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ACOUSTIC PRESSURE OF A SYSTEM OF CONCENTRIC ANNULAR SOURCES IN A PARALLEL-PIPED LAYER OF A GASEOUS MEDIUM*

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In this paper, an expression was derived for the acoustic pressure distribution in the near and far fields, radiated by a system of planar concentric annular sources. The propagation of the pressure wave was considered for a parallel-piped layer, bounded by rigid baffles, filled with a lossless gaseous medium. It was assumed that the system of sources, with known axially-symmetric vibration velocity distribution, was on one of the planar and rigid baffles. Linear phenomena dependent sinusoidally on time were analysed.

By solving the Neumann boundary problem by means of the method of Hankel transforms of the zeroth order, an integral expression was obtained for the acoustic pressure distribution in a parallel-piped layer. The pressure, expressed by an integral in the complex variable plane, was represented in the form of a series of residua at the poles of the subintegral function, giving a formula, convenient for practical calculations and easy to interpret, in the form of a series of normal waves. The theoretical analysis of the acoustic pressure distribution was supported by numerical examples, for which curves of acoustic pressure were plotted as a function of the distance from the source.

1. Introduction

This paper is concerned with investigations of the acoustic properties of a system of concentric annular sources, consisting in the determination of the acoustic field distribution in a parallel-piped layer.

Most of the previous studies on the wave generation and propagation were concerned with analyses of the acoustic pressure distribution in the far field for the whole space or for a half-space. In the latter case, it was assumed that the source of the acoustic wave was in a planar and rigid baffle.

* This investigation was carried out within the problem MR.I.24

Considering these problems, McLachlan [4] carried out a detailed analysis of the problem of the directional characteristic of a planar annular piston with uniform vibration velocity distribution.

A similar expression for the directional characteristic of a single circular ring was given by Thompson [7].

A detailed analysis of the pressure distribution in the far field, radiated by a membrane or a circular plate, excited to axially-symmetric or axially-nonsymmetric self vibration, was carried out in paper [6]. This analysis was an attempt to determine the way in which the field distribution is affected by the individual elements of the vibrating surface of the membrane or plate, i.e. a system of planar concentric annular sources.

More complex acoustic phenomena occur for generation of acoustic waves in a layered medium, and in the simplest case-in a parallel-piped layer.

Within this range of problems, deep theoretical research was carried out by Brekhovskikh [1], by analysing both electromagnetic waves and acoustic waves generated by point sources.

The present paper refers to papers [6] and [1]. The object of the analysis carried out in this theoretical study is the investigation of the pressure distribution in the near and far field, radiated by a system of planar concentric annular sources. The wave propagation was considered for a parallel-piped layer, filled with a lossless gaseous medium, bounded by rigid baffles. Linear phenomena dependent sinusoidally on time were analysed on the assumption that the system of sources was on one of the planar and rigid baffles.

Assuming knowledge of the vibration velocity distribution on the surface of the system of sources, the Neumann boundary problem was solved and an integral expression obtained for the acoustic pressure distribution in the parallel-piped layer. In view of the axially-symmetric vibration velocity distribution assumed here, the general method of Hankel transforms of the zeroth order was used. The pressure, expressed by an integral in the complex variable plane, was represented in the form of a series of residua at the poles of the subintegral function, giving a formula, convenient for practical calculations and easy to interpret, for the acoustic pressure in the form of a series of the so-called normal waves.

In the numerical example, calculations were made for the acoustic pressure radiated by a circular membrane, excited to axially-symmetric vibration, i.e. by a system of planar concentric annular sources. The surface of the membrane was divided in such a way that the annular surfaces were bounded by nodal circles for the respective vibration modes. In view of the axially-symmetric vibration assumed here, all the points on the surface of any of the rings vibrate in phase, whereas those of the adjacent rings vibrate in antiphase.

The theoretical analysis carried out in this paper for the acoustic field radiated by a planar system of annular sources in a parallel-piped layer was supported by numerical examples, for which curves of pressure were plotted as a function of the distance from the source.

2. Expression for the acoustic pressure in integral form

We consider the problem of acoustic wave propagation in a parallel-piped layer of a homogeneous gaseous medium with the rest density ϱ_0 and the sound wave velocity c_0. The parallel-piped layer of the medium is bounded by planar, rigid baffles spaced at the distance h. The sound source is a vibrating system of a finite number of concentrically situated planar circular rings, placed on the plane $z = 0$, which is the rigid baffle.

