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NON-INVASIVE ULTRASONIC METHOD FOR THE BLOOD FLOW AND PRESSURE MEASUREMENTS TO EVALUATE THE HEMODYNAMIC PROPERTIES OF THE CEREBRO-VASCULAR SYSTEM

TADEUSZ POWAŁOWSKI, BOGUMIŁ PEŃSKO

Institute of Fundamental Technological Research, Polish Academy of Sciences
(00–049 Warsaw, ul. Świętokrzyska 21)

This paper presents a method for using ultrasonic measurements of the blood velocity and the motion of the walls of the blood vessel in simultaneous, noninvasive determination of the blood pressure and blood flow rate. The measurements were carried out in carotid arteries in people aged 25 to 40. On the basis of the results obtained, the vessel impedance was determined by the method of discrete Fourier transform. Analysis of the distribution of the impedance modulus and phase permitted the calculation of the parameters of the substitute cerebro-vascular model, such as resistance, inertia and compliance. By applying the model assumed the authors performed a computer simulation of the blood flow in the common carotid artery on the basis of the pressure determined from the movements of the walls of the artery under study. The results obtained coincide with the flows measured by the Doppler method in healthy persons.

1. Introduction

The description of the properties of man's vascular system requires the knowledge of a number of hemodynamic quantities, such as blood flow, blood pressure, vascular resistance, elasticity of blood vessel walls, capacity and resistance of the vascular bed. In 1959 McDonald and Taylor [13] suggested that the determination of the input vessel impedance might be the simplest way of describing these properties. The investigations carried out in a large number of laboratories in the world ever since in order to determine the impedance have confirmed this suggestion [1, 3, 12, 14, 22].

The input impedance of the vascular system is defined as the ratio between the blood pressure inside the vessel and the flow rate of blood flowing through this vessel, measured at the same time and over the same vessel cross — section. When assuming the vascular system investigated as linear, the impedance Z can be determined by the Fourier transform of the time dependent pressure P and the blood flow Q, according to the formula

$$Z(2\pi nf) = P(2\pi nf)/Q(2\pi nf), \tag{1}$$

where $n = 0, 1, 2, \ldots$ and f is the frequency of the heart.

In the literature, the vessel impedance is most frequently represented in the form of modulus and phase as functions of frequency. Mutual relationships are sought among the modulus and phase distributions and the properties of the vascular system. In addition, the vessel impedance can contain information about the reflection sites of the waves of the pressure and flow at the branching points of blood vessels or their narrowings or blockages, which can be of significant values in diagnosing the patency of blood vessel in regions inaccessible to the currently used measurement methods.

The determination of the vessel impedance requires the simultaneous measurement of the instantaneous values of the blood flow and pressure. It is possible to carry out noninvasive measurements of the blood velocity in vessels by using ultrasonic Doppler methods [4, 7]. However, the pressure measurement offers considerable problems. It requires the introduction of a pressure transducer into the vessel studied. This restricts greatly the experimental possibilities, limiting practically the investigations to animals and very large vessels, for the introduction of the pressure transducer into the vessel perturbs the blood velocity measured simultaneously.

It would be possible to achieve a further extension of studies on the vessel impedance by finding noninvasive methods proposed by the authors, consisting in the determination of the blood pressure from the measurement of the displacement of the blood vessel walls [15]. At the Ultrasonic Department, Institute of Fundamental Technological Research, Polish Academy of Sciences in Warsaw an ultrasonic device was constructed for the simultaneous measurement of the blood vessel wall movement and the velocity of blood flowing through these vessels. This device was applied in measurements in man's carotid arteries. A computer, connected on line to the measurement device, calculates the blood pressure and blood flow rate, and the vessel impedance.

