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CALCULATION OF THE SOUND-ABSORBING PANELS BASED ON THE PRINCIPLE OF ELECTROACOUSTIC ANALOGIES AND CONSIDERING THE RAYLEIGH IMPEDANCE

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This paper reports on calculations of the parameters of a bothsided perforated sound-absorbing system, based on the "electric model" and considering the Rayleigh impedance. Both the absorption coefficient for perpendicular incidence and the reverberation coefficient over a broad frequency range were calculated.

1. Introduction

This paper reports on calculations of bothsided perforated sound-absorbing panels, based on the electroacoustic analogies applied in acoustics [1, 3–5, 7]. The calculations were based on a new "electric model", taking into account the Rayleigh impedance both at the input and the output of the perforated system. The results obtained in the form of a curve of the absorption coefficient versus frequency are comparable to the characteristic of series-made sound-absorbing panels. Analysis of this is given in the Conclusions.

2. Theoretical basis of the calculations

A bothsided perforated sound — absorbing panel filled with some sound-absorbing material can be represented in the form of an equivalent electric system, whose schematic diagram is shown in Fig. 1. It was considered in the calculations that this sound-absorbing system has n holes per 1 m² of the plate, therefore, the quantities m, r and L define, respectively, the equivalent impedance for n holes connected in parallel. According to the system of electroacoustic

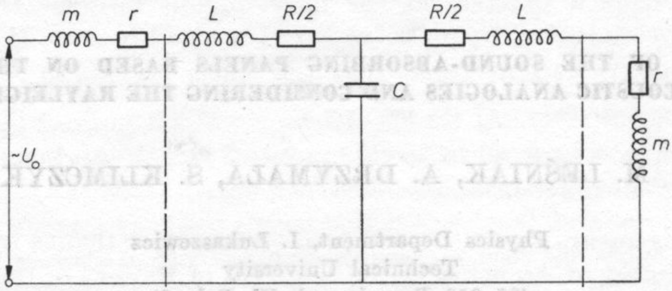

Fig. 1. An equivalent electric system of the two-sided perforated sound-absorbing panel

analogies accepted in acoustics, L corresponds to the acoustic mass of the air in the holes, R — to the resistance of the ports and C — to the acoustic compliance of a chamber of the system. The calculations were carried out by taking the quantities characterizing the plates as concentrated constants, which, for sizes occurring in practice, is valid up to about 8 kHz, with further increase of the frequency the approximation becomes increasingly "gross". By referring all the quantities to unit area and considering the existence of n holes connected in parallel, the following substitutions can be applied [1, 3–5, 7]:

$$R = R_{ac}d, \qquad C = \frac{D}{\varrho c^2}, \qquad L = \frac{\varrho l}{p}, \tag{1}$$

where d is the thickness of the layer of sound-absorbing material, R_{ac} is the flow resistance of the sound-absorbing material, l is the thickness of the perforated plate, D is the distance between the perforated plates and p is the perforation coefficient of the plates, defined as the ratio of the surface area of the holes in unit surface of the plate to this unit surface:

$$p = \frac{nS}{1\text{m}^2}, \tag{2}$$

where n is the number of ports per 1 m², S is the surface area of a port, ϱc is the acoustic resistance of the air, ϱ is the density of the air and c is the sound speed.

This system also considers the Rayleigh impedances r and m resulting from the diffraction effects at the holes. Practically over the whole working frequency range of soundabsorbing panels, and quite certainly for the approximations applied here, it can be assumed that the air vibrates in a port as a rigid piston. The Rayleigh impedance for a single port [4] is given by the formula

$$Z' = \frac{\varrho c}{S}\left[1-\frac{J_1(2ka)}{ka}+i\frac{S(2ka)}{ka}\right], \tag{3}$$

where J_1 is a Bessel function of the first order, S is a Struwe function of the first order, k is a wave number and a is the radius of the port.

