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INVESTIGATIONS OF THE DIRECTIONAL PROPERTIES OF ACOUSTIC FIELD IN FINITE CUBOIDAL SPACES (GEOMETRICAL FIELD ANALYSIS)

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The paper presents an analysis of the properties of the acoustic field in a cuboidal space with the aid of the geometrical method using computer techniques. The assumptions and a brief description of the method together with the results of calculations for a space of fixed dimensions are presented. The studied properties of the acoustic field are in disagreement with results obtained by the statistical method. The directional properties of field, expressed in terms of energy, have been described as a directional characteristic of the equivalent source. Examples of plotted characteristic curves are given and the effect of various parameters on their form is discussed.

1. Introduction

Complicated physical phenomena, occurring during the propagation of acoustic waves in bounded spaces, have not yet been described satisfactorily by mathematical relations. The preferably used methods of the analysis — the wave and statistical ones — are based on the simplifying assumptions which idealize the conditions. Consequently, the results obtained from such an analysis are an idealized approach to reality. The adopted simplifying assumptions either restrict the number of cases to which the method can be applied with a sufficiently small error or result in difficult for a quantative determination errors. The assumptions of the wave method limit its application range to the regions of cuboidal or other regular form. On the other hand, the assumptions of the statistical method disregard the shape of the space, the arrangement of surfaces with different absorption coefficients, the position of the sound source and observation point, as well the decrease of the energy density due to the spherical shape of the propagating wave. These are essential simplifications since practice has shown that especially the shape of an interior and the distribution of sound absorbing materials can perceptibly affect the properties of the acoustic field. The effect of the above-mentioned factors on properties of the acoustic field is accounted for in the geometrical method of the field analysis in bounded spaces, which is based on the principles of geometrical optics. It seems that simplifying assumptions adopted in this method allow for its wider application and relatively easy estimation of errors. The application of the geometrical-graphical method is hindered by the tedious determination of image sources of higher orders and the summation of energy of individual waves reaching the observation point. This problem can be easily solved by the use of computers and only such approach to the geometrical method will enable us to take advantage of its possibilities. Straszewicz [4], dealing with the geometrical method of the field analysis, made a brief evaluation of methods used for the analysis of the acoustic field and showed the superiority of the geometrical method over the statistical and wave methods. Basing on results of the analysis of the distribution of image sources in two-dimensional areas, obtained without the use of a computer, he has come to a number of interesting conclusions. This result suggests a need for further development of the method using computers.

2. Computational techniques in geometrical method of field analysis

The method of field analysis, known as the geometrical method, is not defined uniquely. In the early days of the architectural acoustics the geometrical method used to be defined as a method which used the laws of statistics. This method is now referred to as a statistical method. Kuttruff [2], carrying out the acoustic field analysis employing a method called by him the geometrical method, takes advantage of the mirror reflexion of the sound waves and introduces the simplifying assumptions characteristic for this method, but in the obtained relations, defining the reverberation time value, he is applying the laws of statistics. Thus his method can be termed as the geometrical and statistical method of the acoustic field analysis. The method presented below is a "pure" geometrical method of the acoustic field analysis in a bounded space, i.e. it is based exclusively on the principle of the geometrical optics. Simplifying assumptions of the method can be divided into two groups: the general – related to the very essence of the method – and the additional detailed assumptions related to a certain way of its realization.

The general assumptions for the geometrical method are the following:

1. Sound waves are replaced by "sound rays" propagating from a determined point, i.e. from a sound source. The "rays" obey the same laws of propagation as do the light rays.

2. The dimensions of the bounded space are large in comparison with the wavelength.

Additional assumptions of the presented geometrical method are:

1. The space is bounded by surfaces with a determined absorption coefficient $a \in (0, 1)$ which is independent of the angle of incidence of a "sound ray".

2. The sound source is a point source which emits a spike pulse or a very narrow noise band.

3. The change in the energy density within the investigated space results

from the propagation of a spherical wave and the absorption on the boundaries of the space.

4. Absorption of energy by the medium is neglected.

5. Diffraction and phase relationships, during propagation and reflexion, are also neglected.

6. The only interference effect of the signals reaching the observation point is the addition of the energy of waves.

As it has already been found in the previous investigations, additional simplifying assumptions are the result of the approach to the method involving the use of a simple procedure algorithm enabling the definition of certain parameters of the acoustic field. A different approach to the method and the use of another algorithm can extent the domain of applications.

Each acoustic wave reaching a given observation point, after being reflected from the surface enclosing the space, is determined uniquely by its pressure, the phase, the direction and the reverberation time. Of these four quantities only the sound pressure and the reverberation time will be described in the presented method. Below, a procedure algorithm used for this purpose is presented.

1. The determination of the position of the sound source and observation point.

2. The calculation of the distance of the image sources from a given observation point.

3. Theoretical calculations of an echogram for the purpose of determing the amplitude-time characteristic of reflections, i.e. the room impulse response

$$k(t) = \sum_n A_n \delta(t-t_n), \tag{1}$$

where A_n denotes the value of the pressure of the n-th reflection reaching the observation point after the time t_n.

