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ULTRASONIC WAVE PROPAGATION ALONG THE SURFACE OF A ROD IMMERSED IN A LIQUID

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This paper describes the wave phenomena occurring in a needle used for puncture of body organs, under the simplifying assumption that this needle is an ideal elastic cylinder immersed in an ideal liquid.

Investigations carried out by the author using an echo method showed that the velocity of the wave propagating in the needle immersed in water is close to the velocity of the wave propagating in water.

The author has analysed the propagation of waves along a cylindrical rod of infinite length immersed in a liquid, solving the wave equations for displacement potentials in the rod surrounding liquid, taking into consideration the boundary conditions on the rod surface. It was found that it is possible for the velocity of the propagating wave to be lower than the velocity of the wave in water, with the wave being guided by the rod and the surrounding liquid layer. The characteristic equation obtained was solved numerically for a 1.5 mm diameter steel rod immersed in water at wave frequencies of 3 and 5 MHz. Stress distributions, acoustic pressure and the propagating wave displacements were determined. It can be concluded from the character of the wave that it is a surface wave.

The results obtained can be used as the first approximation to the problem of wave propagation along a needle in the case where the needle wall thickness and the frequency are adequately large.

Notation

a — rod radius
A, B, C — constants
c_0, c_g — phase velocity and group velocity of wave along the rod, respectively
C_L, C_T — velocities of longitudinal and transverse waves, respectively, in a solid medium
C_W — wave velocity in liquid
f — frequency
$H_n^{(2)}$ — Hankel function of the second kind of order n

J_n — Bessel function of the n-th order
I_n — modified Bessel function of the first kind of order n
k_0 — wave number, see formula (15a)
k_L, k_T, k_W — see formulae (22)-(24)
K_n — modified Bessel function of the second kind of order n
L, T, W — see formulae (39), (40) and (41)
p — acoustic pressure
r, θ, z — cylindrical coordinates
t — time
$\boldsymbol{u}$ — vector of displacement in the rod
u_r, u_z — components of vector $\boldsymbol{u}$
$\boldsymbol{U_W}$ — vector of displacement in liquid
u_{Wr}, u_{Wz} — components of vector $\boldsymbol{U_W}$
$\boldsymbol{W}$ — vector potential of displacement
W_θ — component of vector $\boldsymbol{W}$
λ, μ — Lamé constants
ϱ, ϱ_W — densities of rod and liquid, respectively
σ_{zz}, σ_{rr} — normal components of stress
τ_{rz} — tangential component of stress
φ, χ — scalar potentials of displacement in rod and liquid, respectively
ψ — see formula (9)
ω — angular frequency.

1. Introduction

The puncture of various body organs with a needle was recently introduced into ultrasonic medical diagnostics, with the direction and also sometimes the depth of puncture being ultrasonically controlled. The above method has been used in obstetrics in amniocentosis to investigate genetically conditioned foetus deformations, in phthisiology in punctures to remove fluids from pleura, in oncology in histopathological investigations of tissues suspected of cancer development, and for localization and puncture of blood vessels [1, 7, 6, 13]. In these cases ultrasonic waves of frequencies usually between 2 and 8 MHz are introduced along a steel needle of 2 mm diameter into the interior of the patient's body which from the viewpoint of its elastic properties is comparable to a liquid.

The puncture of the body is achieved with a needle whose hole is filled with a metal rod which is removed after the puncture has been made. The needle itself is in turn placed in a hole concentrically made in the centre of a disc-shaped piezoelectric transducer (Fig. 1). By means of the ultrasonic

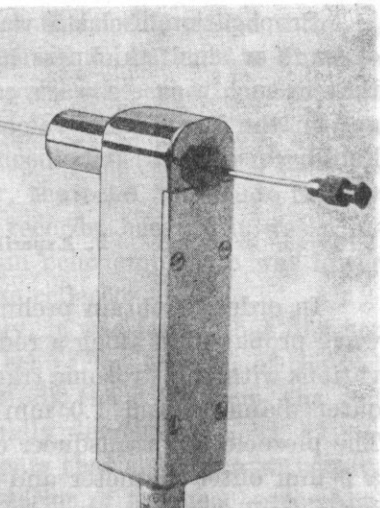

Fig. 1. An ultrasonic transducer for ultrasonically guided punctures [6]

beam radiated by the echoscope or ultrasonograph head the biological structure to be investigated is localized and the needle is subsequently entered along the ultrasonic beam into the patient's body, the needle being inserted through the hole in the piezoelectric transducer (Fig. 2).

