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MULTIPLE-LOUDSPEAKER SYSTEMS

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In this paper it is shown that a multiple-loudspeaker system composed of N similar loudspeakers can be replaced by N independent, similar single-loudspeaker systems. This provides the possibility of applying known methods of analysis and synthesis to multiple-loudspeaker systems. The efficiency of the multiple-loudspeaker system is calculated and measured results of the efficiency are given. Standardized transmittance functions for multiple-loudspeaker systems are discussed. The resonance frequencies of multiple-loudspeaker systems are determined.

Notation

B	induction in the gap of a loudspeaker magnet, T,
c	sound velocity in air, in normal conditions ($c = 345$ m/s),
C_{AB}	acoustic compliance of air in enclosure, m^5/N,
C_{AP}	acoustic compliance of the passive membrane suspension, m^5/N,
C_{AS}	acoustic compliance of the loudspeaker membrane suspension, m^5/N,
e_g	open-circuit output voltage of source, V,
$G(s)$	system transfer function,
i_g	current of source, A,
l	length of winding of the voice coil in the magnetic field, m,
M_{AA}	total acoustic mass of the co-vibrating medium, $\mathrm{kg/m}^4$,
M_{AD}	acoustic mass of the membrane and voice coil of the loudspeaker, $\mathrm{kg/m}^4$,
M_{AP}	acoustic mass of the passive membrane or air in the opening, $\mathrm{kg/m}^4$,
M_{AS}	acoustic mass of the membrane and voice coil together with the co-operating mass of medium, $\mathrm{kg/m}^4$,
M_{AT}	acoustic mass of the membrane and voice coil together with the mass of the co-vibrating medium of one loudspeaker in the multiple-loudspeaker system, $\mathrm{kg/m}^4$,
Q_T	resultant magnification at resonance, considering all resistances,
R_{AB}	acoustic resistance of the loss in the opening or the passive membrane in an enclosure, $\mathrm{kg/m^4s}$,
R_{AL}	acoustic resistance of loss in the gap, $\mathrm{kg/m^4s}$,
R_{AP}	acoustic resistance in the opening or passive membrane, $\mathrm{kg/m^4s}$,
R_{AS}	acoustic resistance of the loss of the suspension of loudspeaker membrane, $\mathrm{kg/m^4s}$,
R_E	resistance of the voice coil of loudspeaker, Ω,
R_g	output resistance of the source, Ω,

S_D	active surface of the loudspeaker membrane, m^2,
$T_S = 1/\omega_S$	time constant of the resonance circuit, s,
U_B	volume velocity of the membrane in an enclosure, m^3/s,
U_D	volume velocity of the loudspeaker membrane, m^3/s,
U_L	volume veolocity in the loss gap, m^3/s,
U_0	resultant volume velocity, m^3/s,
U_P	volume velocity of the passive membrane or air in the opening, m^3/s,
η_0	reference efficiency,
ϱ_0	air density in normal conditions, kg/m^3,
ω_S	resonance frequency of the system, 1/s.

1. Introduction

A set of loudspeakers together with a suitable network in an enclosure of any type represents a *loudspeaker system*. The influence of the enclosure on properties of the system is significant only within the range of low frequencies. At medium and high frequencies the characteristics of the system do not depend essentially on the type of enclosure, but only on the loudspeakers used [3]. Thus the most important problem encountered during the design of a loudspeaker system is to ensure a proper cooperation of the loudspeaker with its enclosure at the lower frequencies.

Known methods of analysis and synthesis of loudspeaker systems (e.g. [5-8] permit to produce systems of various types which satisfy the design criteria with sufficient practical accuracy. In papers devoted to this problem prominence is given to those systems in which only one loudspeaker is operating in a given frequency range. The purpose of this paper is to show that it is also possible to use the methods mentioned above for the analysis and design of multiple-loudspeaker systems.