If the room impulse response is determined, then it is possible to evaluate the response $s'(t)$ for a given signal $s(t)$:

$$s'(t) = \int_{-\infty}^{+\infty} s(X)\,k(t-X)\,dX = \sum_n A_n s(t-t_n). \tag{2}$$

4. The determination of the reflected waves pressure level L_{sc} and of the total pressure level L with the direct wave being at a given observation point

$$L_{sc} = 10\log\int_{t=0}^{\infty} s'(t)\,dt = 10\log\sum_{t=t_1}^{t_n}\Big[\sum_n A_n^2\,\delta(t-t_n)\Big], \tag{3}$$

$$L_{sc} = 10\log\int_{t=t_2}^{\infty} s'(t)\,dt = 10\log\sum_{t=t_1}^{t_n}\Big[\sum_n A_n^2\,\delta(t-t_n)\Big], \tag{4}$$

where t_1 is the arrival time of the direct wave and t_2 is the arrival time of the first reflected wave.

3. Subject of analysis

In this paper the properties of the acoustic field in a cuboidal space will be analyzed with the aid of the geometrical method using computers.

Since the laws of the propagation of a wave propagating directly from the sound source are precisely defined and can be introduced at any moment into the analysis, the main emphasis will be placed on the properties of the acoustic field of the reflected waves [4]. The main parameter defining the field in the method used is the spatial distribution of the reflected waves pressure. An analysis of this distribution will constitute an essential part of this paper.

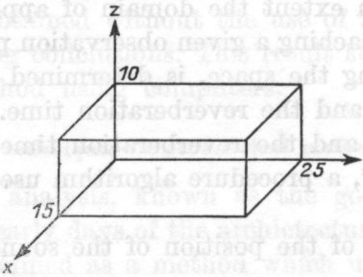


Fig. 1. The position of the tested enclosed space in the coordinate system

The limitation of the range of analysis to only one form of the cuboidal space has resulted in such advantages as clear form of the results and the possibility of systematic investigation of the changes of the field parameters as functions of other parameters such as the distance between the source and observation point, their arrangement within the space, and also the absorption coefficient at various surfaces. In the cuboidal space the distribution of the image sources and its changes under the influence of the above-mentioned parameters are easy to predict and, since this distribution determines the properties of the acoustic field, one can discuss without difficulty the results obtained by computation. Furthermore, the use of the geometrical method for the cuboidal spaces has enabled to evolve a simple and concise computation program, e.g. in comparison with a program for the region of any shape [6].

In the examples of the use of computers in the geometrical method of the field analysis, as cited in the literature [5, 7], preference is given to the "ray method" while the forms of the analyzed spaces are rather simple for reasons mentioned before. The properties of the acoustic field – found for the cuboidal spaces – can, to some approximation, be extended to the acoustic field in regions of different form, but the distribution of the apparent sources in a given case should be accounted for, at least qualitatively. Obviously, for the accurate calculations, a suitably elaborated program is a necessity.

4. Outline of the program

The program has been written in the FORTRAN language; the calculations were performed on CYBER CDC 6000 computer. Below the block diagram of the program is given.

Data

↓

Calculation of one coordinate determining the distance from the observation point as well as the number of reflections from either of two walls perpendicular to the axis on which the coordinate is calculated

↓

Calculation of the distance of the sound source from the observation point, the approach time of the reflected wave to the observation point, the losses arising from reflection as well as pressure of subsequent reflected waves, i.e. of echoes

↓

Arrangement of results in sequence of the growth arrival time of echoes

↓

Addition of the energies of echoes, the difference of the arrival times being C

↓

Calculation of the total pressure level L as well as of the pressure level of the sum of reflected waves L_{sc}

↓

output of the result

The input data of the program are: dimensions of the bounded space, coordinates of the sound source and observation point, absorption coefficients of individual surfaces, the time interval C over which energy of reflection is summed, pressure level of the direct wave at a distance of 1 m from the sound sorce, the order of reflections $\leqslant 10$ to be considered. The results of calculations are: a full theoretical echogram containing all the echoes and giving the sequence of reflections from particular walls, an abbreviated echogram, i.e. the echogram obtained after the summing up the energies of the reflected waves which reach the observation point in sufficiently small time intervals, the reflected waves pressure level L_{sc} and the total pressure level L at the observation point.

5. The results of investigations and their interpretation

The calculations have been performed for a cuboidal space of dimensions $15 \times 25 \times 10$ m, with a point sound source radiating the wave with a pressure level 100 dB at a distance of 1 m.

To limit the computation, time reflections up to the 7-th order were taken into account and this seems to be sufficient considering that the last reflected wave reaching the observation point after approximately 500 ms has a pressure level lower by 35-40 dB than the first one. The time interval C is equal to 0,1 ms and this practically ensures the summation of the waves reaching the observation point on the paths of equal length. The position of the sound source and observation point and the surface absorption coefficient are changed in the calculations.

The computations yield interesting results on essential spatial properties of the acoustic field which do not agree with the results of the analysis performed by statistical method, namely:

1. variability of the reflected waves pressure level L_{sc} as a function of the distance from the source r;

2. relation between the reflected waves pressure level L_{sc} and the position of the sound source and observation point in the space at a constant distance r from the source;

3. relations between the reflected waves pressure level L_{sc} and the arrangement of boundary surfaces with different absorption coefficients.