For $ka \ll 1$, in the series expansion of both Bessel and Struwe functions, the terms of higher orders can be neglected. On this assumption, from [2], the Bessel and Struwe functions can be given in the following way:

$$J_1(2ka) = ka-\tfrac{1}{2}k^3a^3, \qquad S(2ka) = \frac{8k^2a^2}{3\pi}. \tag{4}$$

Substitution of $J_1(2\ ka)$ and $S(2\ ka)$ in (3) and consideration that $Z' = r_1+i\omega m_1$ give

$$r_1 = \frac{\varrho\omega^2a^2}{2cS}, \quad m_1 = \frac{8a\varrho}{3\pi p}. \tag{5}$$

Since the sound-absorbing system considered contains n holes connected in parallel, the resultant Rayleigh impedance, referred to unit area, will be

$$r = \frac{\varrho\omega^2a^2}{2cp}, \quad m = \frac{8a\varrho}{3\pi p}. \tag{6}$$

The aim of the calculations is to find the parameters of the system on the assumption that the absorption coefficient reaches a maximum for a predetermined resonance frequency. This coefficient is defined as the ratio between the energy absorbed by the system and the energy incident on the system. The energy incident on the system (P_1) is partly absorbed (P_2), partly reflected (P_3), and part of the energy is transmitted through the system (P_4) – according to the diagram shown in Fig. 2.

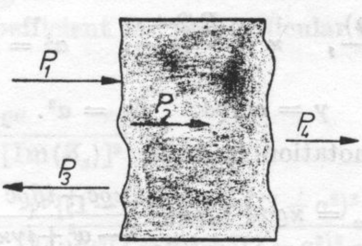

Fig. 2. A schematic representation of the distribution of energy incided on the absorbing material

By taking into account this notation, the absorption coefficient is defined in the following way:

$$\eta = \frac{P_2}{P_1}. \tag{7}$$

Applying the notation in Fig. 2, it can be written that

$$P_2 = P_1 - P_3 - P_4. \tag{8}$$

From that point of view,

$$\eta = \frac{P_1 - P_3 - P_4}{P_1} = 1 - \frac{P_3}{P_1} - \frac{P_4}{P_1}. \tag{9}$$

We begin with a plane wave for perpendicular incidence on the boundary between two media, and then the second term of expression (9) becomes [6, 7]

$$\frac{P_3}{P_1} = \left| \frac{\varrho c - Z_a}{\varrho c + Z_a} \right|^2, \tag{10}$$

where Z_a is the acoustic impedance of the sound-absorbing system, whose equivalent diagram is shown in Fig. 1.

Expression (10) is the definition of the reflection coefficient. After separating the real and imaginary parts of Z_a, the reflection coefficient becomes [6]

$$\frac{P_3}{P_1} = \frac{[\varrho c - \mathrm{Re}(Z_a)]^2 + [\mathrm{Im}(Z_a)]^2}{[\varrho c + \mathrm{Re}(Z_a)]^2 + [\mathrm{Im}(Z_a)]^2}; \tag{11}$$

or, by combining the first two terms of expression (9), there is

$$1 - \frac{P_3}{P_1} = \frac{4\,\mathrm{Re}(Z_a)\varrho c}{[\varrho c + \mathrm{Re}(Z_a)]^2 + [\mathrm{Im}(Z_a)]^2}. \tag{12}$$

The acoustic impedance for a given sound-absorbing system was calculated by means of an equivalent electric system:

$$Z_a = r + R/2 + i\omega(L+m) + \frac{r + R/2 + i\omega(L+m)}{1 - \omega^2 C(L+m) + i\omega C(R/2 + r)}. \tag{13}$$

For simplification, the following notation will be introduced:

$$\beta = \frac{\omega(L+m)}{\varrho c}, \quad \varkappa = \frac{R/2 + r}{\varrho c}, \quad \alpha^2 = \omega^2 (L+m) C,$$

$$\gamma = \omega C \varrho c, \quad \beta\gamma = \alpha^2. \tag{14}$$

Introduction of this notation leads to

$$Z_a = \varkappa\varrho c + i\beta\varrho c + \frac{\varkappa\varrho c + i\beta\varrho c}{1 - \alpha^2 + i\gamma\varkappa}. \tag{15}$$

Some simple transformations lead, respectively, to

$$\mathrm{Re}(Z_a) = \varkappa\varrho c + \frac{\varkappa\varrho c(1-\alpha^2) + \beta\gamma\varkappa\varrho c}{(1-\alpha^2)^2 + \gamma^2\varkappa^2}, \tag{16a}$$

$$\mathrm{Im}(Z_a) = \beta\varrho c + \frac{\beta\varrho c(1-\alpha^2) - \gamma\varkappa^2\varrho c}{(1-\alpha^2)^2 + \gamma^2\varkappa^2}. \tag{16b}$$

