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THE AEROSOL PARTICLE DRIFT IN A STANDING WAVE FIELD

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This study considers the equation of motion of an aerosol particle in a standing wave field under the influence of drift and resistance forces in the Stokes and Oseen approximation. It evaluates the possibility of simplifying the equation of motion, taking into account the drift related to the radiation pressure, periodic viscosity changes and the asymmetry of motion of particle vibrating in a standing wave field. It gives the general properties of the solutions of motion equations for considerable attenuation and formulates the applicability of approximation which consists in neglecting the inertia of a particle.

1. Introduction

Although the method of cleaning gases acoustically has long been known, its use in the industry still remains limited. The main reason for this are the high investment and exploitation costs of the proper technical installations. Despite this, the acoustical cleaning remains a method which is irreplaceable in some special cases, such as the clearing gases with high temperatures and chemically active. The aerosol of particles which are difficult to electrify or explosive can also be cleaned acoustically, excluding the use of electrofilters. Recently, this method has been applied in the nuclear technology [1].

The organizations engaged in the protection of the natural environment point out that the aerosols and suspensions of fine-particle dust, with diameters smaller than a few micrometers [2], are particularly dangerous for Nature and man's health. In this case, acoustic methods appear to be very useful; the forces which act in a standing wave field are most effective for exactly such particle sizes. Moreover, in the process of acoustic coagulation the mean particle size increases, decreasing the toxicity of the aerosol and facilitating its further cleaning by traditional method.

To lower the costs of applying the acoustic method to clean gases, research efforts were undertaken in the following main directions:

1) the investigation of new, highly efficient sources of acoustic energy;

2) the development of acoustic field configuration, e.g., the superposition of a standing wave and a nonsinusoidal travelling wave [3];

3) the search for the optimum parameters of the acoustic field depending on the type of aerosol, in particular on its dispersion.

This study is a contribution to research in the second and third of fields mentioned above. It considers the motion of aerosol particles caused by the drift forces in a standing acoustic wave field. These forces, a consequence of interaction between the particle and the vibrating medium, result from such phenomena as the radiation pressures, the asymmetric vibration motion of the particle or periodic changes in the viscosity of the gas. On the other hand, different kinds of drift have a common property: the forces applied on the particle depend in the same way on its position with respect to the loops and nodes of the standing wave and are proportional to the density of the wave energy. However, the drift forces depend very strongly on the particle size. So far, the problem of the contributions of the different drift forces in the complex process of aerosol coagulation has not been investigated. Studies on the drift-driven transport mostly considered the radiation drift, which a detailed analysis found to be weakest. No study was also carried out on the character of the trajectory of a particle moving under the effect of the drift forces, or on the role of the friction forces in the general equation of motion.

This paper is devoted to the problem of the equation of motion of a particle, considering the drift and resistance forces caused by viscosity. It evaluates the effect of the drift forces of different types depending on the particle size. It also defines the applicability range of the approximation consisting in neglecting the nonlinear term in the friction force. It also analyzes the general properties of the equation of motion of the aerosol particle in the case of large attenuation constants, corresponding for typical values of the drift forces to particles with radii less than 10^{-5}m. Finally, it defines the applicability criterion of the so-called King—St. Clair approximation, most widely applied in the literature to describe the motion, consisting in the assumption of equilibrium between the drift and the Stokes viscosity forces, neglecting the term representing inertia. This approximation is of particular significance, since it makes possible analytical estimation of the time constants of the particle transport to points of stable equilibrium in a standing wave field. Methods for determining these constants will be presented in another study [4].

Below, consideration is given only to the problem of the motion of a single aerosol particle. This means that the effect of interaction of particles is neglected, i.e., the process leading to coagulation itself, namely the fact that smaller particles link to form larger aggregates. Causing particles to gather near points of stable equilibrium (minima of the potential of the drift forces), the phenomena caused by the drift forces assist in a way the elementary acts of coagulation by increasing the concentration of particles near the nodes or loops of the standing wave. The ultimate coagulation can then occur through an orthokinetic process if the standing wave is combined with the saw-tooth travelling wave moving along the planes of the loops and nodes. With such a configuration of the field, proposed by Scott [3] and so far the most efficient, the transport under the effect of the drift forces determines the rate of the cleaning process.