The variability of the reflected waves pressure level as a function of the distance of the observation point from the sound source was observed by Straszewicz [1] in the case of non-rectangular shapes of the projections of the interiors. For this purpose he took advantage of the geometrical analysis method without resort to computation. The results of calculations of the reflected waves pressure level L_{sc} obtained with the aid of the geometrical method, as shown in Figs. 1–5, are compared with those obtained by the statistical method [1].

The comparison of these results is complicated because of the fact that in the geometrical method the value of the absorption coefficient a_c should be close to the value of the physical coefficient while in the statistical method — to the value of the reverberation coefficient a_p. The relation between these two coefficients [3] depends on the frequency and absolute value of coefficients. In all comparative calculations performed by the statistical method it has been assumed that

$$\frac{a_p}{a_c} = \begin{cases} 1.5 & \text{for } a_c \leqslant 0.5, \\ 1.2 & \text{for } 0.5 \leqslant a_c \leqslant 0{,}8, \\ 1 & \text{for } a_c > 0.8. \end{cases} \tag{5}$$

The analysis of individual groups of results leads to the conclusions which are summarized below. The reflected waves pressure level L_{sc} is plotted in Fig. 2 as a function of the distance from the source r, for several positions of the source and the pressure level L_{sc} calculated by the statistical method, with the absorption coefficient level being equal at all the surfaces (Fig 2a — $a = 0.1$; Fig. 2b — $a = 0.6$).

Remark 1. The reflected waves pressure level decreases with the growth of the distance from the source.

Remark 2. The rate of changes of the pressure level L_{sc} depends on the position of the sound source; these changes are the faster the closer lies the

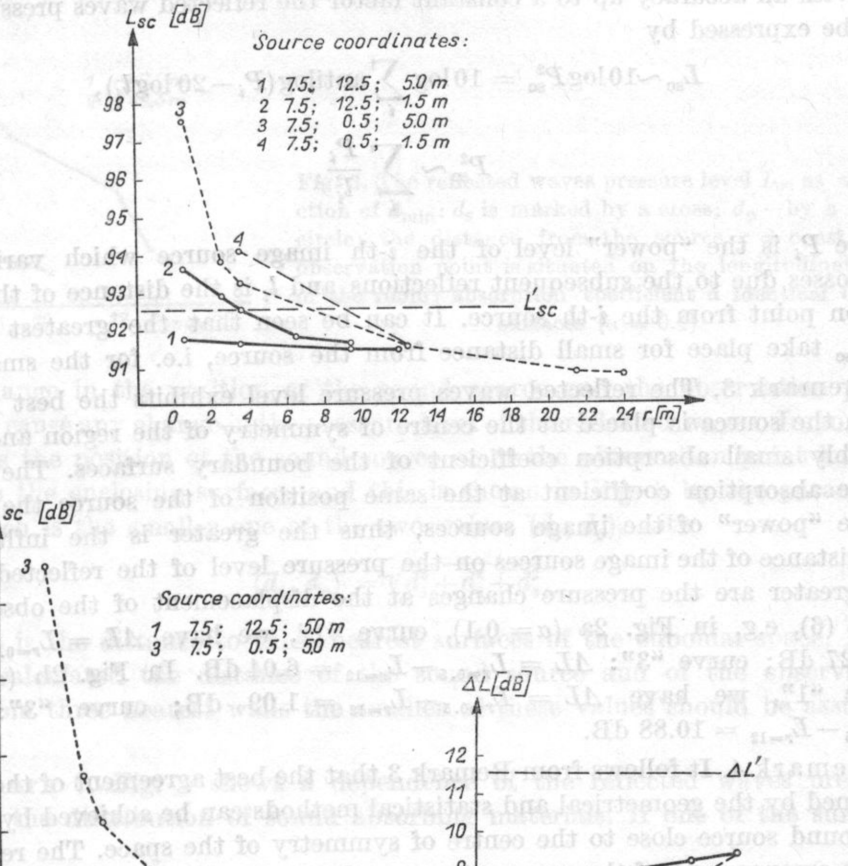


Fig. 2. The reflected waves pressure level L_{sc} as a function of the distance from the source r

The observation point is located on the longitudinal axis of the room; absorption coefficients identical on all surfaces: (a) $\alpha = 0.1$; (b) $\alpha = 0.6$; (c) difference between reflected pressure level at $\alpha = 0.1$ and $\alpha = 0.6$ for two positions of the source (the coordinates as shown in Fig. 1a). For comparison the values calculated by the statistical method are given

sound source or observation point to the boundaries of the space; in a given position of the source the greatest differences in the pressure level of the reflected waves occur for small distances, e.g., in Fig. 2a curve "1" when the source lies in the centre of symmetry of the space $\Delta L = L_{r=0.5} - L_{r=3.5} = 0.07$ dB, and $\Delta L = L_{r=3.5} - L_{r=12} = 0.19$ dB; curve "3", when the source is close to the wall $\Delta L = L_{r=0.5} - L_{r=3.5} = 4.34$ dB, $\Delta L = L_{r=3.5} - L_{r=12} = 1.7$ dB.