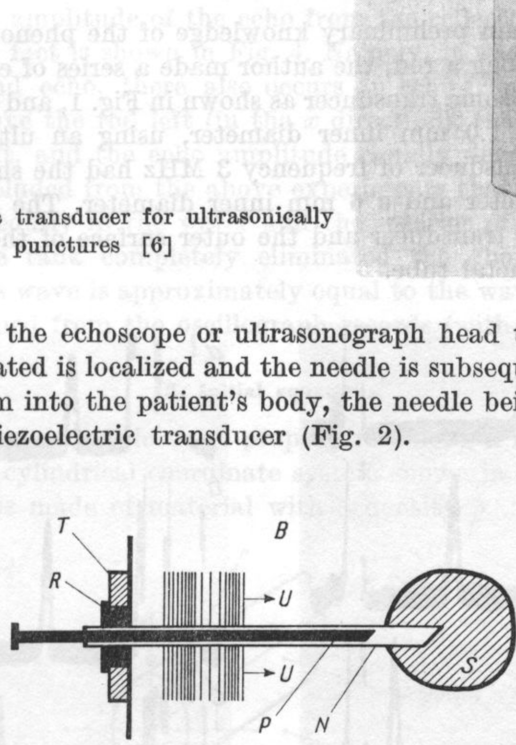

Fig. 2. The principle of a needle guided by an ultrasonic beam

B – a patient's body, S – the biological structure investigated, T – the piezoelectric transducer, R – tube, N – the needle, U – the ultrasonic wave, P – the rod filling the needle

The objective of this investigation is an explanation of the wave phenomena occurring in the needle used for puncture of the body organs, under the simplifying assumption that this needle is an ideal elastic cylinder immersed in an ideal liquid.

The problem of ultrasonic wave propagation along a rod immersed in a liquid was first examined by BJÖRNO and KUMAR [3]. In the present investigation the author uses a similar analysis, the different approach to the problem resulting from the particular conditions imposed by the puncture, which was the primary object of the author's consideration.

Propagation of elastic waves in a liquid along a flat layer of a solid medium of large or small thickness has been investigated by GRABOWSKA [5] showing that in such a case a wave can propagate simultaneously in the solid medium and in the liquid at a velocity slightly lower than the wave velocity in the liquid.

2. Experiments carried out using a needle

In order to obtain preliminary knowledge of the phenomena of ultrasonic wave propagation along a rod, the author made a series of experimental observations with an ultrasonic transducer as shown in Fig. 1, and a needle of 1.5 mm outer diameter and 1.0 mm inner diameter, using an ultrasonic echoscope. The piezoelectric transducer of frequency 3 MHz had the shape of a ring with a 9 mm outer diameter and a 6 mm inner diameter. The space between the inner surface of the transducer and the outer surface of the needle was filled with a removable metal tube.

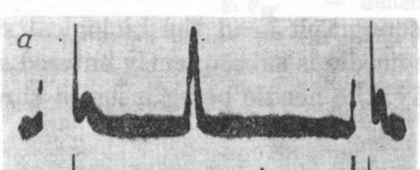

Fig. 3. Oscillographic records of ultrasonic pulses (a, b, c) and the measuring system (d)

T – a piezoelectric transducer, W – water, N – the needle, R – the rod filling the needle, P – a flat reflector of methyl metacrylate, x – direction of motion of the needle and the rod, as shown in (d)

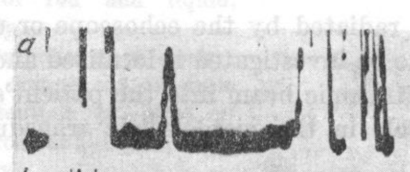

Fig. 4. Oscillographic records of ultrasonic pulses (a, b, c) and the measuring system (d)

The record (c) corresponds to the position of the needle and the rod, as shown in (d). Notation as in Fig. 3

Figure 3d shows the ultrasonic transducer with the needle pulled through, filled with a metal rod, and a flat reflector immersed in water at a distance of 5 cm from the transducer. The transmitted impulse, the echo from the flat reflector (methyl metracrylate) and the echo from the needle end are shown in the oscillograph records above. If we move the needle left (in the x direction), the echo from its end also moves left. However, the echo amplitude rapidly decreases. This is not visible on the oscillograph records, because it was compensated by amplification variation. The maximum echo emplitude was lower by 14 dB than the amplitude of the echo from the reflector.

An interesting fact is shown in Fig. 4. Namely, it was found that distinct from the needle end echo, there also occurs an echo from the end of the rod filling it. If we move the rod left (in the x direction), the echo from the rod end also moves left, and the echo amplitude rapidly decreases.

It can be concluded from the above experiments that there is a wave propagating simultaneously in the water and the interior of the needle; removal of water from the tank completely eliminated the above-mentioned wave. The velocity of the wave is approximately equal to the wave velocity in water, as can be determined from the oscillograph records (with a precision of 5 %).

