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A STUDY OF THE UNDERWATER ACOUSTIC DISTURBANCES PRODUCED BY A SHIP PROPELLER

EUGENIUSZ KOZACZKA

Naval College, Gdynia

The paper discusses briefly the problem of the generation of underwater acoustic disturbances by hydrodynamic sources (e. g. a ship propeller).

The results of the investigations carried out in a hydrodynamic channel are presented. They refer to the following characteristics: the amplitude spectrum of the disturbances, the propeller performance (characteristic), and the acoustic pressure level as a function of the propeller speed at given velocities of movement relative to the undisturbed water. The results of investigations performed in an anechoic basin are described . They are illustrated by the amplitude spectra of the acoustic disturbances recorded using a narrow-band analyser with a bandwidth of 3.16 Hz. In addition, the level of the acoustic underwater disturbances as a function of the propeller rotational speed is discussed.

Having in mind certain limitations imposed upon the experiments performed in a hydrodynamic channel and an anechoic basin the measurements of acoustic underwater disturbances at the marine conditions were carried. The electrically driven ship propeller was used. The results discussed in this paper concern a propeller operating in the presence of cavitation.

In addition to the results of these investigations a method is also presented for studying the acoustic effects produced by a ship propeller working under various conditions.

Notation

A/A_0 – area coefficient,
A – area of the ship propeller blades [m^2],
A_0 – area of the propeller disc [m^2],
c – sound velocity [m/s],
D – ship propeller diameter [m],
H/D – propeller pitch factor,
F_1 – i-th component of the volume force density,
H – propeller pitch,
f – frequency [Hz],
i – index $i = 1, 2, 3$,
J – effective pitch propeller advance coefficient,
j – index $j = 1, 2, 3$,
J_{mz} – Bessel function of the first kind order mz,

K_m — propeller torque coefficient,
K_T — propeller thrust coefficient,
M — torque applied to the propeller [Nm],
m — order of harmonic,
n — propeller speed r. p.s.,
η — ship propeller efficiency,
p — sound pressure,
p_m — sound pressure of the m-th harmonic [N/m^2],
m — order of harmonic of the sound pressure,
Q — volume rate of the mass source [kg/m$^3 \cdot$ s],
R_0 — radius of the ship propeller [m],
R_e — effective radius of the ship propeller [m],
ϱ — density of the medium [kg/m^3],
ω — angular velocity i/s,
T — thrust of the ship propeller,
T_{ij} — tensor of stress [kg/m $\cdot$ m^2],
t — time [s],
v — velocity of the stream flowing on to the propeller [m/s],
Z — number of blades of the ship propeller,
X_i — coordinates,
V — volume of a propeller blade.

The generation of acoustic underwater disturbances by a ship propeller received little attention in the literature on underwater acoustics. The problem is complicated and has not as yet been comprehensively described either theoretically or experimentally.

The papers on the generation of acoustic disturbances by systems of the "propeller type" are well known, but mostly refer to a gaseous medium.

Among the more interesting papers in this field the publications by Gutin [1, 2], Lighthill [3, 4], Garrick and Watkins [5, 6] and other authors [7, 8, 9, 14, 15] should be mentioned. As regards experimental work, there is virtually no publication on the generation of acoustic disturbances by a ship propeller. The paper by Aleksandrov [10] deals only in a fragmentary form with this problem.

The process of the generation of underwater acoustic disturbances can be divided into two ranges. The first range is concerned with propeller operation without cavitation, while the other range is concerned with propeller operation in the presence of cavitation.

The mechanics of the generation of acoustic disturbances in the first range is related to the hydrodynamic effect of a propeller upon the ambient medium. The acoustic radiation is related, inter alia, to such phenomenon as the action of the propeller blades on the medium, which generates the thrust force. The effect of the generation of the thrust force and the periodic changes in the volume (section 1) define the value of the acoustic pressure produced by the propeller.

It should be stressed that even in the absence of cavitation, the propeller operation produces phenomena which are very complex hydromechanically.

A precise analysis of the flow around the propeller has not yet been made even for an incompressible fluid model. The formation of swelling and detachment areas, the formation of vortices and vortex lines make the interpretation of the phenomena observed during propeller operation rather difficult. In addition, the mutual effect of the propeller blades (the blade cascade effect) also contributes to the complexity of the phenomena. It is for these reasons that it is not easy to give an exact mathematical description of the acoustic phenomena encountered during propeller operation. Under actual conditions the ship propeller is working in non-uniform fluid velocity field and this, in the author's opinion has a fundamental influence on the magnitude of the acoustic effects. It is worth to mention that even for a uniform field velocity the theoretical description is rather hard to formulate.

Thus an experimental study of the acoustic effects related to ship propeller operation is at present more attractive.

Cavitation on a propeller constitutes an additional source of underwater acoustic disturbances which, by their nature, differ from those which exist in the sub-cavitation range.

The growth and collapse of air or gas bubbles produce acoustic effects. The bubbles, when distributed within the area of propeller operation, can be considered as point sources of shock waves. Furthermore, the formation of air bubbles brings about a rapid growth of non-uniformity in the velocity field, and also in the medium (two-phase medium). This makes the description of the phenomenon investigated even more difficult.

