta_w, \tag{13}$$

where β_l is the elementary relative admittance of the horn outlet, while β_w is the elementary relative admittance of the outlet opening of the horn seen from the outside. β_l is obtained by substituting $\alpha = \alpha_l$ in (12), where α_l is the dimensionless abscissa of the horn outlet. β_w can be achieved by rearranging the well-known

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* At this stage of considerations we will use the notion of admittance in place of impedance in order to simplify used formulae.

Rayleigh formula for elementary acoustic relative impedance of a circular piston in an infinite baffle. Thus, introducing abbreviated notations, condition (13) has the following form

$$\operatorname{ctgh}[\pi(\varepsilon - i\sigma_l)] = Q + iW, \tag{14}$$

where

$$\sigma_l = \delta - \frac{\sqrt{\mu^2 - \frac{1}{4}}}{\pi}\alpha_l, \tag{15}$$

$$Q = \left(1 - \frac{1}{4\mu^2}\right)^{-1/2} \frac{w^2 - 2wI_{1(w)}}{[w - 2I_{1(w)}]^2 + 4[S_{1(w)}]^2}, \tag{16}$$

$$W = \left(1 - \frac{1}{4\mu}\right)^{-1/2} \frac{2wS_{1(w)}}{[w - 2I_{1(w)}]^2 + 4[S_{1(w)}]^2} - \frac{1}{2\sqrt{\mu^2 - \frac{1}{4}}} \tag{17}$$

$I_{1(w)}$ and $S_{1(w)}$ denote Bessel and Struve functions of the first order, while

$$w = kd_w \tag{18}$$

where k is a wave number and d_w is the diameter of the horn outlet.

Rather arduous rearrangements allow us to determine ε and σ_l from (14):

$$2\pi\varepsilon = \operatorname{arctgh}[2Q/(1 + Q^2 + W^2)] \tag{19}$$

$$2\pi\sigma_l = \operatorname{arctgh}[2W/(Q^2 + W^2 - 1)]. \tag{20}$$

Then, having σ_l we can determine δ from (15)

$$\delta = \sigma_l + \frac{\sqrt{\mu^2 - \frac{1}{4}}}{\pi}\alpha_l. \tag{21}$$

When we include (21) in the imaginary part of the expression under the hyperbolic cotangent in formula (12), we reach a conclusion that the phase shift between the wave reflected from the horn outlet and the incident wave, is a linear function in terms of position on the horn axis:

$$\sigma_{(\alpha)} = \sigma_l + \frac{\sqrt{\mu^2 - \frac{1}{4}}}{\pi}(\alpha_l - \alpha). \tag{22}$$

Whereas, quantity ε, which characterizes the amplitude ratio of the reflected and incident wave, is constant (formula (19)) and depends solely on the impedance at the horn outlet.

At present we will consider cases of $\mu = \mu_{gr}$ and $\mu < \mu_{gr}$. Two methods can be applied to the limiting case, $\mu = \mu_{gr}$ (i.e. $K = 0$). The first one consists in a rather complicated rearrangement of formula (12) with repeated application of the de l'Hospital principle; while the second one is based on the solution of the wave equation. In the case of $K = 0$ this solution has the following form:

$$d^2F/d\alpha^2 = 0, \tag{23}$$

Both methods lead to the same final result

$$\beta = i[1-2/(C+\alpha)], \tag{24}$$

where C is expressed by

$$C = 2\left\{\frac{i[w^2-2wI_{1(w)}]-2wS_{1(w)}}{[w-2I_{1(w)}]^2+4[S_{1(w)}]^2}+1\right\}^{-1} - \alpha_l. \tag{25}$$

In the case of $\mu < \mu_{gr}$, in formula (12) we accept

$$K = -i\sqrt{\frac{1}{4}-\mu^2} \tag{26}$$

and reach

$$\beta = \frac{-i\sqrt{\frac{1}{4}-\mu^2}}{\mu}\operatorname{ctgh}\left\{\pi\left[\left(\varepsilon+\frac{\sqrt{\frac{1}{4}-\mu^2}}{\pi}\alpha\right)-i\delta\right]\right\}+\frac{i}{2\mu}. \tag{27}$$

Introducing $\alpha = 0$ in formulae (12), (22), (24) and (27) we achieve the admittance and thus – the impedance of the horn inlet. This is the most useful quantity in the analysis of wave radiation conditions in the structural system of a generator.

To conclude our considerations of the case of $T = 1$ we will present a numerical example and calculate the elementary relative impedance of the horn outlet in terms of frequency on the basis of formulae (12), (24) and (27). The following dimensions of the horn were accepted for calculations:

- width of the horn channel at the inlet $h_0 = 1.5\cdot 10^{-3}$ m,
- inside diameter of the ring-shaped horn channel $d_0 = 10^{-1}$ m,
- diameter of the horn outlet $d_w = 2\cdot 10^{-1}$ m,
- horn length $l = 15\cdot 10^{-2}$ m.

These dimensions are typical for designs of axial flow generators.

Diagrams in Figs. 3 and 4 illustrate calculation results. Oscillations caused by the reflection of the wave from the horn outlet can be seen in frequency characteristics of real and imaginary parts of the inlet impedance. In the range of low frequencies oscillations of the inlet impedance are considerable and can influence the acoustic power emission and efficiency of the generator.