With an accuracy up to a constant factor the reflected waves pressure level can be expressed by

$$L_{sc} \sim 10\log P_{sc}^2 = 10\log\sum_i \text{antilog}(P_i - 20\log l_i), \tag{6}$$

$$P_{sc}^2 \sim \sum_i \frac{P_i}{l_i^2}, \tag{6a}$$

where P_i is the "power" level of the i-th image source which varies with the losses due to the subsequent reflections and l_i is the distance of the observation point from the i-th source. It can be seen that the greatest changes of L_{sc} take place for small distance from the source, i.e. for the smallest l_i.

Remark 3. The reflected waves pressure level exhibits the best stability when the source is placed at the centre of symmetry of the region and at the possibly small absorption coefficient of the boundary surfaces. The greater is the absorption coefficient at the same position of the source the smaller is the "power" of the image sources, thus the greater is the influence of the distance of the image sources on the pressure level of the reflected waves, the greater are the pressure changes at the displacement of the observation point (6), e.g. in Fig. 2a ($\alpha = 0.1$), curve "1", we have $\Delta L = L_{r=0.5} - L_{r=12} = 0{,}27$ dB; curve "3": $\Delta L = L_{r=0.5} - L_{r=12} = 6.04$ dB. In Fig. 2b ($\alpha = 0.6$), curve "1", we have $\Delta L = L_{r=0.5} - L_{r=12} = 1.09$ dB; curve "3": $\Delta L = L_{r=0.5} - L_{r=12} = 10.88$ dB.

Remark 4. It follows from Remark 3 that the best agreement of the results obtained by the geometrical and statistical methods can be achieved by placing the sound source close to the centre of symmetry of the space. The regularity of the arrangement of the apparent sources brings about that the displacement of the observation point leads to only minor changes of the value of L_{sc}.

Remark 5. The increase of absorption coefficients on the boundary surfaces causes faster changes of L_{sc} as a function of the distance r (for explanation see item 3). Fig. 2c shows the value of the difference of levels ΔL of the reflected waves as a function of the distance from the source, with absorption coefficients $\alpha = 0.1$ and $\alpha = 0.6$ for two positions of the source. It can be seen that the rate of changes of ΔL decreases when the source is moving away, with their increasing absolute value, i.e., the increase of the absorption coefficient causes the greater changes the greater is their distance from the source.

Remark 6. With various positions of the source and the constant distance of the observation point, with identical absorption coefficients on the surfaces, the reflected waves pressure level is the smaller the greater is the distance of the sound source or observation point from the boundary surface. Considering the symmetry of the spatial structure of the network of apparent sources for a cuboidal space, it is possible to interchange a position of the sound source and the observation point.

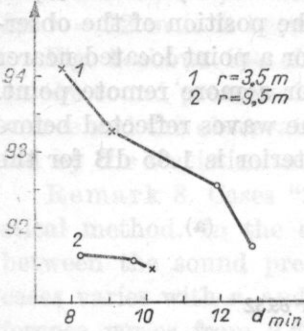


Fig. 3. The reflected waves pressure level L_{sc} as a function of $d_{\min}$: d_s is marked by a cross; d_p – by a small circle; the distance from the source r = const; the observation point is situated on the longitudinal axis of the room; absorption coefficient α identical on all surfaces ($\alpha = 0.1$)

A change in the position of the sound source and the observation point does not cause any change in the pressure level of the reflected waves. Important for L_{sc} is the position of the sound source or of the observation point located closer to the enclosing surfaces and this is shown in Fig. 3 by the parameter $d_{\min}$ which is the smaller one of the two values (d_s, d_p), with

$$(d_s, d_p) = \sqrt{d_1^2 + d_2^2 + d_3^2}, \tag{7}$$

where d_i is the distance to three nearest surfaces of the cuboidal space. After having calculated the distance of the sound source and of the observation point from three nearest walls the smaller of these values should be assumed as $d_{\min}$.

Remark 7. Fig. 4 shows a dependence of the reflected waves pressure level on the distribution of sound absorbing materials. If one of the surfaces

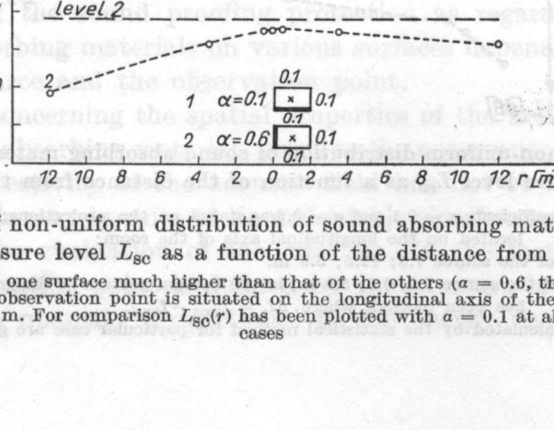


Fig. 4. The effect of non-uniform distribution of sound absorbing materials on the reflected waves pressure level L_{sc} as a function of the distance from the source r

