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THE EFFECT OF THE SHAPE OF THE HOLES OF THE ROTOR AND STATOR ON THE ACOUSTIC PARAMETERS OF A DYNAMIC AXIAL GENERATOR

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The paper presents the results of investigations concerning choice of the optimal shape (from the viewpoint of acoustical efficiency) of the holes of the rotor and stator of a dynamic axial generator, the horn and pressure chamber of which are common to all the stator channels.

Compared to the sound power and the efficiency for a generator operating with the rotor preferred hitherto a doubling of the values of the parameters was achieved for a generator which has an approximate rectangular function for the time-dependence of the active surface area of the inlet holes of stator channels.

1. Introduction

ALLEN and WATTERS [2, 3], MILNE [6] and WYRZYKOWSKI [12] have dealt in their papers with the effect of the shape of the rotor and the stator on the acoustic parameters of an axial generator.

Allen and Watters have undertaken an attempt to develop a dynamic generator producing a sinusoidal wave. Hitherto the work by both of these authors has been purely experimental. The investigations involved the hole systems (arrangement) of the rotor and the stator of the axial generator.

Milne and Wyrzykowski have theoretically considered the effect of the shape of the holes of the rotor and stator on the acoustic parameters of a dynamic generator. The latter author provided a solution to this problem for a dynamic axial generator, the stator of which is formed by the inlet holes of the catenoidal horns, with the acoustic impedance of the pressure chamber being negligibly small compared to the impedance of the horn inlet. He has confined his considerations to the frequency range, in which the impedance of the horn inlet can be regarded as independent of the frequency. The paper by Milne is concerned with a particular case of these considerations.

A different version of the design of a dynamic axial generator considered by Wyrzykowski was in practice [4]. However, it is the most feasible to use several horns, so that the optimal working range of these generators is confined to comparatively low frequencies.

At higher frequencies, of the order of kHz, dynamic axial generators of a different design were used [1]. They are provided with a separate stator with channels which at one end fit into a common horn with an annular cross-section, while at the other end they lead into a common pressure chamber. The pressure chamber of such a generator does not usually contain any sound-absorbing elements.

The paper is the second one dealing with a dynamic axial generator of the above-mentioned design.

In the first paper [9] a theoretical model of the generator was presented. The model showed agreement with experiment over the range of frequencies for which it is possible to propagate exclusively plane waves in the acoustic system of the generator, and for air pressure excess in the pressure chamber, small compared to the ambient pressure.

In this paper the results of investigations aimed at increasing the acoustic efficiency of the generator are presented. The stimulus for the work was the suggestion by Jones [4] and Wyrzykowski [13] that dynamic generators producing a rectangular wave can have twice the acoustic efficiency of those producing a wave which has a sinusoidal variation in time.

2. Criteria for the choice of hole shape

The author has previously shown [9] that while operating a dynamic axial generator with a catenoidal horn common to all the stator channels, it is not possible to create a state in which the acoustic impedance of the inlet hole of the stator channel is real and independent of the frequency. For this reason it is not possible to apply the analysis of Wyrzykowski [12] to the above-mentioned design of a dynamic axial generator. The necessity has thus arisen of carrying out investigations aimed at the determination of the optimum shape of the holes of the rotor and stator of this design of a dynamic axial generator (Fig. 1) for maximizing the acoustical efficiency.

The following criteria for the choice of the shape of these holes have been adopted:

— the dependence of the active field of the inlet hole surface of the stator channel on the time, $S(t)$, should as closely as possible, approximate a rectangular function with a duty factor (being the ratio of the pulse duration of the function $S(t)$ to its period) equal to 0.5;

– the shape of the holes of the rotor and stator should be as simple as possible in design.