A multiple-loudspeaker system is produced by placing in any enclosure two or more loudspeakers destined for operation in a given frequency range. The use of different loudspeakers for joint operation is not advisable: if the loudspeakers differ in their resonance frequencies, the resultant characteristics will be worse than the characteristics of a loudspeaker transmitting a broader band. On the other hand, if the sound powers of the loudspeakers differ considerably, then the sound intensity increase will be small in relation to that produced by a loudspeaker of higher power, and the cost of the system will increase considerably.

2. Equivalent diagrams

Fig. 1 shows an equivalent diagram for a multiple-loudspeaker system with the voice coils connected in parallel. No consideration has been given in this diagram to the negligible [9] inductances of voice coils or to the radiation resistances. It is the diagram of a generalized system [2, 5] since it contains all the mechanical and acoustical elements of closed enclosures with an opening with a passive membrane (the membrane which is used in loudspeakers being suspended from a flexible spring in the enclosure opening), with a loss gap, and also infinitely large acoustic baffles. It is possible to obtain any specific system from it by assigning the suitable values to the elements.

Since the loudspeakers are similar, we have

$$R_{E1} = R_{E2} = \ldots = R_{EN} = R_E, \quad C_{AS1} = C_{AS2} = \ldots = C_{ASN} = C_{AS},$$

$$R_{AS1} = R_{AS2} = \ldots = R_{ASN} = R_{AT}, \quad Bl_1 = Bl_2 = \ldots = Bl_N = Bl,$$

$$M_{AT1} = M_{AT2} = \ldots = M_{ATN} = M_{AT}, \quad S_{D1} = S_{D2} = \ldots = S_{DN} = S_D$$

as well as

$$i_{g1} = i_{g2} = \ldots = i_{gN} = i_g/N, \quad U_{D1} = U_{D2} = \ldots = U_{DN} = U_D/N.$$

It is thus possible to represent the multiple loudspeaker, shown in Fig. 1, in the form of N independent, similar single-loudspeaker systems. Fig. 2 shows the equivalent diagram of a single-loudspeaker system obtained this way.

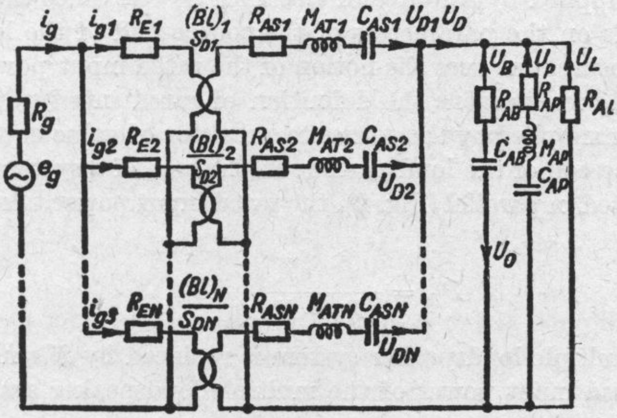

Fig. 1. Equivalent diagram of a generalized loudspeaker system with the loudspeakers connected in parallel

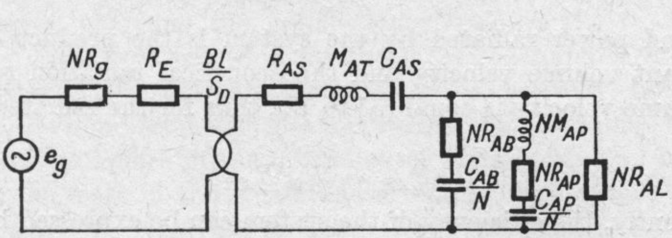

Fig. 2. Equivalent diagram of the single-loudspeaker system obtained from a multiple-loudspeaker system

The acoustic equivalent diagram of a multiple-loudspeaker system reduced to a single-loudspeaker system is shown in Fig. 3.

This reasoning also applies to the analysis of systems with loudspeakers connected in series or in a mixed manner.

