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A GEOMETRICAL-NUMERICAL METHOD FOR THE DETERMINATION OF THE ACOUSTIC FIELD PROPERTIES RELATED TO THE DIRECTIONS OF REFLECTED WAVES*

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A method for theoretical determination of the field in enclosures is presented. To analyse the directivity of the field, the spatial distributions of the directions of the reflected waves and their energy are calculated using the geometrical-numerical method. Diffusion coefficients of the directions, energy and ray energy are introduced. These coefficients characterize in general the directional properties of the field at a given point and determine in detail the values of energy reaching the observation point from different parts of the space. An algorithm for a computer program and the results of numerical calculations for a rectangular room which illustrate the effect of the different parameters of the source-room-observation point system on the directional properties of the field are given.

1. Introduction

The methods for the evaluation of the field directivity used to date have mainly been based on measurement results [1], while theory has so far failed to provide effective tools for this purpose [2]. Such possibilities are provided by the geometrical method for field analysis which uses a computer and consists in the determination of an array of image sources [3] and in further processing of data which result from the shape of the array. Paper [3] proposed an energy approach to directivity, which defines only the mutual relations among the energies of waves reaching different points of the field but does not give information of the directions from which the waves reach a given point.

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When application, and not the cognitive aspects, is taken into account, this energy approach can in general be considered sufficient in noise control but is insufficient in architectural acoustics in which it is necessary to know also the directions from which the energy reaches a point. In the geometrical-numerical method for field analysis this problem can be solved in a rather simple way. When an array of image sources has been determined for a given source, it is possible to construct an algorithm for the determination of the directions from which particular reflected waves reach the observation point. The question arises as to whether it is necessary to determine each direction in the form of a versor or directional cosines, or whether it is sufficient to determine the number of directions in which the waves propagate in successive parts of the space. The determination of the directional cosines of all waves which reach the observation point would permit theoretical development of a directional-energy image of reflections analogous to the saw-tooth shape obtained from measurements using a directional microphone [2]. Such an image — in particular when given in spatial form — is clear and easy to interpret for one point. It is, however, more difficult to interpret it for several points. It seems therefore useful to gather information of all reflected waves in larger groups which include the waves which reach the observation point from a certain part of the space. This gives information in more compact form, which it is thus easier to use further without additional processing. When the number of the parts of the space in which the directional properties are determined decreases, in the limits information is gained which is expressed by one number for a given observation point. Such generalized information makes it easier to compare or evaluate, but also makes difficult the work towards the introduction of desired changes, since this requires detailed information. It seems that a reasonable compromise is the giving of generalized information with possible access — as the need arises — to detailed information. Such a solution has been assumed in the development of theoretical determination of the directivity of the field using a computer.

2. The method for the determination of the field directivity

Since the present method is based on geometrical analysis of the field using image sources, it satisfies the same assumptions as the analysis does:

(1) acoustic waves — represented in the form of sound rays — radiated by point image sources reach the observation point;

(2) the energy of particular sound rays sums up at the observation point.

Conceptually the method is based on the following two assumptions:

(1) the space is divided into parts in which some parameters of the field are calculated;

(2) the directional properties of the field in a point are determined by the number of the directions in which the waves propagate in successive parts of

the space, by the energy reaching the observation point and by the energy of rays in the same parts of the space.

The centre of the coordinate system xyz is placed at the observation point. The system divides the entire space into eight parts by the planes $x = 0$, $y = 0$ and $z = 0$ (Fig. 1). The number of rays K_i which reach the observation point, the energy E_i carried by these rays and the mean energy corresponding to a ray, $p_i = E_i/K_i$, are determined for each ith octant. From the above values the deviations D_{ki} and D_{ei} of these quantities from the mean values for the entire space are determined from the relations

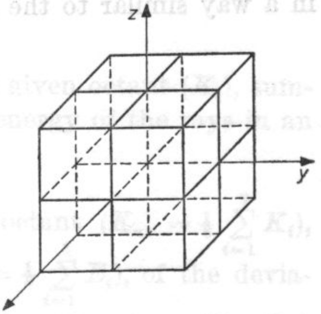

Fig. 1. The division of the space into octants by the planes $x = 0$, $y = 0$ and $z = 0$

$$D_{ki} = \frac{K_i - K_{\text{av}}}{K_{\text{av}}}, \quad \text{where } K_{\text{av}} = \frac{1}{8}\sum_{i=1}^{8} K_i; \tag{1}$$

$$D_{ei} = \frac{E_i - E_{\text{av}}}{E_{\text{av}}}, \quad \text{where } E_{\text{av}} = \frac{1}{8}\sum_{i=1}^{8} E_i. \tag{2}$$