The third term of expression (9) $-P_4/P_1$ — can also be calculated by applying an equivalent electric system. The power of the incident wave is calculated as the power of the plane wave, and, in order to retain the logical continuity of reasoning, in keeping with the principle of electroacoustic analogies, the amplitude of the accoustic pressure p_0 can be replaced by U_0. Thus, this expression can be given in the form

$$\frac{P_4}{P_1} = \frac{|I_3|^2 r}{U_0^2/\varrho c} = \frac{|I_3|^2 r \varrho c}{U_0^2}, \tag{17}$$

where I_3 is the intensity of the current flowing through the elements r and m (Fig. 1) and U_0 is the *rms* value of the tension.

I_3 now remains to be calculated. This can be done by taking advantage of the Kirchhoff laws in reference to the circuit shown in Fig. 1.

$$I_3 = \frac{U_0 \dfrac{1}{i\omega C}}{\left[\dfrac{R}{2} + r + i\omega(L+m) + \dfrac{1}{i\omega C}\right]^2 - \left[\dfrac{1}{i\omega C}\right]^2}. \tag{18}$$

By using the notation in (14), expression (18) becomes

$$I_3 = \frac{i\gamma U_0 \dfrac{1}{\varrho c}}{[i\gamma\varkappa + (1-\alpha^2)]^2 - 1}. \tag{19}$$

Substitution of (19) in (17) and transformations give

$$\frac{P_4}{P_1} = \frac{\gamma^2\{(1-\alpha^2)^2\varkappa^2\gamma^2 - [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2}{\{4\gamma^2\varkappa^2(1-\alpha^2)^2 + [(1-\alpha^2)^2 - 1 + \gamma^2\varkappa^2)]^2\}^2} \frac{r}{\varrho c}. \tag{20}$$

Finally, the absorption coefficient for perpendicular incidence is given by the relationship

$$\eta_\perp = \frac{4\,\mathrm{Re}(Z_a)\varrho c}{[\varrho c + \mathrm{Re}(Z_a)]^2 + [\mathrm{Im}(Z_a)]^2} - $$

$$- \frac{\gamma^2\{(1-\alpha^2)^2\gamma^2\varkappa^2 - [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}}{\{4\gamma^2\varkappa^2(1-\alpha)^2 + [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}^2} \frac{r}{\varrho c}. \tag{21}$$

The above formula permits the absorption coefficient for perpendicular incidence to be calculated as a function of frequency with predetermined parameters of the sound-absorbing system. The absorption coefficient depends on the angle of the wave incidence on the surface of the sound-absorbing material. Because of this, the sound absorption coefficient for a plane wave incident at an angle φ on the surface of the material was called the directional absorption coefficient. The dependence of the absorption coefficient on the wave incidence angle can be given in the following way [6]:

$$\frac{P_3}{P_1} = \frac{[\varrho c - \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi}{[\varrho c + \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi} \tag{22}$$

or by calculating

$$1 - \frac{P_3}{P_1} = \frac{4\ \mathrm{Re}(Z_a)\varrho c\cos\varphi}{[\varrho c + \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi}. \tag{23}$$

The expression of the directional coefficient becomes then

$$\eta_\varphi = \frac{4\ \mathrm{Re}(Z_a)\varrho c\cos\varphi}{[\varrho c + \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi} - \frac{\gamma^2\{(1-\alpha^2)^2\varkappa^2\gamma^2 - [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2}{\{4\gamma^2\varkappa^2(1-\alpha^2)^2 + [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}^2}\frac{r}{\varrho c}. \tag{24}$$