2. Relationship between the blood pressure and the displacement of the vessel wall

The magnitude of changes in the diameter of the blood vessel as a function of blood pressure depends on the mechanical properties of the vessel, its shape and size, and on the position of the vessel with respect to the heart. The determination of the functional relationship between the pressure and the wall displacement requires the assumption of some, of necessity, simplified mechanical model of the blood vessel. Most frequently, large vessels are regarded as axially symetric cylinders with thin walls built of a homogeneous, elastic and incompressible material exhibiting transverse isotropy. Such cylinders are subjected to loads which also show axial symmetry, and the only effect of this loading is a change in the radial component of strain. Investigations carried out in a large number of centres on the properties of blood vessel have confirmed the justifiability of using the above simplifications [1, 2, 6, 16].

The direct application of the onedimensional Hooke law in the direction of the vessel radius leads to the simplest relationship between changes in the radius and changes in the pressure

$$\Delta P = E_P \frac{\Delta R}{R_0}, \tag{2}$$

where ΔP is the pressure difference causing a change in the vessel radius R_0 by a value ΔR, E_p is the elastic modulus "corresponding" to Young's modulus.

It follows from formula (2) that the dependence of E_p on the pressure or radius signifies a deviation from the Hooke law, i.e. elastic nonlinearity. E_p has repeatedly been observed to vary [8, 17]. There have been attempts to "improve" the modulus E_p by taking into account finite wall thickness, introducing the Poisson's ratio etc. [1,2,8].

With time variable dynamic loads, vessel walls can show viscoelastic properties. For sinusoidal load changes, the complex elastic modulus E^* can be introduced in the form

$$E^* = E_{\text{dyn}} + j\eta\omega, \tag{3}$$

where E_{dyn} is a dynamic elastic modulus and η is the coefficient of viscosity. Investigations carried out by a large number of authors have shown that the phase lag between the pressure and the radius change does not exceed 10° and that the inequality $\eta\omega \leqslant 0.1\, E_{\text{dyn}}$ is valid [8,17]. This permits the viscoelastic effect to be neglected in the simplified model.

A successive approximation of the wall reaction to the internal pressure is the assumption that the pressure change is in the direct proportion to the relative change in the vessel volume [18]

$$\Delta P = K \frac{\Delta V}{V}, \tag{4}$$

where V is the volume of a vessel section and K is the elastic modulus. When the vessel cross-section is circular, then formula (4) becomes

$$P - P_0 = K \frac{R^2 - R_0^2}{R_0^2}, \tag{5}$$

where R_0 is the vessel radius at the pressure P_0 and R is the vessel radius at the pressure P.

In addition to the attempts to describe the relationship between the vessel radius and the pressure, mentioned above, in the literature [19, 21] there has also been the suggestion that this relationship is described by an exponential function.

In view of the large variety of descriptions of the relationship between the pressure and the vessel radius, the authors undertook an attempt to carry out their own evaluation of this function. On the basis of mathematical analysis introduced by GREEN [10] for an elastic system with axial symmetry subjected to large-amplitude strains and of SIMON's experimental results [19], the following dependence was assumed:

$$P(R) = P_0 \exp(\gamma R^2). \tag{6}$$

The parameters P_0 and γ can be determined by substituting in formula (6) the values of P and R in systole (P_s, R_s) and in diastole (P_d, R_d). Relationship (6) becomes then

$$P(R) = P_d \exp\left[\left(\frac{R^2 - R_d^2}{R_s^2 - R_d^2}\right) \ln \frac{P_s}{P_d}\right]. \tag{7}$$

Fig. 1 shows the function $P(R)$ calculated from the above relationship. For comparison, experimental data given by SIMON for canine abdominal aorta and the results of numerical calculations carried out on the basis of dependencies presented by GREEN are also given. In the calculations, the values $R_d = 4.4$ mm, $P_d = 46$ mmHg, $R_s = 5.2$ mm and $P_s = 199$ mmHg were assumed. It can be seen that the exponential function (formula (7)) approximates very well the experimental results and those of the numerical calculations.

It should be emphasized that in the case of small ratios $\Delta P/\Delta R$ all the relationships between the pressure and the vessel radius, mentioned above, give results close to each other. However, in view of the limited reaction of the vessel walls to large pressures, the authors assumed the exponential function as the more probable physiological dependence compared with a linear or quadratic function.