2. The drift forces in a standing wave field

If the acoustic potential of the standing wave is described by the function

$$\Phi(x, t) = \Phi_0 \cos kx \cos \omega t, \tag{1}$$

the deflection x_g of particles of the gaseous medium, as a function of the position x and the time t, is given by the equation

$$x_g = x_0 \sin kx \sin \omega t. \tag{2}$$

In the planes $kx = n\pi, n = 0, \pm 1, \pm 2, \ldots$, these deflections are permanently equal to zero; therefore, they are nodes of the standing wave. The planes with equations $kx = (n+\frac{1}{2}\pi), n = 0, \pm 1, \pm 2, \ldots$, correspond to the nodes. In an acoustic field of this type aerosol particles are acted upon by forces which do not decay on average in time, directing these particles to the closest planes of nodes of loops. It appears that, irrespective of the mechanism of the occurrence of such forces, they can be described by the formula

$$F_D(x) = F_0 \sin 2kx. \tag{3}$$

The forces of this type are called the drift forces. Therefore the potential related to their action has the form

$$U_D(x) = F_0(2k)^{-1} \cos 2kx. \tag{4}$$

The position of the potential minima depends on the sign of the constant F_0, describing the maximum value of the drift force. For $F_0 > 0$ the minima of potential (4) coincide with the value $2kx = (2n+1)\pi$, giving the equation of the planes of loops. In turn, for $F_D < 0$ the drift forces gather aerosol particles at the nodes of the standing wave. It is clear, however, that the sign of the constant F_0 has no effect on the kinetics of the process of particle transport.

The review of the drift forces should begin with the so-called radiation drift related to the radiation pressure. Calculating the difference between the momentum of the incident wave and that of a wave scattered on a sphere, Westervelt [5] gave for the drift of this type an expression of the force in the form

$$F_{DR} = \frac{8}{3}\pi k r^3 \mu_g^2 \bar{E} \sin 2kx, \tag{5}$$

where $\bar{E}$ denotes the density of the acoustic wave energy and μ_g is the so-called coefficient of the particle flow,

$$\mu_g^2 = \omega^2 \tau^2 (1 + \omega^2 \tau^2)^{-1}, \tag{6}$$

the value of which, very strongly dependent on the frequency ω, describes the degree of participation of an aerosol particle in the oscillation motion of the medium and plays a fundamental in the theory of coagulation microprocesses. In turn,

$$\tau = m_p (6\pi r \eta)^{-1}, \tag{7}$$

called the relaxation time, which is the quotient of the particle mass m_p and the coefficient of the Stokes resistance connected to the viscosity of the medium η, characterizes the effect of the resistance on the motion.

The radiation drift was studied earlier by KING [6] and later also by GORKOV [7]. In each of the cases the theoretical considerations led the authors to slightly different numerical coefficients characterizing the intensity of the radiation drift. The validity of formula (5) given above was finally established by experimental research [8].

Over many years, even in relatively recent studies [9], [10] the radiation drift was regarded as the fundamental effect which concentrates particles in a standing wave field. However, as early as 1950, WESTERVELT [11] predicted theoretically the existence of another type of drift. It is connected with periodic changes in the viscosity of the medium, which result in turn from periodic changes in temperature in a standing wave field. The averaging in time of the Stokes force gives a non vanishing component which causes a drift which will be called the viscosity drift below, and the L-type drift in short. The force of the viscosity drift is expressed by the formula [11]

$$F_{DL} = 3\pi(\varkappa-3)\eta r(\varrho_g c)^{-1}\mu_g^2 \bar{E}\sin 2kx. \tag{8}$$

The constant $\varkappa$ is here the exponent of the adiabate whose value for air can be assumed as 1.4, and ϱ_g is the density of the gas. In this case, the constant F_0, which characterizes in the general formula (5) the amplitude of the drift force, is negative, therefore, the L-type drift concentrates particles at the loops of a standing wave. It will be shown below that its action on particles can be considerably stronger than the radiation drift.

Another type of drift is characteristics only of a standing wave. It results from the fact that in such a wave the vibration amplitude of the gaseous medium depends on position being greater in the area of loops. In view of their inertia, aerosol particles do not keep pace with the motion of the medium and are affected by variable forces over the time of their oscillation. Thus, their motion is asymmetrical, causing a constant motion towards the loops. The component of this motion which does not decay after averaging in time is called the asymmetric drift, and will be called the A-type drift below. The expression of the force of the asymmetric drift was given for the first time by DUCHIN [12]; however, his original study contained certain faults, including probably printers' errors which were partly removed in the monograph work [13]. A detailed analysis of the problem indicates that the correct expression of the force of the asymmetric drift has the form

$$F_{DA} = \frac{\pi}{3}kr^3\mu_p[(3+\frac{9}{2}b)\mu_p - \frac{9}{2}b(b+1)\mu_g]\bar{E}\sin 2kx, \tag{9}$$

where $b = r^{-1}(2\eta/\omega\varrho_g)^{\frac{1}{2}}$ and μ_p is the coefficient of particle entrainment known from orthokinetic theory [13]

$$\mu_p = (1+\omega^2\tau^2)^{-\frac{1}{2}}, \tag{10}$$

which is the coefficient of proportionality between the vibration amplitudes of the medium and the aerosol particle. Along with the flow coefficient introduced in formula (6), it satisfies the relation $\mu_p^2+\mu_g^2=1$.

