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INTERACTION OF A TWO-LAYERED HALF-CYLINDRICAL SHELL WITH ACOUSTIC MEDIUM

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Some numerical results of investigations into the coupling between the acoustic field inside a two-layered half-cylindrical shell and the vibrations of the containing structure are presented in this paper. An analytical approach has been used to find resonant frequencies of the system as a whole.

1. Introduction

Interaction effects that exist between the structure and the enclosed acoustic medium have been receiving increasing attention during past years. Such effects can cause resonant frequencies of the whole structure to be considerably different from these in vacuum.

2. Equations of motion

The differential equations of motion of a double-layered cylindrical shell were derived by MARKUŠ [1]. With an internal fluid enclosed they can be written in the following form [2]

$$\begin{gathered}RA_1(\partial^2 u/\partial x^2)-A_{1\nu}\partial w/\partial x+1/2R(A_1-A_{1\nu})\partial^2 u/\partial\varphi^2+\\+(1/2)(A_1+A_{1\nu})\partial^2 v/\partial x\partial\varphi-Rm_0\partial^2 u/\partial t^2=0,\\ [D_{1\nu}/4R+(1/2)(A_1+A_{1\nu})]\partial^2 u/\partial x\partial\varphi+RP(\partial^2 v/\partial x^2)+\\+(A_1/R+B_1/R^3)(\partial^2 v/\partial\varphi^2)-A_1/R(\partial w/\partial\varphi)-\\-Rm_0(\partial^2 v/\partial t^2)=0,\\ A_{1\nu}\partial u/\partial x+(1/R)A_1(\partial v/\partial\varphi)-(1/R)A_1 w+D_{1\nu}(\partial^2 w/\partial x^2)-RB_1(\partial^4 w/\partial x^4)+\\+(2/R^2)(\partial^4 w/\partial x^2\partial\varphi^2)+(1/R^4)(\partial^4 w/\partial\varphi^4)-Rm_0(\partial^2 w/\partial t^2)=-R\varrho(\partial\Phi/\partial t)\big|_{r=a},\end{gathered}\tag{1}$$

where (u, v, w) are the components of displacement of the shell in the axial, circumferential and radial directions, respectively; r, φ and x are cylindrical coordinates as shown in Fig. 1; h_1, h_2 – thicknesses of separate layers; E_i, ν_i, $i = 1, 2$ – Young's moduli and Poisson's constants of the inner and outer layer, respectively.

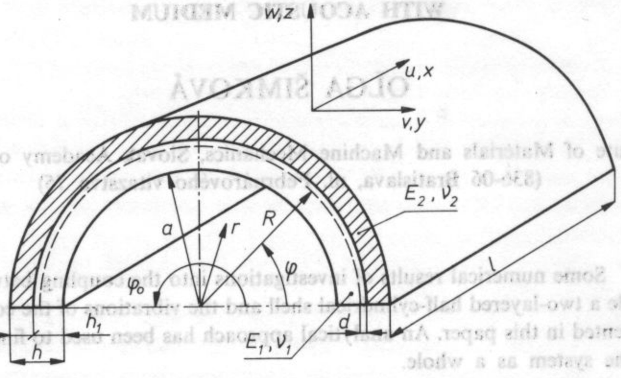

Fig. 1. Geometry and co-ordinate system of a two-layered shell

$$S_i = E_i(1-\nu_i^2),\ i = 1, 2; \quad A_1 = S_1 h_1 + S_2 h_2; \quad A_{1\nu} = S_1 h_1 \nu_1 + S_2 h_2 \nu_2;$$

$$B_1 = (1/3)(S_1 h_1^3 + S_2 h_2^3) - d(S_1 h_1^2 - S_2 h_2^2) + d^2(S_1 h_1 + S_2 h_2);$$

$$B_{1\nu} = (1/3)(S_1 h_1^3 \nu_1 + S_2 h_2^3 \nu_2) - d(S_1 h_1^2 \nu_1 - S_2 h_2^2 \nu_2) + d^2(S_1 h_1 \nu_1 + S_2 h_2 \nu_2);$$

$$D_{1\nu} = (S_1 h_1^2 \nu_1 - S_2 h_2^2 \nu_2) - 2d(S_1 h_1 \nu_1 + S_2 h_2 \nu_2);$$

$$d = (1/2)(S_1 h_1^2 - S_2 h_2^2)/(S_1 h_1 + S_2 h_2);$$

$$P = (1/2)(A_1 - A_{1\nu}) - 3D_{1\nu}/4R + 1/R^2(B_1 - B_{1\nu});$$

m_0 – mass per unit length of the shell; t – time; R – equivalent radius of the shell; $a = R - h_1 + d$; ϱ – density of the fluid; Φ – velocity potential of the fluid.

