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PHASE VELOCITY OF A PRESSURE WAVE ON THE AXIS OF AN ACOUSTIC FIELD OF A CIRCULAR RING IN AN ACOUSTIC BAFFLE

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This paper is concerned with the so-called local velocity of a harmonic pressure wave of a circular ring vibrating with a constant velocity amplitude. The ring is placed in an infinite rigid acoustic baffle. The local propagation velocity was calculated on the axis conducted from the center of the ring perpendicularly to its surface.

The propagation velocity changes from infinity for $z = 0$ (singular point) to a constant value, c_0, for z equal to about 10 times the external radius of the ring.

1. Introduction

Because this paper is a continuation of paper [4] we will not repeat the complete reasoning presented previously. Only several fundamental formulae and definitions will be given.

In a case of an arbitrary harmonic pressure wave, i.e. a wave with an arbitrary amplitude, $A(x_i)$ $i = 1, 2, 3$, and wave front $f(x_i)$, we have

$$p(x_i, t) = A(x_i)e^{i[\omega t - f(x_i)]}. \tag{1}$$

The condition of wave propagation acquires the form:

$$\omega t - f(x_i) = \text{const}. \tag{2}$$

Differentiating both sides of equation (2) with respect to time we obtain:

$$\omega - |\text{grad} f| c = 0 \tag{3}$$

where the so-called local velocity of wave propagation:

$$c = \frac{\omega}{|\text{grad} f|} \tag{4}$$

depends on the position in the acoustic field. It is constant only for a plane wave and an elementary spherical wave, as in both cases we have:

$$|\mathrm{grad} f| = k_0 = \frac{\omega}{c_0} \tag{5}$$

where c_0 is the material constant — propagation velocity from d'Alambert's equation [1] [2].

Paper [4] has dealt with the local velocity of propagation of an acoustic pressure wave in the near field of a circular piston in a rigid baffle. The wave propagated along the axis of symmetry of the field, marked as the z-axis. It has been proved there that this velocity changes from $2c_0$ on the source, to c_0 for all greater distances from the source (piston), and practically at a distance equal to 5 radii of the piston, c differs from c_0 by less than 1%.

The second case considered was the local velocity in an acoustic field of a cylinder for a zero order wave. Here the ratio c/c_0 depends on the value of $k_0 r$, where r is a polar variable, i.e. the distance from the axis of the cylinder (source). As opposed to the first case, here c is lower than c_0 and with the increase of $k_0 r$ increases from zero (this value does not have physical sense) to c_0, and for $k_0 r \approx 5$ c differs from c_0 by less than 1%.

This paper discusses an acoustic system, in which the local velocity can be arbitrarily great and theoretically decreases from ∞ to c_0. Such a system is ensured by a circular ring placed in an infinite rigid baffle. The field is considered on the z-axis, which is the axis of symmetry of the system and is drawn from the source (ring) perpendicularily to its surface. It will be proved that when $z \to 0$, then the local velocity approaches infinity.

2. Calculation of the phase velocity on the axis of the near field of a circular ring

The formula used for the acoustic pressure on axis z, directed as it has been given in the introduction, is given in accordance to Stenzel [3], [5]. Denoting the internal and external radius of the ring by a_1 and a_2, respectively, the velocity amplitude on the ring by u_0 and the rest density of the medium by ϱ_0, we obtain:

$$p = 2 u_0 \varrho_0 c_0 \sin\left[\frac{k_0}{2}\left(\sqrt{z^2+a_2^2} - \sqrt{z^2+a_1^2}\right)\right] e^{i\left[\omega t + \frac{\pi}{2} - \frac{k_0}{2}\left(\sqrt{z^2+a_2^2} + \sqrt{z^2+a_1^2}\right)\right]} \tag{6}$$

The condition of wave propagation (phase stability when z and t are changed) has the following form:

$$\omega t + \frac{\pi}{2} - \frac{k_0}{2}\left(\sqrt{z^2+a_2^2} + \sqrt{z^2+a_1^2}\right) = \mathrm{const.} \tag{7}$$

Differentiating both sides of equation (7) with respect to time, we have:

$$\omega - \frac{k_0}{2}\left(\frac{z}{\sqrt{z^2+a_2^2}} + \frac{z}{\sqrt{z^2+a_1^2}}\right)\frac{dz}{dt} = 0. \tag{8}$$