Sometimes there is also a strong hydroelastic effect which induces strong torsional vibrations of the propeller blade. These vibrations are the source of the acoustic disturbances commonly referred to "propeller singing". This phenomenon occurs only for some propellers over certain ranges of speed. This paper presents the results of investigations of propeller operation in the subcavitation range. They constitute a particular part of the study of the underwater acoustic effects associated with the operation of ship propellers.

1. The mathematical and physical description of the generation of underwater acoustic disturbances by ship propeller

A mathematical description of a ship propeller operating under normal conditions has not so far been elaborated. The mathematical model of an aircraft propeller operating in a uniform velocity field developed by GUTIN [1] can be used only for the qualitative description of the operation of a ship propeller. Although the problem considered in this paper would appear similar, the results obtained by calculating the sound pressure are several orders of magnitude smaller than the values obtained experimentally [9]. Since no formulae have so far been derived which would permit a clear and simple interpretation of the phenomena, the results obtained by Gutin are presented below to explain some of the mechanics of sound generation by a ship propeller.

The problem of the radiation of sound by a hydrodynamic source is described by a non-uniform partial differential equation of hyperbolic type (sometimes referred to as the Lighthill equation) which is of the form:

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\partial Q}{\partial t} + \frac{\partial F_i}{\partial x_i} - \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where p is the sound pressure, Q is the volume rate of the mass source, F_i is the component of the body force acting in the i-th direction, T_{ij} is the tensor of the stress, and c is the sound velocity.

The third term on the right-hand side of Eq. (1) describes sound generation by turbulent flow and will not be extensively considered here, since such effects are not perceptible in the investigations carried out.

However, the first term characterizes the time changes of the mass velocity in the space of the action of the propeller blades upon the surrounding medium. It is related to the periodical displacement, by the propeller blades, of a certain volume of fluid which in turn fills the space left by the blades. The resulting radiation is monopolar in its character (this means that a source of such a type can be replaced with a source of zero order). Such disturbances generated by a screw propeller are sometimes called the "volumetric noise" of the screw [16].

The second term of equation (1) is related to the change of body forces accompanying propeller operation. The body forces occurring in the field of a ship propeller are primarily the thrust force and the force from the propeller torque. During rotation there is a periodical change of the body forces in space. This type of generation results in dipole radiation and is called the „propeller force noise" [16].

Since equation (1) is too general to be useful in studying a relationship between the sound pressure and the quantities describing an acoustic field source, we shall use the relationship derived by GUTIN [2, 9] for a clearer interpretation of the propeller action. This relation defines, for a given observation point, the value of the sound pressure associated with the parameters of propeller operation:

$$|p_m| = \left\{\frac{m\omega z}{2\pi cR}\left| -T\frac{x}{R} + M\frac{c}{\omega R_e^2}\right| + \frac{\dot{\varrho}(m\omega z)^2}{2\pi R} V\right\} J_{mz}\left(\frac{k_m R_e}{R}\, y\right), \tag{2}$$

where p_m is the pressure of the m-th harmonic component of the sound pressure induced by the propeller, ω is the angular velocity of the propeller, z is the number of propeller blades, T is the thrust of the propeller, X is the coordinate along the axis of revolution of the propeller, R is the distance of the observation point from the centre of the propeller, M is the torque applied to the propeller, R_e is the effective radius of the propeller $R_e = (0.7 - 0.8)R_0$, R_0

is the radius of the propeller $R_0 = D/2$, Y is the coordinate axis perpendicular to the X axis, J_{mz} is a Bessel function of the first kind of order mz, k_m is the wave number, $k_m = m\omega z/c$, x, y are the coordinates of the observation point, V is the volume of the propeller blades, ϱ is the density of medium and c is the velocity of sound in the medium.

Formula (2) is obtained from the solution of equation (1) by substituting appropriate expressions for the first and second terms on the right-hand side of the equation. These terms are related to the working conditions of the propeller and to its geometry [9]. The method of solving the equation is given in the Appendix.

Formula (2) describes the propeller operation for a water stream flowing on to the propeller with a uniform velocity field. It can be seen from formula (2) that the value of the sound pressure is related to the rotational speed of the propeller and the number of blades. It also depends on the load of the propeller in terms of thrust and torque and on the radius of the propeller. As it has already been stated, this formula describes ship propeller operation only qualitatively, since the numerical results are not in quantitative agreement with the experimental data [9].

More complex mathematical models of the generation of acoustic underwater disturbances by a ship propeller use the circulation model of propeller operation [9]. However these models are very complicated, and their practicability is limited because the actual distribution of the velocity of the stream flowing onto the propeller is not always known. Consequently it is not possible to determine the circulation of the velocity on the propeller blades.

The investigation of the acoustic underwater disturbances generated by propellers can be carried out under the following conditions:

1. in hydrodynamic channels with models of propellers;
2. in anechoic water basins with the propeller operating but stationary;
3. in anechoic water basins with the propeller operating but in motion;
4. at sea, on real propellers.

2. Investigations of propeller operation in a hydrodynamic channel

The propeller operation is, in principle, described by the three following coefficients:

1. The propeller thrust coefficient

$$K_T = \frac{T}{\varrho n^2 D^4}, \tag{3}$$

where T is the propeller thrust, n is the rotational speed, D is the propeller diameter, and ϱ is the density of the medium.

2. The coefficient of propeller torque load

$$K_M = \frac{M}{\varrho n^2 D^5}, \tag{4}$$

where M is the torque applied to the propeller.