When scattered sound, i.e. sound composed of waves travelling in all directions, is incident on the material, the absorption coefficient has some mean value and is called the reverberation coefficient of sound absorption. This coefficient can be given, depending on the directional absorption coefficient η_φ for any incidence angle φ, in the following way [7]:

$$\bar{\eta} = \int_0^{\pi/2} \eta_\varphi \sin 2\varphi d\varphi. \tag{25}$$

Substitution of (24) in (25) gives the following expression of the reverberation absorption coefficient:

$$\bar{\eta} = \int_0^{\pi/2} \left\{ \frac{4\ \mathrm{Re}(Z_a)\varrho c\cos\varphi}{[\varrho c + \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi} - \frac{\gamma^2\{(1-\alpha^2)^2\gamma^2\varkappa^2 - [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}}{\{4\gamma^2\varkappa^2(1-\alpha^2)^2 + [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}^2}\frac{r}{\varrho c} \right\} \sin 2\varphi d\varphi \tag{26}$$

or

$$\bar{\eta} = \int_0^{\pi/2} \frac{4\ \mathrm{Re}(Z_a)\varrho c\cos\varphi}{[\varrho c + \mathrm{Re}(Z_a)\cos\varphi]^2 + [\mathrm{Im}(Z_a)]^2\cos^2\varphi} \sin 2\varphi d\varphi - \frac{\gamma^2\{(1-\alpha^2)^2\gamma^2\varkappa^2 - [(1-\alpha^2)^2 - [1+\gamma^2\varkappa^2)]^2\}}{\{4\gamma^2\varkappa^2(1-\alpha^2)^2 + [(1-\alpha^2)^2 - (1+\gamma^2\varkappa^2)]^2\}^2}\frac{r}{\varrho c}. \tag{27}$$

Finally, after calculating the integral, the reverberation coefficient is given by the formula

$$\eta = \frac{8\varrho c \operatorname{Re}(Z_a)}{|\operatorname{Re}(Z_a)|^2+|\operatorname{Im}(Z_a)|^2}\left\{1-\frac{\varrho c \operatorname{Re}(Z_a)}{[\operatorname{Re}(Z_a)]^2+[\operatorname{Im}(Z_a)]^2}\times\right.$$

$$\times\left[\ln\left([\varrho c+\operatorname{Re}(Z_a)]^2+[\operatorname{Im}(Z_a)]\right)^2-\ln(\varrho c)^2\right]+\frac{\varrho c}{\operatorname{Im}(Z_a)}\times$$

$$\times\left(\frac{2[\operatorname{Re}(Z_a)]^2}{[\operatorname{Re}(Z_a)]^2+[\operatorname{Im}(Z_a)]^2}-1\right)\left(\operatorname{arctg}\frac{[\operatorname{Re}(Z_a)]^2+[\operatorname{Im}(Z_a)]^2+\varrho c\operatorname{Re}(Z_a)}{\varrho c\operatorname{Im}(Z_a)}-\right.$$

$$\left.\left.-\operatorname{arctg}\frac{\operatorname{Re}(Z_a)}{\operatorname{Im}(Z_a)}\right)\right\}-\frac{\gamma^2\{(1-\alpha^2)^2\gamma^2\varkappa^2-[(1-\alpha^2)^2-(1+\varkappa^2\gamma^2)]^2\}}{\{4\gamma^2\varkappa^2(1-\alpha^2)^2+[(1-\gamma^2\varkappa^2)-(1+\gamma^2\varkappa^2)]^2\}^2}\,\frac{r}{c}. \quad (28)$$

Just as (21), formula (28) considers only a real part of the absorption coefficient. These expressions permit the calculation of the absorption coefficient as a function of frequency. They also make it possible to calculate the parameters of the system on the assumption that the absorption coefficient reaches a maximum for a predetermined resonance frequency. The resonance frequency is matched to a frequency at which in the noise spectrum there is the maximum accoustic pressure level. Expression (21) reaches the highest value when the following conditions are satisfied:

$$\operatorname{Re}(Z_a) = \varrho c, \quad \operatorname{Im}(Z_a) = 0. \quad (29)$$

By taking into account these conditions (29), one can calculate the resonance frequency or the parameters of the soundabsorbing system, and the value of the absorption coefficient for the resonance frequency. In order to do this, from the condition $\operatorname{Im}(Z_a) = 0$, it is obtained from equation (16b) that

$$\beta\varrho c[(1-\alpha^2)^2+\varkappa^2\gamma^2]+\beta\varrho c(1-\alpha^2)-\gamma^2\varkappa^2 = 0. \quad (30)$$

From condition (14), i.e., that $\beta = \alpha^2/\gamma$, equation (30) becomes

$$\alpha^6-3\alpha^4+\alpha^2(2+\varkappa^2\gamma^2)-\varkappa^2\gamma^2 = 0. \quad (31)$$