Apart from those mentioned above, one should mention a drift connected with distortions of a sinusoidal acoustic wave. WESTERVELT [15] derived an expression of the drift force of this type, taking into account the amplitude and phase of the first harmonic in a travelling wave. The estimation of the magnitude of the action of this drift on aerosol particles, carried out in study [4], indicates that for the amplitude of the first harmonic equal to half the amplitude of the fundamental wave, this drift acts a force of the same order of magnitude, as was discussed above. Since, however, the possibility of constructing a device applying a distorted standing wave does not seem real, this problem will be neglected here, to concentrate instead on phenomena typical for a sinusoidal standing wave.

3. Analysis of the equation of motion

Considering the motion of an aerosol particle effected by the drift forces, let us now write the equation of motion of this particle, taking into account the resistance forces on the part of the medium,

$$m_p\frac{d^2x}{dt^2} = -6\pi\eta r\frac{dx}{dt} - \frac{9}{4}\pi r^2\varrho_g\frac{dx}{dt}\left|\frac{dx}{dt}\right| + F_0\sin 2kx. \tag{11}$$

The first term on the right side of the equation represents the Stokes force related to the viscosity of the medium, whereas the second one represents the nonlinear Oseen correction which is significant for large Reynolds numbers. The above simple differential equation, which is nonlinear in view of the last two terms, has no elementary solution. Therefore, further attempts will be made to investigate the general properties of solutions of this equation, taking into account such values of its coefficients as result from the practice of acoustic coagulation of aerosols.

By dividing both sides by the particle mass, equation (11) can be rewritten in the form

$$\frac{d^2x}{dt^2}+\frac{1}{\tau}\frac{dx}{dt}+\frac{27\varrho_g}{16r\varrho_p}\frac{dx}{dt}\left|\frac{dx}{dt}\right| - \frac{F_0}{m_p}\sin 2kx = 0. \tag{12}$$

Now, the effect of the Stokes force is characterized by the relaxation time τ, which was earlier introduced by equation (7). The term representing the drift force is accompanied by the coefficient F_0/m_p denoting the acceleration of the particle in the absence of the friction forces at points of the maximum drift force. Depending on the value of this coefficient, the other terms of the equation of motion, namely the first (inertial) the linear and nonlinear terms representing friction, would affect the particle motion to a lesser or greater extent. Therefore, it is purposeful to introduce the quantity

$$A_D = \frac{F_0}{m_p}, \tag{13}$$

which by analogy to other interactions, can be called the intensity of the drift force field. The calculation of the value of this quantity will make possible above all to compare the efficiency of particular kinds of drift discussed in the previous chapter. To compare the efficiency of various types of drift, previous studies usually applied the comparison of the values of the force amplitudes [13]. It can be seen from (12) that the effect of a force causing motion of a particle on the trajectory is rather represented by the constant A_D. An additional aspect of the problem is the complex dependence of the drift forces on the particle radius. Expressions (5), (8) and (9) depend explicitly on r in the form of power functions and by the coefficients of entrainment and flow, which are strongly dependent on the relaxation time, which is in turn a function of the particle radius. So one should expect the strong variation of the drift forces as a function of r, and, in particular, distinct maxima. Maxima of the quantity A_D which is the coefficient of the principal equation (12) usually occur somewhere else as a function of the radius, since the particle mass varies as r^3. Moreover, by calculating the quantity A_D, with the dimension of acceleration, it is possible to evaluate the part of the Earth's acceleration in the motion of aerosol particles. It is essential in problems where the drift and fall of particles in the gravitational field are considered simultaneously [6], [9].