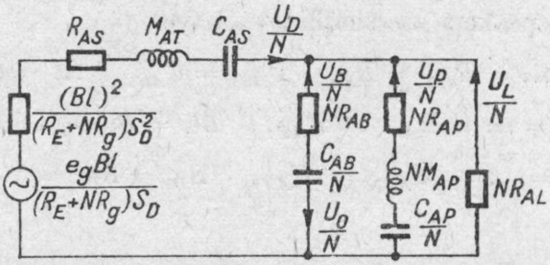

Fig. 3. Acoustic equivalent diagram of a generalized multiple-loudspeaker system with the loudspeakers connected in parallel

3. Efficiency

The *efficiency* of a loudspeaker system is the ratio of the radiated sound power to that supplied by a source of electrical power. Since the electric power supplied depends on the parameters of the source and of the loudspeaker and also on the enclosure, one uses the notion of the rated input power of the loudspeaker. For practical reasons the definition of rated input power accepted is that of the power supplied by the source to a resistor of value equal to the resistance of the voice coil of the loudspeaker. In the case of a system with the loudspeakers connected in parallel (Fig. 2), the rated input power takes the following form:

$$P_R = \frac{e_g^2}{(R_E+NR_g)^2} R_E. \tag{1}$$

Since the multiple-loudspeaker system is replaced by N single-loudspeaker systems, the rated input power of the multiple-loudspeaker system is N times higher:

$$P_{RN} = N\frac{e_g^2}{(R_E+NR_g)^2} R_E. \tag{2}$$

The sound power radiated by the system is the product of the square of the resultant volume velocity and the acoustical radiation resistance. The resultant volume velocity is equal [5] to U_0, thus for the radiated sound power we have

$$P_{AR} = |U_0|^2 R_{AR}. \tag{3}$$

Consequently, the efficiency of the system can be expressed by the relation

$$\eta = \frac{P_{AR}}{P_{RN}} = \frac{R_{AR}(R_E+NR_g)^2}{NR_E}\,\frac{|U_0|^2}{e_g^2}. \tag{4}$$

As has been shown in [1], the acoustic resistance of loudspeaker radiation in the listening room is equal to the resistance of a piston vibrating in an infinite acoustic baffle and it amounts to

$$R_{AR} = \frac{\varrho_0 \omega^2}{2\pi c}. \tag{5}$$

From the equivalent diagram of Fig. 3 we can evaluate U_0/N:

$$\frac{U_0}{N} = \frac{e_g Bl}{(R_E + NR_g) S_D j\omega M_{AT}} G(j\omega). \tag{6}$$

The function $G(j\omega)$, which represents a normalized transmittance for the system, corresponds to the transmittance function of a high-pass filter, of an order and type dependent on the type of enclosure. Equation (6) is at the same time a formula of definition for the function $G(j\omega)$. Substituting equations (5) and (6) into equation (4) we obtain the formula for the frequency efficiency characteristics of the system:

$$\eta = \frac{\varrho_0 (Bl)^2 N}{2\pi c R_E S_D^2 M_{AT}^2} |G(j\omega)|^2. \tag{7}$$

Over the frequency range, for which $|G(j\omega)| = 1$, the efficiency of loudspeaker systems does not depend on the enclosure and represents the reference efficiency:

$$\eta_0 = \frac{\varrho_0 (Bl)^2 N}{2\pi c R_E S_D^2 M_{AT}^2}. \tag{8}$$

This formula does not contain either the acoustic compliance of the suspension C_{AS} or the acoustic loss resistance of the membrane suspension R_{AS} – parameters which show the greatest scatter during measurements of various loudspeakers of the same type. On the other hand, these elements have an effect on the standardized transmittance function in the proximity of the resonance frequencies of the loudspeakers and on the resonance frequencies proper, without, however, changing the limit value $|G(j\omega)| = 1$.