We consider linear phenomena dependent sinusoidally on time. It is assumed that the vibration velocity distributions on the surface of the rings are axially-symmetric, assumed to be known. Thus, the value of the normal component of the vibration velocity is known, $\boldsymbol{n}\boldsymbol{v} = -v$, where $\boldsymbol{n}$ is a unitary vector normal to the surface of the source, directed in a direction opposite from that of the velocity vector $\boldsymbol{v}$. The vibrating surface of the sth circular ring, arbitrarily chosen from the system of sources, is bounded by circles with the radii r_s and r_{s-1}, with $r_{s-1} < r_s$.

Within the parallel-piped layer, filled by a gaseous medium with the density ϱ_0, for the acoustic potential $\Phi_s(r, z)\exp(i\omega t)$, whose source is the sth vibrating circular ring, the Helmholtz equation

$$\Delta\Phi_s(r, z) + k^2\Phi_s(r, z) = 0 \tag{1}$$

is valid. The quantity $k = \omega/c_0$ is the wave number, ω is the angular frequency. One should find such a solution of equation (1) for the region $\{0 \leqslant z \leqslant h, 0 \leqslant r < \infty\}$, which satisfies the inhomogeneous Neumann boundary condition

$$\frac{\partial\Phi_s(r, z)}{\partial z}\bigg|_{z=0} = \begin{cases} -v_s(r) & \text{for the sth ring} \\ 0 & \text{beyond the ring,} \end{cases} \tag{2}$$

and the homogeneous Neumann boundary condition

$$\frac{\partial\Phi_s(r, z)}{\partial z}\bigg|_{z=h} = 0, \tag{3}$$

where $v_s(r)$ is the vibration velocity distribution function assumed to be known.

Following the general method of Hankel transforms of the zeroth order, the solution is sought in the form

$$\Phi_s(r, z) = \int_0^\infty g(\tau, z) J_0(\tau r)\tau d\tau, \tag{4}$$

where

$$g(\tau, z) = \int_0^\infty \Phi_s(r, z) J_0(\tau r) r dr, \tag{5}$$

J_0 is a Bessel function of the zeroth order, while the complex parameter τ is a propagation constant in the radial direction. Equation (1) can become

$$\frac{d^2 g(\tau, z)}{dz^2} + (k^2 - \tau^2) g(\tau, z) = 0, \tag{6}$$

with the following solution:

$$g(\tau, z) = A \exp(-i\gamma z) + B \exp(i\gamma z), \tag{7}$$

where A and B are integration constants, γ is a propagation constant towards the axis z and $k^2 = \gamma^2 + \tau^2$.

Relation (2) is replaced by

$$\left. \frac{dg(\tau, z)}{dz} \right|_{\substack{z=0 \\ r_{s-1} \leqslant r \leqslant r_s}} = -W_s(\tau), \tag{8}$$

where

$$W_s(\tau) = \int_{r_{s-1}}^{r_s} v_s(r_0) J_0(\tau r_0) r_0 dr_0 \tag{9}$$

is a characteristic function of the sth ring, which is the set of points $\{r_{s-1} \leqslant r \leqslant r_s, 0 \leqslant \varphi \leqslant 2\pi\}$.

The integration constants A and B can be determined from relations (3) and (8). This gives

$$g(\tau, z) = -W_s(\tau) \frac{\cos\gamma(h-z)}{\gamma \sin\gamma h}. \tag{10}$$

The use of inverse Hankel transformation from formula (4) and consideration that for phenomena sinusoidally dependent on time the dependence of pressure on the potential is linear: $p(r) = ik\varrho_0 c_0 \Phi(r)$ give the sought solution in integral form for the acoustic pressure generated by the sth ring, in the form

$$p(r, z) = -ik\varrho_0 c_0 \int_0^\infty \frac{\cos\gamma(h-z)}{\gamma \sin\gamma h} W_s(\tau) J_0(\tau r) \tau d\tau. \tag{11}$$

3. Expression for the acoustic pressure in the form of a series of normal waves

When the distance at which a point of the field is, is much larger than the linear dimensions of the source and than the length of the acoustic wave radiated, it is convenient to transform formula (11) to a form in which the integral is calculated within the limits $(-\infty, +\infty)$. The following dependence can be used [8]:

$$J_0(u) = \frac{1}{2}\,[H_0^{(2)}(u) - H_0^{(2)}(-u)], \tag{12}$$

where $H_0^{(2)}$ is a cylindrical Hankel function of the zeroth order, of the second kind, satisfying the radiation condition of the time dependence $\exp(i\omega t)$. The substitution of (12) in (11) and the taking advantage of the evenness property of the characteristic function $W_s(\tau)$ with respect to the variable τ (see definition (9)) give

$$p(r, z) = \varrho_0 c_0 \frac{-ik}{2} \int_{-\infty}^{+\infty} \frac{\cos\gamma(h-z)}{\gamma \sin\gamma h}\, W_s(\tau) H_0^{(2)}(\tau r)\,\tau d\tau. \tag{13}$$