Applying the expressions of the drift forces of different kinds as given in the previous point, it is now possible to write the corresponding field intensities

radiation drift

$$A_{DR} = 2k\varrho_p^{-1}\mu_g^2\bar{E}, \tag{14}$$

viscosity drift

$$A_{DL} = \frac{9}{4}(\varkappa-3)\eta r^{-2}(\varrho_p\varrho_g c)^{-1}\mu_g^2\bar{E}, \tag{15}$$

asymmetric drift

$$A_{DA} = 4k\varrho_p^{-1}\mu_p\left[\left(3+\frac{9}{2}b\right)\mu_p - \frac{9}{2}b\left(b+1\right)\mu_g\right]\bar{E}. \tag{16}$$

In calculating the numerical values it is assummed that $\bar{E} = 100 J/m^3$, $\varrho_p = 10^3$ kg/m^3, $\varrho_g = 1.2$ kg/m^3, $\eta = 1.85\cdot10^{-5}$ Ns/m^2 and $c = 340$m/s. The plots of the maximum intensities of the field of force of different types of drift are shown in Figs. 1 and 2, where the respective frequencies of 1 KHz and 10 KHz were assumed. The plots made for particle radii within the interval 10^{-7}–10^{-4} m on a double logarithmic scale. This makes it easy to read the values of intensity for a density of the wave energy other than the given one, since the quantities represented in plots depend on E in a linear way and an increase or decrease in this value by one order of magnitude causes the same change in the value of the intensity drift fields.

Analysis of the plots indicates that particular kinds of drift dominate various intervals of variation of the particle radius. Thus, for the largest particles, with radii of the order of 10^{-4} m, the radiation drift is the strongest, whereas the smallest

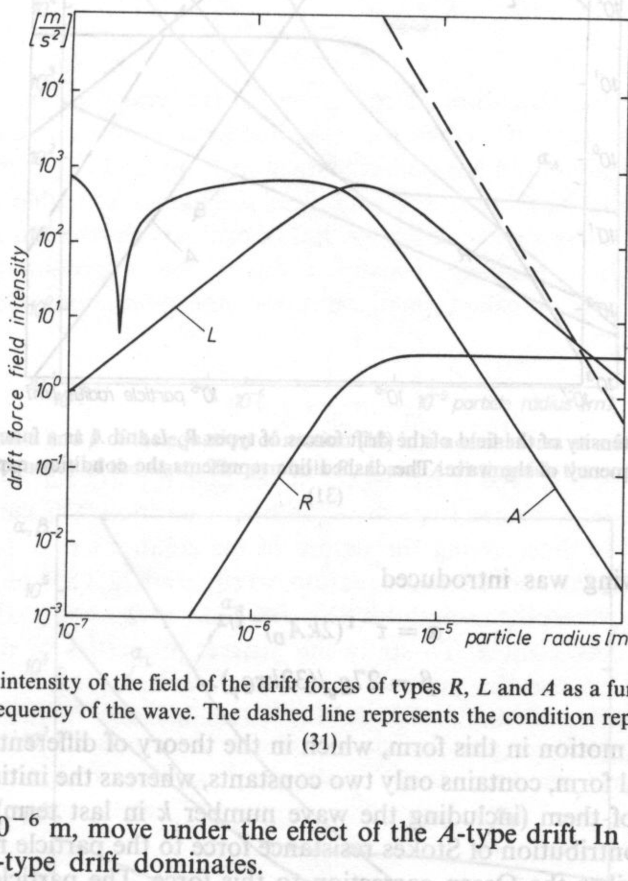

Fig. 1. Plots of the intensity of the field of the drift forces of types R, L and A as a function of the particle radius at 1 KHz frequency of the wave. The dashed line represents the condition represented by formula (31)

particles, $r < 10^{-6}$ m, move under the effect of the A-type drift. In the intermediate interval, the L-type drift dominates.

Let us now evaluate the other components of the basic equation of motion (12). To estimate their effect on the character of the solution, let us reduce to a minimum the number of constants in the equation by replacing the position and time by the nondimensional variables

$$y = \pi - 2kx, \qquad \theta = (2kA_D)^{\frac{1}{2}} t. \tag{17}$$

For the new variables the equation of motion becomes

$$\frac{d^2 y}{d\theta} + \alpha \frac{dy}{d\theta} + \beta \frac{dy}{d\theta}\left|\frac{dy}{d\theta}\right| + \sin y = 0, \tag{18}$$