A necessary condition for meeting the first of these criteria is an acceptance of the equality of the times for which the inlet holes of the stator channels are fully opened and fully closed by the rotor. Initially it can be said that the time of opening (or closing) of the inlet holes of the stator channels by the rotor should be sufficiently small compared to the time the holes are fully open (or closed). The problem of the definition of an acceptable value for the ratio of these two times still remains. A reduction in the ratio leads to a reduction in the number of holes which can be arranged on the circumference of the stator channel thus increasing the mismatch of the impedance of the horn inlet to the wave impedance of the stator channel [9]. Finally, instead of increasing the sound power and efficiency of the generator we could have diminished the values of these quantities.

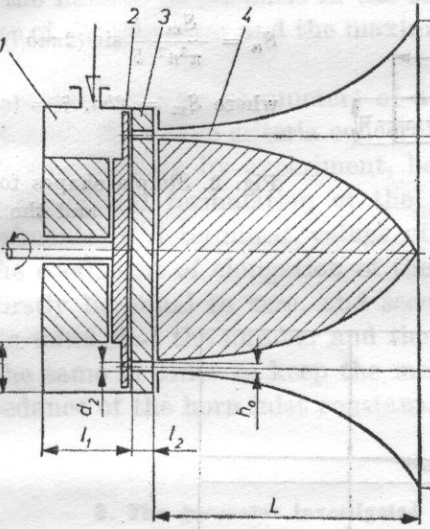


Fig. 1. Diagram of the acoustic system of a dynamic axial generator with horn and pressure chamber common to all the stator channels

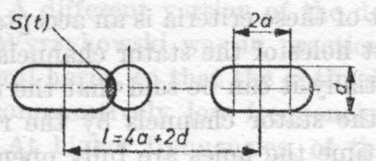
1 – pressure chamber, *2* – rotor, *3* – stator, *4* – horn

Taking into consideration the second of the above criteria we can on the basis of paper [12] define, for the rotor and stator shapes shown in Fig. 2, the curve of variation of the ratio φ_1 (the amplitude of the first harmonic of the function $S(t)$ to the maximum values S_m of this function) with the coefficient of elongation, of these holes. The coefficient of elongation is defined as the ratio of the time the inlet hole of the stator channel is fully open (or closed), to the time of opening or closing of the hole by the revolving rotor of the generator.

The results obtained are presented in Fig. 3. From the figure one can conclude that if it is assumed that the times of the inlet holes of the stator channels being fully open and fully closed by the rotor are equal, for $\xi > 0.5$ the

Coefficients of expansion of the function $S(t)$ into a Fourier series:

a)



$$S_n = \frac{2S_m}{\pi^2 n^2}\frac{l}{d}\left[I_1(2\pi nd/l)\sin(2\pi na/l) + S_1(2\pi nd/l)\times \cos(2\pi na/l)\right],$$

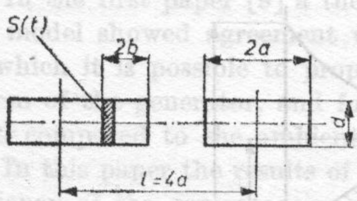
where $S_m = \pi d^2/4$, $\xi = 2a/d$

b)

$$S_n = \frac{4S_m}{\pi^2 n^2}\frac{l}{d} I_1(\pi nd/l)\sin(2\pi na/l),$$

where $S_m \pi d^2/4$, $\xi = (2a-d)/d$; $I_1(z)$ and $S_1(z)$ are Bessel and Struve functions of the first order, respectively

c)



$$S_n = \frac{S_m}{\pi^2 n^2}\frac{l}{b}\sin(2\pi na/l)\sin(2\pi nb/l),$$

where $S_m = 2bd$, $\xi = (a-b)/b$, $n = 0,1,2,\ldots$

Fig. 2. Simple shapes for the holes of the rotor and the stator

Fig. 3. The dependence of the ratio of the first harmonic of the function $S(t)$ to the maximum value S_m of this function, on the coefficient of elongation ξ for holes shaped as in Fig. 2

shape of these holes is not really essential. The value φ_1 is then constantly greater than 0.5. On the other hand if $\xi > 2$, the value of φ_1 is constant and equal to 0.63, being independent of both the shape of holes of the rotor and stator, and the value of the coefficient of elongation. From the above discussion two conclusions can be drawn:

– in approximating the time variations of the active area of the inlet hole of the stator channel by a rectangular function, the shape of the holes of the rotor and the stator is of little practical importance. Thus a shape should be chosen that is simple to realize in practice, e.g. the one shown in Fig. 2a,

– if possible, it is advisable to admit a value for the coefficient of elongation of these holes as near as possible to two.