Integration will be carried out over the real axis, where the singular point $\tau = 0$, in integrating for the transition from negative real values to positive ones, is bypassed along a small half-circle underneath, since $H_0^{(2)}(\tau r)$ has a logarithmic singularity at the point $\tau = 0$.

Expression (13) can be represented in the form of a series of residua at the poles of the subintegral function. To achieve this, it is possible to use Jordan's lemma and Cauchy's residua theorem [2], closing the integration path in the lower halfplane of the complex variable τ (Fig. 1).

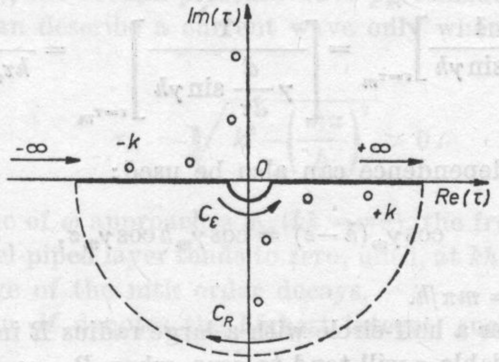

Fig. 1. Integration path including poles of the subintegral function (13) in the lower half-plane of the complex variable

The poles of the subintegral function (13) are at the points defined by the roots of the equation $\sin\gamma h = 0$, whose solutions are $\gamma h = m\pi$; $m = 0, \pm 1, \pm 2, \ldots$ In the neighbourhood of the points, defined by a root corresponding to $m = 0$,

$$\frac{1}{\gamma \sin\gamma h} \simeq \frac{1}{h(k^2 - \tau^2)}. \tag{14}$$

Expression (14) has poles of the first order at the points $\tau_0 = k$, $\tau_0 = -k$. In general, when $m = 0, \pm 1, \pm 2, \ldots$, from the expression $\gamma h = m\pi$, for the poles $\tau_m = \pm\sqrt{k^2-(m\pi/h)^2}$.

It will be easier to analyse integral (13) when it is considered that in real conditions in a gaseous medium there is the phenomenon of absorption of propagating waves. In view of this, it is assumed initially that the wave number k has a low negative value of the imaginary component $k = k_0 - i\delta$; k_0, $\delta > 0$.

The integration will thus include the poles in the lower half-plane of the complex variable τ, i.e. those for which $\tau_0 = k$, $\tau_m = \sqrt{k^2-(m\pi/h)^2}$. The integration gives as a result a final expression in which it will be possible, in the limits, to pass from the value of δ to zero.

The residuum of the subintegral function (13) can be calculated, i.e.

$$\operatorname{Res}[F(\tau)]_{\tau=\tau_m} = \tau_m \cos\gamma_m(h-z) W_s(\tau_m) H_0^{(2)}(\tau_m r) \operatorname{Res}\left[\frac{1}{\gamma \sin\gamma h}\right]_{\tau=\tau_m}. \tag{15}$$

For the pole $m = 0$, considering relation (14),

$$\operatorname{Res}\left[\frac{1}{\gamma\sin\gamma h}\right]_{\tau=k} = \operatorname{Res}\left[\frac{1}{k(h-\tau)(k+\tau)}\right]_{\tau=k} = -\frac{1}{2kh}. \tag{16}$$

For the poles $m = 1, 2, \ldots$

$$\operatorname{Res}\left[\frac{1}{\gamma\sin\gamma h}\right]_{\tau=\tau_m} = \left[\frac{1}{\gamma\dfrac{d}{d\tau}\sin\gamma h}\right]_{\tau=\tau_m} = \frac{-1}{h\tau_m\cos\gamma_m h}. \tag{17}$$

The following dependence can also be used:

$$\cos\gamma_m(h-z) = \cos\gamma_m h \cos\gamma_m z, \tag{18}$$

where $\gamma = 0$, $\gamma_m = m\pi/h$.

The integral over a half-circle with a large radius R in the lower half-plane of the complex variable τ will tend to zero, when $R\to\infty$. For very large R the function $H_0^{(2)}(\tau r)$ will tend to zero, when $r \neq 0$. The value of the integral over the half-circle C_s with a small radius ε in the limits for $\varepsilon\to 0$ will also tend to zero.