3. Initial equations

Let us consider ultrasonic wave propagation along a rod of circular cross-section, as in the cylindrical coordinate system shown in Fig. 5. The rod with a diameter of $2a$ is made of material with a density ϱ, and the velocities of

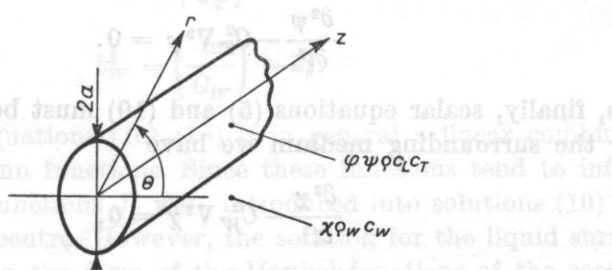

Fig. 5. The cylindrical coordinate system assumed in the analysis

longitudinal and transverse waves in the material are c_L and c_T, respectively. The rod is immersed in a liquid with a density ϱ_W, where the ultrasonic wave velocity is c_W.

The displacement vector in the rod is the sum

$$u = u_L + u_T, \tag{1}$$

where

$$\operatorname{curl} u_L = 0, \tag{2}$$

$$\operatorname{div} u_T = 0 \tag{3}$$

thus

$$u = \operatorname{grad}\varphi + \operatorname{curl} W, \tag{4}$$

potentials φ and $\overline{W}$ satisfying the scalar and vector wave equations:

$$\frac{\partial^2 \varphi}{\partial t^2} - C_L^2 \nabla^2 \varphi = 0, \tag{5}$$

$$\frac{\partial^2 W}{\partial t^2} - C_T^2 \nabla^2 W = 0. \tag{6}$$

Because of rotational symmetry only the component W_θ of the vector potential is different from zero while $W_r = W_z = 0$. Thus, expanding (4), we can write

$$u_r = \frac{\partial \varphi}{\partial r} - \frac{\partial W_\theta}{\partial z} \tag{7}$$

$$u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial (r W_\theta)}{r \partial r}. \tag{8}$$

Equation (6) can be reduced to a scalar equation, introducing [10] a new scalar quantity ψ from the relation

$$W_\theta = -\frac{\partial \psi}{\partial r}, \tag{9}$$

where the quantity ψ satisfies the wave equation

$$\frac{\partial^2 \psi}{\partial t^2} - C_T^2 \nabla^2 \psi = 0. \tag{10}$$

Thus, finally, scalar equations (5) and (10) must be satisfied for the rod, while for the surrounding medium we have

$$\frac{\partial^2 \chi}{\partial t^2} - C_W^2 \nabla^2 \chi = 0, \tag{11}$$

where χ is the scalar potential of displacement u_W in the liquid in accordance with the relation

$$U_W = \operatorname{grad}\chi. \tag{12}$$

The solutions of equations (5), (10) and (11) can be assumed in the form of waves travelling in the direction z,

$$\varphi = \varphi_0(r)\, e^{j(\omega t - k_0 z)}, \tag{13}$$

$$\psi = \psi_0(r)\, e^{j(\omega t - k_0 z)}, \tag{14}$$

$$\chi = \chi_0(r)\, e^{j(\omega t - k_0 z)}, \tag{15}$$

where $k_0 = \omega / c_0$, $\omega = 2\pi f$, f — frequency, and c_0 — phase velocity of a wave travelling along the z-axis.

Substituting (13), (14) and (15) into the wave equations (5), (10) and (11) we obtain the Bessel equations of the zero order:

$$\frac{\partial^2\varphi_0(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_0(r)}{\partial r}+\left[\left(\frac{\omega}{C_L}\right)^2-k_0^2\right]\varphi_0(r)=0, \tag{16}$$

$$\frac{\partial^2\psi_0(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_0(r)}{\partial r}+\left[\left(\frac{\omega}{C_T}\right)^2-k_0^2\right]\psi_0(r)=0, \tag{17}$$

$$\frac{\partial^2\chi_0(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\chi_0(r)}{\partial r}+\left[\left(\frac{\omega}{C_W}\right)^2-k_0^2\right]\chi_0(r)=0. \tag{18}$$

The solutions of these equations can be given [11] in the form

$$\varphi = AJ_0(k_L r)\,e^{j(\omega t-k_0 z)}, \tag{19}$$

$$\psi = CJ_0(k_T r)\,e^{j(\omega t-k_0 z)}, \tag{20}$$

$$\chi = BH_0^{(2)}(k_W r)\,e^{j(\omega t-k_0 z)}, \tag{21}$$

where J_0 is the Bessel function of zero order, $H_0^{(2)}$ — the Hankel function of the second kind of zero order, and

$$k_L^2=\left(\frac{\omega}{C_L}\right)^2-k_0^2, \tag{22}$$

$$k_T^2=\left(\frac{\omega}{C_T}\right)^2-k_0^2, \tag{23}$$

$$k_W^2=\left(\frac{\omega}{C_W}\right)^2-k_0^2. \tag{24}$$

The solution of equations (16)-(18) is in general a linear combination of the Bessel and Neumann functions. Since these functions tend to infinity for $r\to 0$, only the Bessel functions J_0 were introduced into solutions (19) and (20) which include the rod centre. However, the solution for the liquid surrounding the rod was assumed in the form of the Hankel functions of the second kind which represents a wave travelling with an increasing value of r.