3. Advance coefficient

$$J = \frac{v}{nD}, \tag{5}$$

where v is the velocity of the undisturbed water stream.

The so called efficiency coefficient, of the form

$$\eta = \frac{K_T}{K_M} \cdot \frac{J}{2\pi} \tag{6}$$

is also often used.

The characteristics of the investigated propeller are shown in Fig. 1. The parameters of this propeller are summarized in Table 1.

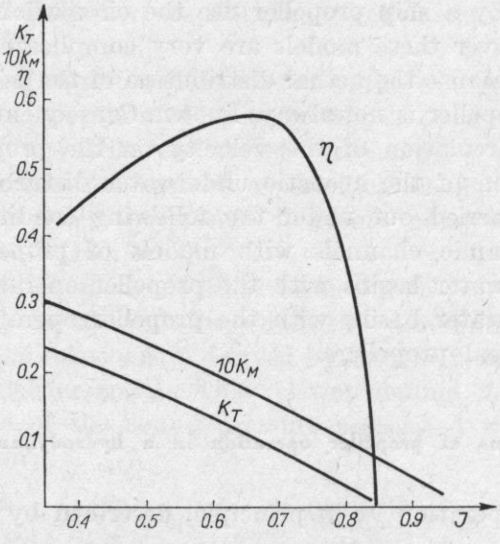

Fig. 1. Characteristics of ship propeller No. 1

Table 1

Propeller No.	Outer diameter D[m]	Number of blades z	Surface coefficient A/A_0	Pitch coefficient H/D	Direction of revolution
1	0.17	5	0.75	0.7	left

The investigations were carried out in a hydrodynamic channel on a model of a five-blade propeller [1, 12, 13].

The measuring system is shown schematically in Fig. 2

A number of measurements were made (Fig. 3) to determine the spectra of the acoustic underwater disturbances produced by the propeller rotating at a speed of $n = 20$ r. p. s. One can see here a distinct band centred at a frequency of $f_0 = 100$ Hz.

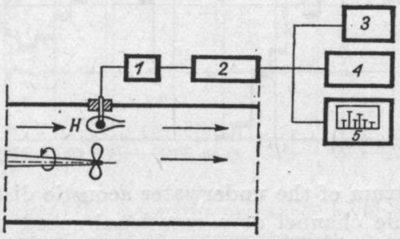

Fig. 2. Diagram of the system for measuring the acoustic disturbances produced by a ship propeller in a hydrodynamic channel

H – the measuring hydrophone; *1* – type 2626 charge amplifier; *2* – type 2606 measuring amplifier; *3* – type 2010 spectrum analyser; *4* – type 7001 magnetic tape recorder; *5* – type 3347 Brüel & Kjaer real time 1/3 octave analyser with display unit

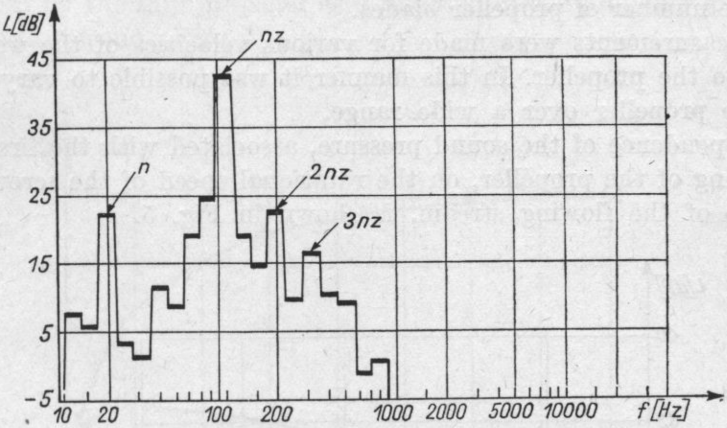

Fig. 3. The amplitude spectrum of the underwater acoustic disturbances of propeller No. 1, operating in a hydrodynamic channel at a rotational speed $n = 20$ r. p. s. Flow rate of water stream, $v = 1.2$ m/s

The spectra of acoustic underwater disturbances generated by the ship propeller rotating at a speed of 40 r. p. s. are shown in Fig. 4.

It can be seen from Figs. 3 and 4 that the level in the frequency band corresponding to the fundamental driving has a considerable effect on the total level of the acoustic underwater disturbances which are generated by the ship propeller. The frequency of this driving can be related to the parame-

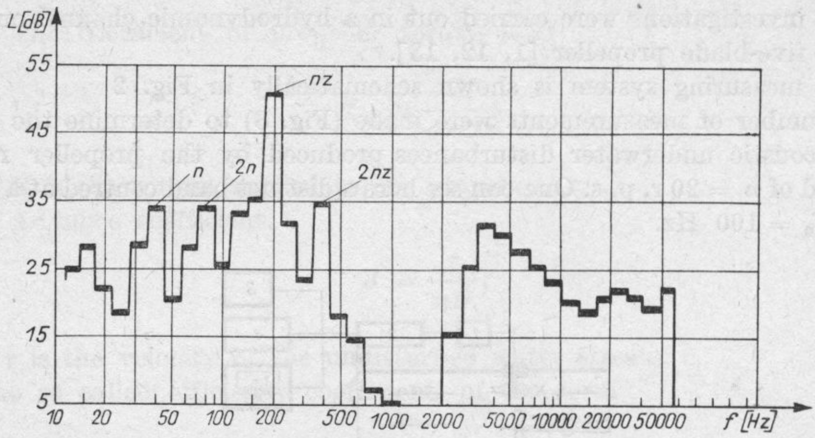

Fig. 4. The amplitude spectrum of the underwater acoustic disturbances of propeller No. 1 operating in a hydrodynamic channel at a rotational speed of $n = 40$ r. p. s. Flow rate of water stream, $v = 2.4$ m/s

ters of the propeller operation by the following formula

$$f_m = m\,n\,z, \tag{7}$$

where m is the number of the harmonic 1, 2, ..., n is the propeller speed [r.p.s.] and z is the number of propeller blades.