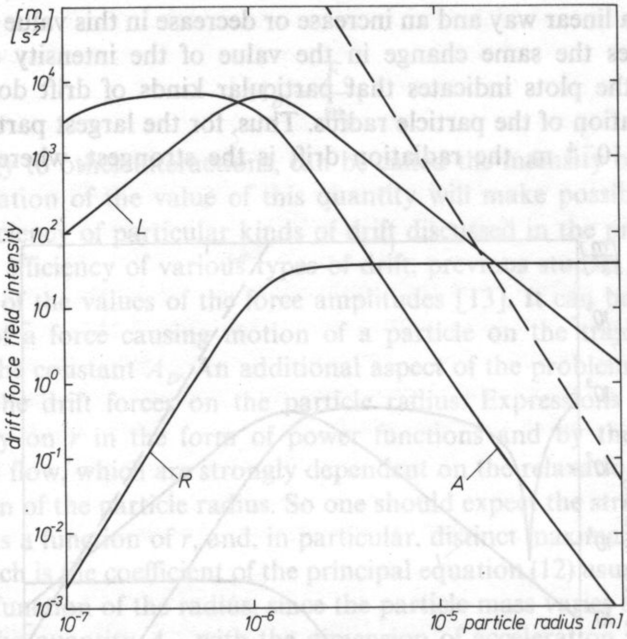

Fig. 2. Plots of the intensity of the field of the drift forces of types R, L and A as a function of the particle radius at 10KHz frequency of the wave. The dashed line represents the condition expressed by formula (31)

where the following was introduced

$$\alpha = \tau^{-1}(2kA_D)^{-1/2}, \tag{19}$$

$$\beta = 27\varrho_g/(32kr\varrho_p). \tag{20}$$

The equation of motion in this form, which in the theory of differential equations is called the normal form, contains only two constants, whereas the initial equation (12) contained four of them (including the wave number k in last term). The constant represents the contribution of Stokes resistance force to the particle motion, and the constant β describes the Oseen correction to this force. The particle mass and the drift force, and also the wavelength, are normalized in a way in this equation making it easier to analyze the effect of the two dissipation terms on the solution.

In general, the constants α and β depend on the parameters characterizing the particle and the wave; moreover, the constant α depends on the intensity of the drift force field, and, therefore, on the kind of drift. Assuming that same numerical values which were used in calculating the quantity A_D, one can estimate the constants α and β, and, thus, evaluate the two terms representing friction in the equation of motion.

Figs. 3 and 4 show plots of the constants of equation (18) as a functions of the particle radius for the respective frequencies of 1 KHz and 10 KHz. Analysis of these plots justifies one to state that the nonlinear friction term, represented by the

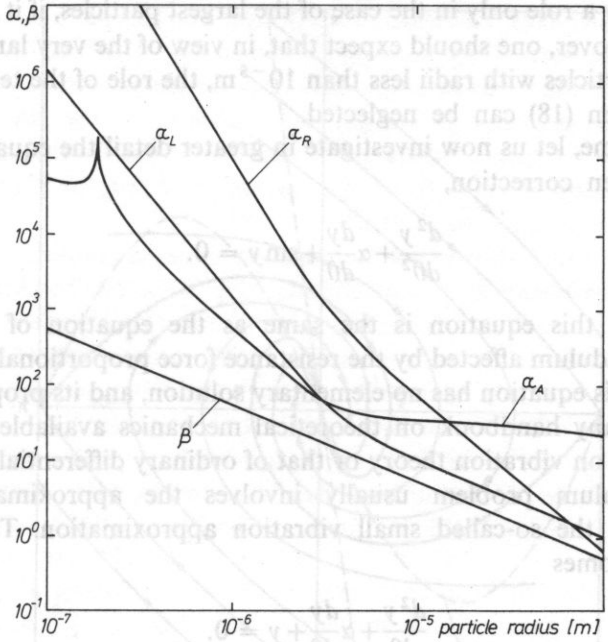

Fig. 3. The constants α and β of the equation of motion (18) in a normal form as a function of the particle radius at 1 KHz frequency of the wave. The symbols R, L and A distinguish the considered types of drift

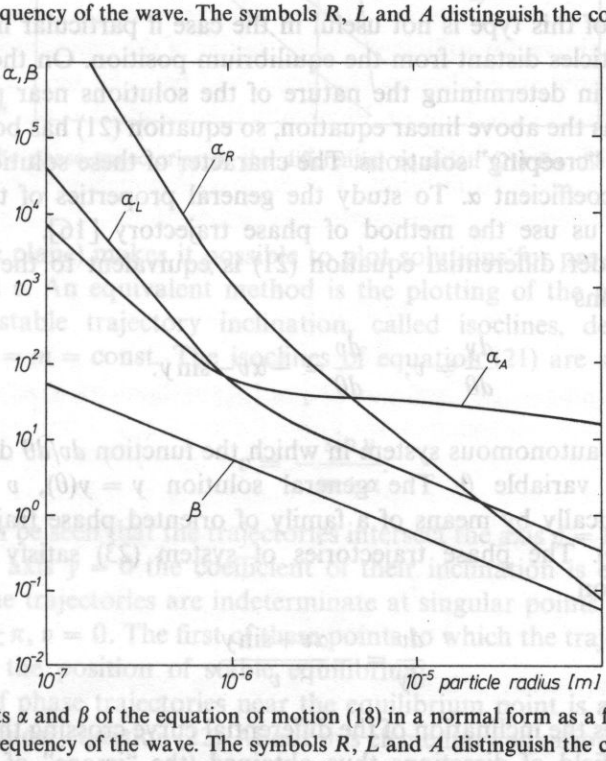

Fig. 4. The constants α and β of the equation of motion (18) in a normal form as a function of the particle radius at 10 KHz frequency of the wave. The symbols R, L and A distinguish the considered types of the drift