3. Measurement equipment

An ultrasonic device containing a bi-directional C.W. Doppler flowmeter and an echo system was constructed for the simultaneous measurement of the blood velocity and the instaneous diameter of the blood vessel. The parameters of the device were selected from the point of view of applications in measurements in carotid arteries. The ultrasonic probes, connected with the flowmeter and the pulse system, were focussed in tissue at depth between 1 and 3 cm. In the measurement of the blood velocity, the frequency of the transmitted ultrasonic wave is 4.5 MHz, while the frequency used in the measurement of the diameter of the vessel is 6.75 MHz. The resolution of the pulse system in tissue is 0.5 mm and permits the measurement of the internal diameter of the carotid artery. The measurement device contains a digital unit for tracing and measuring the motion of the blood vessel walls [9]. The accuracy of mapping the vessel wall displacements is 0.03 mm.

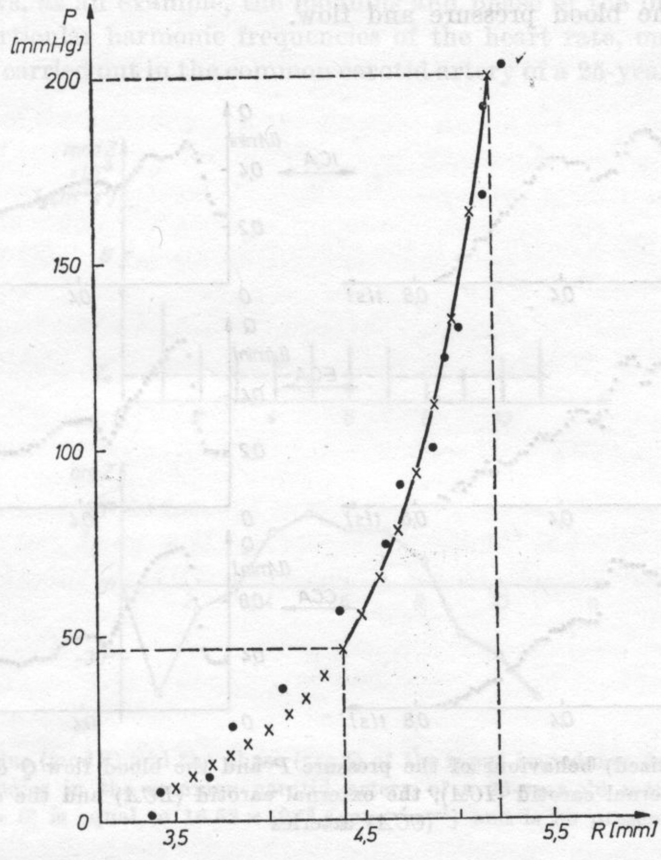

Fig. 1 The blood pressure versus the vessel radius (•) experimental points, according to SIMON, for canine abdominal aorta, (×) the results of numerical calculations, solid line — dependence from formula (6)

The device is equipped on line with a computer (MERA 60 — a Polish equivalent of PDP—11), which on the basis of the measured blood velocity, vessel diameter and the displacements of its wall, calculates the blood flow rate and the accompanying changes in the blood pressure. Each time the calculated results were averaged from three cycles of the heart rate.

The pressure (formula (7)), determined from displacements of the vessel walls, in the carotid arteries was calibrated in absolute values by the systole pressure P_s and the diastole one P_d measured in the brachial artery. The quantities P_s and P_d were measured with a cuff in a lying patient, at the height of the carotid artery.

Fig. 2 shows, as an example, the blood pressure and flow determined by the authors using the device described in the internal (*ICA*), external (*ECA*) and common (*CCA*) carotid arteries of a 40-year-old man. It is possible to see a difference between the external and internal carotid arteries in the form of changes in the blood pressure and flow.