The value of the deviation D_{ki} can fall in the range $[-1, 7]$, where $D_{ki} = -1$ no ray reaches the observation point from the ith octant; $-1 < D_{ki} < 0$ fewer rays than follows from their mean number reach the observation point from the ith octant; $D_{ki} = 0$ the mean number of rays reach the observation point from the ith octant (when for all i $D_{ki} = 0$, the distribution of directions is uniform in the entire space); $0 < D_{ki} < 7$ more rays than follows from their mean number reach the observation point from the ith octant; $D_{ki} = 7$ all the rays concentrate in the ith octant.

Analogous interpretation is valid for the energy deviation D_{ei} in the ith octant.

In defining quantitatively the spatial distribution of directions and energies, the measured values of K_i and E_i and their deviations permit the distribution to be shaped at a given point by changing the parameters of the source-room-observation point system. Such detailed information of the spatial directivity in the form of several dozens of numbers for one observation point are little useful in comparing the directional properties of the field at different points. In view of this, the diffusion coefficients — of direction d_k and of energy d_e — which characterize the directional properties in a more compact way have been introduced. The diffusion coefficients average the directional properties for all octants, expressing these by two numbers for a given point, and thus permit comparison of different points of the field. These coefficients are defined in a way similar to the directional diffusion coefficients [4],

$$d_k = 1 - \frac{\frac{1}{8}\sum_{i=1}^{8} |K_i - K_{\mathrm{av}}|}{K_{\mathrm{av}}}, \tag{3}$$

$$d_e = 1 - \frac{\frac{1}{8}\sum_{i=1}^{8} |E_i - E_{\mathrm{av}}|}{E_{\mathrm{av}}}. \tag{4}$$

The diffusion coefficients d_k and d_e take values from the range $[-0.75, 1]$; d_k(or d_e) $= -0.75$ corresponds to the field concentrated in one octant, while d_k(or d_e) $= 1$ corresponds to a dispersed field, i.e. one with a uniform distribution of directions and energies (with an accuracy of an octant). When the space is divided into more than eight parts, the range will extend, tending on the left side to -1 with $i \to \infty$.

The present computational method for the determination of the directional properties of the field permits rather easy calculation of the diffusion coefficients of directions and energy for a given point, i.e. permits a general evaluation of the directivity. It at the same time gives detailed data on particular octants of the space, thus permitting the desired shaping of the spatial distribution of directions and energies by choosing the parameters of the source-room-observation point system.

3. A short description of the programme

The programme was written in FORTRAN, while calculations were performed on a CYBER CDC 6000 system computer. A flow chart of the program is given below for a rectangular room.

data

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calculation of one coordinate defining the distance of the image source from the observation point and of the number of reflections from each of the two walls perpendicular to the axis along which the coordinate is calculated

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calculation of the distance between the image source and the observation point, of propagation and reflection losses and of the sound pressure level of reflected waves

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determination of the sign of the coordinates x, y, z in a new coordinate system, related to the observation point and subordination of the reflected wave to a certain octant

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counting of waves reaching the observation point in a given octant (K_i), summing up of their energies (E_i) and calculation of the energy of the rays in an octant ($p_i = E_i/K_i$)

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calculation of the mean number of reflections in an octant ($K_{\text{av}} = \frac{1}{8}\sum_{i=1}^{8} K_i$), of the mean energy corresponding to the octant ($E_{\text{av}} = \frac{1}{8}\sum_{i=1}^{8} E_i$), of the deviations of the number of directions and of energy for the ith octant (D_{ki}, D_{ei})

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calculation of the diffusion coefficients of direction (d_k) and of energy (d_e)