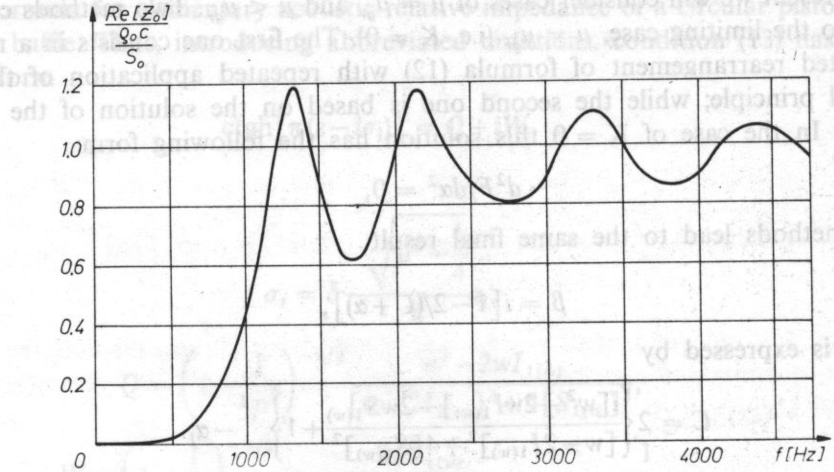

Fig. 3. Inlet acoustic resistance of the generator horn with an exponential profile

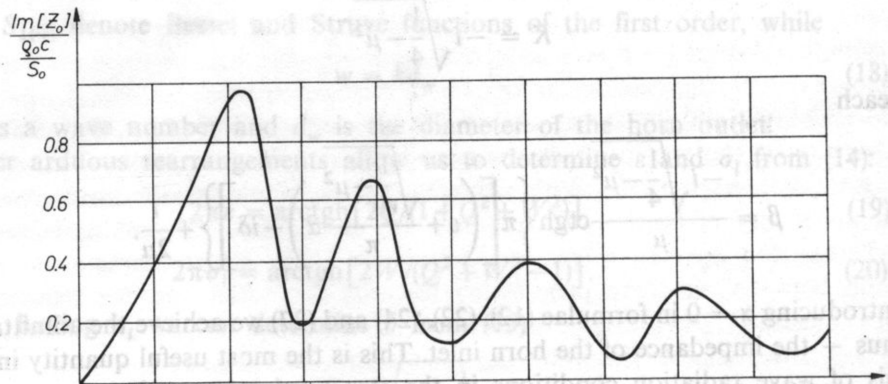

Fig. 4. Inlet acoustic reactance of the generator horn with an exponential profile

Now we will consider the case of $T \in [0, 1)$. It results from (4) that the frequency delimiting periodic ($\mu^2 \geqslant V_{(\alpha)}$) and aperiodic ($\mu^2 < V_{(\alpha)}$) solutions is equal to

$$\mu_{gr} = = \sqrt{1/2-(1/4)\,\mathrm{tgh}^2(\alpha+\Omega)}. \tag{28}$$

We can see from formula (28) that the cut-off frequency is a function of the position on the axis of the horn, when values from interval $[\mu_{gr1}, \mu_{gr2}]$ are accepted. While,

$$\mu_{gr_1} = \sqrt{1/2-(1/4)\,\mathrm{tgh}^2(\alpha_l+\Omega)}, \tag{29}$$

$$\mu_{gr_2} = \sqrt{1/2-(1/4)\,\mathrm{tgh}^2\,\Omega}. \tag{30}$$

This is the so-called broadening of the cut-off frequency of a horn and it has been described in [17]. An approximate method of solving the wave equation of a horn, which was given in [18] will be applied in order to calculate the admittance. This method consists in the approximation of function $V_{(\alpha)}$ by a broken line. Horn $T \in [0, 1)$ is considered to be a polyadic horn and every unit corresponds to one segment of the broken line. A rearrangement of formulae given in [18] results in an expression for the elementary relative impedance of the n-unit of the horn

$$\beta_n = \frac{-i}{\mu}\left\{\frac{\sqrt{C_n(\alpha+b_n)}\,[I_{-\frac{2}{3}}(\frac{2}{3}\xi^{3/2}) - D_n I_{\frac{2}{3}}(\frac{2}{3}\xi^{3/2})]}{I_{\frac{1}{3}}(\frac{2}{3}\xi^{3/2}) + D_n I_{-\frac{1}{3}}(\frac{2}{3}\xi^{3/2})} - \frac{1}{2}\mathrm{tgh}(\alpha+\Omega)\right\}, \tag{31}$$

where C_n — slope of the n-segment of the broken line, $b_n = (\mu^2 - V_{(\alpha n-1)})/C_n$, $V_{(\alpha n-1)}$ denotes the value of function $V_{(\alpha)}$ at the inlet of the n-unit of the horn, D_n — constant derived by equating the outlet admittance of the n-unit and the inlet admittance of the $(n+1)$-unit of the horn.