The measurements were made for various velocities of the water stream flowing onto the propeller. In this manner it was possible to vary the thrust load of the propeller over a wide range.

The dependence of the sound pressure, associated with the first harmonic of the driving of the propeller, on the rotational speed of the screw, for high velocities v of the flowing stream, is shown in Fig. 5.

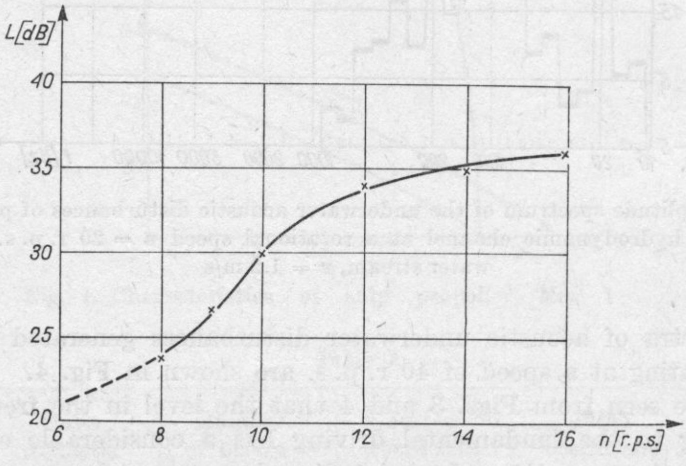

Fig. 5. The relationship between the acoustic pressure level and the rotational speed of the propeller. Flow rate of water stream, $v = 1$ m/s

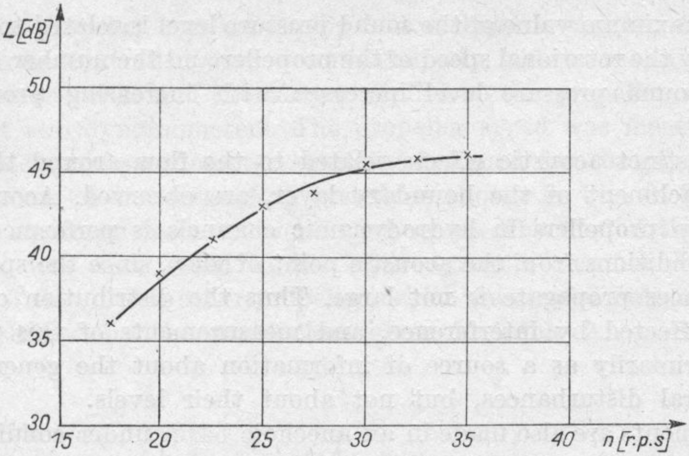

Fig. 6. The relationship between the acoustic pressure level and the rotational speed of the propeller. Flow rate of water stream: $v = 2$ m/s

In order to relate the acoustic effects to the load of the propeller, that is to the thrust and torque, use has been made of relations (3) (5), and of the graph presented in Fig. 1. On this basis a graph was plotted of the sound pressure produced by the ship propeller as a function of the propeller load in terms of thrust and torque (Fig. 7).

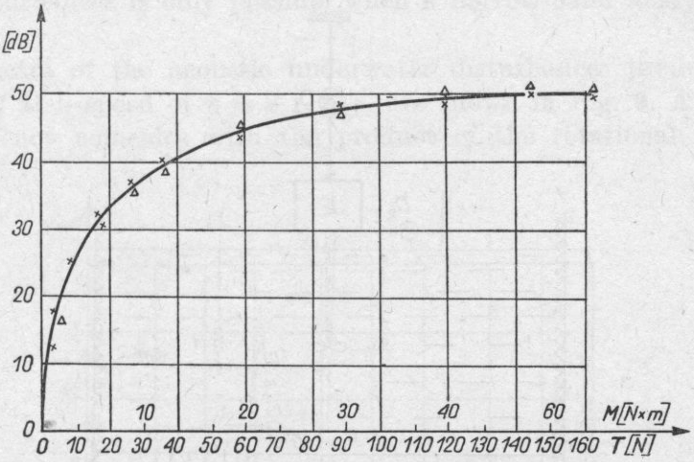

Fig. 7. The relationship between the acoustic pressure level L and the propeller load. T is the thrust propeller and M – the torque

On the basis of the investigations carried out on the generation of acoustic underwater disturbances by a model of the ship propeller in a hydrodynamic channel the following conclusions can be drawn.

1. The maximum value of the sound pressure level is related to the driving determined by the rotational speed of the propeller and the number of its blades.