Absorption coefficient of one surface much higher than that of the others ($\alpha = 0.6$, the others $\alpha = 0.1$ as stated on the projection); the observation point is situated on the longitudinal axis of the room; the coordinates of the source 7.5; 12.5; 5.0 m. For comparison $L_{sc}(r)$ has been plotted with $\alpha = 0.1$ at all surfaces and L'_{sc} for bot cases has the sound absorption coefficient greater than the others, at the equal distance from the source, the value of L_{sc} depends on the position of the observation point relative to this surface : when $r = 12$ m, for a point located nearer the absorbing surface L_{sc} is smaller by 0.6 dB than for a more remote point. The difference between the sound pressures levels of the waves reflected before and after the change in the absorbing power of the interior is 1.65 dB for the

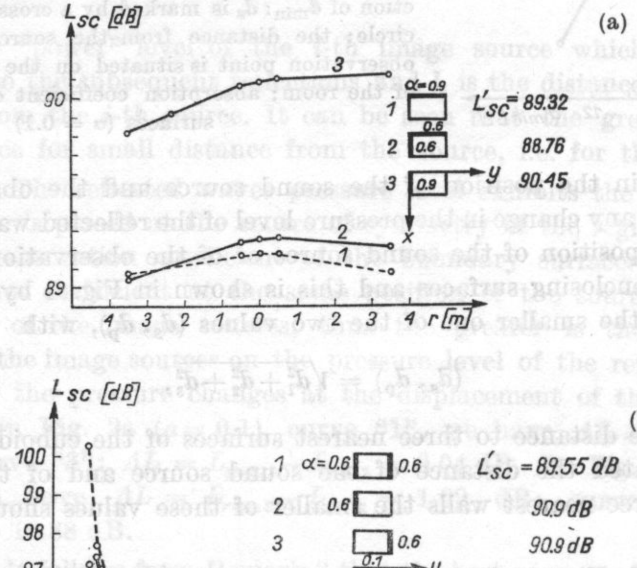


Fig. 5. The effect of non-uniform distribution of sound absorbing materials on the reflected waves pressure level L_{sc} as a function of the distance from the source r

Surface absorption coefficients $\alpha = 0.1$ and $\alpha = 0.6$ as stated on the projections; the observation point is located on the longitudinal axis of the room:

(a) the coordinates of the source 7.5; 12.5; 5.0 m.

(b) the coordinates of the source 7.5; 0.5; 5.0 m; in the bottom portion of the graph the course of $\Delta L(r)$ has been plotted, where $\Delta L = L_{SC3} - L_{SC2}$;

The values of L_{SC} calculated by the statistical method for particular case are given statistical where as in the geometrical method, it differs for various directions: for $r = 12$ m it is between 0.96 and 1.61 dB.

Fig. 5 shows the effect of the arrangement of sound absorbing materials on the sound pressure level of the reflected waves at the same position of the source and the observation point. For comparison the values of L_{sc} are calculated by the statistical method.

Remark 8. Cases "2" and "3" are equivalent in calculations by the statistical method. In the calculations by the geometrical method the difference between the sound pressure levels of the reflected waves ΔL for these two cases varies with r, and this is shown in the bottom part of Fig. 5a: this difference varies from 2.64 dB for $r = 0.5$ m to 0.65 dB for $r = 12$ m.

Remark 9. If the source and the observation point are at equal distance from opposite walls, then the use of sound absorbing materials on each of either walls gives the same result in the case of consideration of L_{sc}, e.g., in Fig. 5a curves "2" and "3" for $r = 24$ m, $L_{sc} = \text{const}$.

Remark 10. For non-uniform distribution of sound absorbing materials the sound pressure level of the reflected waves may not be the highest at the source point as is the case of the uniform distribution (Fig. 3, curve "1"). Although for the source point the sum of distances from the apparent sources is the smallest, their different "power", being determined by the distribution of sound absorbing materials, causes the displacement of the point having a maximum sound pressure level of the reflected waves towards the surface with a smaller absorbtion coefficient (Fig. 5b).

Remark 11. Using the statistical method for curves "1" and "2", it can be seen from Fig. 5b that a lower level of L_{sc} can be obtained by using a material with a smaller sound absorption coefficient on a larger area ("2"). On the other hand, basing on results obtained by the geometrical method, it is evident that for certain parts of the interior it is advisable — from the viewpoint of reducing L_{sc} — to use on a smaller surface a material with a greater coefficient ("1"). The total absorption for the mentioned curves "1" and "2" is comparable and amounts to 420 and 460 m^2, respectively.

From the foregoing discussion practical conclusions can be drawn, e.g. from the viewpoint of the sound proofing protection as regards the efficiency of using sound absorbing materials on various surfaces depending on the position of the sound source and the observation point.

All remarks concerning the spatial properties of the field for various distributions of sound absorbing materials point to the occurrence of some directional features. The directivity of the acoustic field has not so far been defined in an inobjectionable way in spite of its undeniable significance for the perception of sounds and the evaluation of the interiors. In the method used the field at a given observation point is defined by the pressure level of the reflected waves, i. e. by a quantity proportional to the energy density. Therefore, an approach in terms of power to the directivity of the acoustic field has been proposed. Taking into account the distribution of energy in the field of the reflected waves, the action of an omnidirectional sound source in an enclosed space can be replaced by the action of a source with a determined direction characteristics in free space. In other words, the effect of the system geometry (the space enclosed by the surfaces with determined absorption coefficients, the omnidirectional source, the observation point) on the distribution of energy in the field of the reflected waves becomes transformed into the directional characteristic of the sound source radiating in free space. Such approach does not provide the information about the direction of the energy reaching a given observation point, but only about mutual relations of the energy of the wave reaching various points of the field.