Quantity ξ, which is found in the argument of the Bessel function in formula (31), is expressed by the following expression

$$\xi = C_n^{1/3}(\alpha + b_n). \tag{32}$$

Formula (31) is valid for $\xi > 0$. Rearranging general relationships given in [18] we reach an expression for the admittance of horns under consideration for a case of $\xi = 0$ and $\xi < 0$:

for $\xi = 0$

$$\beta_n = \frac{-i}{\mu}\left[\frac{(3C_n)^{1/3}\Gamma(\frac{2}{3})}{D_n\Gamma(\frac{1}{3})} - \frac{1}{2}\mathrm{tgh}(\alpha+\Omega)\right], \tag{33}$$

where Γ is the Euler function:

and for $\xi < 0$

$$\beta_n = \frac{-i}{\mu}\left\{\frac{-i\sqrt{C_n|\alpha+b_n|}\,[I_{-\frac{2}{3}}(-i\frac{2}{3}|\xi|^{3/2}) - D_n I_{\frac{2}{3}}(-i\frac{2}{3}|\xi|^{3/2})]}{I_{\frac{1}{3}}(-i\frac{2}{3}|\xi|^{3/2}) + D_n I_{-\frac{1}{3}}(-i\frac{2}{3}|\xi|^{3/2})} - \frac{1}{2}\mathrm{tgh}(\alpha+\Omega)\right\}. \tag{34}$$

The admittance of the horn inlet should be calculated in several stages, beginning from the outlet and ending at the inlet. First of all, the inlet admittance of the end unit has to be found by accepting it as equal to the outlet load of the preceeding unit, then the inlet admittance of this unit has to be found, etc. In numerical calculations based on formulae (31)–(34) a catenoidal horn ($T = 0$), with dimensions identical to those of the exponential horn analysed before, was considered. As in case of $T = 1$ it was accepted that the horn $T = 0$ is loaded at the outlet with the impedance of a circular piston with the diameter equal to the diameter of the horn outlet, which vibrates in an infinite baffle. Calculation results are illustrated in Figs. 5 and 6. Diagrams of real and imaginary parts of the elementary relative impedance of the horn outlet in terms of frequency exhibit oscillations due to the reflection of a wave from the outlet. A comparison between Figs. 5, 6 and Figs. 3, 4 shows that a change of the horn profile from exponential to catenoidal at constant dimensions of its inlet, outlet and length, leads to significant quantitative changes in frequency characteristics of impedance. In particular the average level of the real part of the relative impedance becomes higher with a simultaneous increase of the first maximum with respect to the other ones. On the other hand resonant and antiresonant frequencies of the horn undergo only slight changes.

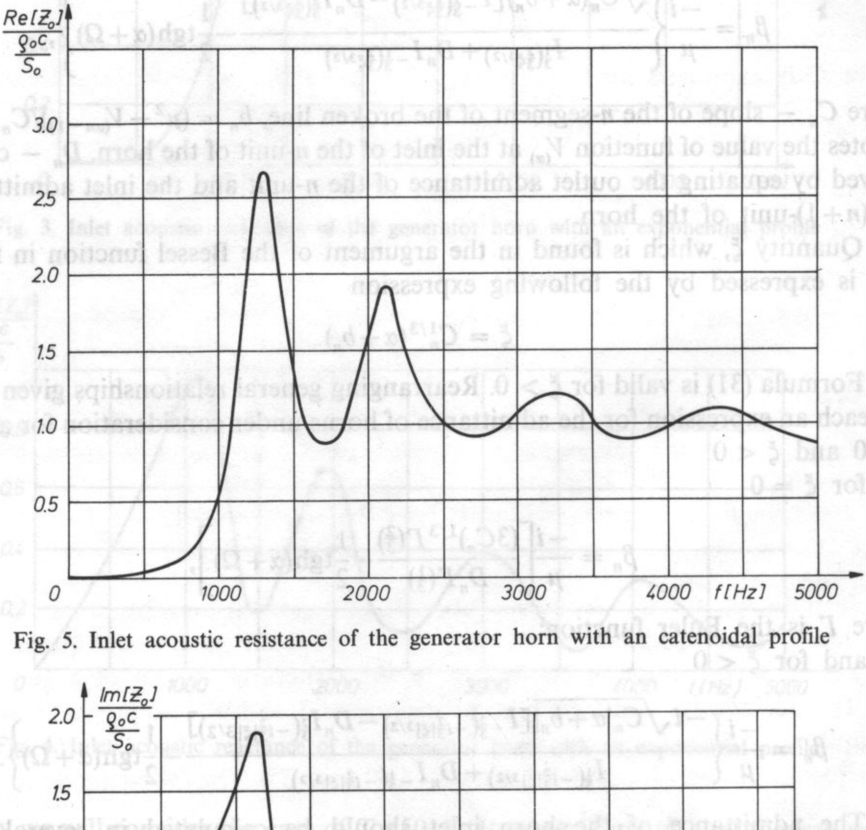

Fig. 5. Inlet acoustic resistance of the generator horn with an catenoidal profile

