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THE EFFECT OF THE MODULATION TRANSFER FUNCTION ON THE IMAGE IN AN ACOUSTIC MICROSCOPE

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This paper considers the effect of the diffraction phenomenon and of errors in the geometry of the system on the quality of images obtained using a scanning acoustic microscope working in transmission. An analysis of image formation is presented, permitting the derivation of the formula for the modulation transfer function (MTF). The effect of nonaxial and out-of-parallel elements of the system on the symmetry of this function with respect to zero frequency is determined. The paper also shows microscopic images obtained from numerical simulation for two selected objects with different magnitude of deviation from the symmetry of the function MTF.

The shape of the function MTF for a system working at a relatively low frequency of 3 MHz was also determined experimentally and compared with the shape expected according to the theoretical formula derived in this paper. Satisfactory agreement was observed.

1. Introduction

The development of the acoustic microscope significantly extended the possibility of using ultrasonic waves for investigation of small biological objects, surfaces of materials and integrated elements. Taking advantage of the relatively large transmittivity of ultrasonic waves even at very high frequencies, it is now possible to construct visualization devices with resolution comparable to that of light microscopes (1 μm).

Interpretation of acoustic images is to a large extent related to their quality described by the modulation transfer function for a given acoustic system. This function describes how the process of image formation affects changes in the amplitude of spatial frequencies involved in the functions describing the object. The shape of the MTF gives an insight into the fidelity with which information contained in the image is reproduced. It depends on diffraction effects, lens transmission, aberration, irregular attenuation and on the geometry of the acoustic system.

The aim of the present paper is to describe theoretically the formation of an image in an acoustic microscope working in transmission and to discuss the effect of the frequency transmission function on the resultant image.

2. Image formation in an acoustic scanning microscope

The acoustical part of the scanning microscope can with good approximation be regarded as a system of two thin lenses working in a confocal setting (Fig. 1). The lenses can be assumed to be thin (which, as a matter of fact, they are not), since this only neglects spherical aberration. In practice this aberration is very low, since it depends not only on the numerical aperture of the lens but also on the inverse of the square of the refraction coefficient [4] which is very large ($n = 7.4$) in the case of materials used in this imaging technique.

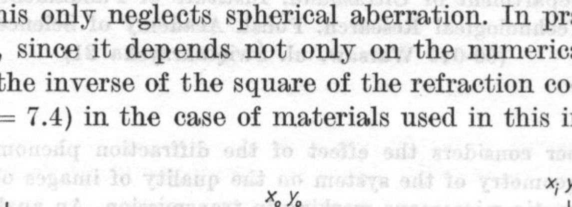

Fig. 1. A schematic diagram of the system of the acoustic scanning microscope

It can be assumed that a plane continuous ultrasonic wave with the length λ is incident on the lens L_1 and is focussed in the plane of the object. The object is scanned in the plane $X_0 Y_0$. The origin of the coordinate system related to the plane lies in the focus of two lenses. Let the function $T(x_0, y_0)$ describe the effect of the object on the ultrasonic wave propagating through it, determining the transmittivity of the amplitude by the object. When the pressure distribution preceding the passage of the wave through the object is defined as $U(x_0 y_0)$, the wave leaving the object can be written as [1]

$$U_p(x_0, x_0) = U(x_0, y_0)\, T(x_0, y_0). \tag{1}$$

The independence of transmission from the incidence angle of the wave is assumed here.

The wave U_p is subsequently collected by the lens L_2 and as a pseudo-plane wave falls onto the receiver, a transducer placed in the plane $X_i Y_i$. The response of the transducer is proportional to an integral over its surface of the pressure field of the wave multiplied by the response function of the transducer. The response function of the transducer corresponds to the pressure distribution on the surface of the transducer caused by supplying to its electrodes a voltage with unitary amplitude. The voltage collected from the transducer serves in controlling the brightness of the corresponding points on the screen of the receiver. The position of these points should correspond to the values of the coordinates $(\bar{x}, \bar{y})$ which the centre of the system X_0Y_0 has in the coordinates XY related to the object being scanned (Fig. 2).