The input data of the programme are: the dimensions of the room, the coordinates of the sound source and of the observation point, the absorption coefficients of the walls, the sound pressure level of the direct wave at a distance of 1 m from the source, the coefficient of energy attenuation in the air and the order of the reflections considered. The calculated results are given in two tables. Table 1 gives the values of the number of waves reaching the observation point K_j, of their summary energy E_j, of the level of this energy L_{ej} and of the energy of the rays p_j. The index j takes values from 1 to 27, since the waves reaching the observation point along the axes Ox, Oy and Oz of the system and the planes xy, xz and yz are treated individually. The a priori subordination of the axes and planes to specific octants, since it masks some information, prevents effective control of the directivity of the field by varying the input data. In addition it would be incorrect to calculate the values of the means and deviations for each of 27 parts of the space, since each of these parts is treated as equivalent, irrespective of whether it is an octant or half-axis. These results, which are detailed information, are therefore only an intermediate stage. Table 2 gives the final results, i.e. the above-mentioned quantities K_i, E_i, L_{ei}, p_i, D_{ki} and D_{ei} calculated for particular octants and the diffusion coefficients d_k and d_e for a given observation point. The waves which reach the observation point in the planes xy, xz and yz are divided between the two octants adjacent to them, while the waves which propagate along the axis are separated among the four adjacent octants. The octants are numbered in the following way: from 1 to 4 for $z > 0$ counter-clockwise, where octant 1 lies between the planes determined by the positive half-axes of the system; from 5 to 8 for $z < 0$, where octant 5 lies underneath octant 1. The programme is so built that it does not limit the order of reflections considered. The restriction on the order results from the time limit only and thus from calculation cost.

4. Calculated results and their interpretation

Calculations were performed for a rectangular room with dimensions 15 ×25 ×10 m, with an active point sound source which radiated a wave of 100 dB sound pressure level at a distance of 1 m. Fig. 2 shows the position of the enclosure in the coordinate system. The coefficient of energy attenuation $m = 0.004$ was taken. The values of the absorption coefficients of the walls, α, varied from 0.1-0.6. The calculations took into account reflected waves up to the seventh order.

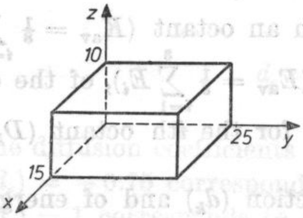

Fig. 2. The position of the room in the coordinate system

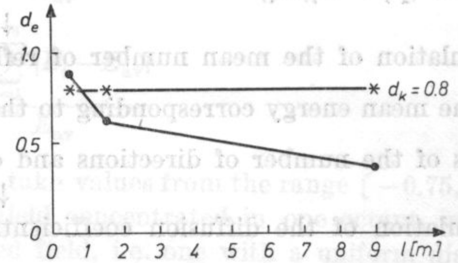

Fig. 3. The effect of the distance of the observation point from the wall on the energy uniformity of the field expressed by the diffusion coefficient of energy d_e

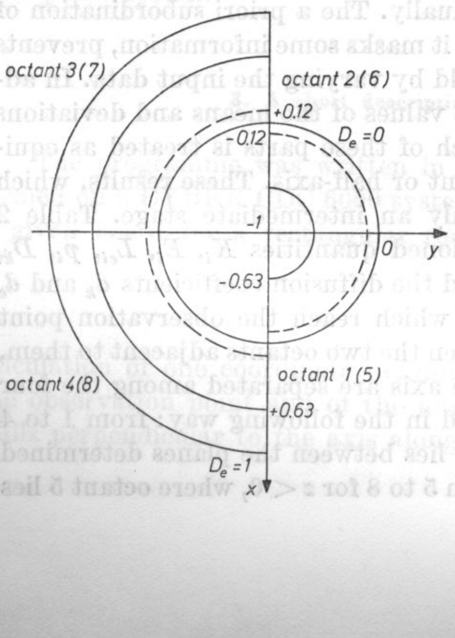

Fig. 4. The energy deviation D_e in octants for two positions of the observation point: far from one of the walls and close to it. Calculations were performed for the following parameters: the absorption coefficients of the walls $\alpha = 0.1$; the coordinates of the source placed in the centre of symmetry of the area (7.5; 12.5; 5.0 m); a solid line represents D_e calculated for an observation point with the coordinates (7.5; 16.0; 5.0 m) a dashed line expresses that for the coordinates (7.5; 24.5; 5.0 m). The solid circle marks 0 scale for $D_e = 0$

Investigations were performed on the directional properties of the field defined by the diffusion coefficients of direction and energy by always varying only one parameter of the source-room-observation point system so that it would be possible to interpret the results in a clear way.

