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THE EQUIVALENT AREA AND ACTIVE MASS OF MICROPHONE MEMBRANES

RUFIN MAKAREWICZ, JACEK KONIECZNY

Chair of Acoustics, A. Mickiewicz University (Poznań)

The paper presents a proposed method for determining the equivalent area and active mass of microphone membranes. It is based on an energetic definition of these parameters and assumes a flat piston. It requires a knowledge of the phase distribution and also of the displacement amplitude and velocity over the whole surface of the real membrane. The methodology of measuring these quantities is the subject of a separate publication. In these considerations the effect of the wavelength, the direction of incidence and the effect of the membrane shape on the active parameters are taken into account. Indirectly, the manner of mounting the membrane at its border has also been considered.

1. Introduction

The equivalent diagrams of electroacoustic transducers are generally constructed in the form of analogue systems with lumped parameters. For the analysis of microphones the active parameters of the membrane are determined by means of lumped parameters, including the equivalent area and the active mass.

In this paper analytical relations are derived which permit the determination of both parameters assuming that the shape of the membrane, its surface density, the phase, displacement amplitude and velocity distributions are known.

The theoretical arguments used to obtain the relations mentioned above were based on the equivalence of the work of the acoustic field and the kinetic energy of the real membrane, assuming a flat piston. This approach is not new but, to the authors' knowledge, the equivalent area and the active mass have not been consistently defined up to the present time, according to the assumed equivalence with the simultaneous assumption that the phase distribution, the displacement amplitudes and velocities are given *a priori*. (It is assumed very frequently for example that the equivalent area of a membrane is its cross-section in a plane normal to the axis of symmetry).

2. The equivalent area of a membrane

The *equivalent area* S_c of a membrane is the area of a vibrating flat piston with an amplitude of vibration (A_m) equal to the maximum amplitude of displacement of the membrane, normal to its surface. It is of such an area that the mechanical force exerted upon it by an evenly distributed acoustic pressure field does the same work as the forces actually acting upon the membrane.

This definition differs from the definition of the equivalent area formulated by Żyszkowski [5] in that instead of the formulation «equal to the maximum displacement of the membrane normal to its surface ...» there is the expresion «... equal to the amplitude at the centre of the membrane». This difference can be explained by the fact that the latter considers a particular case of a vibrating membrane: that for which the amplitude decreases with distance from the axis of symmetry of the membrane. In the general case which will be considered in this paper, the maximum normal displacement amplitude need not occur at the centre of the membrane.

The proposed definition uses the normal component of the displacement amplitude because, when the viscosity of the air is neglected, the work to be performed by the tangential component of the displacement of the membrane is equal to zero.

It is assumed that the shape of the fixed membrane is described by the function

$$z = f(r, \varphi), \tag{1}$$

and the orientation of any element δS of the membrane is determined by the angle of inclination $\alpha(r, \varphi)$ of the tangent at the point (r, φ, z) (Fig. 1).

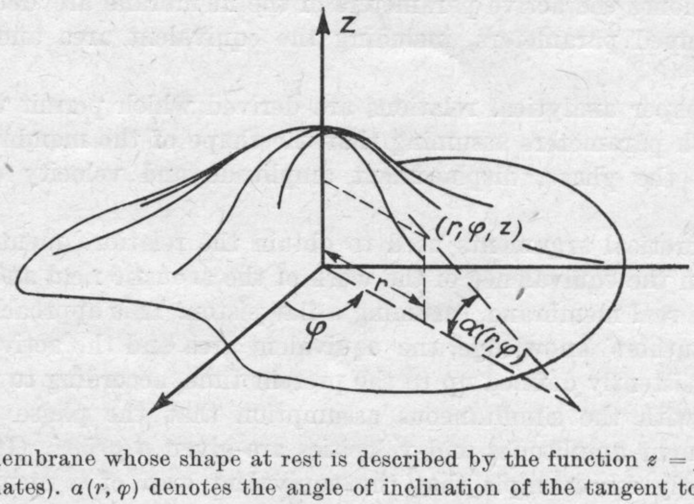

Fig. 1. The membrane whose shape at rest is described by the function $z = f(r, \varphi)$ (in cylindrical coordinates). $\alpha(r, \varphi)$ denotes the angle of inclination of the tangent to the membrane at the point (r, φ, z)

The border of the membrane will be assumed to be in the plane (r,φ) and is described by $r = R(\varphi)$ (Fig. 2). This delineates the boundary of the area defined by the function z (formula (1)).

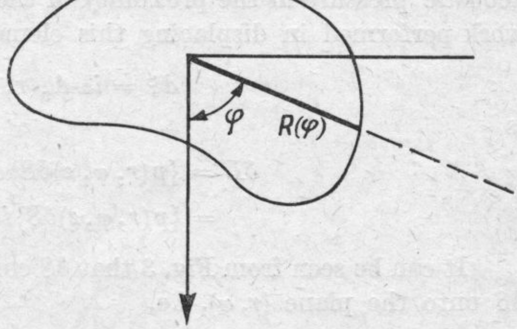

Fig. 2. The border of the membrane in the plane (r, φ) described by the function $r = R(\varphi)$

Let us calculate the work performed by the vibrating membrane. In the general case the displacement amplitude A of the element δS depends on its position: $A = A(r, \varphi)$. (The dependence on z can be neglected because of the dependence $z \leftrightarrow (r, \varphi)$ via formula (1).)