As can be seen from equation (8) the increased number of loudspeakers makes the system more efficient. This increase is, however, not proportional to N, since as the number of loudspeakers increases so also does the mass M_{AT} which appears as a square in the denominator of the formula. The mass M_{AT} is the sum of the mass of the membrane of one loudspeaker M_{AD} and of the mass of the whole co-vibrating medium M'_{AA} referred to one loudspeaker:

$$M_{AT} = M_{AD} + NM'_{AA}. \tag{9}$$

The mass of the co-vibrating medium is inversely proportional to the square root of the surface area of the vibrating piston [10],

$$M_{AA} = \frac{k}{\sqrt{S_D}}, \tag{10}$$

k being the coefficient of proportionality.

Since the multiple-loudspeaker system contains N pistons with a total surface NS_D, the total acoustic mass of the co-vibrating medium M'_{AA} is expressed by the formula

$$M'_{AA} = \frac{k}{\sqrt{NS_D}} = \frac{M_{AA}}{\sqrt{N}}. \tag{11}$$

Introducing (11) into (9) we obtain an expression defining the acoustic mass of a loudspeaker operating in a multiple-loudspeaker system:

$$M_{AT} = M_{AD} + \sqrt{N}\, M_{AA}. \tag{12}$$

If the system contains only one loudspeaker, the acoustic mass is the mass M_{AS} and amounts to

$$M_{AS} = M_{AD} + M_{AA}. \tag{13}$$

Thus the acoustic mass M_{AT} of a multiple loudspeaker, expressed in terms of the acoustic mass of a single-loudspeaker system M_{AS}, is described by the relation

$$M_{AT} = M_{AS} + (\sqrt{N} - 1) M_{AA}. \tag{14}$$

It can be seen that the co-vibrating mass of each loudspeaker in the multiple loudspeaker is higher than the co-vibrating mass of the single-loudspeaker system.

Another limitation on the increase of efficiency of a multiple-loudspeaker system can be accounted for by the fact that increasing the number of loudspeakers necessarily increases the distance between them.

In order to determine the real dependence of the reference efficiency of the multiple loudspeaker on the number of loudspeakers, measurements were carried out. The sound power radiation by a loudspeaker system with one, two and three loudspeakers was measured in an anechoic chamber, the electric power supplied to the loudspeakers being in all cases constant. The power measurement was performed by an intermediate method of measuring the acoustic pressure on a hypothetical spherical surface, at the centre of which the system was located. The measurements indicate that the mean ratio of the efficiency of systems with one, two and three loudspeakers in the frequency range up to 1000 Hz is

$$\eta_1 : \eta_2 : \eta_3 = 1 : \sqrt{2} : 2, \tag{15}$$

the scatter of individual results not exceeding 10%.

It was also found that the mutual arrangement of the loudspeakers on the front plate of an enclosure of dimensions 0.32 m × 0.54 m did not affect the accuracy of the proportion (15).

SATHYANARAYANA [4] has carried out similar investigations by measuring the efficiency of a column speaker composed of loudspeakers with individual enclosures. Although the measurements were made only at the resonance frequency, his results were in agreement with relation (15).

Thus, the expressions describing the efficiency of the loudspeaker system with the number of loudspeakers $N \leqslant 3$ can take the following form:

$$\eta_0 = 2^{(N-1)/2} \frac{\varrho_0 (Bl)^2}{\pi c R_E S_D^2 M_{AS}^2}. \tag{16}$$

It should be noted that in this relation it was possible to replace the heavily determinable M_{AT} by M_{AS}, thus considerably simplifying the measurements.

The efficiency of loudspeaker systems analyzed here pertain to loudspeakers with the voice coils connected in parallel. A similar procedure for different connections of the voice coils leads to an identical relationship for the reference efficiency, but the functions $G(j\omega)$ are different.

4. Standardized transmittance function

As has been shown in section 2, a multiple-loudspeaker system can be reduced to a single-loudspeaker system with corresponding parameters. The reference efficiency of these systems does not depend on the method of connecting the loudspeakers while the transmittance functions, although of a similar type, have different coefficients.