The use of Cauchy's residua theorem indicates that integral (13) is equal to the sum of the remainders of the subintegral function at the poles, multiplied by the factor $-2\pi i$. There emerges the following expression for the acoustic pressure, in the form of a series of normal waves:

$$p(r, z) = p_0(r) + \sum_{m=1}^{\infty} p_m(r, z), \tag{19}$$

where

$$p_0(r) = \varrho_0 c_0 \frac{\pi k}{2h} W_s(k) H_0^{(2)}(kr), \tag{20}$$

$$p_m(r, z) = \varrho_0 c_0 \frac{\pi k}{h} \cos \frac{m\pi z}{h} W_s(\tau_m) H_0^{(2)}(r\tau_m). \tag{21}$$

Each of the components $p_m(r, z)$ in expression (21) is suitable to describe the pressure wave, propagating towards the increasing values of the radial variable r and the standing wave towards the variable z.

When τ_m tends to zero, the amplitude of the mth normal wave tends to infinity, in view of the infinite value of the Hankel function. This specific case corresponds to a resonance at which the pressure value is in theory infinitely large. This follows from the analysis carried out for the acoustic wave propagation in a lossless medium. The parameter $\gamma_m = m(\pi/h)$ is called the critical wave number (the cut-off wave number), since it determines the wave frequency ω_m at which free propagation of a normal wave of the mth order decays, i.e. the frequency at which τ_m takes a zero value. This occurs when $\gamma_m = k$, i.e. when $kh = m\pi$.

For such frequencies ω (the wave frequencies in a free space) at which $\omega > \omega_m(k = m\pi/h)$, the normal pressure wave p_m considered propagates freely. Expression (21) can describe a current wave only when τ_m is a real, positive quantity, i.e.

$$\tau_m = \sqrt{k^2 - \left(\frac{m\pi}{h}\right)^2} > 0. \tag{22}$$

When the value of ω approaches $\omega_m(kh \to m\pi)$, the frequency of the normal wave in the parallel-piped layer tends to zero, until, at $kh = m\pi$, the free propagation of the wave of the mth order decays.

In a case when M denotes the highest integer, such for which

$$M\pi < kh \leqslant (M+1)\pi, \tag{23}$$

then M will define the order of the highest normal pressure wave, p_m, which can propagate in the parallel-piped layer with a prescribed value of the interference parameter kh.

For $\omega < \omega_m(kh < m\pi)$ the normal pressure wave p_m considered is damped along the propagation direction. In this case $m > M$, whereas the phase of the square root is assumed to be $-\pi/2$, i.e.

$$\tau_m = -i|\tau_m|, \tag{24}$$

where

$$|\tau_m| = \sqrt{\left(\frac{m\pi}{h}\right)^2 - k^2} > 0. \tag{24a}$$

The characteristic function $W(\tau_m)$, defined by formula (9), is even with respect to the parameter τ_m, which takes real values for $m < M$ and purely imaginary ones for $m > M$. It is thus a real function, i.e. $W(\tau_m) = W^*(\tau_m)$. For real τ_m $(m < M)$ [8]

$$H_0^{(2)}(r\tau_m) = J_0(r\tau_m) - iN_0(r\tau_m), \tag{25}$$

while for purely imaginary $\tau_m (m > M)$

$$H_0^{(2)}(-ir|\tau_m|) = \frac{2i}{\pi} K_0(r|\tau_m|), \tag{26}$$

where N_0 is a Neumann function of the zeroth order and K is a MacDonald function of the zeroth order.

After separating the real component $p' = Re(p)$ and the imaginary one $p'' = \mathrm{Im}(p)$, the acoustic pressure (19) can be written in the following way:

$$p = p' + ip'', \tag{27}$$

where

$$p'(r, z) = \varrho_0 c_0 \frac{\pi k}{2h} \sum_{m=0}^{M} \varepsilon_m \cos\left(\frac{m\pi z}{hW}\right) W_s(\tau_m) J_0(r\tau_m), \tag{28}$$

$$p''(r, z) = \varrho_0 c_0 \frac{\pi k}{2h} \left\{ -\sum_{m=0}^{M} \varepsilon_m \cos\left(\frac{m\pi z}{h}\right) W_s(\tau_m) N_0(r\tau_m) + \right.$$

$$\left. + \frac{2}{\pi} \sum_{m=M+1}^{\infty} \varepsilon_m \cos\left(\frac{m\pi z}{h}\right) W_s(-i|\tau_m|) K_0(r|\tau_m|) \right\}, \tag{29}$$

$$\varepsilon_m = 1 \text{ for } m = 0 \quad \text{and } \varepsilon_m = 2 \text{ for } m \geqslant 1.$$

In expression (29) the infinite series $(M+1 \leqslant m < \infty)$ can be replaced in specific numerical calculations by a finite number of terms $M' - (M+1)$. Summation is then carried out over the index m from $m = M+1$ to the value $m = M'$, dependent on the magnitude of the parameters ka, kh and kr.