2. The sound pressure level increases with increasing propeller speed and load.

3. No distinct acoustic effects related to the flow around the propeller, e. g. the detachment of the boundary layer, are observed. Acoustic investigation of ship propellers in hydrodynamic channels is performed under unfavourable conditions from the acoustic point of view, since the space in which the disturbances propagate is not large. Thus the distribution of the sound pressure is affected by interference, and measurements of this type can be considered primarily as a source of information about the general structure of the spectral disturbances, but not about their levels.

Measurements are also made in an anechoic basin under conditions of stationary propeller operation. The conditions in the anechoic basin are more favourable acoustically, but less favourable from the point of view of the propeller dynamics (as compared to the investigations carried out in a hydrodynamic channel), since the flow rate of the water is too low.

3. Investigation of a propeller in an anechoic basin

Investigations of the acoustic effects associated with ship propeller operation in an anechoic basin were carried out with the propeller operating but stationary.

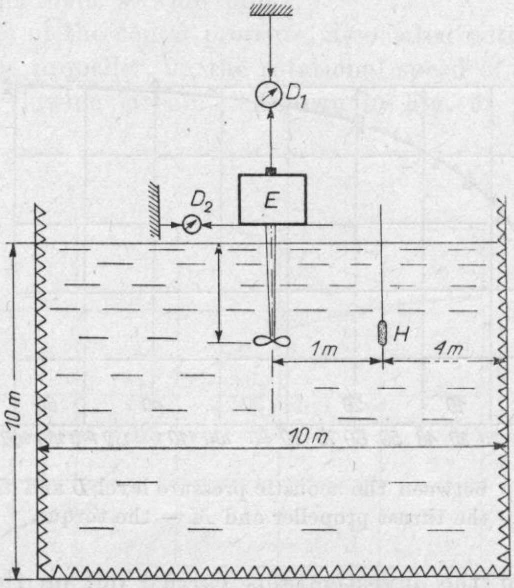

Fig. 8. Diagram of a system for measuring underwater acoustic disturbances produced by a ship propeller in an anechoic basin

D_1 – dynamometer for measuring the thrust force, D_2 – dynamometer for measuring the torque, E – eletric motor, H – hydrophone

The ship propeller was driven by a d. c. electric motor, which enabled the speed to be controlled over a comparatively wide range. The thrust produced by the propeller and the torque applied to the propeller were measured by means of two dynamometers. The propeller speed was measured with the aid of an electronic speedometer specially designed for the purpose.

The scheme of the measuring system is shown in Fig. 8. Two ship propellers were tested on the same measuring stand. The first of them is the propeller which was also tested in the hydrodynamic channel. Its parameters are given in Table 1. The other propeller, which was tested only in the anechoic basin is described by the parameters given in Table 2.

Table 2

Propeller No	Outer diameter D[m]	Number of blades Z	Surface coefficient A/A_0	Pitch coefficient	Direction of revolution
2	0.5	3	0.35	0.45	left

The spectral analysis of the underwater acoustic disturbances was carried out using a heterodyne analyser, with a bandwidth of $\Delta f = 3.16$ Hz.

It should be noted that for this type of investigation a fixed bandwidth narrow-band analyser should be used. This results from the fact that the disturbances have a discrete character and exact identification of the frequencies related to these disturbances is only possible when a narrow-band analysis is performed.

The spectra of the acoustic underwater disturbances produced by propeller. No. 2 at a speed of $v = 8$ r. p. s. are shown in Fig. 9. A spectral line whose frequency coincides with the product of the rotational speed of the

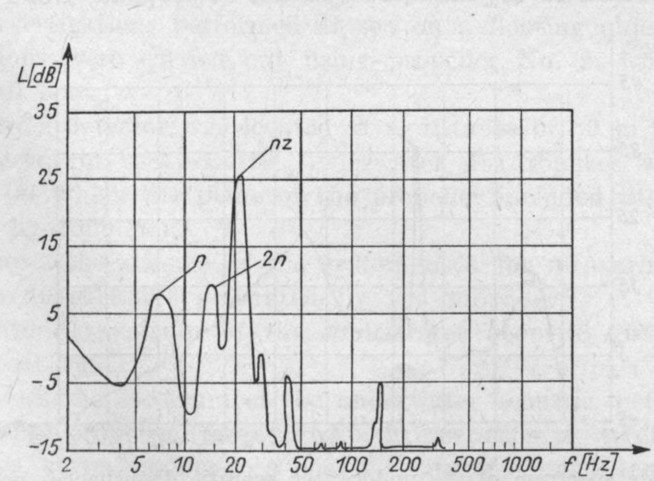

Fig. 9. Amplitude spectrum of underwater acoustic disturbances produced by propeller No. 2 rotating at a speed of $n = 8$ r. p. s.

propeller and the number of blades can be clearly seen. This line has a level which predominates over the whole spectrum. The level defines the total level of the disturbances generated by the propeller.

Fig. 10 shows the dependence of the sound pressure produced by propeller No. 2 as a function of the propeller speed. The level increases with increasing propeller speed, until the cavitation bubbles begin to appear on the propeller.