The velocity potential Φ satisfies the wave equation

$$\nabla^2 \Phi - (1/c_0^2)(\partial^2 \Phi/\partial t^2) = 0, \tag{2}$$

where ∇^2 is the Laplacian operator in the form

$$(1/r)(\partial/\partial r)\left[r(\partial/\partial r)\right] + (1/r^2)(\partial^2/\partial \varphi^2) + (\partial^2/\partial x^2)$$

and c_0 is the velocity of sound in the fluid. It is assumed that the shell and the fluid remain in contact and so

$$\partial w/\partial t = -\partial \Phi/\partial r\big|_{r=a}. \tag{3}$$

3. Solution of the problem

The task is to find resonant frequencies ω of the system described by equations (1)–(3). The normal modes are harmonic functions in the axial and circumferential directions, and Bessel functions in the radial direction.

Considering a simply-supported half cylindrical shell with the length l, solutions are taken to be in the form

$$
\begin{aligned}
u &= A\cos\lambda^* x \sin n\varphi e^{-i\omega t},\\
v &= B\sin\lambda^* x \cos n\varphi e^{-i\omega t},\\
w &= C\sin\lambda^* x \sin n\varphi e^{-i\omega t},\\
\Phi &= DJ_n(\beta r)\sin\lambda^* x \sin n\varphi e^{-i\omega t},
\end{aligned}
\tag{4}
$$

where $J_n(\beta r)$ — Bessel function of the first kind and order n, $\lambda^* = m\pi/l$, $m = 1, 2, 3, \ldots$ bending mode number, $n = k\pi/\varphi_0$, $k = 1, 2, 3, \ldots$ circumferential mode number, ($\varphi_0 = \pi$ for a half-cylindrical shell).

Substituting solutions (4) in equations (2) and (3) we have

$$\beta^2 + \lambda^{*2} = \omega^2/c_0^2, \tag{5}$$

$$i\omega C = D\beta J_n'(\beta a). \tag{6}$$

The following relations between A, B and C can be obtained, from equations (1), using equation (6) to eliminate D:

$$
\begin{aligned}
A(H_1 - \Omega^2) + BH_2 + CH_3 &= 0,\\
AH_4 + B(H_5 - \Omega^2) + CH_6 &= 0,\\
AH_7 + BH_8 + C\{H_9 - \Omega^2[1 + K/F_n(\xi)]\} &= 0,
\end{aligned}
\tag{7}
$$

where

$$H_1 = \lambda^2 + (1/2)[(A_1 - A_{1\nu})/A_1]n^2, \quad H_2 = (1/2)[(A_1 + A_{1\nu})/A_1]\lambda n,$$

$$H_3 = H_7 = (A_{1\nu}/A_1)\lambda, \quad H_4 = \lambda n[D_{1\nu}/4A_1R + (1/2)(A_1 + A_{1\nu})/A_1],$$

$$H_5 = (P/A_1)\lambda^2 + n^2[1 + B_1/(A_1R^2)], \quad H_6 = H_8 = n,$$

$$H_9 = 1 + (D_{1\nu}/A_1R)\lambda^2 + B_1/(A_1R^2)(\lambda^2 + n^2)^2, \quad \lambda = \lambda^* R, \quad \Omega = \omega^2 m_0 R^2 (1/A_1),$$

$$\beta = \sqrt{(1/R^2)(\Omega^2 A_1/(m_0 c_0^2) - \lambda^2)}, \quad K = \varrho a/m_0, \quad \xi = \beta a,$$

$$F_n(\xi) = [J_n'(\xi)/J_n(\xi)]\xi.$$

Equations (7) are valid simultaneously, when the determinant of coefficients vanishes. This condition can be written as

$$\Omega^6 - K_2(\Omega)\Omega^4 + K_1(\Omega)\Omega^2 - K_0(\Omega) = 0, \tag{8}$$

where

$$K_2(\Omega) = H_5 + H_1 + \mu H_9,$$

$$K_1(\Omega) = H_1 H_5 - H_2 H_4 + \mu(H_1 H_9 + H_5 H_9 - H_3 H_7 - H_6 H_8),$$