4. Acoustic pressure for much larger distances with respect to the length of the wave radiated

In a specific case, when the distance r of a point of the field is much large than the length of the wave radiated, λ, and when $\tau_m \neq 0$, the Bessel, Neumann and MacDonald functions in expressions (28) and (29) can be replaced by their asymptotic representations:

$$J_0(r\tau_m) \simeq \sqrt{\frac{2}{\pi r\tau_m}} \cos\left(r\tau_m - \frac{\pi}{4}\right),$$

$$N_0(r\tau_m) \simeq \sqrt{\frac{2}{\pi r\tau_m}} \sin\left(r\tau_m - \frac{\pi}{4}\right),$$

$$K_0(r|\tau_m|) \simeq \sqrt{\frac{\pi}{2r|\tau_m|}} \exp(-r|\tau_m|). \tag{30}$$

From the practical point of view, it is very important to be able to predict the pressure distribution about the source, when the distance r of a point of the pressure studied is larger than the linear dimensions of the source. When, in addition, in series (28) and (29) $r \geqslant h$, then in practical calculations only the finite number of M terms can be considered, for which $m\lambda < 2h (m\pi < kh)$. Consideration of dependencies (30), (28) and (29) gives then

$$p(r, z) = \varrho_0 c_0 \frac{k}{h} \sqrt{\frac{\pi}{2r}} \exp\left(i\frac{\pi}{4}\right) \sum_{m=0}^{M} \varepsilon_m \cos\left(\frac{m\pi z}{h}\right) W_s(\tau_m) \frac{\exp(-ir\tau_m)}{\sqrt{\tau_m}}.$$

5. Characteristic function

Bearing in mind the practical applications, the surface vibration velocity distribution can be assumed to be the same as that which occurs in the case of a circular membrane excited to axially-symmetric vibration. Such dimensions of the individual annular surfaces are assumed that they are bounded by circles corresponding to nodal circles for the $(0, n)$ mode of the vibrating circular membrane. Since the object of the analysis is axially-symmetric vibration, then all the points on the surface of any of the rings vibrate in phase, while all the two adjacent annular surfaces vibrate in antiphase.

For phenomena harmonic in time, the distribution of self vibration velocity for the $(0, n)$ axially-symmetric mode is expressed by the formula [3]

$$v_n(r) = v_{0n} J_0\left(\frac{r}{a} q_n\right), \tag{32}$$

where $q_n = a(\omega_n/c_M)$ is a root of the equation $J_0(q_n) = 0$, ω_n is the angular frequency of self vibration, c_M is the wave propagation velocity on the membrane, v_{0n} is the maximum vibration velocity. The radii r_s of the nodal circles for the $(0, n)$ mode of vibration, as found from the equation

$$J_0\left(\frac{r_s}{a}\right) q_n = 0, \tag{33}$$

are $r_s = a(q_s/q_n)$, $s = 1, 2, \ldots, n-1$; with, for $s = n$, $r_n = a$ being the radius of the membrane, i.e. of the external nodal circle (see [5]).

The characteristic function (9)

$$W_s(\tau) = v_{0n} \int\limits_{aq_{s-1}/q_n}^{aq_s/q_n} J_0\left(\frac{r_0}{a} q_n\right) J_0(\tau r_0) r_0 dr_0 \tag{34}$$

for the sth ring, being the set of points $\{a(q_{s-1}/q_n) \leqslant r \leqslant a(q_s/q_n),\ 0 \leqslant \varphi \leqslant 2\pi$, after considering the integral property [8]

$$\int\limits_0^u wJ_0(hw)J_0(lw)\,dw = \frac{u}{h^2-l^2}\{hJ_1(hu)J_0(lu) - lJ_0(hu)J_1(lu)\}, \tag{35}$$

is

$$W_s(\tau) = \frac{v_{0n}a^2}{q_n^2-(a\tau)^2}\left\{q_sJ_1(q_s)J_0\left(\frac{q_s}{q_n}a\tau\right) - q_{s-1}J_1(q_{s-1})J_0\left(\frac{q_{s-1}}{q_n}a\tau\right)\right., \tag{36}$$

where $J_0(q_s) = J_0(q_{s-1}) = 0$.