4. Boundary conditions

On the rod surface ($r = a$) boundary conditions in the form of equality of normal stresses in the rod σ_{rr} and of acoustic pressure p (in the liquid) must be satisfied, while tangential stress τ_{rz} on the rod surface must be equal to zero. The first condition takes [9] the form

$$\sigma_{rr}=\lambda\left[\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}\right]+2\mu\frac{\partial u_r}{\partial r}=-p \qquad \text{for } r=a \tag{25}$$

where

$$p = -\varrho_W \frac{\partial^2 \chi}{\partial t^2}, \tag{26}$$

and λ and μ are the Lamé constants. The second derivative, instead of the first one, appears in formula (26) because, in accordance with formula (12), the function is the potential of displacement, and not that of velocity, as is usually assumed in acoustics.

The second condition for disappearance of tangent stresses takes [9] the form

$$\tau_{rz} = \mu\left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right] = 0 \quad \text{for } r = a. \tag{27}$$

Subsequently we shall introduce the third boundary condition in the form of equality of the radial displacements in the rod (u_r) and in the liquid (u_{wr}) on the boundaries of the rod and the liquid:

$$u_r = u_{wr} \quad \text{for } r = a. \tag{28}$$

The components of displacements in the rod and displacements in the liquid can be expressed by means of functions of φ, ψ, χ, on the basis of relations (7)-(9) and (12):

$$u_r = \frac{\partial \varphi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z}, \tag{29}$$

$$u_z = \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{r \partial r} - \frac{\partial^2 \psi}{\partial r^2}, \tag{30}$$

$$u_{wr} = \frac{\partial \chi}{\partial r}, \quad u_{wz} = \frac{\partial \chi}{\partial z}. \tag{31}$$

Substituting formulae (29) and (31) into (25), (27) and (28) we obtain the three ultimate boundary conditions

$$A\left[\frac{1}{2}(k_0^2 - k_T^2) J_0(k_L a) + \frac{k_T}{a} J_1(k_L a)\right] + Cjk_0 k_T [k_T J_0(k_T a) - \frac{1}{a} J_1(k_T a)] +$$

$$+ B \frac{\omega^2 \varrho_W}{2\varrho C_T^2} H_0^{(2)}(k_W a) = 0, \tag{32}$$

$$A[2jk_L k_0 J_1(k_L a)] + C[(k_0^2 k_T - k_T^3) J_1(k_T a)] = 0, \tag{33}$$

$$-A[k_L J_1(k_L a)] + C[jk_0 k_T J_1(k_T a)] + Bk_W H_1^{(2)}(k_W a) = 0, \tag{34}$$

where J_1 and $H_1^{(2)}$ are the Bessel and Hankel functions of the first order, respectively.

5. The characteristic equation

Eliminating quantities A, B and C from equations (32)-(34), after numerous transformations we obtain the characteristic equation for the problem under consideration:

$$\frac{\varrho_W}{4\varrho}\left(\frac{\omega}{C_T}\right)^4 \frac{1}{k_W}\frac{H_0^{(2)}(k_W a)}{H_1^{(2)}(k_W a)}$$

$$= k_0^2 k_T \frac{J_0(k_T a)}{J_1(k_T a)} - \frac{1}{2a}\left(\frac{\omega}{C_T}\right)^2 + \left[\frac{1}{2}\left(\frac{\omega}{C_T}\right)^2 - k_0^2\right]^2 \frac{1}{k_L}\frac{J_0(k_L a)}{J_1(k_L a)}. \tag{35}$$

In the case where a vacuum surrounds the rod instead of a liquid we have $\varrho_W = 0$ and the left-hand side of equation (35) disappears. Thus we obtain the characteristic equation for a cylinder, as given by REDWOOD [10] after POCHHAMMER and CHREE.

When solving the characteristic equation, only real constants of propagation k_0 will be considered. In the general case, a finite number of imaginary propagation constants, satisfying the equation, occurs for each given frequency. They are analogous to propagation constants occurring in acoustic waveguides below the cut-off frequency, when the vibration modes investigated do not propagate, being completely attenuated ($Re(k_0 a) = 0$, $Im(k_0 a) \neq 0$). We can also neglect the case of complex propagation constants k_0, since the imaginary component of these constants represents spatial wave attenuation. The waves of this type will in practice decrease their amplitude until totally attenuated [14].

We investigate the case where the ultrasonic wave is guided along the rod and is not attenuated in the rod, nor re-radiated by the rod to the liquid.