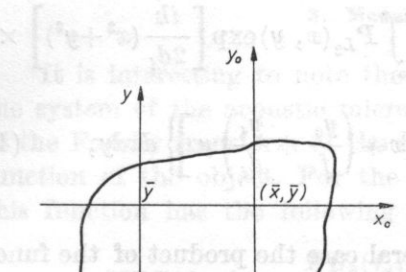

Fig. 2. Coordinate systems related to the object being scanned and to the axis of symmetry of the acoustic system

$\bar{x}, \bar{y}$ – the coordinates of the centre of the system related to the axis in coordinates related to the object, $t(x, y)$ – the object in coordinates related to the object, $T(x_0, y_0) = t(\bar{x}+x_0, \bar{y}+y_0)$ – the object in coordinates related to the axis of the system

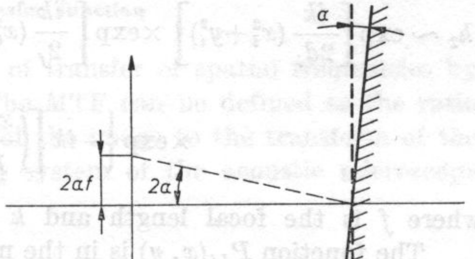

Fig. 3. A schematic diagram of the system fo measurement of the MTF

Using the known principle of image formation [1], the introduction of the function $h_3(x_0, y_0)$ permits the formulation of the following integral expression for the spatial distribution of voltage which constitutes an image of the object

$$U_0(\bar{x}, \bar{y}) = \iint_{\infty} h_3(x_0 y_0)\, U(x_0, y_0)\, t(\bar{x}+x_0, \bar{y}+y_0)\, dx_0 dy_0,$$
$$t(\bar{x}+x_0, \bar{y}+y_0) := T(x_0, y_0), \tag{2}$$

where the function $h_3(x_0, y_0)$ defines the magnitude of the voltage appearing on the transducer caused by the placing of the point source of the wave in the point (x_0, y_0). The function $h_3(x_0, y_0)$ can be found to be

$$h_3(x_0, y_0) = \iint_{\infty} h_2(x_i, y_i, x_0, y_0)\, P_p(x_i, y_i)\, S(x_i, y_i)\, dx_i dy_i, \tag{3}$$

where h_2 describes the pressure distribution in the plane of the transducer, P_p defines the aperture of the transducer and S is its response function. It seems justified to assume that the beam of the incident energy is completely focussed on the surface of the transducer, which permits it to take $P_p(x_i, y_i) = 1$. The function h_2 can be found by placing the point source in a point with the coordinates x_0, y_0. Consideration of the phase shift caused by the lens and the use of the Fresnel diffraction formulae permit the determination of the pressure distribution from this source in the plane of the transducer [1]. For the present system this distribution is given by the formula

$$h_2 \sim \exp\left[\frac{ik}{2d_i}(x_i^2+y_i^2)\right] \times \exp\left[\frac{ik}{2_f}(x_0^2+y_0^2)\right] \iint\limits_{\infty} P_{L_2}(x, y) \exp\left[\frac{ik}{2d_i}(x^2+y^2)\right] \times$$
$$\times \exp\left\{-ik\left[\left(\frac{x_0}{f}+\frac{x_i}{d_i}\right)x+\left(\frac{y_0}{f}+\frac{y_i}{d_i}\right)y\right]\right\} dxdy, \tag{4}$$

where f is the focal length and $k = 2\pi/\lambda$.

The function $P_{Li}(x, y)$ is in the most general case the product of the function of the lens aperture L_i $(i = 1, 2)$ and of some functional dependent on the lens transmission, irregular attenuation and spherical aberration. Assuming that $S = \text{const}$, which signifies that the transducer radiates a plane wave, integration can be made over dx_i and dy_i in formula (3).

This gives in effect

$$h_3(x_0, y_0) \sim \exp\left[\frac{ik}{2f}(x_0^2+y_0^2)\right] FP_{L_2}\left(\frac{x_0}{\lambda_f}, \frac{y_0}{\lambda_f}\right), \tag{5}$$

where $FP(x, y)$ corresponds to a Fourier transform of the function P made at a point with the coordinates (x, y). The smallness of the area of non-zero value of FP_{L2} [1] permits one to take the oscillating factor as constant.