The use of holes in the rotor and stator of the generator, for which the coefficient of elongation has a value greater than unity brings about a considerable decrease in the number of channels in the stator, and thus diminishes both the sound power of the generator and the maximum frequency of the wave it produces.

The evaluation of the acoustic parameters of a generator, the rotor and stator holes of which meet the above criteria concerning the shape of the holes, can at present be performed only by experiment, because of basic difficulties encountered in the theoretical formulation of the problem. This paper will thus compare experimentally determined values of the acoustic parameters of the generator. The coefficient of elongation of the rotor and stator holes in the generator will firstly be equal to zero, and secondly very close to unity. It should be borne in mind that the number and the surface area of the stator channel should be the same in order to keep the matching conditions of these channels to the impedance of the horn inlet constant.

3. The generator investigated

The investigations involved the dynamic axial generator [8-10] the design of which is shown schematically in Fig. 1. The generator has a catenoidal horn common to all the stator channels with an annular cross-section. The pressure chamber is also common to all these channels and does not contain any sound-absorbing elements. The generator stator has 50 channels of circular cross-section, which are arranged evenly on the circumference of a circle, 100 mm in diameter. The generator is equipped with replaceable rotors with holes shaped as shown in Fig. 4. Both rotors satisfy the requirement of having the holes of the stator channel open and shut for equal times. The coefficient of elongation, ξ for the first of the rotors (Fig. 4a), which approximates by rectangular function time variations of the active surface area of the inlet hole of the stator channel, is equal to unity, while for the second classical one (Fig. 4b) it is equal to zero.

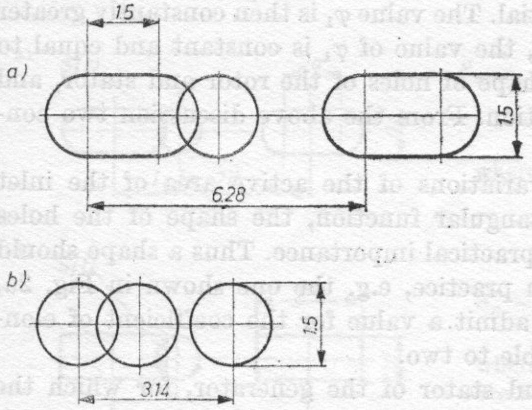


Fig. 4. The shapes and geometrical dimensions of the holes of the rotor and stator of the dynamic axial generator tested experimentally

4. Coefficient of acoustic efficiency

The preferred mostly used measure of acoustic efficiency of a dynamic generator is the ratio of the sound power radiated by the generator to the power required for the compression of the air mass taken by the generator from the ambient to the supply pressure [1-4, 8, 9, 11]. It is called the mechanical-acoustical coefficient of the generator efficiency (the acoustic efficiency), and expresses the effectiveness of the process of transforming the energy of compressed air into acoustic energy. Accordingly, for the determination of the acoustic efficiency one has to know the sound power radiated by the generator, and the power used to supply the generator with compressed air.