The function $H_0^{(2)}$, occurring in the assumed solution (21), corresponds to the radiation of wave by the rod to the liquid along the axis r. This follows from the asymptotic value of this function which for high values of r takes [11] the form

$$H_0^2(x) \cong \sqrt{\frac{2}{\pi x}}\, e^{-j(x-\pi/4)}\left[1 - \frac{1}{j\cdot 8x} + \dots\right]. \tag{36}$$

However, in the case where the phase velocity c_0 of wave is lower than the velocity c_W, in accordance with (24) the wave number k_W becomes an imaginary quantity and the Hankel function passes into a modified Bessel function of the second kind $K_n(x)$, in accordance [2, 8] with the relation

$$K_n(x) \cong \frac{1}{2}\pi\cdot j^{-n-1} H_n^2(-jx). \tag{37}$$

Of all cylindrical functions only this function decreases monotonically to zero for the argument tending to infinity, and this occurs only when the

Hankel function argument is a negative imaginary quantity [8], because [2] for the modified Bessel function $K_n(x)$ we have

$$K_n(x) \cong \sqrt{\frac{\pi}{2x}}\, e^{-x}\left[1+\frac{\mu-1}{8x}+\ldots\right], \tag{38}$$

where $\mu = 4n^2$ for $x \gg n$.

When $c_0 < c_W$, the wave numbers k_W, k_L, k_T are imaginary:

$$k_W = \pm j\frac{\omega}{C_W}\sqrt{\frac{C_W^2}{c_0^2}-1} = \pm jW, \tag{39}$$

$$k_L = \pm j\frac{\omega}{C_L}\sqrt{\frac{C_L^2}{c_0^2}-1} = \pm jL, \tag{40}$$

$$k_T = \pm j\frac{\omega}{C_T}\sqrt{\frac{C_T^2}{c_0^2}-1} = \pm jT. \tag{41}$$

Assuming, from the above discussion, a negative value of the imaginary wave number k_W (see (39)), we obtain from (37)

$$\frac{H_0^{(2)}(-jWa)}{H_1^{(2)}(-jWa)} = -j\frac{K_0(Wa)}{K_1(Wa)}. \tag{42}$$

Similarly for imaginary arguments: the Bessel functions of the first kind pass into the modified Bessel functions of the first kind, in accordance [12] with the relation

$$I_n(x) = j^{-n}J_n(jx). \tag{43}$$

It should be noted [12] that

$$J_0(jx) = J_0(-jx) \tag{44}$$

and

$$J_1(jx) = -J_1(-jx). \tag{45}$$

Taking into consideration relations (43)-(45) and also (42), we obtain for both positive and negative imaginary wave numbers k_L, k_T (see (40) and (41)) a new form of the characteristic equation (35):

$$\frac{\varrho_W}{4\varrho}\left(\frac{\omega}{C_T}\right)^4\frac{1}{W}\frac{K_0(Wa)}{K_1(Wa)}$$

$$= k_0^2T\frac{I_0(Ta)}{I_1(Ta)} - \frac{1}{2a}\left(\frac{\omega}{C_T}\right)^2 - \left[\frac{1}{2}\left(\frac{\omega}{C_T}\right)^2 - k_0^2\right]^2\frac{1}{L}\frac{I_0(La)}{I_1(La)}. \tag{46}$$

Such an involved form of this equation makes its solution very difficult, because the quantities T, L and W depend on k_0 through relations (39)-(41) and (22)-(24). Therefore equation (46) was solved taking into consideration the fact experimentally determined in Section 2 that the velocity of the wave discussed propagating along the rod is very close to that of wave in water. In such a case only the value of W depends essentially on the value of c_0, while the value of c_0 has little influence on the values of L, T and k_0. Thus, assuming $c_0 = c_W$ in formulae (40), (41) and (15a), we can solve the characteristic equation (46) for the quantity W.

Under such conditions, equation (46) was numerically solved assuming a frequency $f = 3$ MHz, a rod diameter equal to $2a = 1.5$ mm, a rod made of steel in which longitudinal and transverse waves velocities are equal to $c_L = 5.9$ km/s and $c_T = 3.23$ km/s, respectively, and the wave velocity in water equal to 1.48 km/s. Equation (46) is satisfied for wave numbers equal to $k_W = 0.18\ \text{cm}^{-1}$, $k_T = \pm j\cdot 113\ \text{cm}^{-1}$, $k_L = \pm j\cdot 123\ \text{cm}^{-1}$. Having the wave number k_W, the phase velocity of wave travelling along the rod was determined. Being equal to $c_0 = 1.479998$ km/s, it is thus only slightly lower than the wave velocity in water without a rod. Other possible solutions of the characteristic equation were not examined.