constant, can play a role only in the case of the largest particles, if it is assumed that $|dy/d\theta| < 1$. Moreover, one should expect that, in view of the very large values of the constant α for particles with radii less than 10^{-5} m, the role of the term representing inertia in equation (18) can be neglected.

In the meantime, let us now investigate in greater detail the equation of motion, refusing the Oseen correction,

$$\frac{d^2 y}{d\theta^2}+\alpha\frac{dy}{d\theta}+\sin y = 0. \tag{21}$$

In formal terms, this equation is the same as the equation of motion of the mathematical pendulum affected by the resistance force proportional to the velocity. However, even this equation has no elementary solution, and its properties have not been studied in any handbook on theoretical mechanics available to the present author, nor those on vibration theory or that of ordinary differential equations. The analogous pendulum problem usually involves the approximation $\sin y = y$, corresponding to the so-called small vibration approximation. The equation of motion then becomes

$$\frac{d^2 y}{d\theta}+\alpha\frac{dy}{d\theta}+y = 0. \tag{22}$$

The linearization of this type is not useful in the case if particular interest is in the trajectories of particles distant from the equilibrium position. On the other hand, it may prove useful in determining the nature of the solutions near points of stable equilibrium. Just as the above linear equation, so equation (21) has both quasi-periodic and aperiodic, "creeping" solutions. The character of these solutions depends on the value of the coefficient α. To study the general properties of the solutions of equation (21), let us use the method of phase trajectory [16].

The second-order differential equation (21) is equivalent to the system of two first-order equations

$$\frac{dy}{d\theta} = v; \quad \frac{dv}{d\theta} = -\alpha v-\sin y. \tag{23}$$

This is a so-called autonomous system in which the function $dv/d\theta$ does not depend explicitly on the variable θ. The general solution $y = y(\theta)$, $v = v(\theta)$ can be represented graphically by means of a family of oriented phase trajectories on the phase plane (y, v). The phase trajectories of system (23) satisfy the first-order differential equation

$$\frac{dv}{dy} = -\frac{\alpha v+\sin y}{v}, \tag{24}$$

which subordinates the inclination of the differential curve crossing this point to each point (y, v). The field of directions thus obtained (the "image" of the differential

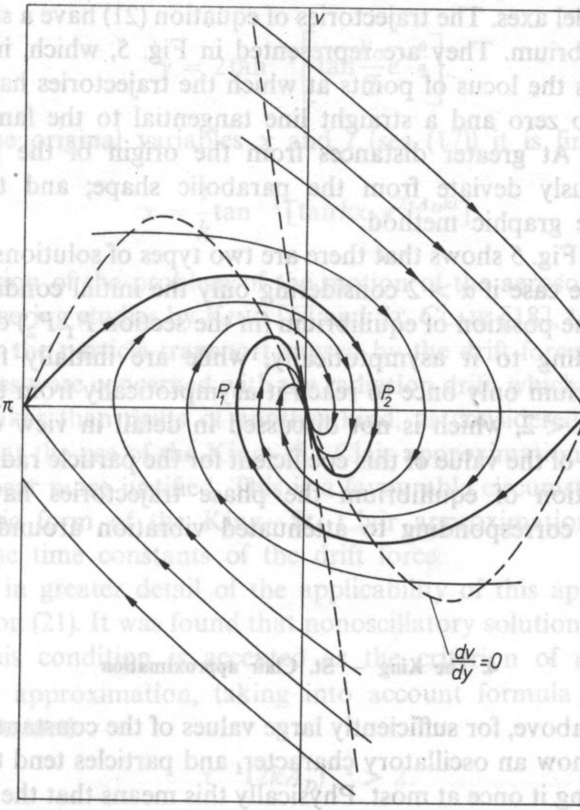

Fig. 5. The phase trajectories of the differential equation (21) for the case $\alpha > 2$

equation on the plane) makes it possible to plot solutions for pre-determined initial values of y and v. An equivalent method is the plotting of the geometric locus of points of the stable trajectory inclination, called isoclines, determined by the equation $dv/dy = m =$ const. The isoclines of equation (21) are sinusoids with the equation

$$v = -\frac{\sin y}{m+\alpha}. \tag{25}$$

In general, it can be seen that the trajectories intersect the axis $v = 0$ perpendicularly, whereas on the axis $y = 0$ the coefficient of their inclination is equal to $-\alpha$. The inclination of the trajectories are indeterminate at singular points, which are $y = 0$, $v = 0$ and $y = \pm\pi$, $v = 0$. The first of these points to which the trajectories converge, corresponds to the position of stable equilibrium.