In a way analogous to (4) it is possible to determine the quantity $U(x_0, y_0)$, i.e. the pressure distribution from the transmitting transducer determined just before the scanning plane. It can be assumed that a plane wave is incident on L_1, which is equivalent to the placement of the point source in infinity $(f \to \infty)$. Adequate substitution in formula (4) $(d_i = f,\ P_{L2} = P_{L1})$ gives

$$U(x_0, y_0) \sim \exp\left[\frac{ik}{2f}(x_0^2+y_0^2)\right] FP_{L1}\left(\frac{x_0}{\lambda f}, \frac{y_0}{\lambda f}\right). \tag{6}$$

For the same reasons as in (5) the oscillating factor before FP_{L1} in (6) can be regarded as constant. Insertion of (5) and (6) into (2) now gives an expression describing an image of the object $t(x, y)$, i.e. the voltage distribution taken from the transducer, depending on the position of the object with respect to the focus of the system.

$$U_0(\bar{x},\bar{y}) \sim \iint_{\infty} FP_{L1}\left(\frac{x_0}{\lambda f},\frac{y_0}{\lambda f}\right) FP_{L2}\left(\frac{x_0}{\lambda f},\frac{y_0}{\lambda f}\right) t(\bar{x}+x_0,\bar{y}+y_0)\,dx_0dy_0$$

$$\Leftrightarrow U_0(\bar{x},\bar{y}) \sim t(\bar{x},\bar{y}) * \left(FP_{L1}\cdot FP_{L2}\left(\frac{\bar{x}}{\lambda f},\frac{\bar{y}}{\lambda f}\right)\right), \tag{7}$$

where $*$ signifies the operation of correlation of the function.

3. Modulation transfer function

It is interesting to note the manner of transfer of spatial frequencies by the system of the acoustic microscope. The MTF can be defined as the ratio of the Fourier transform of the function of the image to the transform of the function of the object. For the scanning system of the acoustic microscope this function has the following form

$$MTF(f_x,f_y) = \frac{FU_0(f_x,f_y)}{Ft(f_x,f_y)} \sim P_{L1}(\lambda ff_x,\lambda ff_y)\times P_{L2}(\lambda ff_x,\lambda ff_y)$$

$$= \iint_{\infty} P_{L1}(x,y)P_{L2}(\lambda ff_x-x,\lambda ff_y-y)\,dxdy, \tag{8}$$

where $\times$ signifies convolution and f_x, f_y are spatial frequencies.

It is particularly easy to determine an explicit form of the MTF in the case of two identical lenses with a circular aperture, without aberration and transmission independent of the incidence angle. In this case the MTF is a convolution of two identical functions of the form

$$\mathrm{circ}\left(\frac{\sqrt{f_x^2+f_y^2}}{l/2\lambda f}\right) := \begin{cases}1 & \text{for } f_x^2+f_y^2 \leqslant (l/2\lambda f)^2;\\ 0 & \text{for } f_x^2+f_x^2 > (l/2\lambda f)^2,\end{cases}$$

where l is the diameter of the lens, λ is the wavelength in the medium surrounding the lens and f is the focal length of the lens.

The operation of convolution gives

$$\mathrm{MTF}(\varrho) = \begin{cases}\frac{2}{\pi}\left[\arccos\left(\frac{\varrho\lambda f}{l}\right)-\frac{\varrho\lambda f}{l}\left(1-\frac{\varrho\lambda f}{l}\right)^{1/2}\right] & \text{for } \varrho \leqslant \frac{1}{\lambda f},\\ 0 \quad \text{for } \varrho > \frac{l}{\lambda f},\end{cases} \tag{9}$$

$\varrho = (f_x^2+f_y^2)^{1/2}$ is the radius in the frequency space. Thus, in an ideal case, considering only the diffraction phenomena, the maximum frequency carried by the microscope system is equal to $l/\lambda f$.

4. Measurement of the modulation transfer function

The value of the MTF resulting from formula (8) derived here will now be compared with the values obtained experimentally using the method proposed in paper [5]. This method takes advantage of the fact that theoretically the system in Fig. 1 is equivalent to a system in which a reflecting plate would be placed in the focal plane of the lenses and in which the transducer 1 and the lens L_1 would work in a transmit-receive arrangement (Fig. 3). Under the assumption of non-dependence of the reflected amplitude on the incidence angle, this plate can be described as $t(x, y) = \text{const}$.

The use of formula (7) defining the relation between the image and the object gives the following expression for the image of the plate

$$U_0(\bar{x}, \bar{y}) = \iint\limits_{\infty} FP_{L1}\left(\frac{x_0}{\lambda f}, \frac{y_0}{\lambda f}\right) FP_{L2}\left(\frac{x_0}{\lambda f}\frac{y_0}{\lambda f}\right) \text{const}\, dx_0 dy_0. \tag{10}$$

Since this quantity is not dependent on the coordinates $\bar{x}, \bar{y}$, only one point of the image can be considered further. The Parseval equation [2] permits the transformation of expression (10) to the form

$$U_0 \sim \iint\limits_{\infty} P_{l1}(x, y) P_{L1}(-x, -y)\, dx dy. \tag{11}$$

It can now be assumed that the plate was deflected from the X axis by a small angle α_x. This causes a change in the path of the reflected beam by $2\alpha_x$. It can be assumed for small deflections that the pressure field of the beam reflected before the lens L_1 is shifted along the X axis by $2\alpha_x f$ with respect to the field for $\alpha_x = 0$. This is equivalent to a shift of the lens L_1 working in the collecting mode by the same value.