The group velocity c_g of the wave propagating along the rod is equal to the phase velocity c_0,

$$c_g = \frac{d\omega}{dk_0} = \frac{d\omega}{d(\omega/c_0)} \cong c_0, \tag{46a}$$

in the range of the considered frequencies of 3-5 MHz, because the phase velocity c_0 is in this case practically constant. This can be concluded from the numerical computations carried out, since in this frequency range its value changes only at the fifth decimal place.

6. Wave distribution inside and outside the rod

The distribution of displacements in the rod and the surrounding liquid occurring for the wave type considered appears to be of interest. Taking into consideration relations (37), (43) and (44), solutions (19)-(21) take the following form:

$$\varphi = AI_0(Lr)e^{j(\omega t - k_0 z)}, \tag{47}$$

$$\psi = CI_0(Tr)e^{j(\omega t - k_0 z)}, \tag{48}$$

$$\chi = B\frac{2}{\pi}jK_0(Wr)e^{j(\omega t - k_0 z)}. \tag{49}$$

From relation (33) we find the ratio A/C,

$$\frac{A}{C} = \frac{k_T(k_T^2 - k_0^2)}{2jk_L k_0}\,\frac{J_1(k_T a)}{J_1(k_L a)}, \tag{50}$$

which for imaginary values of wave numbers, in accordance with (40), (41) and (43)-(45) takes the following form:

$$\frac{A}{C} = \pm j\,\frac{T(T^2 + k_0^2)}{2Lk_0}\,\frac{I_1(Ta)}{I_1(La)}. \tag{51}$$

It can be readily checked that condition (33), from which the ratio A/C was determined, is satisfied only for the positive sign on the right-hand side of formula (51). Therefore positive and negative values should be at the same time assumed for the imaginary wave numbers (40) and (41).

Similarly, from relation (34) we obtain a value for the ratio B/C equal to

$$\frac{B}{C} = -j\frac{k_T(k_T^2-k_0^2)}{2k_0 k_W}\frac{J_1(k_T a)}{H_1^{(2)}(k_W a)}, \tag{52}$$

which, after taking into consideration (37), (40), (41), (43)-(45), takes the form

$$\frac{B}{C} = \frac{\pi T(k_0^2-T^2)}{4k_0 W}\frac{I_1(Ta)}{K_1(Wa)}. \tag{53}$$

The value of (53) does not depend on the sign of the imaginary wave number $k_T = \pm jT$. Earlier we assumed a negative sign for the wave number $k_W = \pm jW$. Substituting the values of the potential (47) and (48) into formulae (29) and (30), and taking into consideration the relation

$$\frac{dI_n(x)}{dx} = I_{n-1}(x) - \frac{n}{x}I_n(x); \quad I_0'(x) = I_1(x), \tag{54}$$

we obtain displacements related to a constant coefficient C:

$$\frac{u_r}{C} = \left[\frac{A}{C}LI_1(Lr) - jk_0 TJ_1(Tr)\right]e^{j(\omega t-k_0 z)}, \tag{55}$$

$$\frac{u_z}{C} = \left[-jk_0\frac{A}{C}I_0(Lr) - T^2 I_0(Tr)\right]e^{j(\omega t-k_0 z)}. \tag{56}$$

The stress σ_{rr} in the rod will be determined on the basis of formula (25). Taking into consideration the values obtained for dispacements u_r and u_z, we finally obtain

$$\frac{\sigma_{rr}}{C} = \frac{A\varrho}{C}\left\{\left\{[L^2C_L^2 - k_0^2(C_L^2-2C_T^2)]I_0(Lr) - 2C_T^2 L\frac{I_1(Lr)}{r}\right\} + \right.$$
$$\left. + j\cdot 2k_0 C_T^2\varrho T\left\{\frac{I_1(Tr)}{r} - TI_0(Tr)\right\}\right\}e^{j(\omega t-k_0 z)}. \tag{57}$$

The stresses σ_{zz} in the rod will be determined [9] from formula

$$\sigma_{zz} = \lambda\left[\frac{u_r}{r} - \frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial z}\right] + 2\mu\frac{\partial u_z}{\partial z} \tag{58}$$

whence, taking into account (55), (56) and the relations

$$\lambda = \varrho(C_L^2 - 2C_T^2), \tag{59}$$

$$\mu = \varrho C_T^2, \tag{60}$$

we finally get

$$\frac{\sigma_{zz}}{C} = \frac{A}{C}\varrho\{[L^2(C_L^2 - C_T^2) - k_0^2 C_L^2]I_0(Lr) + jk_0 T^2 \varrho C_T^2 I_0(Tr)\} e^{j(\omega t - k_0 z)}. \qquad (61)$$

The tangential stresses τ_{rz} will be determined on the basis of relation (27). Taking into consideration (55), (56) and (60), we obtain

$$\frac{\tau_{rz}}{C} = \varrho C_T^2\left[-2jk_0\frac{A}{C}LI_1(Lr) - T(k_0^2 + T^2)I_1(Tr)\right\} e^{j(\omega t - k_0 z)}. \qquad (62)$$

The value of the acoustic pressure in the liquid is determined from formulae (26) and (49). Thus we have

$$\frac{p}{C} = j\frac{B}{C}\varrho_W\omega^2\frac{2}{\pi}K_0(Wr)e^{j(\omega t - k_0 z)}. \qquad (63)$$

The values of displacements and stresses in the rod and the acoustic pressure in the liquid, calculated on the basis of the above formulae for the case investigated, are shown in Figs. 6 and 7, while comparable values, calculated for a frequency of 5 MHz, are shown in Figs. 8 and 9.