The shape of phase trajectories near the equilibrium point is approximately the same as for the linear equation (22). About this elementary equation, it is known [17] that for $\alpha > 2$ its trajectories are parabolic in shape, tangential to a common straight line and with parallel axes. The trajectories of equation (21) have a similar shape near the point of equilibrium. They are represented in Fig. 5, which, in addition, mark a sinusoid which is the locus of points at which the trajectories have an inclination coefficient equal to zero and a straight line tangential to the family of parabolas mentioned above. At greater distances from the origin of the phase plane, the trajectories obviously deviate from the parabolic shape; and their course was determined by the graphic method.

The analysis of Fig. 5 shows that there are two types of solutions of the nonlinear equation (21) in the case if $\alpha > 2$ considering only the initial condition $y_0 = 0$. The particles close to the position of equilibrium (in the section $P_1 P_2$) do not go beyond this position, tending to it asymptotically, while are initially further cross the position of equilibrium only once to reach it asymptotically from the opposite side.

In the case if $\alpha < 2$, which is not discussed in detail in view of the previously performed analysis of the value of this coefficient for the particle radii of interest here, close to the position of equilibrium the phase trajectories have the shape of tightening spirals, corresponding to attenuated vibration around the position of equilibrium.

4. The King – St. Clair approximation

As was shown above, for sufficiently large values of the constant in equation (21), solutions do not show an oscillatory character, and particles tend to the position of equilibrium, crossing it once at most. Physically this means that the force of inertia is negligible compared with other forces: of friction and that of drift, leading the particle to the position of equilibrium. For large α, in equation (21) it is possible to neglect the second derivative representing the inertia force, therefore it is obtained that

$$\alpha\frac{dy}{d\theta}+\sin y = 0. \tag{26}$$

It is a nonlinear differential equation of the first order with separating variables,

$$\frac{dy}{\sin y} = -\frac{d\theta}{\alpha}. \tag{27}$$

The indeterminate integral on the left side of the equation is elementary,

$$\int\frac{dy}{\sin y} = \ln\tan\frac{y}{2}+\text{const}, \tag{28}$$

therefore, with the initial condition $y(0) = y_0$, the solution is in the form

$$\ln\tan\frac{y}{2} = -\frac{\theta}{\alpha}+\ln\tan\frac{y_0}{2},$$

or

$$y = 2\tan^{-1}\left[\tan\frac{y_0}{2}e^{-\frac{\theta}{\alpha}}\right]. \tag{29}$$

Returning to the original variables x and t (see (17)) it is finally obtained that

$$x = \frac{1}{k}\tan^{-1}[\tan kx_0 e^{2\tau A_D kt}]. \tag{30}$$

The above solution of the problem of the motion of the aerosol particle was given even in the pioneering studies by KING [6] and ST. CLAIR [18], to be applied later in other studies on the particle transport caused by the drift forces [9], [10]. In each case, those studies were concerned with the radiation drift, which, as could be seen, is significantly weaker than drifts of another kind, as considered here. It should be experted then that the use of the King–St. Clair approximation for the other types of drift may appear more justified. This is a favourable circumstance in view of the fact that analytic form of the King–St. Clair approximation permits accurate estimation of the time constants of the drift force.

To evaluate in greater detail of the applicability of this approximation, let us return to equation (21). It was found that nonoscillatory solutions could be obtained for $\alpha > 2$. If this condition is accepted as the criterion of applicability of the King–St. Clair approximation, taking into account formula (19), the following inequality is obtained:

$$\tau^{-1}(2kA_D)^{-\frac{1}{2}} > 2.$$

To connect mutually the more fundamental parameters of the particle and medium, let us also make use of formula (7), representing the relaxation time, and also the relation $k = 2\pi f/c$. Appropriate transformations give the condition

$$A_D < \frac{81\eta^2 c}{64\pi r^4 \varrho_p^2 f}. \tag{31}$$

The above inequality defines for what accelerations of the drift forces the particle motions do not have an oscillatory character and the King–St. Clair approximation can be used. In Figs. 1 and 2, representing the dependence $A_D(r)$ for different drift forces, condition (31) determines an area beneath the straight line marked by dashed line. It can be seen that it is satisfied for almost the whole considered range of particle radii and all drifts except for the radiation drift for $r > 10^{-5}$ m.