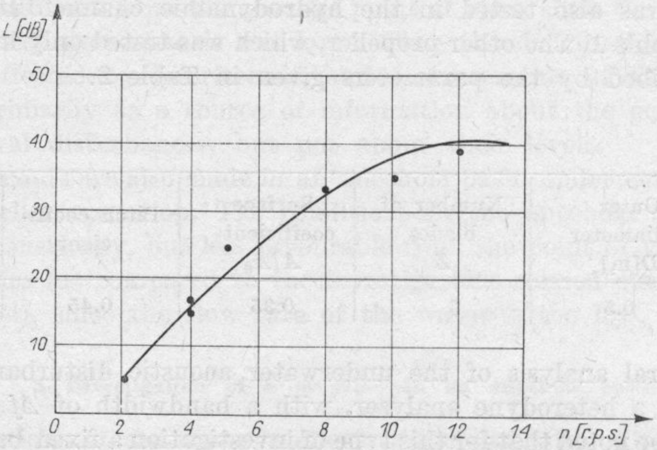

Fig. 10. The level of underwater acoustic disturbances produced by propeller No. 2, vs. the rotational speed

The spectrum of the acoustic underwater disturbances produced by propeller No. 1 is shown in Fig. 11.

Fig. 11. clearly shows the existence of a spectral line with a frequency equal to $f = 150$ Hz. This frequency, as it has already been pointed out is equal to the product of the number of blades of propeller No. 1 ($Z = 5$), and

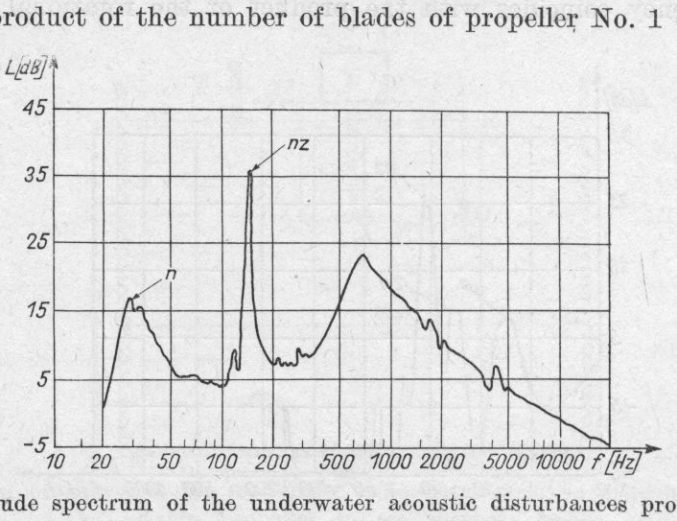

Fig. 11. Amplitude spectrum of the underwater acoustic disturbances produced by propeller No. 1 rotating at a speed of $n = 30$ r. p. s.

the rotational speed $n = 30$. This figure also shows a range of the continuous spectrum which is associated with the cavitation bubbles.

The interpretation of this part of the spectrum is very difficult. The distinct character of cavitation does not permit these effects to be described in greater detail.

It follows from the investigations carried out on the underwater acoustic disturbances generated by a ship propeller in absence of cavitation in an anechoic basin, that the total level of the acoustic disturbances is related to the level of the spectral line of a frequency equal to the product of the propeller speed and the number of propeller blades.

4. Investigations of the propeller at sea

Investigations carried out on the basis of measurement of the acoustic effects produced by a ship propeller in a hydrodynamic channel and in an anechoic basin have certain drawbacks.

In the case of investigations carried out in an hydrodynamic channel, where the free space is comparatively small, the occurrence of a strong water stream produces intense internal noises. It was therefore difficult to measure the magnitude of the acoustic disturbances generated solely by the propeller. On the other hand, the acoustic effects of the propeller in stationary operation in an anechoic basin may be somewhat "deformed" because of the unfavourable hydromechanical conditions for the propeller. Investigations carried out in the two conditions mentioned above are more favourable from the point of view of the practicality and the costs involved, but they provide only general data on the disturbances generated by the propeller. The most realistic ones are investigations performed at sea on a floating object.

Investigations were carried out using propeller No. 2, which was used to drive a small boat.

The measuring detector was located at a distance of 30 m from the propeller. The spectra of the acoustic underwater disturbances were recorded at the moment at which the plane of the propeller included the point of observation (the hydrophone).

Fig. 12 shows the scheme of the system used for measuring the underwater acoustic disturbances generated by the propeller.

The amplitude spectrum of the underwater acoustic disturbances was measured in real time.

Fig. 13 shows the spectrum of the underwater acoustic disturbances produced by the ship propeller (propeller No. 2) operating at a rotational speed of $n = 15$ r. p. s. corresponding to a boat velocity of about 3.1 m/s. The spectrum shown in Fig. 13 is similar to the spectrum shown in Fig. 9, although the component corresponding to the driving input 3 *nz* is significantly enhanced. Besides, the spectrum shown in Fig. 13, is more complicated. This results mainly from the fact of the propeller operating near a rigid body (boat hull), and this causes an increase in the disturbance amplitude, and also introduces certain additional disturbances. Furthermore, the driving system of the propeller, operating in the interior of the boat hull, produces vibrations of the hull plating which also have some effect on the structure of the composite spectrum of the underwater noise of the propeller operating near the boat hull. The non-uniform distribution of the water stream flowing onto the screw has a substantial effect on the disturbance spectrum. Fig. 14 shows the relative changes of the thrust force as a function of the angle of rotation of the propeller induced by the non-uniformity of the velocity field of the water stream flowing onto the propeller.