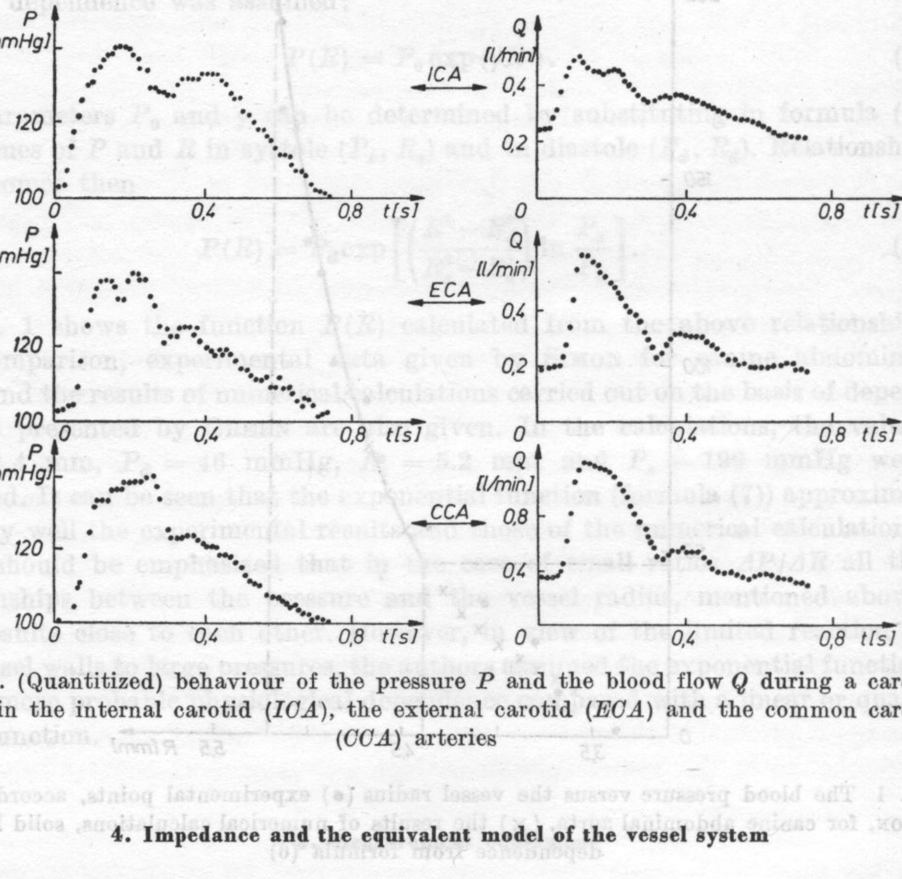

Fig. 2. (Quantitized) behaviour of the pressure P and the blood flow Q during a cardiac cycle in the internal carotid (*ICA*), the external carotid (*ECA*) and the common carotid (*CCA*) arteries

4. Impedance and the equivalent model of the vessel system

The simultaneous measurement of the blood pressure and blood flow in the same cross-section of the vessell permits the determination of the vessel impedance, which is a functional relationship between these quantities (see formula (1)). By using the equipment constructed, the authors carried out measurements of the vessel impedance in the common carotid artery in 10 men aged between 25 and 40 years. In all the cases the investigations were carried out at 2–3 cm before the bifurcation.

The modulus and phase of the vessel impedance were determined by the fast Fourier transform (*FFT*) from the time functions of the blood pressure and flow. For each heart cycle, 64 discrete values of the pressure and flow were used in the calculations. The impedance was determined independently of three successive cardiac cycles, and subsequently the average value was calculated from the results obtained.