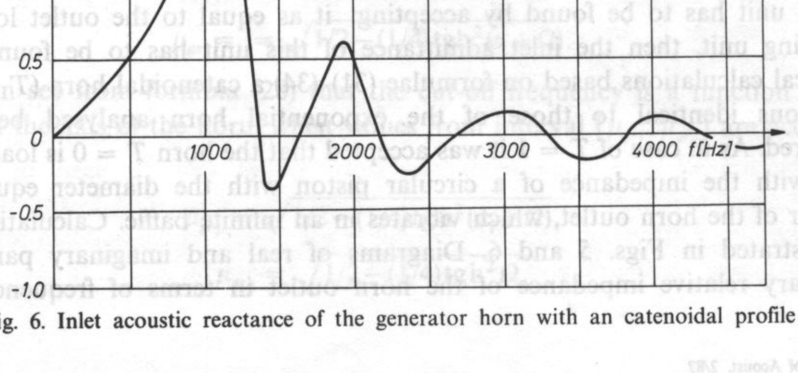

Fig. 6. Inlet acoustic reactance of the generator horn with an catenoidal profile

3. Horn with a stator

The acoustic wave is produced in an axial generator at the inlet of the stator channel. The condition of its radiation to the surroundings depends not only on the horn, but also on other elements of the generator structure (see Fig. 1). In order to analyse this problem in detail we will make use of the generalized theoretical model of an axial dynamic generator [9]. Model [9] accepts that the pressure chamber and the stator channel with the horn are acoustic four-terminal networks in a series connection and the acoustic impedance of the inlet opening of the stator channel is a sum of:

– the input impedance of the pressure chamber Z_1, which loades the inlet of this channel,

– the input impedance of the stator channel Z_2, which can be called the input impedance of the stator-horn system.

It was proved in [9] that Z_1, excluding frequencies close to resonant and antiresonant frequencies of the pressure chamber, can be neglected in calculations of typical axial generators. In such a situation the input impedance of the stator channel with the horn Z_2 is decisive for acoustic power radiation of the generator. The stator channel can be considered to be a cylindrical pipe loaded at the outlet by the impedance of the horn inlet. Hence, the conditions of acoustic power radiation by the generator should improve when the length of these channels is decreased and when their number is increased. In theory best radiation conditions can occur when all openings of the stator join together into a ring-shaped inlet opening of the horn. This corresponds to a situation in which the input impedance of the stator-horn system changes into the impedance of the horn inlet. Therefore, in order to estimate the influence of the stator (as a separate element in the acoustic system of the generator) on the conditions of acoustic power emission, the frequency characteristic of the input impedance of the stator channel together with the horn should be compared with an analogic characteristic of the inlet of the horn itself. In particular frequency characteristics of real parts of both impedances should be compared, because the real part of the relative impedance of the horn inlet, or another matching system, is known in acoustic literature [16] as the so-called coefficient of transmission and characterizes acoustic power transmission.

Let us assume that a plane wave propagates in the stator channel. Then the acoustic impedance of the inlet of the stator channel and horn can be expressed as

$$Z_2 = \frac{\varrho_0 c}{S_k} \frac{Z + i\dfrac{\varrho_0 c}{S_k}\mathrm{tg}(kL_1)}{\dfrac{\varrho_0 c}{S_k} + iZ\,\mathrm{tg}(kL_1)}, \tag{35}$$

where: ϱ_0 – rest density of the medium, c – adiabatic propagation velocity of a sound wave, S_k – cross-sectional area of the stator channel, l_1 – length of stator channel, Z – impedance loading the outlet of stator channel.

In accordance with the theoretical generator model [9] we can note that

$$Z = nZ_0 \tag{36}$$

where n is the number of openings in the stator and Z_0 is the impedance of the horn inlet.

It was accepted in numeric calculations of the input impedance of the stator channel Z_2 that openings in the stator enter the catenoidal horn ($T = 0$) with dimensions as given in paragraph 2. Furthermore, the following parameters were accepted: $n = 50$, $l_1 = 10^{-2}$ m, and the diameter of the stator opening equal to $1.5 \cdot 10^{-3}$ m.

Calculation results of real and imaginary parts of the elementary relative impedance of the inlet of the stator channel and the horn are presented in Figs. 7 and 8. Both diagrams were drawn in the frequency range from zero to four kHz, what corresponds to the range of application of the assumption about the plane wave in the horn. A comparison between diagrams in Figs. 7 and 8 frequency characteristics of the inlet impedance of the horn itself in Figs. 5 and 6 shows that the introduction of a stator as a separate element in the acoustic system of an axial generator does not influence the shape of impedance frequency characteristics, but causes a considerable decrease of its average value. This occurs, because resonant and antiresonant frequencies, and proportions of individual maxima and minima are in both diagrams nearly identical. While, values of real parts of the impedance at a fixed frequency are at least four times smaller for the inlet of the stator-horn system than for the inlet of the horn itself. This is due to the reflection of a wave at the connection of the stator channel outlet with the horn inlet and has to cause a considerable decrease of the acoustic power emitted by the generator. A theoretical power frequency characteristic of a generator with given above dimensions of the