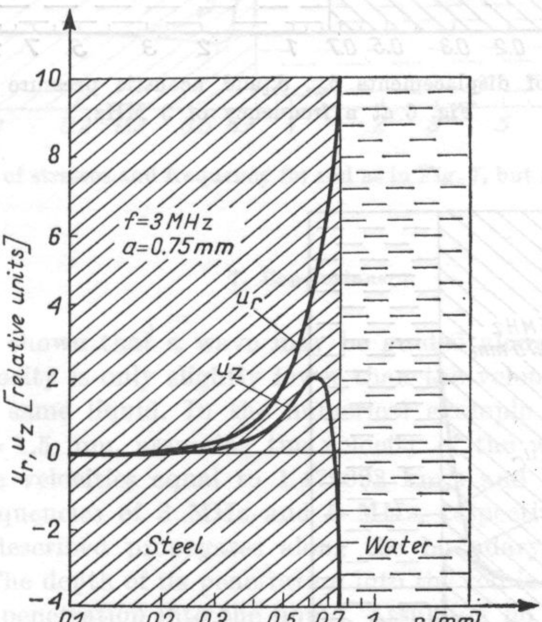

Fig. 6. Distribution of displacements (u_r), (u_z) in the rod with a radius a = 0.75 mm at a frequency of 3 MHz

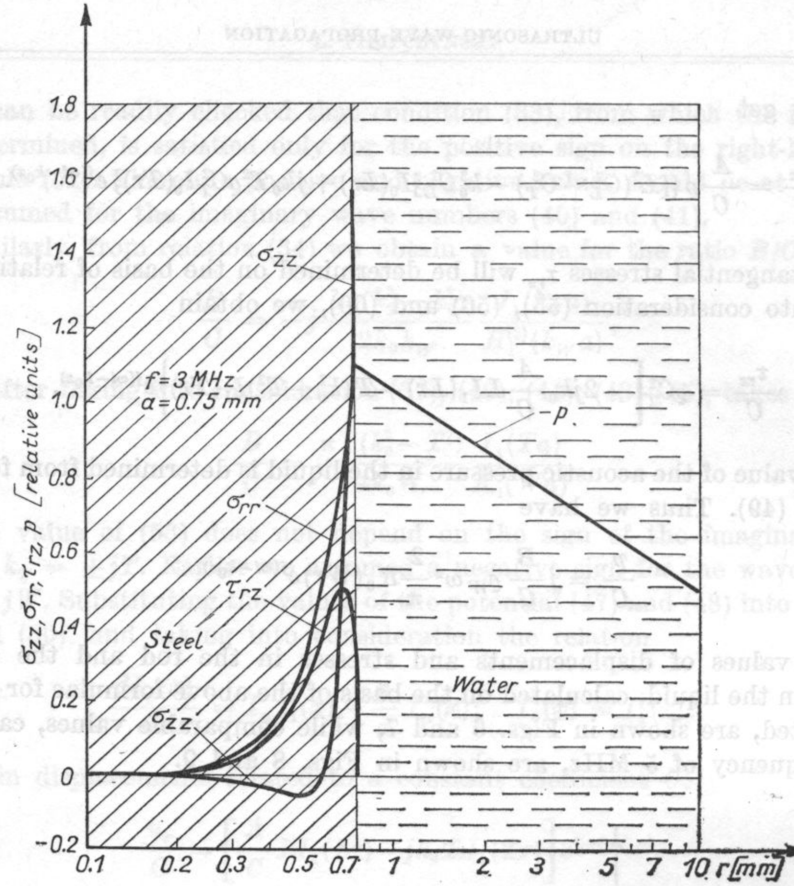

Fig. 7 Distribution of displacements δ_{zz}, $\delta_{rz}$and acoustic pressure p for the rod as in Fig. 6 at a frequency of 3 MHz.