The substitution $\alpha^2 = u$ gives

$$u^3-2u^2+u(2+\varkappa^2\gamma^2)-\varkappa^2\gamma^2 = 0. \quad (32)$$

This equation is satisfied when $u = 1$, i.e. $\alpha^2 = 1$. From the condition obtained, the value of the resonance frequency can be calculated:

$$1 = \alpha^2 = \omega_{\mathrm{res}}^2(L+m)C,$$

hence

$$\omega_{\mathrm{res}} = \sqrt{\frac{1}{C(L+m)}}. \quad (33)$$

When expressing L, m, C by the parameters of the sound-absorbing system, it is possible to calculate the resonance frequency. By dividing the left side of equation (32) by $(u-1)$, it is obtained that

$$u^2-2u+\varkappa^2\gamma^2=0. \tag{34}$$

Solution of equation (34) gives its roots in the form

$$u_{1,2}=1+\sqrt{1-\varkappa^2\gamma^2}. \tag{35}$$

Becouse of the physical sense of $\varkappa$ and γ, the latter condition can be given in the form

$$0<\varkappa\gamma<1. \tag{36}$$

In recapitulating, equation (32) is satisfied when $\alpha^2=1$ for any values of the product $\varkappa\gamma$ and for

$$1<\alpha^2<2 \quad \text{and} \quad 0<\alpha^2<1 \quad \text{when } 0<\varkappa\gamma<1. \tag{37}$$

In turn, from the conditions $\mathrm{Re}(Z_a)=\varrho c$ and $\alpha^2=1$,

$$\gamma^2=\frac{1}{\varkappa(1-\varkappa)}. \tag{38}$$

From (38) and in view of the physical sense, $\varkappa$ must satisfy the condition

$$0<\varkappa<1. \tag{39}$$

Since for $\alpha^2=1$ and the product $\varkappa\gamma$ no condition is imposed, γ can take any values, certainly positive ones, which can be seen from expression (38).

The conditions obtained for $\varkappa$ and γ permit the calculation of the parameters of the sound-absorbing system.

3. Numerical calculations

It follows from conditions (33) and (37) that there can be three resonance frequencies: one for any $\varkappa\gamma$ and two frequencies for the condition $0<\varkappa\gamma<1$. By carrying out calculations for typical sound-absorbing barriers it can be found that $\varkappa\gamma$ must always be larger than 1. From that point of this, condition (37) is eliminated and one resonance frequency remains, which can be calculated from condition (33). The parameters of the system for a predetermined frequency at which in the noise spectrum there is the maximum acoustic pressure level, i.e. for the resonance frequency, can be calculated from dependencies (1), (6), (14) and (38),

$$\left(\frac{c}{2\pi f}\right)^2=\left(\frac{1}{p}+\frac{8a}{3\pi p}\right)D, \tag{40a}$$

$$\varkappa = \frac{R_{ac}d + \dfrac{\varrho a^2 \omega^2}{cp}}{2\varrho c}, \tag{40b}$$

$$\left(\frac{2\pi f}{c}\right)^2 = \frac{1}{D^2 \varkappa (1-\varkappa)}. \tag{40c}$$

In practice, it is most convenient to assume that $D = d$. From relations (40), it is possible to calculate the parameters of the system, by assuming, e.g., a typical tin perforation and typical values of the transmission resistance.

Knowledge of the parameters of the sound-absorbing system permits the calculation of the absorption coeffient for perpendicular incidence as a function of frequency, based on expression (21), and of the reverberation coefficient, from expression (28).

Fig. 3 shows the dependence of the absorption coefficient for perpendicular incidence, $\eta_{\perp}$ (curve 1), and of the reverberation coefficient, $\bar{\eta}$ (curve 2), as a func-