If we neglect the influence of the viscosity of the air, then the force acting on the element δS will only be related to the normal component of the deflection amplitude $A_n(r, \varphi)$ (Fig. 3).

When a wave of angular frequency ω falls onto the membrane, then the instantaneous value of the displacement of the element δS from its equilibrium

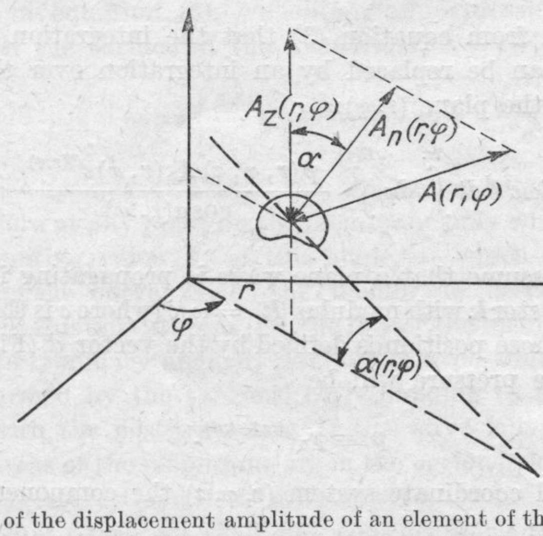

Fig. 3. Components of the displacement amplitude of an element of the membrane δS located at the point (r, φ, z), in the normal direction $A_n(r, \varphi)$ and in a direction $A_z(r, \varphi)$ parallel to the z-axis

position in a normal direction (Fig. 3) is a function of the form

$$\xi = A_n(r, \varphi)\, e^{i\{\omega t + \theta(r,\varphi)\}},$$

where $\theta(r, \varphi)$ defines the phase of the displacement of the element δS. If the acoustic pressure in the proximity of the element δS is denoted by p, then the work performed in displacing this element through a distance

$$d\xi = i\omega A_n(r, \varphi)\, e^{i\{\omega t + \theta(r,\varphi)\}}$$

is

$$\begin{aligned}\delta L &= \{p(r, \varphi, z)\delta S\} d\xi = \\ &= \{p(r, \varphi, z)\delta S\} i\omega A_n(r, \varphi)\, e^{i\{\omega t + \theta(r,\varphi)\}}\end{aligned} \tag{2}$$

It can be seen from Fig. 3 that $\delta S \cos a(r, \varphi)$ is the projection of the surface δS onto the plane (r, φ), i.e.

$$\delta S \cos a(r, \varphi) = r\, dr\, d\varphi,$$

whence

$$\delta S = \frac{r\, dr\, d\varphi}{\cos a(r, \varphi)}. \tag{3}$$

The work dL done by forces acting on the whole membrane during a time dt is the sum of the components δL obtained for all the individual elements δS. Thus expression (2) must be integrated over the whole surface of the membrane S:

$$dL = i\omega e^{i\omega t} dt \iint_S p(r, \varphi, z) A_n(r, \varphi)\, e^{i\theta(r,\varphi)} \delta S.$$

It can be seen from equation (3) that the integration over the surface of the membrane can be replaced by an integration over the area which is its projection onto the plane (r, φ):

$$dL = i\omega e^{i\omega t} dt \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{p(r, \varphi, z) A_n(r, \varphi)\, e^{i\theta(r,\varphi)}}{\cos a(r, \varphi)}\, r\, dr. \tag{4}$$

If we further assume that a plane wave is propagating in a direction defined by the wave vector $\boldsymbol{k}$ with modulus $|\boldsymbol{k}| = \omega/c$ (where c is the sound velocity), then at the point whose position is defined by the vector $\boldsymbol{d}$ (Fig. 4) the instantaneous value of the pressure will be

$$p = p_0 e^{i\{\omega t + \boldsymbol{k}\boldsymbol{d}\}}. \tag{5}$$

In a cylindrical coordinate system (r, φ, z) the components of the vector $\boldsymbol{d}$ are the following:

$$d_x = r\cos\varphi, \quad d_y = r\sin\varphi, \quad d_z = z.$$

Using the notation of Fig. 4 the components of the vector $\boldsymbol{k}$ take the form

$$k_x = \frac{\omega}{c} \sin\delta \cos\gamma, \quad k_y = \frac{\omega}{c} \sin\delta \sin\gamma, \quad k_z = \frac{\omega}{c} \cos\delta.$$