If we have a cuboidal space in which the position of the sound source and of the observation point are defined and if we have a spatial polar coordinates system (r, φ, θ), whose centre lies in the source point, and thus the coordinates of the source are $(0, 0, 0)$, the coordinates of the observation point (r, φ, θ) and the coordinates of the reference point $(r_0, \varphi_0, \theta_0)$, then the pressure at the observation point is p_φ, at the reference point — p_0. The power directivity of the field is defined by the directivity coefficient of the equivalent source q,

$$q(r, \varphi, \theta) = \frac{P_\varphi}{P_0}, \tag{8}$$

where P_φ denotes the power of the omnidirectional source which at the observation point in a free space produces the pressure p_φ, P_0 is the power of the omnidirectional source which at the reference point in free space produces the pressure p_0. By determining two of the three coordinates of the observation point one can obtain suitable directional characteristics of the equivalent source,

$$q(\varphi)_{\substack{r=\mathrm{const}\\ \theta=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(r, 0, \theta)}, \quad q(\theta)_{\substack{r=\mathrm{const}\\ \varphi=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(r, \varphi, 0)},$$

$$q(r)_{\substack{\varphi=\mathrm{const}\\ \theta=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(0, 0, 0)}, \tag{9}$$

where p is the value of the sound pressure of the sum of the reflected waves at a point with given coordinates. If for a given observation point (r, φ, θ) the directivity index is greater unity, e.g. $q(\varphi) > 1$, this means that in the direction φ more energy is concentrated than in the direction $(r, 0, \theta)$. To put it the other way, in order to obtain such a pressure level as it is at the point (r, φ, θ) of the investigated field, it is necessary to use the source with a q times greater power than the power of the source permitting to obtain a determined pressure level at the point $(r, 0, \theta)$ in the case for which the source would function in free space. With an arbitrary choice of parameters and of a variable out of (r, φ, θ),

$q > 1$ means a gain of energy. Figs. 6–8 show – in terms of logarithmic measures – the determined directional characteristic of the equivalent source, that is, a directional gain $\boldsymbol{q}$,

$$\boldsymbol{q}(r, \varphi, \theta) = 10\log\frac{P_\varphi}{P_0} = 10\log q(r, \varphi, \theta)\,\mathrm{dB}, \tag{10}$$

that is to say, the directional gain is indicated by $\boldsymbol{q} > 0$ dB. The omnidirectional characteristic is represented by a circle with the power 0 dB; at points located inside the circle a loss of energy ($\boldsymbol{q} < 0$ dB) occurs in relation to the reference point, at points outside the circle there is a gain of energy ($\boldsymbol{q} > 0$ dB). The determined points of the characteristics connected by the straight lines do not show the shape of the characteristic, but merely some tendencies and regularities. The graphical presentation of more accurate characteristic would require the determination of a considerably greater number of points than four, nevertheless within the cuboidal space under consideration one might not expect larger irregularities (even with the determined four points the interpretation of the characteristic would be possible). To facilitate the interpretation of the characteristic, three groups of cases have been established:

1. the sound source is located at the centre of symmetry of the space, different absorption coefficients of boundary surfaces;
2. the sound source is located at any point of the space, equal absorption coefficients of boundary surfaces;
3. the sound source is located at any point of the space, different absorption coefficients of boundary surfaces.