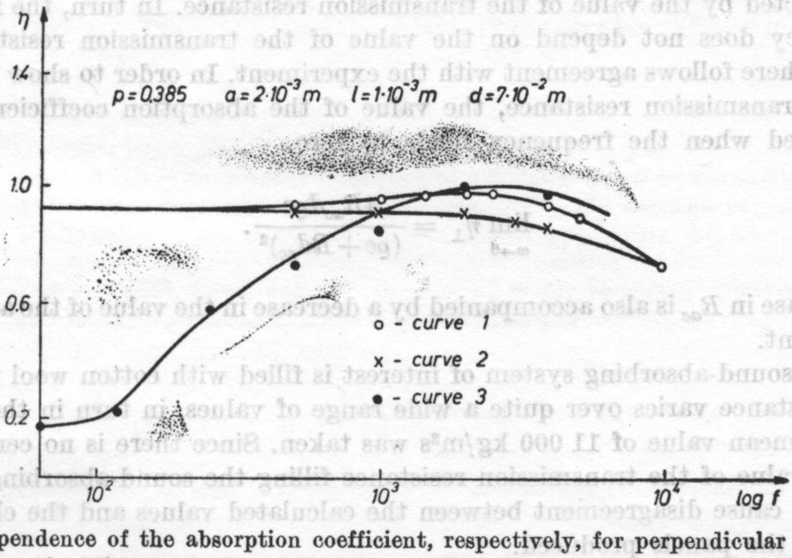

Fig. 3. Dependence of the absorption coefficient, respectively, for perpendicular incidence (curve 1), reverberation (curve 2) and the reverberation absorption coefficient for the system of the "Deska" type (curve 3) versus frequency

tion of frequency. The calculations were carried out for a sound-absorbing system with the tin thickness $l = 0.4 \times 10^{-3}$m, for the typical perforation $p = 0.280$ with the port diameter $2a = 4.5 \times 10^{-3}$ m, by assuming $D = 7 \times 10^{-2}$ m and that the consition $D = d$ is satisfied. Therefore, such a sound-absorbing system was chosen for calculations, since the parameters of sound-absorbing systems of the "Deska" type, produced by the Poznań Factory of Cork Products, are the same. In order to compare the results in Fig. 3, the frequency dependence of the reverberation coefficient of sound absorption (curve 3) was marked for the above-mentioned absorbers of the "Deska" type. These results were drawn from technical data on the product and obtained at a conference with the representatives of the producer.

4. Conclusions

It follows from comparison of these curves that the measured maximum of the absorption coefficient is close to the measured frequency. The calculations indicate that the resonance frequency is equal to 2253 Hz, whereas it follows from the technical data that it is about 3000 Hz. A decidedly, however, lower value is taken by the reverberation absorption coefficient, drawn from the data of the ready panels, than that calculated on the basis of electroacoustic analogies for the part of the low frequencies. It follows from formula (21) that the value of the absorption coefficient from the part of the low frequencies is decidedly affected by the value of the transmission resistance. In turn, the resonance frequency does not depend on the value of the transmission resistance and hence, there follows agreement with the experiment. In order to show the effect of the transmission resistance, the value of the absorption coefficient can be calculated when the frequency tends to zero.

$$\lim_{\omega\to 0} \eta_\perp = \frac{4R_{ac}d\varrho c}{(\varrho c + Rd_{ac})^2}. \tag{41}$$

A decrease in R_{ac} is also accompanied by a decrease in the value of the absorption coefficient.

The sound-absorbing system of interest is filled with cotton wool for which the resistance varies over quite a wide range of values, in turn in the calculations a mean value of 11 000 kg/m³s was taken. Since there is no certainty as to the value of the transmission resistance filling the sound-absorbing system, this can cause disagreement between the calculated values and the characteristics of the panels produced.

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References

[1] A. Drzymała, M. Leśniak, S. Klimczyk, R. Wyrzykowski, *Parameters of sound-absorbing systems for predetermined frequencies* (in Polish), Proceedings of the XXXth OSA, Committee on Acoustics of the Polish Academy of Sciences, Gdańsk.

[2] E. Janke, F. Emde, F. Losch, *Spetsiyalnye funktsi*, Izd. Nauka, Moscow, 1968.

[3] I. Malecki, *Theory of acoustics waves and systems* (in Polish), PWN, Warsaw, 1964.

[4] E. Skudrzyk, *The foundations of acoustics*, Wien-New York, 1971.

[5] R. Wyrzykowski, A. Puch, J. Snakowski, *Acoustic filters* (in Polish), Pedagogical College Press, Rzeszów, 1977.

[6] R. Wyrzykowski, *Linear theory of the acoustic field of gaseous media* (in Polish), Pedagogical College Press, Rzeszów, 1972.

[7] Z. Żyszkowski, *The fundamentals of electroacoustics* (in Polish), WNT, Warsaw, 1966.