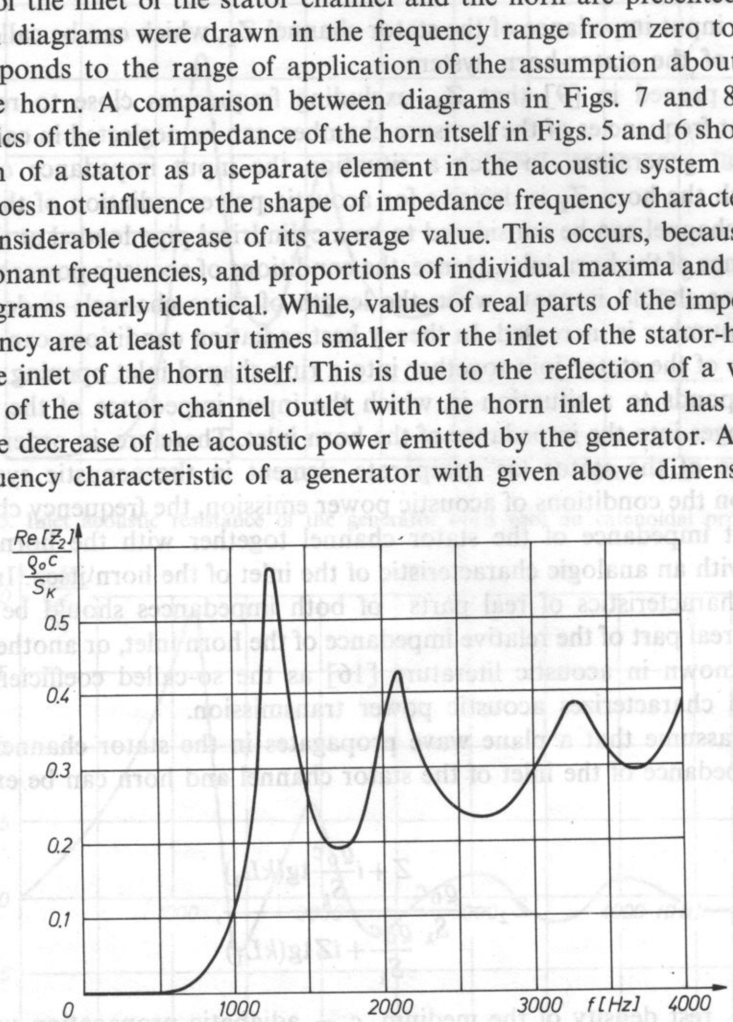

Fig. 7. Inlet acoustic resistance of the stator channel and horn

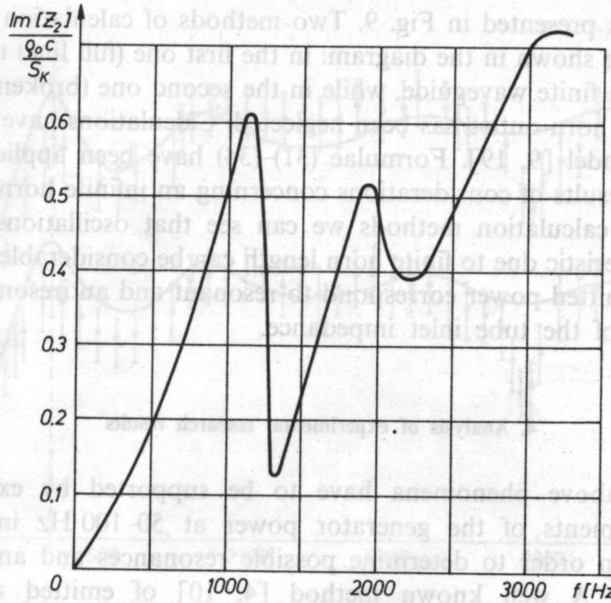

Fig. 8. Inlet acoustic reactance of the stator channel and horn

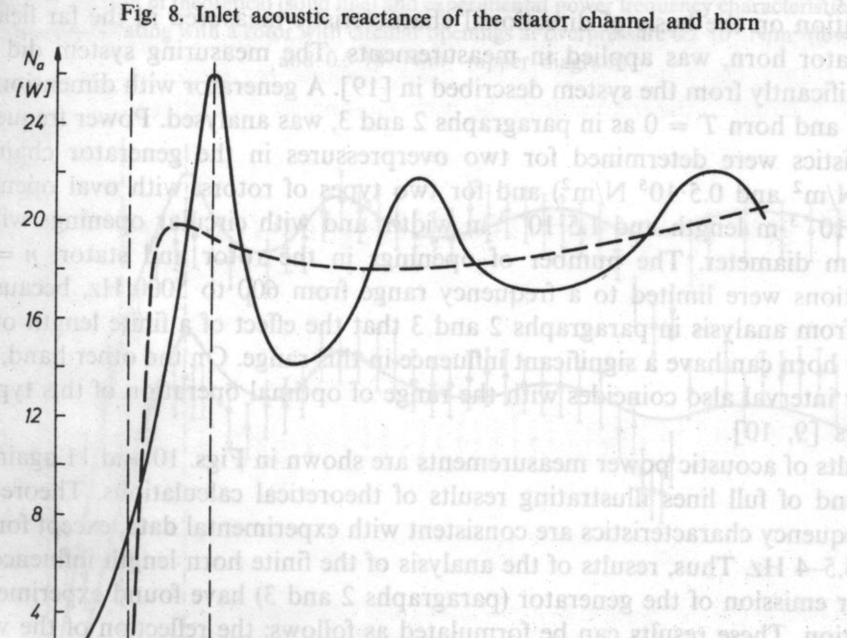

Fig. 9. Comparison of theoretical frequency characteristics of power for a generator collaborating with a catenoidal horn with finite length (solid line) and infinite length (dashed line). Openings in the rotor were taken to be circular and the overpressure in the generator chamber was taken to be equal to $0.5 \cdot 10^5$ N/m^2

stator and horn is presented in Fig. 9. Two methods of calculation of the emitted acoustic power are shown in the diagram: in the first one (full line) the generator is considered to be a finite waveguide, while in the second one (broken line) the wave reflected from the horn outlet has been neglected. Calculations have been based on the theoretical model [9, 19]. Formulae (31)–(34) have been applied to the finite horn, as well as results of considerations concerning an infinite horn, given in [18]. Comparing both calculation methods we can see that oscillations of the power frequency characteristic due to finite horn length can be considerable, while maxima and minima of emitted power correspond to resonant and antiresonant frequencies of the real part of the tube inlet impedance.