$$K_0(\Omega) = \mu(H_1 H_5 H_9 + H_3 H_4 H_8 + H_2 H_6 H_7 - H_3 H_5 H_7 - H_2 H_4 H_9 - H_1 H_6 H_8),$$

$$\mu = F_n(\xi)/(F_n(\xi) + K).$$

4. Numerical results

As an example, the transcendental equation (8) has been solved for the system with parameters as follows:

$$R = 0.5 \text{ m}, \quad E_1 = 2.1\cdot 10^5 \text{ MPa}, \quad E_2 = 10^2 \text{ MPa}, \quad \nu_1 = 0.3, \quad \nu_2 = 0.4;$$
$$\gamma_1 = 7.8\cdot 10^3 \text{ kgm}^{-3}, \quad \gamma_2 = 1.2\cdot 10^3 \text{ kgm}^{-3}, \quad m_0 = \gamma_1 h_1 + \gamma_2 h_2, \quad h_1 = h_2.$$

Three positive real values of frequency parameter Ω were found for any acoustic medium enclosed, but only the lowest value Ω_1 (corresponding to the predominant shell bending mode) has been influenced by the interaction.

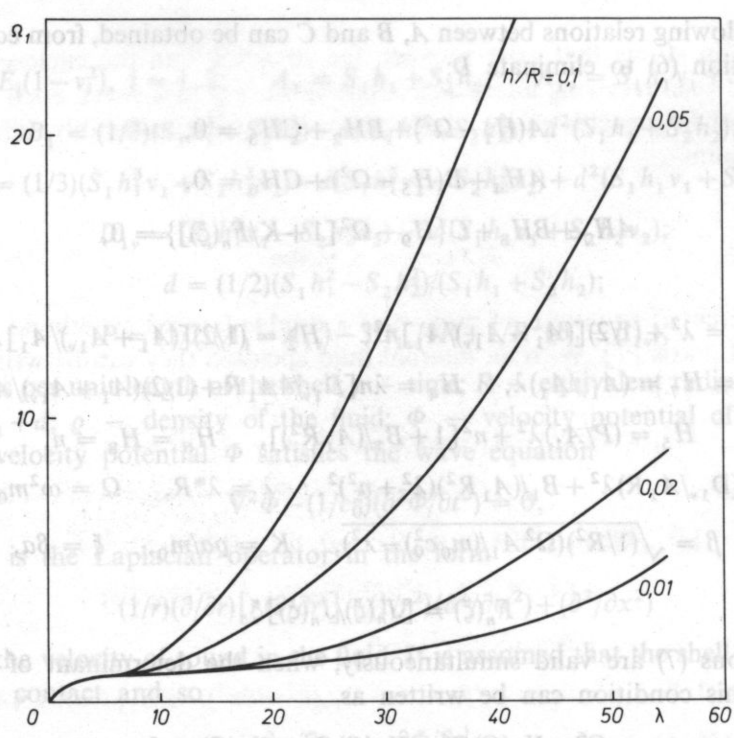

Fig. 2. Ω_1 versus λ for different ratios h/R, $n = 1$ (acoustic medium − air, $\varrho = 1.2$ kgm^{-3}, $c_0 = 343$ ms^{-1})

Values of Ω_1 versus wave number λ for different ratios h/R and for circumferential mode number $n = 1$, with air as the acoustic medium ($\varrho = 1.2$ kgm^{-3}, $c_0 = 343$ ms^{-1}) are plotted in Fig. 2.

Resonant frequencies Ω_1 plotted in Fig. 2 are significantly influenced by the ratio h/R and they are higher for higher values of the wave number λ.

In the research carried out, the same shell as treated above has been analysed in interaction with different acoustic media. Results show only a slight difference between the values of frequency parameters for the coupled system with vacuum ($\varrho = 0$) and those with any gaseous acoustic medium under atmospheric pressure. However, the lowest values of frequency parameters are considerably reduced for any liquid medium. This is illustrated in Fig. 3 for a system with water as an acoustic medium ($\varrho = 1000$ kgm^{-3}, $c_0 = 1500$ ms^{-1}).