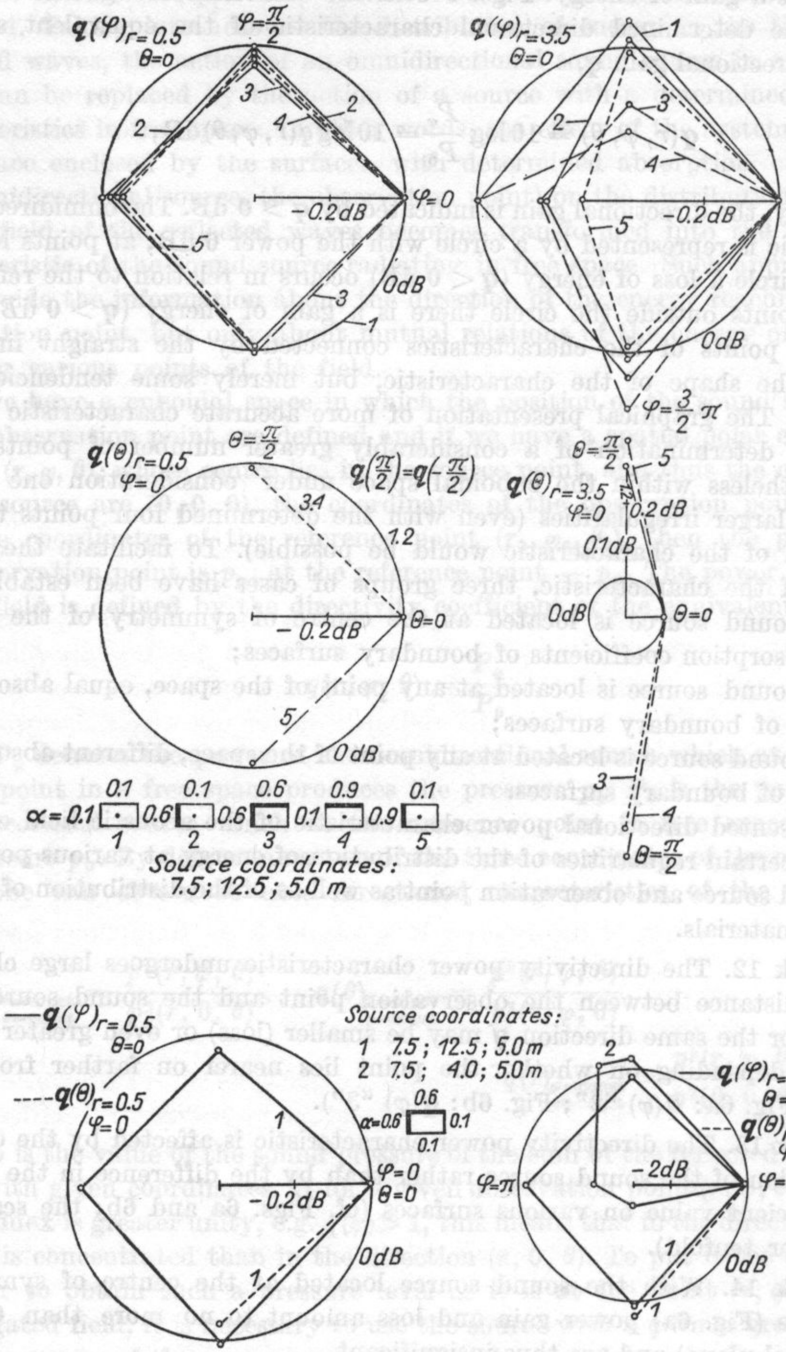
The presented directional power characteristic of the acoustic field enables to observe certain regularities of the distribution of energy at various positions of the sound source and observation point as well as of the distribution of sound absorbing materials.

Remark 12. The directivity power characteristic undergoes large changes when the distance between the observation point and the sound source $\boldsymbol{r}$ is changed. For the same direction $\boldsymbol{q}$ may be smaller (loss) or even greater (gain) than 0 dB depending on whether the point lies nearer on farther from the source (of. Fig. 6a: $\boldsymbol{q}(\varphi)$ "2"; Fig. 6b: $\boldsymbol{q}(\varphi)$ "3").

Remark 13. The directivity power characteristic is affected by the change in the position of the sound source rather than by the difference in the absorption coefficient value on various surfaces (cf. Figs. 6a and 6b; the scales of figures differ tenfold).

Remark 14. With the sound source located at the centre of symmetry of the space (Fig. 6a) power gain and loss amount to no more than 0.2 dB (in horizontal plane) and are thus insignificant.

Remark 15. The gain or loss of power is greater in the direction in which the linear dimension of the space is smaller, i.e. for the space under consideration


(a)
$q(\varphi)_{r=0.5}$ $\theta=0$
$\varphi=\frac{\pi}{2}$
$q(\varphi)_{r=3.5}$ $\theta=0$
$\varphi=0$
$-0.2\,dB$
$0\,dB$
$\varphi=\frac{3}{2}\pi$
$q(\theta)_{r=0.5}$ $\varphi=0$
$\theta=\frac{\pi}{2}$
$q(\frac{\pi}{2})=q(-\frac{\pi}{2})$
$q(\theta)_{r=3.5}$ $\varphi=0$
$\theta=0$
$0.2\,dB$
$0.1\,dB$
$\theta=\frac{\pi}{2}$
1
2
3
4
5
3,4
1,2
$\alpha=0.1$
0.1 0.6 1
0.1 0.6 2
0.6 0.1 3
0.9 0.9 4
0.1 5
Source coordinates:
7.5; 12.5; 5.0 m
$q(\varphi)_{r=0.5}$ $\theta=0$
$q(\theta)_{r=0.5}$ $\varphi=0$
Source coordinates:
1 7.5; 12.5; 5.0 m
2 7.5; 4.0; 5.0 m
$\alpha=0.6$ 0.6 0.1 0.1
$\varphi=0$ $\theta=0$
$\varphi=\pi$
$-2\,dB$
$q(\varphi)_{r=3.5}$ $\theta=0$
$q(\theta)_{r=3.5}$ $\varphi=0$