For loudspeakers, placed in an infinite acoustic baffle (Fig. 4), the transmittance function takes the form

$$G(s) = \frac{s^2 T_S^2}{s^2 T_S^2 + \dfrac{s T_S}{Q_T} + 1}, \tag{17}$$

with $T_S = 1/\omega_S$ being the time constant of the resonant circuit.

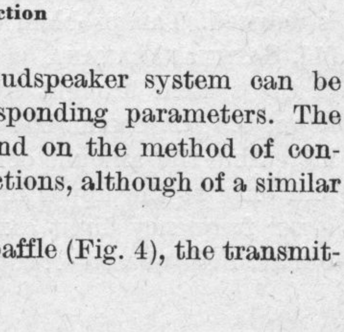

Fig. 4. Equivalent diagram of a multiple-loudspeaker system in an infinite acoustic baffle

The resultant magnification factor Q_T is: for parallel connection

$$Q_{TR} = \omega_S^{-1} C_{AS}^{-1} \left[R_{AS} + \frac{(Bl)^2}{(R_E + NR_g) S_D^2} \right]^{-1}, \tag{18}$$

for series connection

$$Q_{TS} = \omega_S^{-1} C_{AS}^{-1} \left[R_{AS} + \frac{(Bl)^2}{\left(R_E + \frac{R_g}{N} \right) S_D^2} \right]^{-1}. \tag{19}$$

A comparison of formulae (18) and (19) shows that the series connection ensures a better attenuation of the system if $R_g > 0$.

The resonance frequency of the loudspeaker system is expressed by the relation

$$\omega_S' = \frac{1}{\sqrt{C_{AS} M_{AT}}}. \tag{20}$$

Since the mass of the co-vibrating medium increases with the number of cooperating loudspeakers, the resonance frequency of the multiple loudspeaker is reduced. This problem was investigated by SATHYANARAYANA [4] and SMALL [7]. SATHYANARAYANA has shown that the reduction of the resonant frequency of the system brought by the increase of co-vibrating mass did not exceed about 0.5 Hz. SMALL has calculated that the increase of the mass, caused by increasing the amount of the co-vibrating medium, varies from about 2 to 8% and only at the upper limit of this increase an insignificant reduction of the lower frequency limit can be observed. Similar results may be obtained from our estimates based on equation (14). Hence, one can assume that $\omega_S' = \omega_S$.

5. Power limitations

At high frequencies the permissible electric power of the system is limited by the ability to dissipate the heat through voice coils of loudspeakers. The permissible power $P_{E(\max)}$ of the whole system represents the sum of the powers $P_{E(\max)}$ of the loudspeakers included in the system. At low frequencies it is necessary to limit the deflections of the membrane of the loudspeaker so that it does not produce excessive nonlinear distortions. If the permissible electric power as governed by the membrane deflection in one loudspeaker is P_{ER}, then a power N times higher can be supplied to a system consisting of N loudspeakers. However, the maximum sound power will be more than N times higher than the maximum sound power of single-loudspeaker system as a result of the efficiency increase.

6. Conclusion

This paper presents an analysis of loudspeaker systems based on the transformation of a system composed of N similar loudspeakers into N independent, similar single loudspeakers and thus utilizes the methods of analysis of single-loudspeaker systems known from the literature. It was found that the efficiency of the multiple loudspeaker increases with an increasing number of loudspeakers. However, this increase is not proportional to N, because, among other reasons, of the increasing air mass which co-vibrates with the loudspeakers in the multiple-loudspeaker system. The relationship determining the increase of the vibrating mass of a loudspeaker, operating in a multiple-loudspeaker system in relation to the vibrating mass of a single loudspeaker, has been described. The coefficients defining the increase of the efficiency for one, two or three loudspeakers have been found empirically. It was found that the decrease of the resonance frequency, caused by the increase of the co-vibrating mass, is negligible.

The method of the approach to the problem of analysis stated in this paper is based on a transformation into equivalent single-loudspeaker systems. It can be used in the process of synthesis of multiple-loudspeaker systems of various types [3].

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