4. Analysis of experimental research results

Mentioned above phenomena have to be supported by experiment. This requires measurements of the generator power at 50–100 Hz intervals on the frequency scale, in order to determine possible resonances and antiresonances of radiated power*. A well known method [4, 10] of emitted acoustic power determination on the basis of directional characteristic measured in the far field of the generator horn, was applied in measurements. The measuring system did not differ significantly from the system described in [19]. A generator with dimensions of the stator and horn $T = 0$ as in paragraphs 2 and 3, was analysed. Power frequency characteristics were determined for two overpressures in the generator chamber ($0.2 \cdot 10^5$ N/m^2 and $0.5 \cdot 10^5$ N/m^2) and for two types of rotors: with oval openings with a $3 \cdot 10^{-3}$ m length and $1.5 \cdot 10^{-3}$ m width, and with circular openings with a $1.5 \cdot 10^{-3}$ m diameter. The number of openings in the rotor and stator: $n = 50$. Investigations were limited to a frequency range from 600 to 5000 Hz, because it resulted from analysis in paragraphs 2 and 3 that the effect of a finite length of the generator horn can have a significant influence in this range. On the other hand, this frequency interval also coincides with the range of optimal operation of this type of generators [9, 10].

Results of acoustic power measurements are shown in Figs. 10 and 11 against a background of full lines illustrating results of theoretical calculations. Theoretical power frequency characteristics are consistent with experimental data, except for the interval 3.5–4 Hz. Thus, results of the analysis of the finite horn length influence on the power emission of the generator (paragraphs 2 and 3) have found experimental confirmation. These results can be formulated as follows: the reflection of the wave from the horn outlet causes oscillations of the power frequency characteristic, irrespective of the supply pressure and geometry of rotor openings. Oscillations of power correspond to fluctuations of the real part of the impedance of the stator-horn

* Measuring points on power frequency characteristics have been given in previous papers [9, 10, 19] most frequently at 500–1000 Hz intervals.

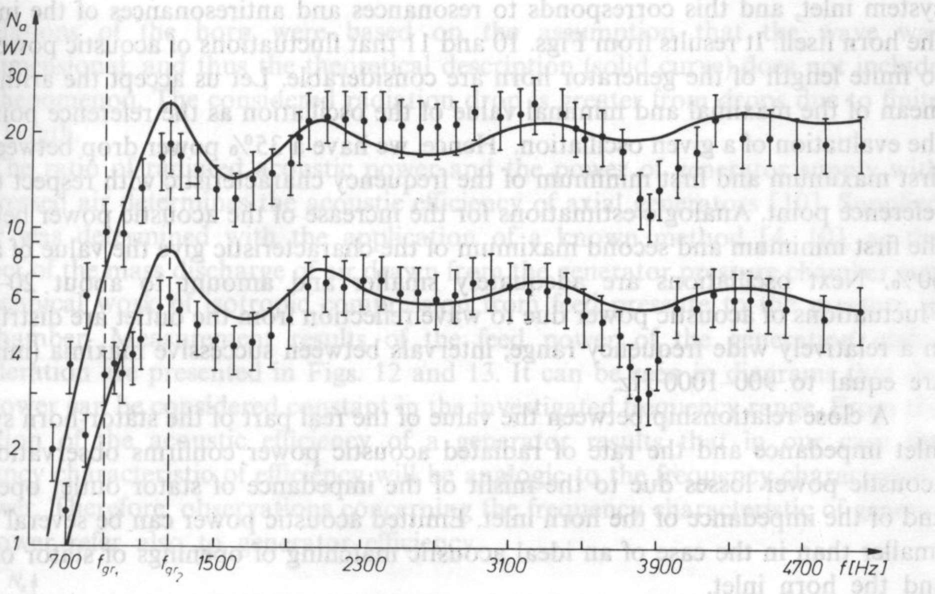

Fig. 10. Comparison of theoretical (solid line) and experimental power frequency characteristics of an axial generator collaborating with a rotor with circular openings at overpressure $0.2 \cdot 10^5$ N/m^2 (lower diagram) and $0.5 \cdot 10^5$ N/m^2 (upper diagram)