When in turn the source of the acoustic field is a system of concentric circular pistons, i.e. when the vibration velocity distribution on the surface of the sth annular piston is uniform, $v_s = v_{0s} = \text{const.}$, then the characteristic function

$$W_s(\tau) = v_{0s} \int\limits_{r_{s-1}}^{r_s} J_0(\tau r_0) r_0 dr_0, \tag{37}$$

after considering the integral property (35) for $l = 0$, is

$$W_s(\tau) = \frac{v_{0s}}{\tau}[r_sJ_1(\tau r_s) - r_{s-1}J_1(\tau r_{s-1})]. \tag{38}$$

In calculating the characteristic function from formulae (36) and (38), the quantity τ should be replaced by τ_m, as defined by relation (22) or (24), depending on whether $m\pi < kh$ or $m\pi > kh$.

6. Numerical example with analysis of results

In analysing the acoustic pressure radiated by a circular membrane for the axially-symmetric vibration mode (0, 2), two vibrating surface elements, separated by the nodal circle $a(q_1/q_2)$, where $a = 2$ cm, $q_1 = 2.4048\ldots$, and $q_2 = 5.5201\ldots$, were distinguished. The central element of the membrane is a circular source, being the set of points $\{0 \leqslant r \leqslant a(q_1/q_2),\ 0 \leqslant \varphi \leqslant 2\pi\}$, the external element is an annular source, being the set of points $\{a(q_1/q_2) \leqslant r \leqslant a,\ 0 \leqslant \varphi \leqslant 2\pi\}$.

In the numerical example the value of the acoustic pressure p radiated by a surface source was referred to the pressure $p_p = \varrho_0 c_0 v_0$. The relative pressure value p/p_0 was represented graphically depending on the dimensionless parameter r/h,

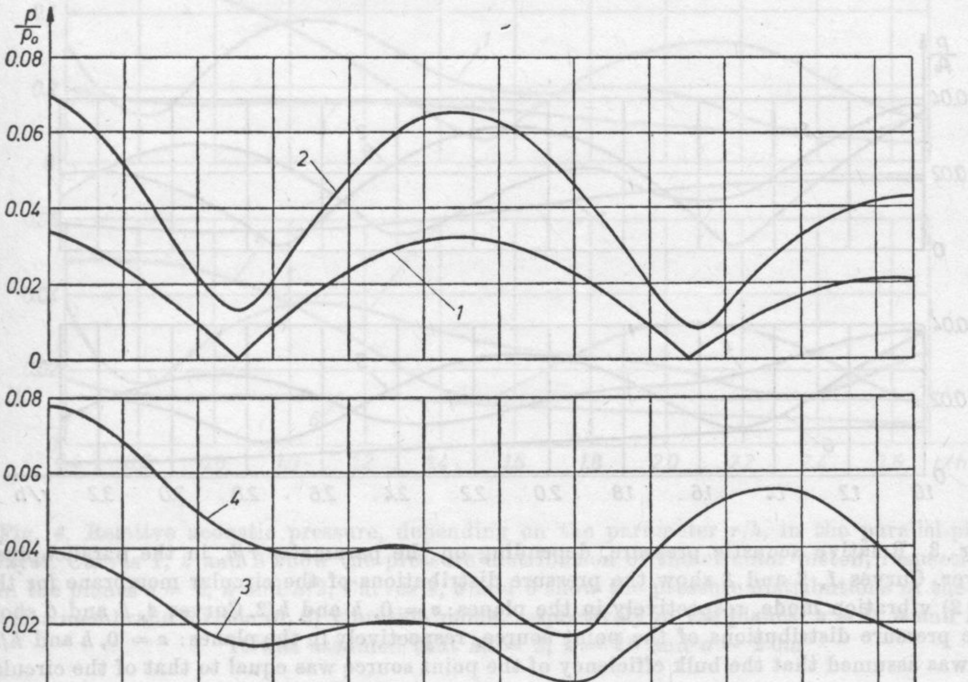

Fig. 2. Relative acoustic pressure p/p_0 of the sources separated on the circular membrane for the (0, 2) vibration mode, depending on the parameter r/h in the parallel-piped layer. It was assumed that $a = 2$ cm, $ka = 2$, $kh = 10$. Pressure distribution in the plane $z = h = 10$ cm: *1* – circular (central) source, curve *2* – annular (external) source. Pressure distribution in the plane of the source ($z = 0$): curve *3* – circular source, curve *4* – annular source

or r/a, with a fixed distance between the baffles, $h = 10$ cm, and the radius $a = 2$ cm.