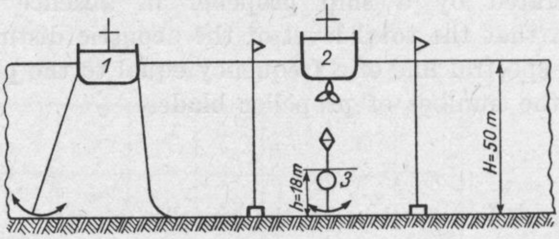

Fig. 12. Diagram of a system for measuring the underwater acoustic disturbances produced by a ship propeller under marine conditions

1 – measuring station; *2* – motor boat driven by an electric motor; *3* – measuring hydrophone

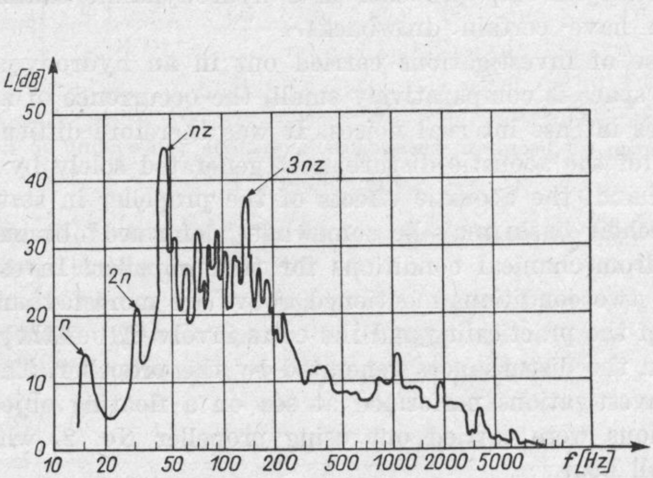

Fig. 13. Amplitude spectrum of the acoustic underwater disturbances produced by ship propeller No. 2, rotating at a speed of $n = 15$ r. p. s.

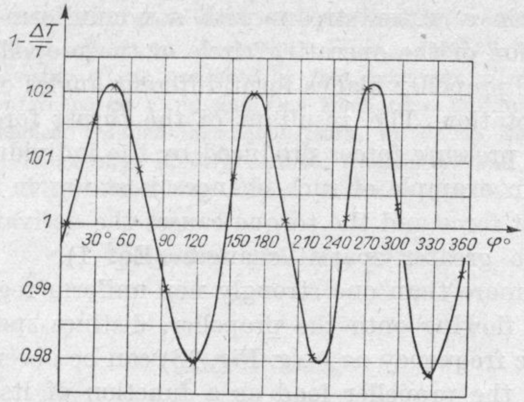

Fig. 14. The relative changes of the loading of the propeller as a function of the angle of rotation

5. Discussion of the results of the acoustic investigations on the ship propeller operation

The investigations of the acoustic effects carried out under different experimental conditions permit one common conclusion to be drawn. The spectrum of the underwater acoustic disturbances produced by a ship propeller, in the subcavitation range, depends on the rotational speed of the propeller and the number of its blades. The level of the disturbance near the frequency nz predominates in all the spectra presented and defines the overall value of the sound pressure level.

The level of the acoustic disturbance produced by the propeller increases as its speed is increased. This level increases with increasing loading of the propeller, as does the acoustic pressure of the m-th harmonic, according to Gutin's formula for aircraft propellers (formula 2). Nevertheless the substitution in that formula of the actual values of the thrust force and the torque, of the geometrical parameters of the propeller and of the medium properties, gives values of the acoustic pressure, e. g. for the first harmonic, which are several tens of decibels smaller than the data obtained experimentally. Thus this formula is not useful for the analytical determination of the acoustic pressure produced by a ship propeller, but may only be used for the qualitative interpretation of the phenomenon.

In the Lighthill equation (formula (1)) the first two terms on the right-hand side of the equation correspond to zero and first order sources. Obviously, this equation is related to the Gutin formula (see Appendix).

In experimental investigations an increase in the sound pressure level can be observed when the ship propeller is operating in fluid stream with a non-uniform velocity field. This applies mainly to the angular distribution, but the changes of fluid velocity with propeller radius have also some effect on the magnitude of the acoustic effects related to the propeller operation. A screw propeller operating in a water stream with a nonuniform velocity field distribution in the region of the operating circle of the propeller, is distinquished by the fact that the propeller blades form different angles of attack depending on the angle of rotation. The resultant of the thrust forces, being the geometrical sum of the pressure forces produced by the individual blades is a fluctuating quantity. An example of such changes is shown in Fig. 14. The fluctuation of the thrust force and the torque causes the derivative of the density of force to undergo greater spatial changes (Eq. 1).

When there is more than one strongly non-uniform region in the velocity field of the stream flowing onto the propeller, distinct spectral lines at multiples of the acoustic frequency *nz* (e. g. Fig. 13) can be observed. Thus it seems that the change of the propeller load as a function of its angle of rotation, (this results from the non-uniform character of the velocity field of the water stream), is an essential factor in determining the overall level of the sound pressure of the underwater acoustic disturbances produced by a screw propeller.

The effect of a rigid body near the propeller e. g. a plate in a plane parallel to the propeller axis causes an additional increase in the acoustic pressure level produced by the propeller.

It was found in the course of the investigations that third octave spectral filters (Fig. 3 and 4) are less useful than narrowband, fixed bandwidth spectrum analysers (Figs. 9, 11, 13.)