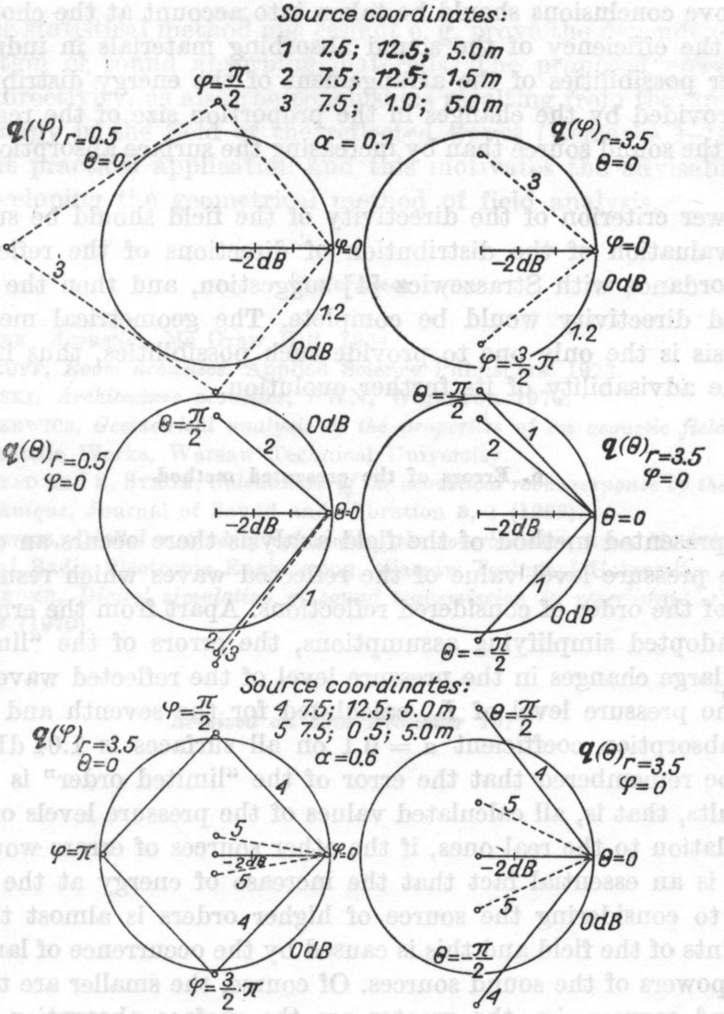


Fig. 6. **The directional characteristics, expressed in terms of logarithmic values of the equivalent source, defining the directional power properties of the field**

(a) The source situated at the centre of symmetry of the space different surface absorption coefficients as stated on projections

(b) The sound source situated at any point of the space enclosing surfaces with equal absorption coefficients $\alpha=0.1$ and $\alpha=0.6$.

(c) The sound source situated at any point of the space enclosing surfaces with different absorption coefficients according to the sketch on the projection

greater changes can be observed on the characteristic $q(\theta)$ than on the characteristic $q(\varphi)$, especially for the source located at the centre of symmetry (Fig. 6a, $q(\varphi)$ and $q(\theta)$).

Remark 16. The change in the directivity power characteristics is the greater the greater is the surface absorption coefficient (Fig. 6b, "1" and "4") as well as the greater are the differences between the coefficient values (Fig. 6a, "2" and "4").

The above conclusions should be taken into account at the choice and evaluation of the efficiency of the sound absorbing materials in industrial halls. E.g. greater possibilities of the arrangement of the energy distribution in the field are provided by the changes in the proportion size of the region and the position of the sound source than by increasing the surface absorption coefficient values.

The power criterion of the directivity of the field should be supplemented with an evaluation of the distribution of directions of the reflected waves, e.g. in accordance with Straszewicz [4] suggestion, and then the information of the field directivity would be complete. The geometrical method of the field analysis is the only one to provide such possibilities, thus it once again justifies the advisability of its further evolution.

6. Errors of the presented method

In the presented method of the field analysis there occurs an error in evaluating the pressure level value of the reflected waves which results from the limitation of the order of considered reflections. Apart from the errors resulting from the adopted simplifying assumptions, the errors of the "limited order" give quite large changes in the pressure level of the reflected waves; the difference in the pressure level of L_{sc} calculated for the seventh and tenth order with the absorption coefficient $\alpha = 0.1$ on all surfaces is 1.04 dB. However, it should be remembered that the error of the "limited order" is common for all the results, that is, all calculated values of the pressure levels of L_{sc} are too small in relation to the real ones, if the other sources of errors would be disregarded. It is an essential fact that the increase of energy at the observation point due to considering the source of higher orders is almost the same for various points of the field and this is caused by the occurrence of large distances and small powers of the sound sources. Of course, the smaller are the "powers" of the sound sources, i.e. the greater are the surface absorption coefficients, the smaller are the errors of the "limited order". For the calculated cases, for which the absorption coefficients are small ($\alpha = 0.1$, $\alpha = 0.6$), these errors are large, but considering their regularity the conclusions drawn from the calculated spatial distribution of the pressure levels of L_{sc} can be considered to be right.

7. Conclusions

The use of computers in the geometrical analysis of acoustic field has enabled us to find the properties of the field which differ from the ones obtained by the statistical method, e. g. the variability of the pressure level of the reflected waves as a function of the distance from the source; also with the use of the statistical method one cannot e. g. prove the dependence of L_{sc} on the distribution of sound absorbing materials. The proposed power criterion of the field directivity, as also the conclusions resulting from the spatial distribution of energy in the field of the reflected waves (Remarks 1–16) can find an important practical application and this motivates the advisability of the work on developing the geometrical method of field analysis.

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