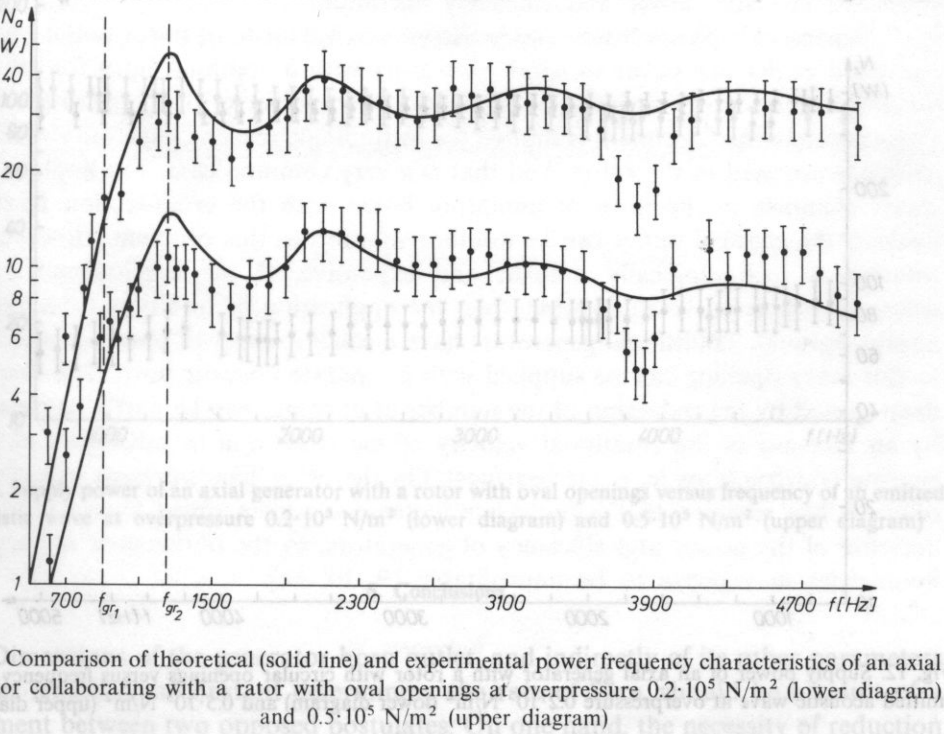

Fig. 11. Comparison of theoretical (solid line) and experimental power frequency characteristics of an axial generator collaborating with a rotor with oval openings at overpressure $0.2 \cdot 10^5$ N/m^2 (lower diagram) and $0.5 \cdot 10^5$ N/m^2 (upper diagram)

system inlet, and this corresponds to resonances and antiresonances of the inlet of the horn itself. It results from Figs. 10 and 11 that fluctuations of acoustic power due to finite length of the generator horn are considerable. Let us accept the arithmetic mean of the maximal and minimal value of the oscillation as the reference point for the evaluation of a given oscillation. Hence, we have a 35% power drop between the first maximum and first minimum of the frequency characteristic with respect to the reference point. Analogic estimations for the increase of the acoustic power between the first minimum and second maximum of the characteristic give the value of about 60%. Next oscillations are adequately smaller and amount to about 20–25%. Fluctuations of acoustic power due to wave reflection from the outlet are distributed in a relatively wide frequency range; intervals between successive maxima (minima) are equal to 900–1000 Hz.

A close relationship between the value of the real part of the stator-horn system inlet impedance and the rate of radiated acoustic power confirms observations of acoustic power losses due to the misfit of the impedance of stator outlet openings and of the impedance of the horn inlet. Emitted acoustic power can be several times smaller than in the case of an ideal acoustic matching of openings of stator outlets and the horn inlet.

Measurements exhibit a minimum of radiation in the interval 3.5–4 kHz, which was not obtained in calculations. According to the author this radiation decrease is caused by transverse wave modes in the horn, because for frequency 3400 Hz the wavelength is equal to the diameter of the horn channel at the outlet ($0.5d_w = 0.1$ m).

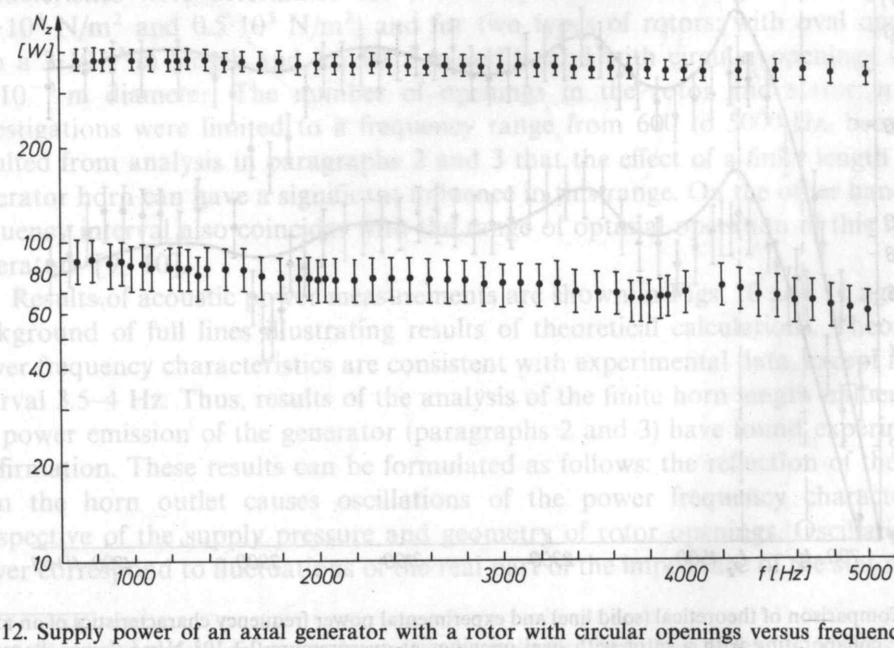

Fig. 12. Supply power of an axial generator with a rotor with circular openings versus frequency of an emitted acoustic wave at overpressure $0.2 \cdot 10^5$ N/m² (lower diagram) and $0.5 \cdot 10^5$ N/m² (upper diagram)

Calculations of the horn were based on the assumption that the wave was one-dimensional, and thus the theoretical description (solid curve) does not include this phenomenon. The considered radiation drop is greater from drops due to finite horn length.