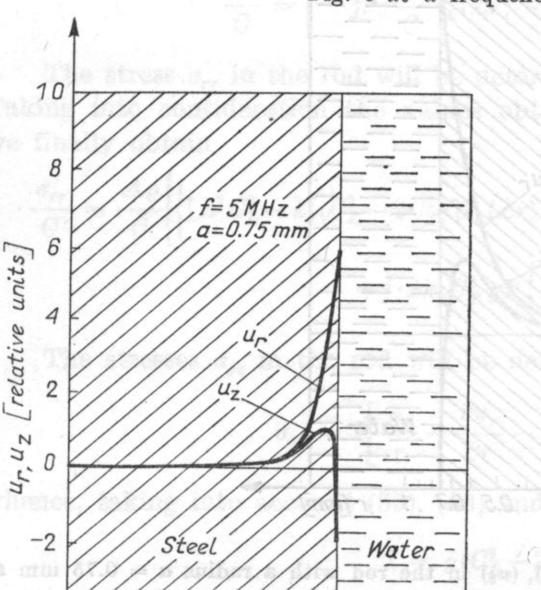

Fig. 8. Distribution of displacement in the rod as in Fig. 6, but at a frequency of 5 MHz

Distributions of acoustic pressure and of stress and displacement components, as in Figs. 6–9, show that the wave penetration depth increases with increasing frequency. It follows from the expressions of the wave that it is of surface type.

The results obtained can be used for a comparison of the wave propagation along a needle in the case where the needle length and the wave frequency are adequately large [4]. In such a case stresses disappear inside the rod, and thus in a spontaneous manner an additional boundary condition of the disappearance of the stresses on the inner needle surface can be satisfied. In general, however, such an approximation may prove unsatisfactory and then the problem of wave propagation along a needle will require a separate analysis.

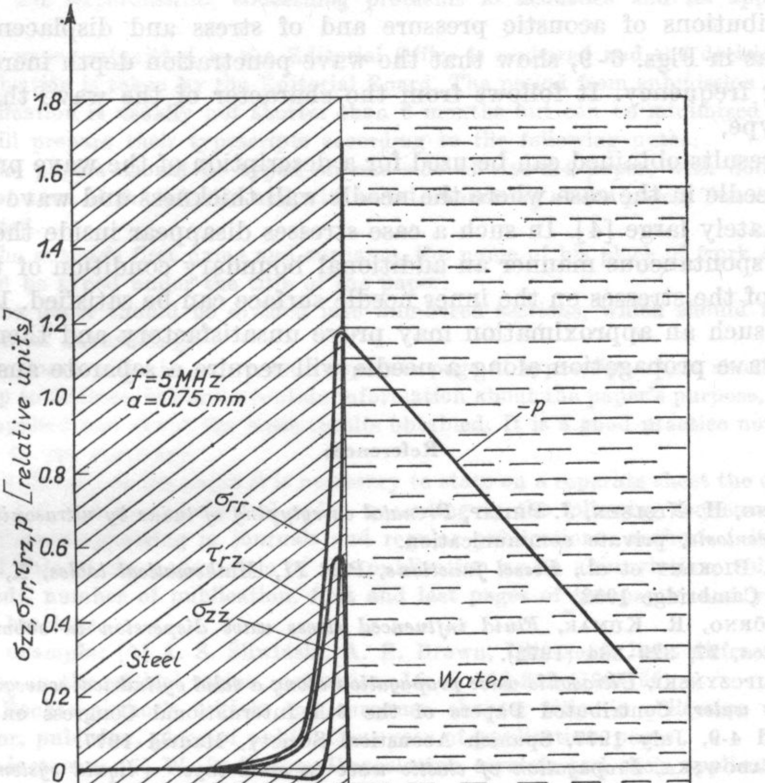

Fig. 9. Distribution of stresses and frequency for rod as in Fig. 7, but at a frequency of 5 MHz

7. Conclusions

It has been shown that a wave may be guided along a rod immersed in liquid whose velocity is only slightly lower than the velocity of the wave propagating in the same liquid. In the numerical example of a steel rod with a diameter $2a = 1.5$ mm, assuming the velocity of the wave in liquid to be 1.48 km/s, wave velocities equal to 1.479998 km/s and 1.479980 km/s were obtained for frequencies of 3 MHz and 5 MHz, respectively.

The wave described propagates along the boundary surface of the rod and the liquid. The depth of its penetration into the rod is considerably smaller than that of its penetration into the liquid. Assuming no propagation loss for both media, the wave propagates without attenuation; the wave number k_0 is real, because this wave is not radiated into the liquid perpendicular the rod axis.

Distributions of acoustic pressure and of stress and displacement components, as in Figs. 6-9, show that the wave penetration depth increases with increasing frequency. It follows from the character of the wave that it is of surface type.

The results obtained can be used for a description of the wave propagation along a needle in the case where the needle wall thickness and wave frequency are adequately large [4]. In such a case stresses disappear inside the rod, and thus in a spontaneous manner an additional boundary condition of the disappearance of the stresses on the inner needle surface can be satisfied. In general, however, such an approximation may prove unsatisfactory and then the problem of wave propagation along a needle will require a separate analysis.

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