In such a case

$$U_0(\alpha_x, \alpha_y = 0) \sim \iint\limits_{\infty} P_{L1}(x, y) P_{L1}(2\alpha_x f - x, -y)\, dx dy. \tag{12}$$

When formula (8) is written for $f_y = 0$ and compared with (12), it becomes evident that [5]

$$\text{MTF}(f_x, 0) \sim U_0(\alpha_x, 0), \quad f\lambda f_x = 2\alpha_x f.$$

It follows therefore that the spatial frequency $f_x = 2\alpha_x/\lambda$ corresponds to the angle α_x. Obviously, an analogous case occurs with deflection from the Y axis and with the frequency f_y. This provides a simple method for the measurement of the MTF of a given system of the acoustic microscope.

MTF measurements using the above method were carried out for a microscope system working at low frequency (3 MHz). A highly damped ceramic transducer gave an almost plane ultrasonic wave which was incident on a polystyrene lens with a focal length in water equal to 210 mm. An aluminium plate whose inclination with respect to the axis of the system could be changed was placed in the focus. A defectoscope was used as a transmit-receive system. Fig. 4 shows the experimentally determined transfer function of spatial frequencies in this system. Fig. 4 also shows the theoretical curve calculated from formula (9) which accounts only for the diffraction effects. A circle with its diameter equal to that of the transducer, $l = 47$ mm, was taken as the active aperture of the lens. The length of the wave at a frequency of 3 MHz in water is 0.5 mm.

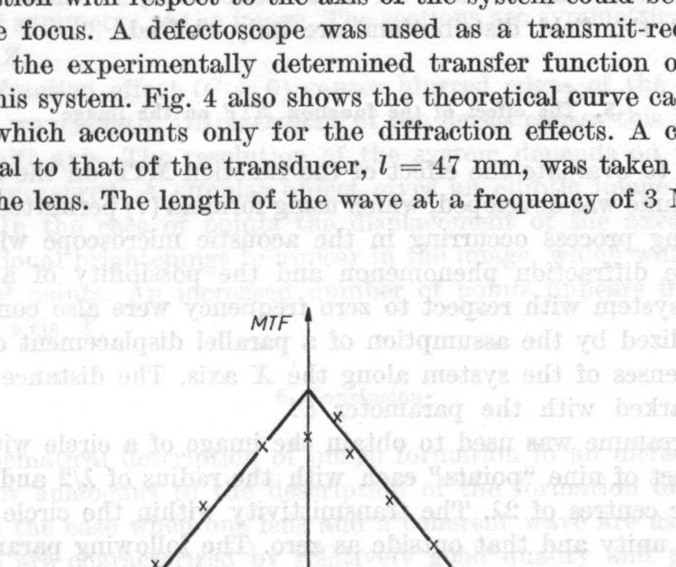

Fig. 4. The curve of MTF for the system of the acoustic microscope working at a frequency of 3 MHz

solid line – theoretical curve, points * correspond to experimentally determined MTF

These curves are only proportional to the measured and calculated MTF functions. Therefore, their comparison with each other is useful only when the comparison involves their total bandwith of the frequency carried and the ratio of amplitudes at different frequencies for the same curve. The measured function MTF is shifted with respect to the origin of the system. This was probably caused by some imprecision in the setting of $\alpha_x = 0$.

In the case of the acoustic microscope working in transmission this effect can be caused by errors in the technology of preparing the lens material and by imprecision in the geometry of the system.

A parallel shift of the axis of the transmitting lens by the distance R with respect to the axis of the receiving lens causes the field incident on this lens to be shifted with respect to it by R from the axial arrangement. Using the expression for the MTF (8) it is possible to calculate that this gives a shift of this function by $R/\lambda f$ in the spatial frequency domain.

A similar case occurs when the transmitting transducer is set so that it radiates a wave at the angle α to the axis of the lens. In this case the use of (8) permits the determination of the shift of MTF by the quantity $2\alpha/\lambda$. Obviously, these relations can only be valid for small deflections, so small that it can be assumed that the field distributions are only shifted.