The curves of the relative acoustic pressure, depending on the parameter r/h, for a separate central element and the external element of the circular membrane are shown in Fig. 2. These curves show that in the variation interval of the parameter r/h under analysis the relative acoustic pressure of the external source exceeds that of the central circular one. This property is satisfied both on the surface of the baffle in which the source is and also on the surface of the baffle at the distance h from the plane of the source. On the baffle at the distance h from the plane of the source there are circles in which the value of the pressure drops to zero (see curve 1 in Fig. 2).

Fig. 3 shows the curves of the relative acoustic pressure, depending on the parameter r/h, radiated by the circular membrane for the (0, 2) vibration mode. This vibrating membrane is a system of two sources: the central circular and

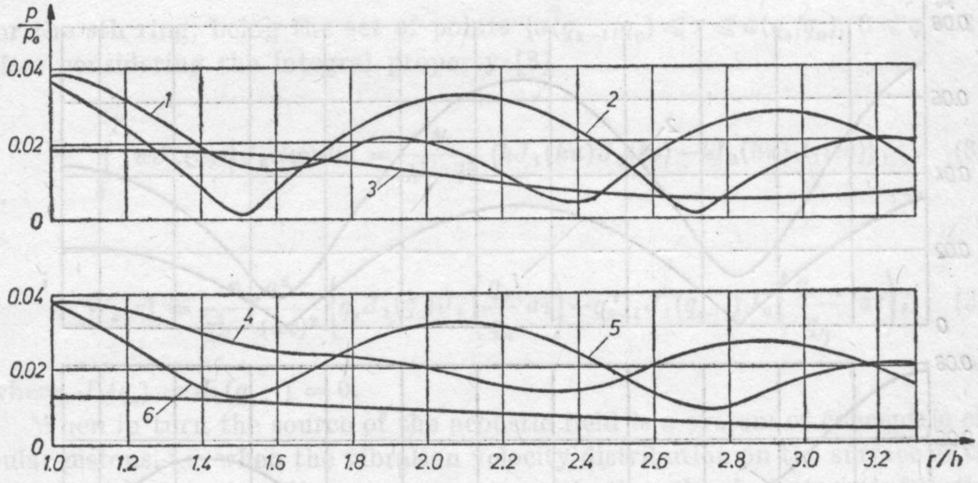

Fig. 3. Relative acoustic pressure, depending on the parameter r/h, in the parallel-piped layer. Curves *1*, *2* and *3* show the pressure distributions of the circular membrane for the (0, 2) vibration mode, respectively in the planes: $z = 0$, h and $h/2$. Curves *4*, *5* and *6* show the pressure distributions of the point source, respectively in the planes: $z = 0$, h and $h/2$. It was assumed that the bulk efficiency of the point source was equal to that of the circular membrane for the (0, 2) vibration mode, $ka = 2$, $kh = 10$, $a = 2$ cm

the external annular ones, separated by a nodal circle with the radius $a(q_1/q_2)$. This figure also shows analogous curves for the point source whose bulk efficiency $Q = 2\pi a^2 v_0 [J_1(q_2)/q_2]$ is the same as that of the circular membrane analysed for the (0,2) vibration mode. For the membrane and the point source separate acoustic pressure curves were plotted depending on the parameter r/h in three planes: in the plane of the source, in a plane at a distance $h/2$ from the plane of the source and in the plane of the baffle situated at the distance h from the plane of the source. These curves show that larger pressure fluctuations occur on the surfaces of the baffles in the case of the point source. The almost monotonous decrease in the acoustic pressure occurs in the plane $z = h/2$ with increasing parameter r/h, both for the point source and the circular membrane (see curves *3* and *6* in Fig. 3).