5. The method for determining the sound power

The direct determination of the total sound power N_a of the generator can be dispensed with on the basis of the experimentally defined distribution of the amplitude of the acoustic pressure in the far field at a constant distance from the horn outlet of the generator. Its value involves the sound power of the harmonic components of the wave radiated by the generator, and also the noise caused by turbulent air flow through some of the elements of the acoustic system of the generator. A selective method was used to single out from the complete signal produced by the generator the necessary components. On the basis of the measurements made it was found that the sound power of the higher harmonics produced by the generator is negligibly small compared to the sound power of the fundamental. For this reason in the following text of the paper the results of the measurement of the sound power of the generator are presented, since although they were obtained for the first harmonic, they can be regarded as representative of the total sound power contained in the discrete components of the wave produced by the generator. The measurements of the acoustic power of the generator were made using the measuring method and apparatus as given in [9].

6. The method for determining the supply power

The power used to supply the generator with compressed air is equal to the power necessary for the compression of the air mass taken per time unit by the generator, from the ambient pressure P_0 to the supply pressure $P_1 = P_0 + P_G$. The determination of its value requires the calculation of the compression work of compressing a unit mass of air in a polytropic process of a particular type, and of the mass air flow to the generator [7]

$$N_z = L_t M_0. \tag{1}$$

The work performed in compressing a unit mass of air is defined as the amount of work done in a comparable compressor equal to the technical work in an isentropic compression

$$L_t = \frac{\varkappa}{\varkappa - 1} RT_1 \left[1 - \left(\frac{P_0}{P_1}\right)^{(\varkappa-1)/\varkappa}\right], \tag{2}$$

where P_1 and T_1 are correspondingly the pressure and temperature of the air in the pressure chamber of the generator, P_0 is the atmospheric pressure, R is the gas constant for air, and $\varkappa$ is the isentropic exponent. The air temperature in the pressure chamber of the generator is measured by means of a thermocouple and compensator (expansion pipe joint), and the excess pressure is measured by a manometer.

The value of the mass air flow M_0 supplying the generator was determined by means of a measuring pipeline equipped with a quadrant orifice plate [5]. The diagram of the compressed air installation supplying individual systems of the generator is shown in Fig. 5. Air is forced by an electrically driven two-stage piston compressor S (output 0.08 m^3/s, at a delivery pressure of $8 \cdot 10^5$ N/m^2) to an equalizing tank Z with a volume of 5 m^3, whence, via the oil separator O, it is fed to

(I) the acoustic system of the generator via a pressure regulator R_G and a regulating valve Z_G which permit precise regulation of the pressure of the air in the pressure chamber and also via the measuring pipeline RP which permits determination of the value of the mass air flow supplying the generator,

(II) the microturbine via a pressure regulator R_T and a regulating valve Z_T which permit precise regulation of the angular velocity of the generator rotor, and thus also of the frequency of the wave produced,

(III) the air bearings via a stop valve Z_L.

The control of the supply parameters of the generator is performed by a set of manometers.

In the measurements performed, the maximum relative error, with which it was possible to determine the value of the supply power of the generator did not exceed 15%.