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APPENDIX

THE DERIVATION OF THE GUTIN FORMULA

Relation (2) used by Gutin [2], which enables the value of the acoustic pressure produced by a blade system to be determined, is obtained by solving equation (1), with the third term of the right-hand side of the equation being neglected.

We shall give the method which permits the determination of the "force noise", the basic component of the sound pressure produced by the propeller. Equation (1) for this problem takes the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) p = \operatorname{div}\vec{F}. \tag{1a}$$

If we present the sound pressure in the form

$$p(x, y, z, t) = p(x, y, z)e^{i\omega t} \tag{2a}$$

and the density of forces as

$$F(x, y, z, t) = F(x, y, z)e^{i\omega t}. \tag{3a}$$

then equation (1a) assumes the form of the non-uniform Helmholtz equation

$$(\nabla^2 + k^2)p(x,y,z) = \operatorname{div}\vec{F}(x,y,z). \tag{4a}$$

When the forces act within a limited area V_0, the solution of equation (4a) can be written in the following form:

$$p(x,y,z) = -\frac{1}{4\pi}\int\left(\frac{\partial Fx}{\partial x_0} + \frac{\partial Fy}{\partial y_0} + \frac{\partial F_2}{\partial z_0}\right)\frac{e^{-ikR}}{R}\,dx_0dy_0dz_0, \tag{5a}$$

where

$$R = \sqrt{(x-x_0)^2+(y-y_0)^2+(2-z_0)^2}.$$

If we have a concentrated force acting at the origin of the system, as is the case of a propeller, then formula (5a) is expressed as follows:

$$p(x,y,z) = -\frac{1}{4\pi}\left(F_x\frac{\partial}{\partial x}+F_y\frac{\partial}{\partial y}+F_z\frac{\partial}{\partial z}\right)\frac{e^{-ikR}}{R}. \tag{6a}$$

Taking into consideration the distribution of the forces acting upon an element of the propeller surface, in the system shown in Fig. 15, we obtain:

$$dT = \frac{A(r)rd\Theta r}{b}, \tag{7a}$$

$$dM = \frac{B(r)rd\Theta}{b}dr, \tag{8a}$$

where $A(r)$, $B(r)$ are the distributions of the hydrodynamic load along the blade radius.

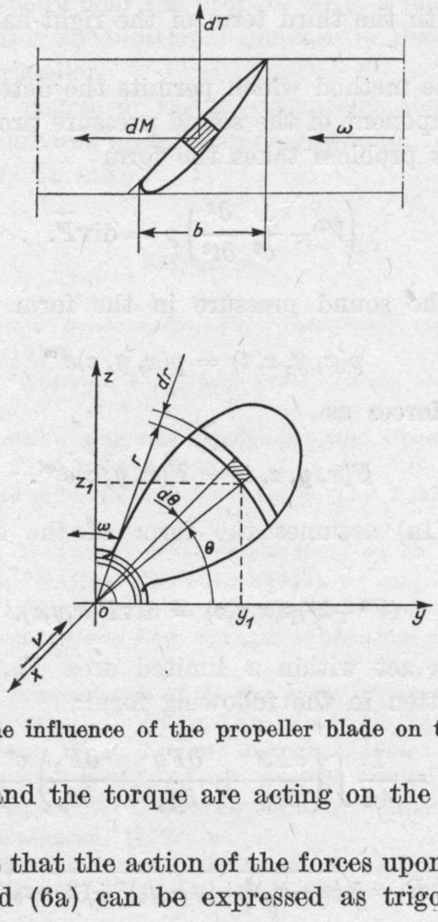

Fig. 15. Scheme of the influence of the propeller blade on the ambient medium

The thrust force and the torque are acting on the medium for the time equal to $\tau = b/r\omega$.

In view of the fact that the action of the forces upon the medium is periodic, equations (5a) and (6a) can be expressed as trigonometric series.

After simplification we obtain the components of the elementary forces acting upon the medium:

$$dF_{mx} = -\frac{1}{\pi}\frac{dT}{dr}\, e^{i(mz\omega t - mz\Theta)} dr d\Theta, \tag{9a}$$

$$dF_{my} = -\frac{1}{\pi r}\frac{dM}{dr}\, e^{i(mz\omega t - mz\Theta)} \sin\Theta dr d\Theta, \tag{10a}$$

$$dF_{mz} = -\frac{1}{\pi r}\frac{dM}{dr}\, e^{i(mz\omega t - mz\Theta)} \cos\Theta dr d\Theta. \tag{11a}$$

Substituting (9a), (10a) and (11a) into (6a) under the assumption that the velocity of the water stream flowing on to the propeller satisfies the relationship $v \ll c$, and after some tedious transformations, we obtain the Gutin formula for the value of the sound pressure of the n-th harmonic in the following form:

$$|p_m| = \frac{m\omega z}{2\pi c R}\left| -T\frac{x}{R} + M\frac{c}{\omega R_e^2}\right| J_{mz}\left(\frac{k_m R_e}{R}\, y\right). \tag{12a}$$

This schematic presentation of the method shows the relationship between equation (1) and the Gutin formula (2). Equation (12a) constitutes part of the relationship (2), and defines the dependence of the sound pressure on the load of the propeller in terms of torque and thrust force.

A similar method is used for the determination of the levels of the harmonics of the sound pressure of the "volume noise" that results from the finite volume of the blades of the ship propeller.

An exact derivation of both formulae is given by Minovich et al. [9].