5. Conclusions

The criterion of applicability of the King–St. Clair approximation derived above can be confronted with the results of numerical calculations. It is most convenient to consider the equation of motion in the normal form (21), since the character of its

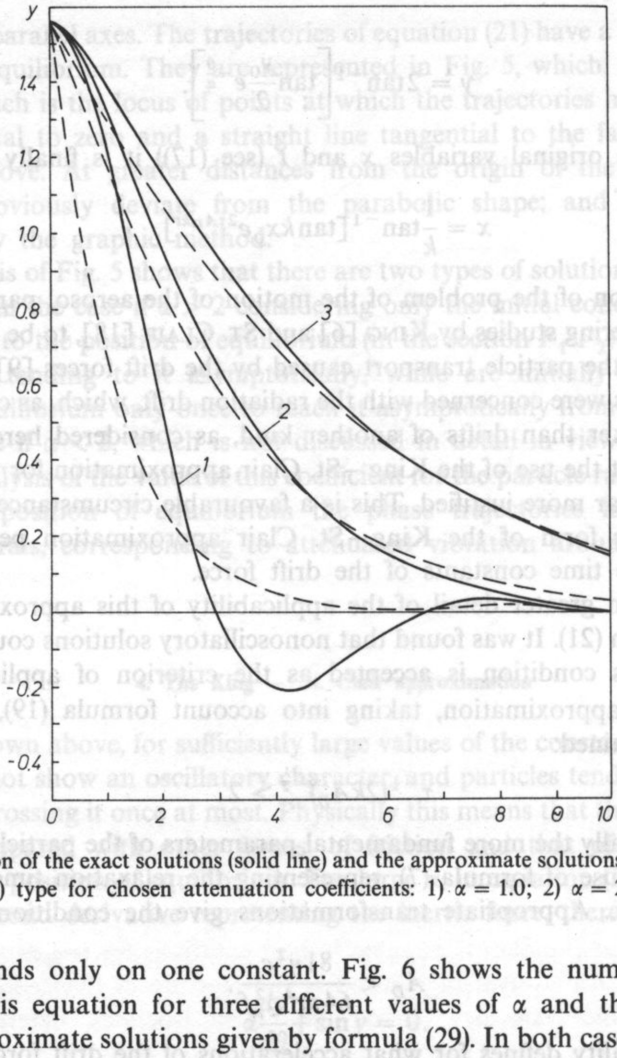

Fig. 6. A comparison of the exact solutions (solid line) and the approximate solutions of the King-St. Clair (dashed line) type for chosen attenuation coefficients: 1) $\alpha = 1.0$; 2) $\alpha = 2.5$; 3) $\alpha = 4.0$

solutions depends only on one constant. Fig. 6 shows the numerically obtained solutions of this equation for three different values of α and the courses of the respective approximate solutions given by formula (29). In both cases it was assumed that $y(0) = \pi/2$, where, moreover, for equation (21) it was put that $\dot{y}(0) = c$ (solution (29) does not depend on the initial velocity, as the first-order integral of the equation). It can be seen that for $\alpha > 2$ approximate solutions are close to the accurate ones. What is, however, more important is that, in each of the presented cases, even for $\alpha = 1$, a particle moving along the trajectory given by the accurate equation, reaches the position of equilibrium earlier than expected by the King – St. Clair formula. Because of this it is possible to predict that the application of the approximate solution gives a beforehand estimation of the constants of the transport phenomenon.

In summing up the earlier considerations, it can be stated moreover, that:

1) the drift forces directing aerosol particles to nodes or loops of the standing

wave, act in the most efficient way on particles with radii of the order of 10^{-5} m and less; with the radiation drift appearing to be the weakest;

2) for particles smaller than 10^{-5} m, it is possible to neglect the nonlinear correction for friction; on the other hand, it does not apply to the region where the radiation drift dominates;

3) within the same interval of the aerosol particle size, in investigating the motion, it is useful to apply the King – St. Clair approximation, making it possible to study the motion analytically. In this case, too, the region of domination of the radiation drift is outside the applicability range of the approximation.

Therefore, the radiation drift, which has been best investigated and most frequently indicated as the reason for the local concentration of aerosol particles acts only on relatively large particles, and the approximations presented here are hardly useful in studies on the motion which it causes. The drift plays, on the other hand, as a mechanism facilitating coagulation, a large role for particles with radii of the order of micrometers when the *L*-type and *A*-type forces dominate.

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