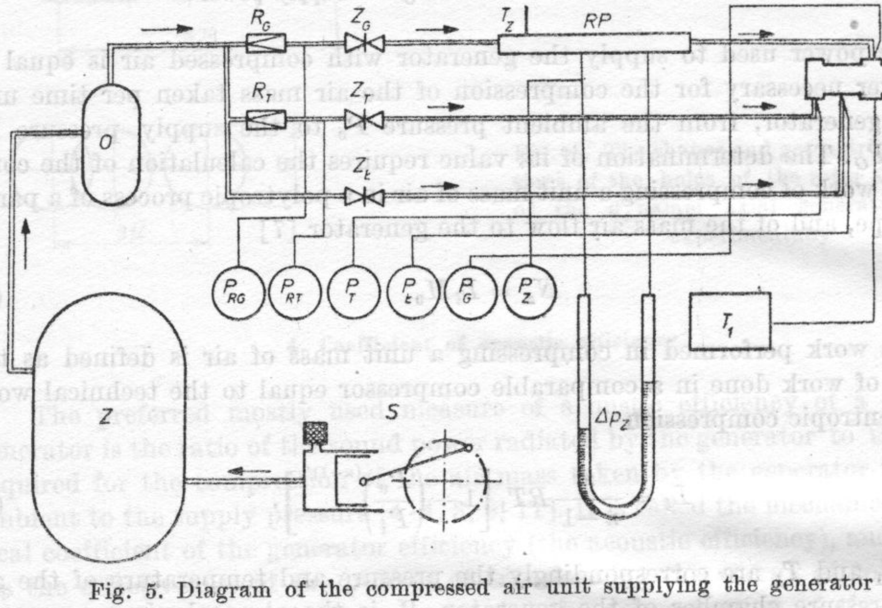


Fig. 5. Diagram of the compressed air unit supplying the generator

Thus, the maximum relative error, with which it was possible to determin the value of the acoustic efficiency of the generator did not exceed 43.1%.

7. Discussion of the results of the measurements and conclusions

It follows from the dependence of the sound power on the frequency of the generator wave presented (in Fig. 6) that, irrespective of the type of rotor with which the generator is actually operating, the sound power decreases above 4000 Hz, and also below about 2000 Hz. From the analysis of a theoretical model of the generator [9] it can be concluded that the decrease of sound power below 2000 Hz is related to the limiting frequency of the horn while that occuring above a frequency of 4000 Hz is caused by the resonance properties of the pressure chamber of the generator. The maximum sound power occurs at those frequencies for which the impedance of the horn inlet is matched to the wave impedance of the stator channel.

It follows from Fig. 7a that with increasing pressure in the pressure chamber of the generator, there occurs an increase in the sound power produced. At higher pressures an increasingly larger fraction of the energy of the compressed air is transformed into the sound power of the noise induced by turbulence in the acoustic system of the generator, which evidently must be realized at the expense of sound power in the harmonic components. This causes a decrease in the sound power of the generator for high values of the air pressure in its pressure chamber.