Fig. 3 shows, as an example, the modulus and phase of the impedance calculated for particular harmonic frequencies of the heart rate, on the basis of measurements carried out in the common carotid artery of a 25-year-old man. In

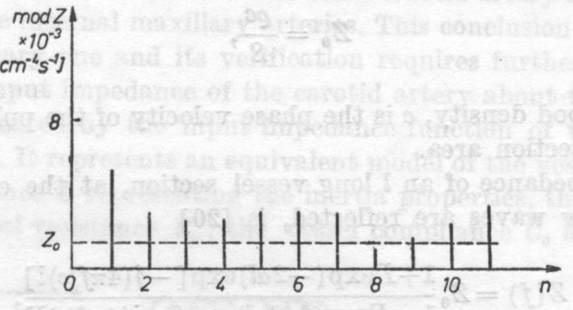

Fig. 3 The modulus (mod Z) and the phase (arg Z) of the input impedance determined for n successive harmonics in the common carotid artery of a 25-year-old man. DC value of mod Z (for $n = 0$) is equal to 16.52×10^{-3} [$\mathrm{gcm}^{-4}\,\mathrm{s}^{-1}$] and is no presented in figure

the interpretation of impedance, 12 first harmonics were considered. The other harmonic frequencies were eliminated, for their values are quotients of components with very small amplitudes, whose share in the spectrum of the signal of the blood pressure and flow velocity is negligibly small.

In all the 10 men examined, it was observed that the modulus of the vessel impedance has two characteristic minima, accompanied by a zero value of the phase. The first minimum occurs between the 3th and 4th harmonics, the other between the 7th and 10th harmonics, depending on the patient. From the viewpoint of the authors, the first minimum is related to the compliance, inertia and resistance of carotid arteries and brain vessels. The negative phase of the impedance for harmonics below a minimum is related to the compliance properties. The positive phase values for harmonics above the minimum corresponds to the inertance properties.

The other impedance minimum is probably the effect of the reflection of the pressure and velocity wave propagating along the carotid arteries. By analysing the phenomenon of the reflection, the authors assumed, just as McDonald [14] did, a simplified model of the vessel system in the form of vessel sections with constant hemodynamic properties. By analogy, these sections can be regarded as sections of a transmission line with the characteristic impedance Z_0. For large blood vessels and higher harmonics, this impedance has a real character and is [18].

$$Z_0 = \frac{\varrho c}{S}, \tag{8}$$

where ϱ is the blood density, c is the phase velocity of the pulse wave and S is the vessel cross-section area.

The input impedance of an l long vessel section, at the end of which the pressure and flow waves are reflected, is [20]

$$Z(f) = Z_0 \frac{1+\Gamma\exp(-2al)\exp[-j(4\pi f/c)l]}{1-\Gamma\exp(-2al)\exp[-j(4\pi f/c)l]}, \tag{9}$$

where Γ is the coefficient of the reflection at the end of the vessel and a is the attenuation coeficient of the blood pressure and flow wave. For real values of Z_0 and Γ, the impedance $Z(f)$ takes (from formula (9)) a real value for a harmonic with the frequency

$$f = \frac{c}{4l} \tag{10}$$

corresponding to 1/4 of the wavelength of the pulse wave in the vessel. In addition, it reaches a minimum value, less than the characteristic impedance, when the reflection cofficient Γ has a positive value.

The above conclusions confirm the author's earlier suggestion that the second minimum of the input impedance for the common carotid artery can be caused by reflection of the pulse wave. Fig. 3 shows that the impedance at the second minimum has a real character (with zero phase) and is lower than the characteristic impedance Z_0 calculated from relationship (8).

In determinig the hypothetic site of reflection the authors assumed that the impedance measured in the common carotid artery before the bifurcation depends on the input impedances of the external and internal carotid arteries. It signifies that the observed minimum of the input impedance of the common carotid artery can be a result of reflection in the external or internal artery. Assuming approximately that the phase velocity c is independent of the frequency, the authors estimated the distance l between the measurement point and the reflection point from dependence (10). The velocity c in this depedence was determined from measurements, on the basis of the formula [18]:

$$c^2 \simeq \frac{1}{\varrho} \frac{P_s - P_d}{S_s - S_d} S_d, \tag{11}$$

where P_s and P_d are systole and diastole pressures and S_s and S_d are the vessel cross-section areas for the two pressures. The distance between the reflection point and the measurement point estimated in this way varied between 15 and 18 cm, depending on the person examined. It suggests that the second minimum of the impedance measured in the common carotid artery can be related to reflection from the bifurcation of the external carotid artery into the superficial temporal and the internal maxillary arteries. This conclusion should be regarded as a preliminary one and its verification requires further studies. The behaviour of the input impedance of the carotid artery about the first minimum can be approximated by the input impedance function of the elastic system shown in Fig. 4a. It represents an equivalent model of the vessel system and includes the inertance L representing the inertia properties, the vessel resistance R_0, the peripheral resistance R_p, the vessel compliance C_0 and the peripheral

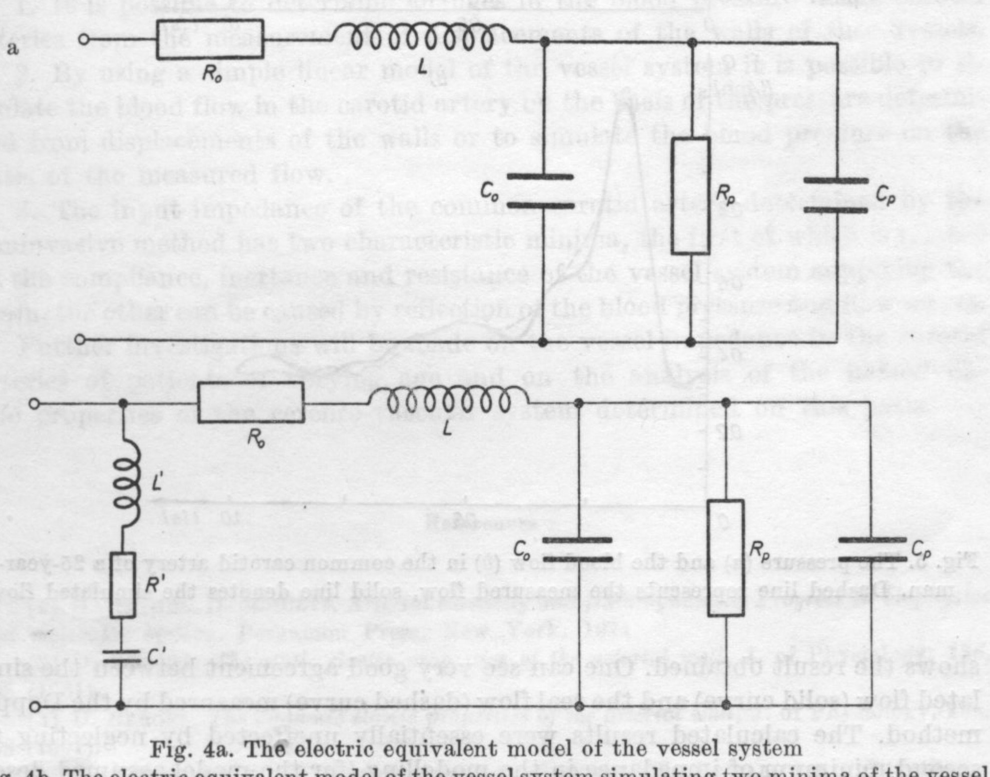

Fig. 4a. The electric equivalent model of the vessel system

Fig. 4b. The electric equivalent model of the vessel system simulating two minima of the vessel impedance compliance C_p. This model is a slight modification of the vessel model proposed by BROEMSER and RANKE [5, 11], which is used by a large number of authors to interpret the impedance measured in the aorta [12, 22].