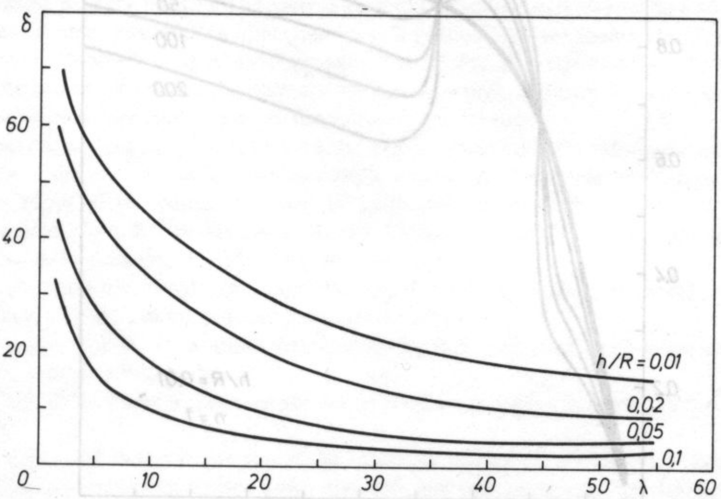

Fig. 3. Relative difference δ versus λ for different ratios h/R, $n = 1$ (acoustic medium – water, $\varrho = 1000$ kgm^{-3}, $c_0 = 1500$ ms^{-1})

The relative difference

$$\delta = \frac{\Omega_{1(\text{vacuum})} - \Omega_{1(\text{water})}}{\Omega_{1(\text{vacuum})}} \cdot 100 \quad [\%]$$

versus the wave number λ for different ratios h/R, $n = 1$ is plotted. The values of δ are lower for higher ratios h/R and they decrease with increasing wave number λ.

Let us pay our attention to air as the acoustic medium, again. A question arises, how the resonant frequency Ω_1 of the coupled system will be influenced by an increase of the wave impedance ϱc_0 of air.

It is well known, that the velocity of sound c_0 in any gaseous medium does not depend on its pressure. The wave impedance of air may thus be increased by pressurizing the structure (increasing the density of air ϱ).

The resonant frequency Ω_1 is presented in Fig. 4 in dependence on the wave number λ for different values of ϱ, for $h/R = 0.01$ and $n = 1$. The thick line stands for uncompressed air ($\varrho = 1.2$ kgm^{-3}). Compressed air tends to re-tune the enclosure in the following way: $\Omega_1' s$ increase for $\lambda < \pi$, then $\Omega_1 s$ decrease and for $\lambda > 4$ they increase again. There is a local maximum at $\lambda = \pi$, what means that an extreme of Ω_1 occurs when the length of the structure is set by an integer multiple of the relevant radius R of the shell. This conclusion holds, of course, for simply supported shells ($\lambda = m\pi R/l$) only.

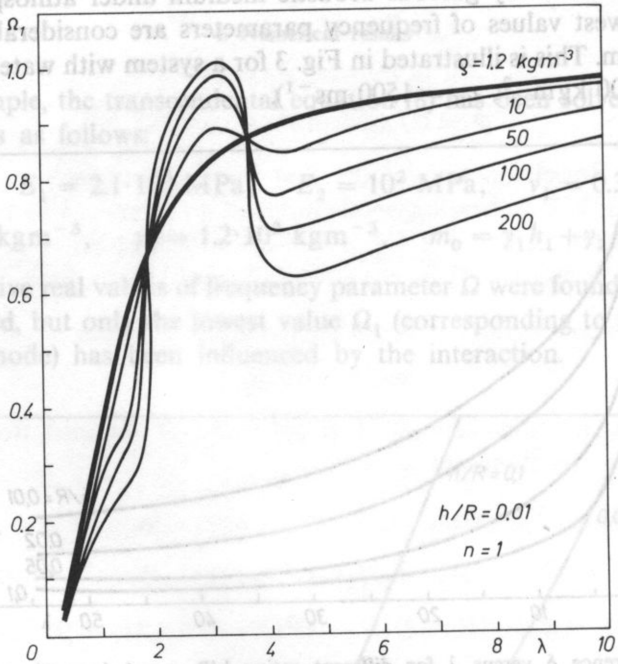

Fig. 4. Ω_1 versus λ for different values of ϱ, $h/R = 0.01$ and $n = 1$ (acoustic medium – compressed air)

5. Conclusion

The following concluding remarks can be yielded from the analysis carried out:

– eigenfrequencies of the system coupled with any gaseous medium under atmospheric pressure enclosed do not differ from those of the structure in vacuum;

– lowest values of the frequency parameter Ω_1 (corresponding to the predominant shell bending mode) are considerably reduced for any liquid medium;

– by pressurizing the enclosure with air inside, the structure may be "re-tuned" to higher, as well as to lower values of eigenfrequencies Ω_1 (depending on the wave number λ).

References

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