5. The effect of the function MTF on the image

In order to evaluate the effect of the function MTF on the image a computer programme was developed, which using formula (7) permitted a simulation of the imaging process occurring in the acoustic microscope with a circular aperture. The diffraction phenomenon and the possibility of a shift of the MTF of the system with respect to zero frequency were also considered. This shift was realized by the assumption of a parallel displacement of the optical axes of the lenses of the system along the X axis. The distance between the axes was marked with the parameter C.

The programme was used to obtain the image of a circle with the radius of 7λ and a set of nine "points" each with the radius of $\lambda/2$ and the distance between their centres of 2λ. The transmittivity within the circle or the point was taken as unity and that outside as zero. The following parameters of the lenses of the microscope system were assumed: the focal length $75\ \lambda$ and the

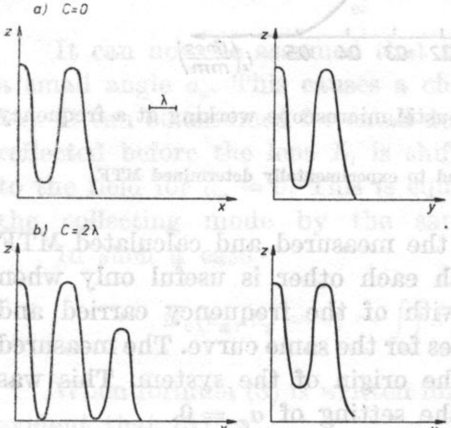

Fig. 5. Sections of the image of a circle obtained for different values of the parameter C

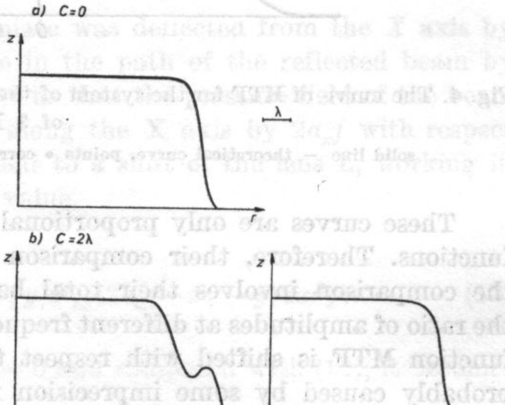

Fig. 6. Sections of images of a system of points obtained for different values of the parameter C

lens diameter 100 λ. Figs. 5 and 6 shows sections of the objects and their images along the axes X and Y. The centre of the coordinate system XY is placed in the centre of symmetry of the image. The sections are symmetrical with respect to the axis Z.

The diffraction effect ($C = 0$) causes blurred edges of the images of the order of the wavelength. Displacement of the lenses causes this blur to extend along the shift axis. The resolution of the system depends on the axis along which it is measured. A circular object gives an elliptic image with irregular brightness. In the case of points the displacement of the axes of the lenses causes additional brightenings to appear in the image, which will be interpreted as additional points. An increased number of points appears in the direction of the shift axis X.

6. Conclusions

A mathematical description of image formation in an ultrasonic scanning microscope is analogous to the description of the formation of the image of an object in the case when one lens and a coherent wave are used [1]. Images of this type are characterized by relatively good quality and particularly by the absence of such misrepresentations as ringing and speckles.

The character of changes in the MTF with increasing spatial frequencies indicates the smoothing properties of imaging of this type. This causes blurred edges of the objects imaged and decreased brightness of details.

In view of errors in the geometry of the acoustic microscope system, its MTF can be nonsymmetrical with respect to the zero spatial frequency. This is a source of numerous misrepresentations in the image. In such cases objects with constant distribution of acoustical parameters show irregular brightness and their edges are differently blurred. This causes a distortion of the image, introduces misinformation and worsens resolution. It seems that the nonsymmetricity of MTF accounts well enough for unexpected darkenings which often occur in images obtained using the acoustic microscope.

The function of the pulse response of the transducer, S, has a large effect on the image. In real systems it is not constant on the surface of the transducer, which was assumed in the present calculations. This can be another source of distortions in the image.

The images obtained numerically for cases of different geometry of the system of the acoustic microscope permit the determination of admissible geometry errors. Coaxiality of the lenses better than the wavelength and out-of-parallelness of the axes of the lenses below $1/3°$ should eliminate one of the major reasons for distortions in the image. Interpretation of acoustical images using the acoustic microscope requires the use of the MTF. It seems that for this purpose it is possible to use an MTF with a shape determined experimentally by means of such a simple and approximate method as the present one.

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