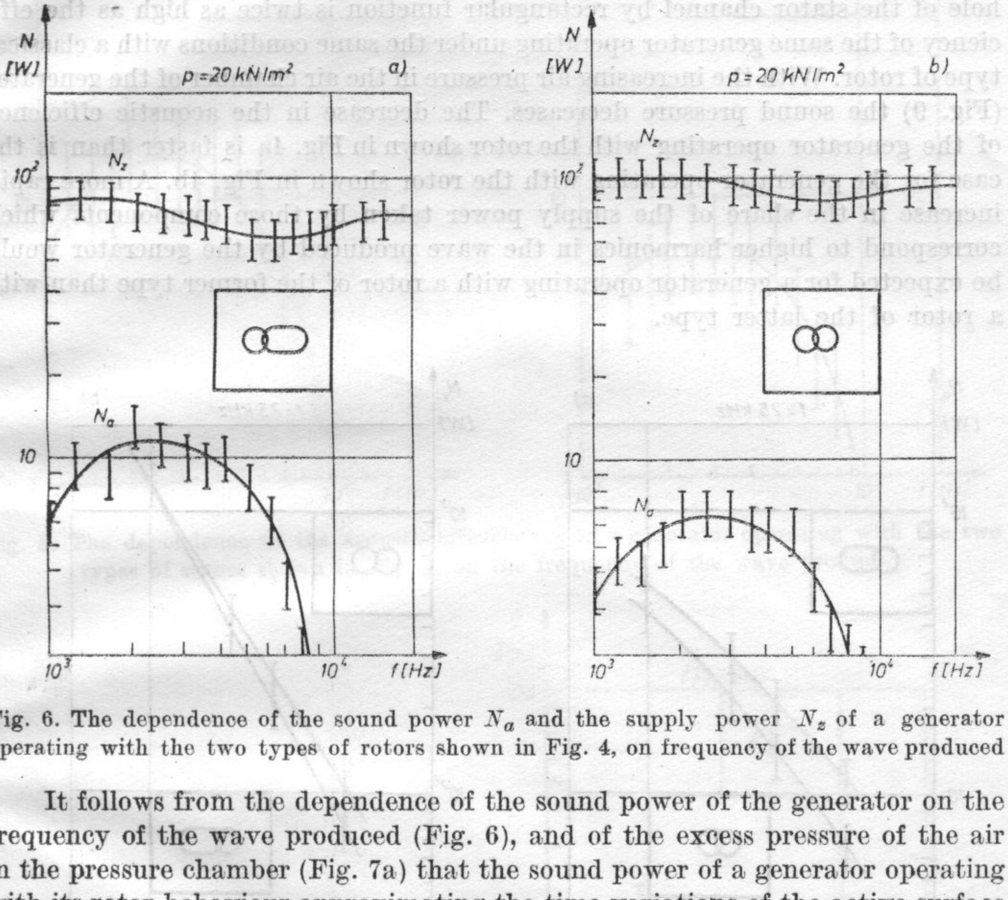


Fig. 6. The dependence of the sound power N_a and the supply power N_z of a generator operating with the two types of rotors shown in Fig. 4, on frequency of the wave produced

It follows from the dependence of the sound power of the generator on the frequency of the wave produced (Fig. 6), and of the excess pressure of the air in the pressure chamber (Fig. 7a) that the sound power of a generator operating with its rotor behaviour approximating the time variations of the active surface area of the inlet hole of the stator channel by a rectangular function (Fig. 4a), is twice as high as the power of a similar generator operating under the same conditions with a classical type of rotor (Fig. 4b).

The supply power of the generator (Fig. 6) shows a local minimum at a frequency of about 8000 Hz. It follows from an analysis of a theoretical model of the generator [9] that such minima occur at the resonance frequencies of the stator channel. The minimum observed in the experiment is for the first of these. With an increase in the air pressure in the pressure chamber of the generator (Fig. 7b), the supply power increased very rapidly. From the dependence of the supply power of the generator on the frequency of the wave produced (Fig. 6), and on the pressure of the air in its pressure chamber (Fig. 7b), it follows that the value of the supply power of a generator operating under given conditions is practically independent of the type of rotor.

From the considerations presented hitherto, and also from Fig. 8 it can be concluded that the acoustic efficiency of a generator operating with a rotor behaviour approximating time variations of the active surface area of the inlet hole of the stator channel by rectangular function is twice as high as the efficiency of the same generator operating under the same conditions with a classical type of rotor. With the increasing air pressure in the air chamber of the generator (Fig. 9) the sound pressure decreases. The decrease in the acoustic efficiency of the generator operating with the rotor shown in Fig. 4a is faster than is the case for the generator operating with the rotor shown in Fig. 4b. A more rapid increase in the share of the supply power taken by those components which correspond to higher harmonics in the wave produced by the generator would be expected for a generator operating with a rotor of the former type than with a rotor of the latter type.

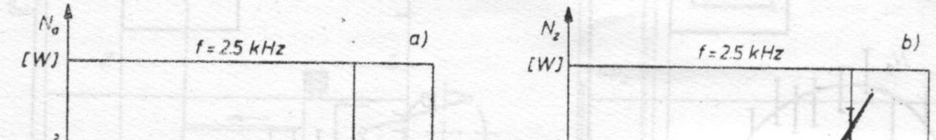


Fig. 7. The dependence of the sound power N_a of a generator operating with the two types of rotors shown in Fig. 4, on the excess air pressure P in its pressure chamber

The maximum value of the acoustic efficiency of a generator with an excess air pressure in its pressure chamber of $0.2 \cdot 10^5$ N/m², and operating with a rotor giving approximately a rectangular function of time variations of the active surface area of the inlet hole the stator channel was (18 ± 8) %. The sound power produced was (12 ± 3) W. Increasing the excess air pressure