Taking into consideration that:

$$\frac{dz}{dt} = c(z) \tag{9}$$

and

$$k_0 = \frac{\omega}{c_0} \tag{10}$$

we obtain

$$\frac{c}{c_0} = \frac{2\sqrt{z^2+a_1^2}\,\sqrt{z^2+a_2^2}}{z\sqrt{z^2+a_1^2}+\sqrt{z+a_2^2}}. \tag{11}$$

It is much more convenient to use formula (11) in a form expressed by the relative distance, z/a_2, and the radii ratio $-$ n, according to formula

$$a_1 = na_2, \quad n < 1. \tag{12}$$

Then we have

$$\frac{c}{c_0} = \frac{2\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}\cdot\sqrt{1+\left(\frac{z}{a_2}\right)^2}}{\frac{z}{a_2}\left(\sqrt{n^2+\left(\frac{z}{a_2}\right)^2}+\sqrt{1+\left(\frac{z}{a_2}\right)^2}\right)} \tag{13}$$

When $n = 0$, the ring changes into a circular piston and then $(a_2 = a)$

$$\frac{c}{c_0} = \frac{2\sqrt{1+\left(\frac{z}{a}\right)^2}}{\frac{z}{a}+\sqrt{1+\left(\frac{z}{a}\right)^2}} \tag{14}$$

in accordance to the result in the paper [4].

When z/a_2 in formula (13) approaches zero, for $n \neq 0$, then $c/c_0 \to \infty$, and thus in the centre of the circular ring, on the baffle, the propagation velocity of the pressure wave has to be infinitely great. This result is only apparently surprising, because the rigidity of the baffle is tantamount to the infinite value of wave resistance. The baffle itself has such a resistance. However, when also the propagation velocity in the medium, in which the wave propagates, approaches infinity when $z/a_2 \to 0$, then we have the continuity of the boundary condition ensured. On the other hand, when $z/a_2 \to \infty$, we have $c/c_0 \to 1$.

Fig. 1 presents values of c/c_0 for various n, including the case of an infinitely thin ring, i.e. when $n = 1$. This case has been considered separately, because substituting $n = 1$, i.e. $a_2 = a_1$, in formula (6) gives the acoustic pressure equal to zero. Thus formula (6) can not be used in such a case. However, further on it will be proved than the result obtained with a different method is the same as if we substituted $n = 1$ in formula (13).

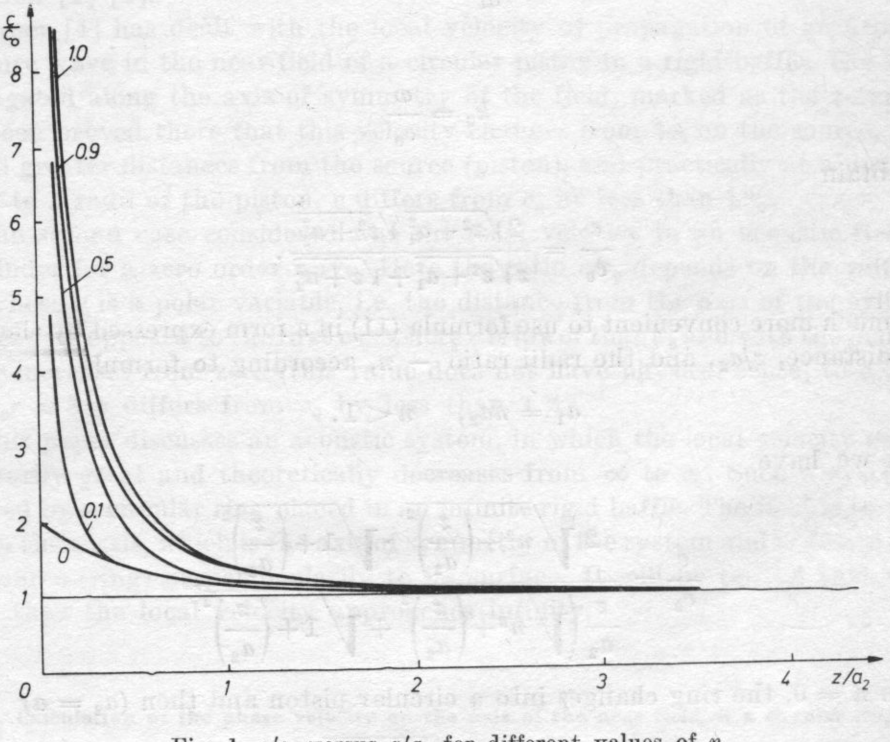

Fig. 1. c/c_0 versus z/a_2 for different values of n

3. Phase velocity on the axis in a case of an infinitely thin ring. i.e. a circumference closely packed with point sources

Even though the case of the near field of an infinitely thin ring is one of the simplest examples of an acoustic field, it has not been mentioned in literature. Only the formula for the far field has been given [1], [2]. Therefore, we will begin with the calculation of the near field.