The effect of the position of the observation point with a constant position of the source at the centre of symmetry of the room. The shift of the observation point along the Oy axis towards the wall does not change the dispersion coefficient of direction but improves, instead, the dispersion of energy (Fig. 3). This can be explained by the effect of high-energy reflections from the close wall. This is also illustrated by Fig. 4 which shows the energy deviation $D_{ei} = \Delta E_i/E_{av}$ for four octants; for the other octants the plot is the same because of the geometrical symmetry of the system. The value of the deviation D_e in a given octant is constant, which is represented in the polar coordinate system in the form of a quadrant: plotted in a solid line for an observation point at a distance of 16 m from the wall $y = 0$, in dashed line for an observation point at a distance of 0.5 m from the opposite wall. The solid circle marks the scale in the form of a circle $D_e = 0$ and $D_e = 1$. The results of the calculations permit a quantitative approach, as opposed to the only qualitative one previously, to the effect of the close wall on the energy at the observation point.

The effect of the symmetry of the position of the observation point with respect to the source placed in the centre of symmetry of the room. Investigations were performed on the cases in which the symmetry of the position of the observation point was maintained with respect to the source, e.g. when the source and the observation point differed only in one coordinates and on the cases when there was no regular position — the source and the observation differing in all the three coordinates. From the point of view of the uniformity of the field, the symmetrical position of the observation point with respect to the source is more convenient, since then both the dispersion coefficients of direction and of energy are closer to 1 than in the case of an arbitrary position of the observation point.

The effect of the geometrical asymmetry of the position of the source and the observation point in the room. Keeping a constant between the source and the observation point the directional properties of the field were compared for a symmetrical position of this pair of points in the area with the source at the centre of symmetry and the observation point on an axis parallel to the wall and for a random position. It was found that the diffusion of directions slightly decreases for a random position of the source and the observation point; the diffusion coefficient of energy, however, increases very greatly, i.e. the energy uniformity of the field decreases (e.g. d_e reduces from 0.37 to -0.05).

The effect of change in the position of the source and the observation point. In view of the geometrical symmetry of the system, the diffusion coefficient of direction, d_k, does not change, whereas the same values of the deviations D_{Ki}

are subordinated to different octants, i.e. the coordinate system undergoes rotation. In turn, the diffusion of energy changes, being greater when the source is closer to the centre of symmetry of the area.

The effect of the arrangement of the sound-absorbing materials. The introduction of differentiated absorption coefficients α of the walls does not change the diffusion of directions but reduces the uniformity of the field ($d_e \searrow$). This second conclusion is rather evident qualitatively but it is essential that quantitative relations are obtained. The magnitude of change in the diffusion coefficient of energy, d_e, depends on two factors: on the absolute change in the coefficient α and on whether the wall of another value of the absorption coefficient is close to or far from the source (Fig. 5). It was also found that the effect of the same change in the coefficient α is stronger for a field of higher nonuniformity, i.e. the diffusion coefficient of energy decreases to a greater extent if it is small before α is changed.

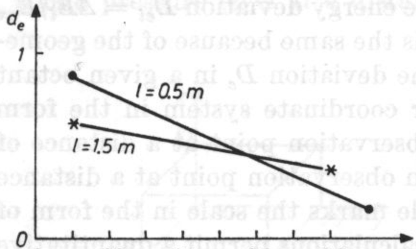

Fig. 5. The dependence of the diffusion coefficient of energy d_e on the variation of the absorption coefficient α of walls for different distances l between the observation point and the walls

The effect of the order of reflections assumed on the calculated directional parameters of the field. In order to determine the error which results from the limitation of the order of reflections up to the seventh one, calculations were performed for the tenth order. The number of reflected waves reaching the observation point increased from 575 to 1561 and the calculation time more than doubled; the diffusion coefficients however, increased only slightly. The differences of the order of hundredths of a part between the values of the diffusion coefficients are so small that it is possible to take as reliable the results calculated when the seventh order is considered.