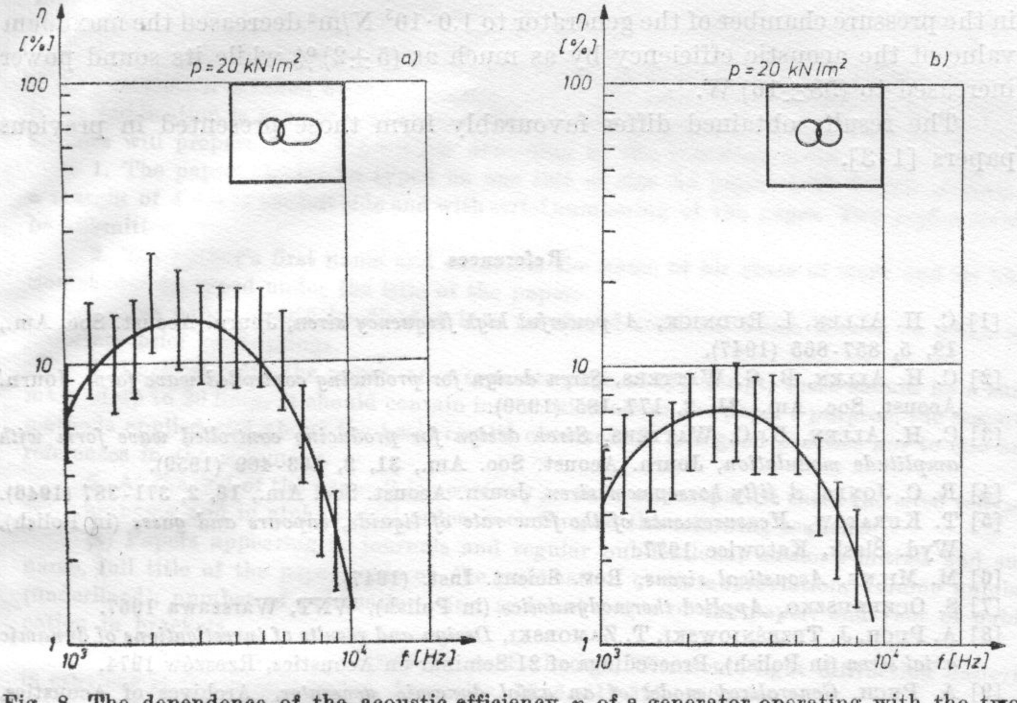


Fig. 8. The dependence of the acoustic efficiency η of a generator operating with the two types of rotors shown in Fig. 4, on the frequency of the wave produced

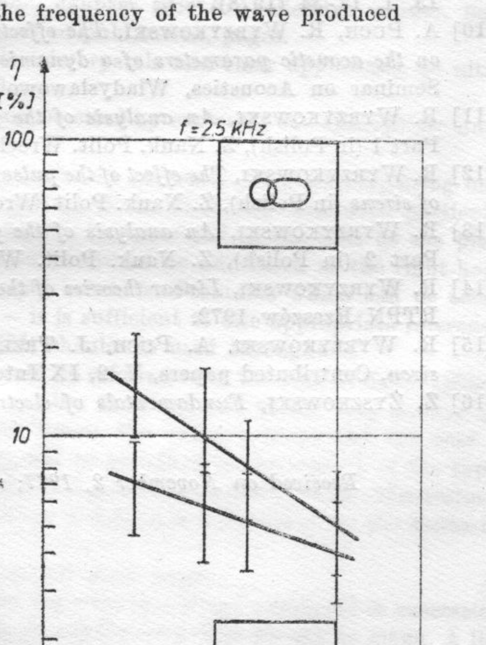


Fig. 9. The dependence of the acoustic efficiency η of a generator operating with the two types of rotors shown in Fig. 4, on the excess air pressure P in its pressure chamber

in the pressure chamber of the generator to $1.0 \cdot 10^5$ N/m² decreased the maximum value of the acoustic efficiency by as much as (5 ± 2) % while its sound power increased to (58 ± 16) W.

The results obtained differ favourably form those presented in previous papers [1-3].

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