According to Fig. 2 we will mark the radius of the ring by a and the velocity amplitude on the ring, calculated formally as the productiveness of the source per unit of arc length, by u_0, and the acoustic pressure on the z-axis will have the following form [1], [2], [4]:

$$p = \frac{k_0 \varrho c_0}{2\pi} u_0 e^{i(\omega t + \pi/2)} \int_L \frac{e^{-ik_0 r}}{r} a d\varphi \tag{15}$$

where L is the length of the circumference of the circle (ring.). As r is a constant value, and we integrate over variable φ from 0 to 2π, then we have

$$p = k_0 \varrho c_0 u_0 e^{i(\omega t + \pi/2 - k_0 r)} \frac{a}{r}. \tag{16}$$

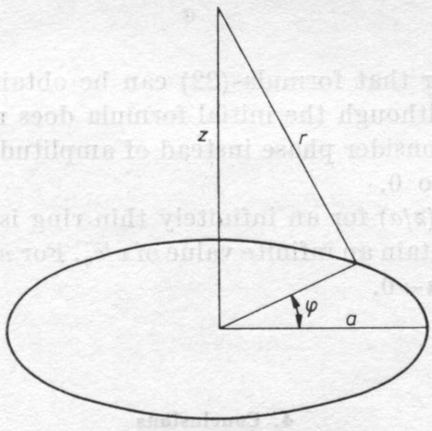

Fig. 2. The geometry of an infinitely thin ring

Naturally, also

$$r = \sqrt{z^2 + a^2}, \tag{17}$$

thus formula (16) will have the following form:

$$p = k_0 \varrho c_0 u_0 \frac{e^{i(\omega t + \pi/2 - k_0 \sqrt{z^2+a^2})}}{\sqrt{z^2+a^2}} \tag{18}$$

The condition of wave propagation is

$$\omega t + \frac{\pi}{2} - k_0 \sqrt{z^2 + a^2} = \text{const}. \tag{19}$$

Differentiating both sides of (19) with respect to time we obtain

$$\omega - k_0 \frac{z}{\sqrt{z^2+a^2}} \frac{dz}{dt} = 0 \tag{20}$$

and taking into consideration that

$$\frac{dz}{dt} = c, \quad k_0 = \frac{\omega}{c_0} \tag{21}$$

finally we acquire the following formula for an infinitely thin ring

$$\frac{c}{c_0} = \frac{\sqrt{1+\left(\frac{z}{a}\right)^2}}{\frac{z}{a}}. \tag{22}$$

It is worth mentioning that formula (22) can be obtained from formula (13) by accepting $n = 1$, although the initial formula does not suit this case. This happens, because we consider phase instead of amplitude, which for $a_2 = a_1$ in formula (6) is equal to 0.

The curve $c/c_0 = f(z/a)$ for an infinitely thin ring is shown in Fig. 1. Also there for $z/a = 0$ we obtain an infinite value of c/c_0. For $n = 1$ the curve ascends most steeply when $z/a \to 0$.

4. Conclusions

An acoustic antenna in the shape of a circular ring radiates a pressure wave, which propagates along axis z (axis of symmetry of the field) with a variable velocity dependent on the position, i.e. on variable z. For $z = 0$ this velocity exhibits a singularity – it is infinite. This means that the central point, $z = 0$, does not vibrate. This corresponds to the boundary condition on the baffle. In a point arbitrarily close to $z = 0$, but at a finite value of z, the local propagation velocity of a wave can be arbitrarily great. For every z this velocity has the greatest value for an infinitely thin ring ($n = 1$) and the smallest value for a circular piston ($n = 0$).

When z increases (as well as z/a_2, where a_2 is the external radius of the ring), then the propagation velocity of a pressure wave decreases to the material velocity, c_0.

These effects practically occur only at relatively small distances from the plane, i.e. up to the value of $z/a_2 \approx 4$, at greater distances they are imperceptible. However, they can play a significant role in large antennas in the shape of a circular ring not only for acoustic waves.

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