The analysis of the calculated results will conclude with an example of the effect of two factors of opposite impact. This effect equilibrates in the calculation of the diffusion coefficient of energy, d_e, but it is, however, not balanced in particular octants. These factors are the shift of the source towards the centre of symmetry of the enclosure, which causes an increase in the diffusion of energy ($d_e \nearrow$), and the introduction of differentiated coefficients α of the walls, which decreases the diffusion of energy ($d_e \searrow$). With a correct selection of parameters the diffusion coefficient d_e may remain constant but the deviations in energy in the octants will vary (Fig. 6). It can be concluded from this example that only an agreement among all the three diffusion coefficients for systems with different parameters makes it possible to assume the identity of the directional properties of the field. The establishment of several parameters and the change in one permit, in turn, a rather precise evaluation of change in directivity.

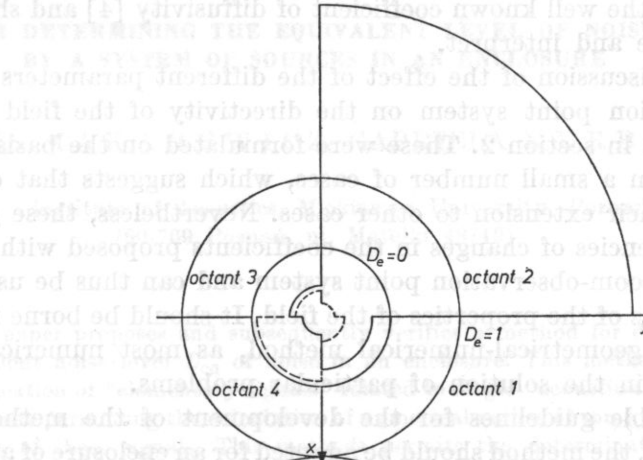

Fig. 6. The values of the energy deviation $D_{ei} = \Delta E_i/E_{av}$ in particular octants for two source room-observation point systems

The diffusion coefficient of energy $d_{e1} = d_{e2} = 0.11$ for both cases, the diffusion coefficient of direction $d_{k1} = d_{k2} = 0.7$, the diffusion coefficient of the energy of the rays with different values: $d_{p1} = 0.37$ and $d_{p2} = 0.35$. A solid line represents D_e calculated for the parameters: the coordinates of the source (7.0; 16.0; 5.5 m); the coordinates of the observation point (7.5; 12.5; 5.0 m); the absorption coefficients of the walls $\alpha = 0.1$. A dashed line represents, respectively, the coordinates (7.5; 12.5; 5.0 m) and (7.0; 16.0; 5.5 m); the absorption coefficient of the wall $y = 0$ is $\alpha = 0.06$, that of the others is $\alpha = 0.1$. The solid circle marks the scale for $D_e = 0$ and $D_e = 1$

5. Conclusion

The present method for the determination of the directional properties of the field expands the theoretical possibilities of application of the geometrical-numerical analysis.

The results of this paper show that it is useful to calculate the diffusion coefficients of direction energy and the energy of rays which express the spatial and directional features of the field. These coefficients, determined numerically, are similar to the well known coefficient of diffusivity [4] and should therefore be easy to use and interpret.

A short discussion of the effect of the different parameters of the source-room-observation point system on the directivity of the field brought some generalizations in section 2. These were formulated on the basis of the results calculated from a small number of cases, which suggests that care should be exercised in their extension to other cases. Nevertheless, these generalizations show the tendencies of changes in the coefficients proposed with given changes in the source-room-observation point system and can thus be useful in further general analysis of the properties of the field. It should be borne in mind, however, that the geometrical-numerical method, as most numerical methods, is useful mostly in the solution of particular problems.

The possible guidelines for the development of the method are readily seen. Above all, the method should be adapted for an enclosure of arbitrary shape. Secondly, it is necessary to introduce a sound source of directional radiation characteristic, which will allow to use the method for the modelling of real cases. Another problem to be solved, which is interesting from the point of view of subjective evaluation of the quality of the field, would be the introduction of time as still another parameter. It is known that in the perception of sound an essential role is played by the arrival times of waves reflected from different directions. This method could be developed toward seeking a relation between the quantities calculated objectively and subjective evaluation. It then would be necessary to introduce a receiver with a directional receiving characteristic the same as the directional characteristic of hearing.

The possibilities of the development of the method outlined above warrant the conclusion that it is purposeful to deepen and extend the geometrical-numerical analysis of the acoustic field.

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