By applying the model shown in Fig. 4*a*, a computer simulation of the blood flow rate in the common carotid artery was carried out on the basis of the pressure determined from displacements of the walls of this artery. Fig. 5

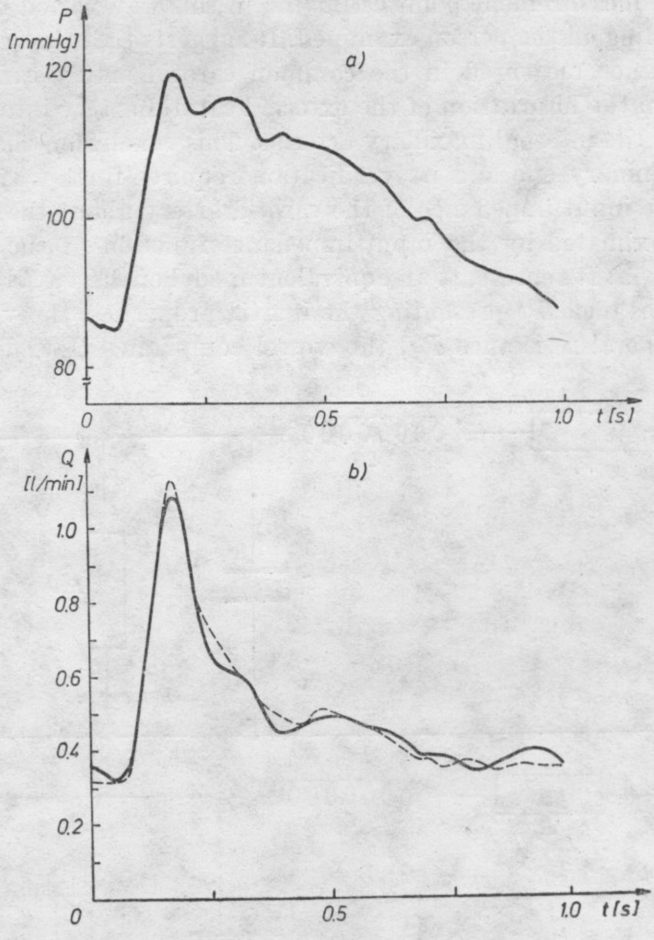

Fig. 5. The pressure (*a*) and the blood flow (*b*) in the common carotid artery of a 25-year-old man. Dashed line represents the measured flow, solid line denotes the simulated flow

shows the result obtained. One can see very good agreement between the simulated flow (solid curve) and the real flow (dashed curve) measured by the Doppler method. The calculated results were essentially uneffected by neglecting the second minimum of impedance in the modelling (for the model assumed describes only the first minimum). It can be explained by the fact that those harmonics for which the impedance takes the second minimum contribute little energy to the blood pressure and flow spectrum.

The second impedance minimum can be simulated by including a series resonance circuit L', R', C' (Fig. 4b) in the equivalent system. The impedance of such a system describes much better the real impedance. However, this model requires further studies and is only an example of a possible successive step towards modelling a vessel system.

The presented example of modelling the blood flow in the common carotid artery was obtained from a 25-year-old man. The parameters of the equivalent system, calculated from the vessel impedance determined for this patient (see Fig. 3), were $R_0 = 2480$ gcm^{-4}s^{-1}, $R_p = 14040$ gcm^{-4}s^{-1}, $L = 56$ gcm^{-4}, $C_p + C_0 = 3.3 \times 10^{-5}$ g^{-1}cm^{4}s^2.

5. Conclusions

The investigation results presented in this paper permit the following conclusions to be drawn.

1. It is possible to determine changes in the blood pressure inside carotid arteries from the measurement of displacements of the walls of thse vessels.

2. By using a simple linear model of the vessel system it is possible to simulate the blood flow in the carotid artery on the basis of the pressure determined from displacements of the walls or to simulate the blood pressure on the basis of the measured flow.

3. The input impedance of the common carotid artery determined by the noninvasive method has two characteristic minima, the first of which is related to the compliance, inertance and resistance of the vessel system supplying the brain, the other can be caused by reflection of the blood pressure and flow waves.

Further investigations will be made on the vessel impedance in the carotid arteries of patients of varying age and on the analysis of the hemodynamic properties of the cerebro-vascular system determined on this basis.

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