As in Fig. 3, acoustic pressure curves, depending on the parameter r/h, were also plotted for the circular membrane in Fig. 4. Curves are also shown for the pressure radiated by a circular piston with the radius $a = 2$ cm. Different

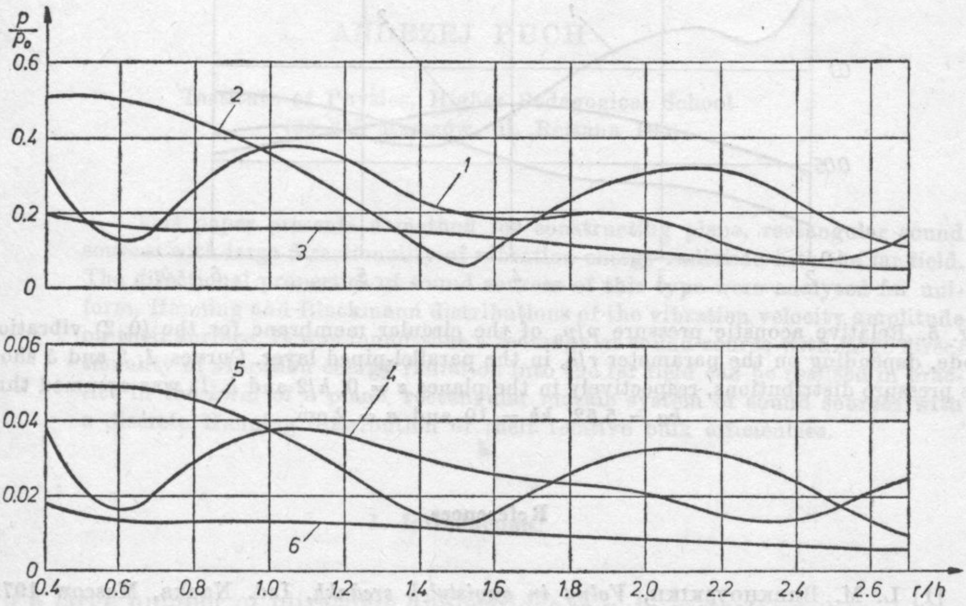

Fig. 4. Relative acoustic pressure, depending on the parameter r/h, in the parallel-piped layer. Curves *1*, *2* and 3 show the pressure distribution of the circular piston, respectively in the planes $z = 0$, h and $h/2$. Curves *4*, *5* and 6 show the pressure distributions of the circular membrane for the (0, 2) vibration mode, respectively in the planes: $z = 0$, h and $h/2$. It was assumed that $ka = 2$, $k = 10$ and $a = 2$ cm

variation intervals of the parameter r/h were assumed for the membrane and the piston (see curves 4,5 and 6 in Figs. 3 and 4). Analysis of the curves (Fig. 4) shows distinct differences among the values of the pressures generated by the membrane and the circular piston for given values of the parameters r/h. It can be assumed with approximation that the value of the pressure from the circular piston is higher by an order of magnitude from that from the circular membrane with a given value of the parameter r/h.

The curves in Figs. 2, 3 and 4 were plotted from calculations for $ka = 2$.

Curves of the relative pressure, depending on the parameter r/a, for the circular membrane are shown in Fig. 5. It was assumed that $ka = 5.52$, which corresponds to the resonance vibration frequency.

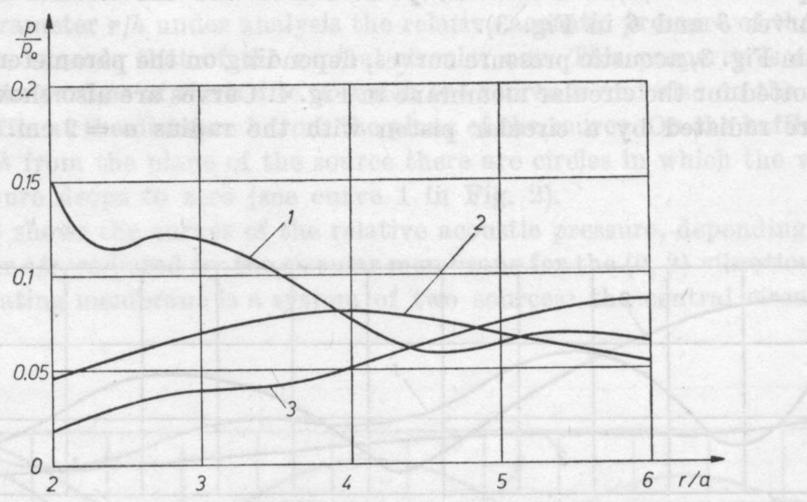

Fig. 5. Relative acoustic pressure p/p_0 of the circular membrane for the (0, 2) vibration mode, depending on the parameter r/a, in the parallel-piped layer. Curves *1*, *2* and *3* show the pressure distributions, respectively in the planes $z = 0$, $h/2$ and h. It was assumed that $ka = 5.52$, $kh = 10$ and $a = 2$ cm

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