The ratio of radiated acoustic power and the power of generator supply with compressed air determines the acoustic efficiency of axial generators [10]. Supplied power was determined with the application of a known method [4, 10], as the product of the mass discharge of air drawn from the generator pressure chamber and the technical work of isotropic compression from feed pressure to the pressure in the chamber. Measurement results of the feed power of the generation under consideration are presented in Figs. 12 and 13. It can be seen in diagrams that the feed power can be considered constant in the investigated frequency range. From the definition of the acoustic efficiency of a generator results that in our case the frequency characteristic of efficiency will be analogic to the frequency characteristic of power. Therefore, observations concerning the frequency characteristic of generated power refer also to generator efficiency.

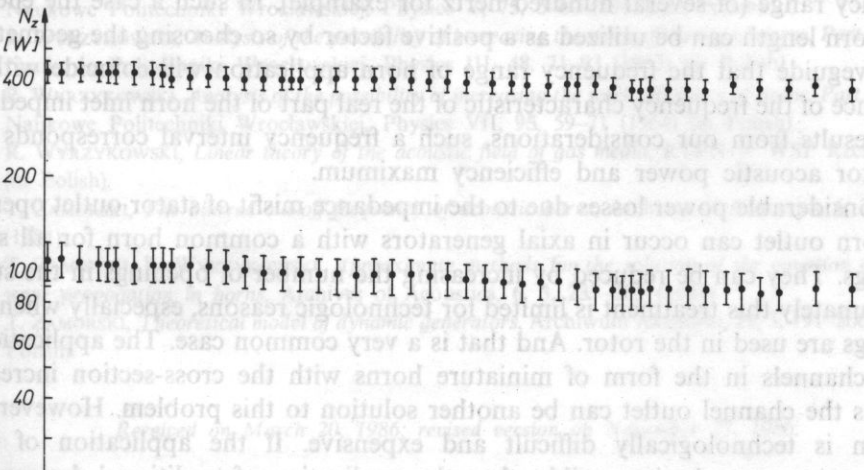

Fig. 13. Supply power of an axial generator with a rotor with oval openings versus frequency of an emitted acoustic wave at overpressure $0.2 \cdot 10^5$ N/m^2 (lower diagram) and $0.5 \cdot 10^5$ N/m^2 (upper diagram)

5. Conclusions

Dimensions of the generator horn outlet, and indirectly of its other parameters such as length or a constant determining the rate of flare, should be a result of a settlement between two opposed postulates. On one hand, the necessity of reduction of wave reflections from the horn outlet requires the diameter of the horn outlet to be large with respect to wavelength, and on the other, the demand for reduction of energy loss due to transverse mode formation requires transverse dimensions of the horn to be small with respect to wavelength. Hence, the application of one horn in the generator can not ensure optimal radiation conditions in the whole frequency range of generator operation. This range can be divided into intervals – each of them requires the application of a different horn. This problem can be solved in practice in two different ways:

– by supplying the generator with a set of replaceable horns, which would be used in adequate intervals of generated frequencies,

– by designing a folding multi-element horn – its elements would be installed or taken off depending on the frequency interval.

If one horn would be used for the whole frequency range of generated waves, then the effect of a finite horn would have an adverse influence, because it would disturb the stability of frequency characteristics of generator power and efficiency. Solutions mentioned above will lead to the application of a given horn in a narrower frequency range (of several hundred hertz for example). In such a case the effect of finite horn length can be utilized as a positive factor by so choosing the geometry of the waveguide that the frequency range of horn application will coincide with the resonance of the frequency characteristic of the real part of the horn inlet impedance. As it results from our considerations, such a frequency interval corresponds with generator acoustic power and efficiency maximum.

Considerable power losses due to the impedance misfit of stator outlet openings and horn outlet can occur in axial generators with a common horn for all stator openings. They can be reduced by increasing the number of openings in the stator. Unfortunately this treatment is limited for technologic reasons, especially when oval openings are used in the rotor. And that is a very common case. The application of stator channels in the form of miniature horns with the cross-section increasing towards the channel outlet can be another solution to this problem. However, this solution is technologically difficult and expensive. If the application of these solutions proves to be impossible, then the application of traditional designs may appear optimal. Traditional generators have a small number of openings and owing to this every opening can be supplied with a separate acoustic horn. The frequency drop caused by the reduction of the number of openings can be partly compensated by an increase of the rotational velocity of the rotor due to utilization of newer bearing constructions (e. g. air bearings). On the other hand, present experimental research has revealed that a frequency increase is accompanied by a considerable decrease of the power and efficiency of generators, so the obtainment of very high frequencies may prove to be unprofitable [9, 10, 14].

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