Using the definition of the scalar product $\boldsymbol{ab} = a_x b_x + a_y b_y + a_z b_z$ we obtain

$$\boldsymbol{kd} = \frac{\omega}{c} (r\cos\varphi \sin\delta \cos\gamma + r\sin\varphi \sin\delta \sin\gamma + z\cos\delta)$$

$$= \frac{\omega}{c} \{r\sin\delta \cos(\varphi - \gamma) + z\cos\delta\}.$$

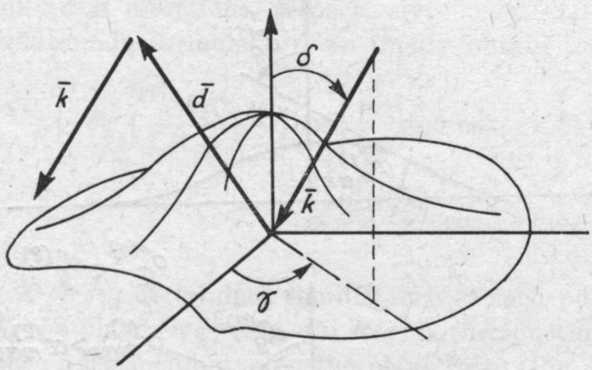

Fig. 4. The angles γ and δ describing the direction of propagation of the plane wave, i.e. the direction of the wave vector $\boldsymbol{k}$

Substituting in equation (5), we obtain an expression for the acoustic pressure acting on the surface of the membrane, $z = f(r, \varphi)$, as

$$p(r, \varphi) = p_0 e^{i\omega t} \exp i \left\{ \frac{\omega r}{c} \sin\delta \cos(\varphi - \gamma) + \frac{\omega f(r, \varphi)}{c} \cos\delta \right\}. \tag{6}$$

It can be seen from Fig. 4 that this expression gives the magnitude of the acoustic pressure at any point on the membrane only when the angle δ is not too large. The limiting value δ_g of this angle, at which equation (6) is still valid, depends on the curvature of the membrane described by the angle $\alpha(r, \varphi)$. The smaller this curvature is (i.e. the flatter the membrane), then greater is the value of the boundary angle δ_g (Fig. 5). It is possible to define δ_g as the angle which is formed by the tangent, corresponding to the maximum value of the angle α, with the positive z-axis. If the wave falls at an angle greater than δ_g, certain areas of the membrane are in the «geometrical shade». To define the value of the pressure there, it is necessary to use the complicated diffraction theory. In this paper we shall not consider such conditions, assuming that the angle of incidence is always smaller than the boundary angle δ_g.

Returning to equation (4), describing the work performed by the real membrane in a time dt, we may substitute the pressure $p(r, \varphi)$ given by equation (6) and obtain

$$dL = i\omega e^{2i\omega t} dt \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{A_n(r, \varphi)}{\cos\alpha(r, \varphi)} \exp i\left\{\omega r/c \sin\delta \cos(\varphi - \gamma) + \frac{\omega f(r, \varphi)}{c} \cos\delta + \theta(r, \varphi)\right\} r\,dr. \quad (7)$$

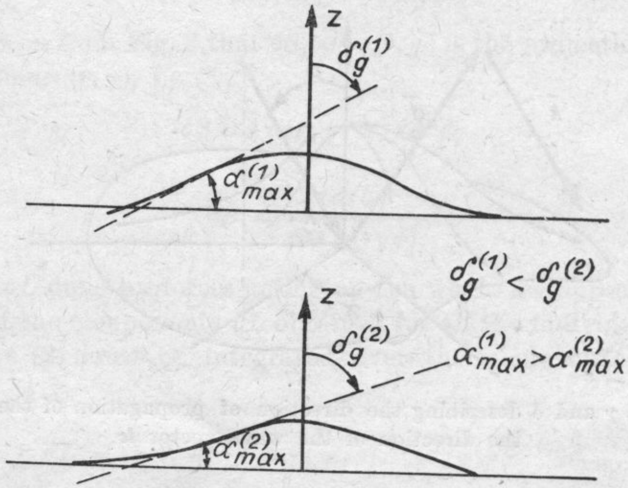

Fig. 5. The effect of the boundary angle δ_g on the maximum value of the angle of inclination of the tangent to the membrane $\alpha_{\max}$

In view of the definition accepted for the equivalent area, we calculate in turn the work performed by the moving flat piston. It will be assumed that this piston has an area S_c (this is the equivalent area we wish to evaluate) and vibrates with an amplitude equal to the maximum displacement amplitude A_m under the action of the incident plane wave of amplitude p_0.

The instantaneous value of the acoustic pressure on its surface is $p = p_0 e^{i\omega t}$, and the instantaneous value of the displacement is $\eta = A_m e^{i\omega t}$. The work done by the forces acting on the piston in time dt is

$$dL = \{p S_c\} d\eta = i\omega p_0 e^{2i\omega t} S_c A_m. \quad (8)$$

According to the definition accepted for the equivalent area S_c, this work should be equal to the work performed by the forces acting on the membrane.

From (7) and (8) we obtain

$$S_c = \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{A_n(r,\varphi)}{A_m \cos\alpha(r,\varphi)} \cos\left\{\frac{\omega r}{c}\sin\delta\cos(\varphi-\gamma) + \frac{\omega f(r,\varphi)}{c}\cos\delta + \theta(r,\varphi)\right\} r\,dr.$$

It can be seen from Fig. 3 that

$$A_n(r,\varphi) = A_z(r,\varphi)\cos\alpha(r,\varphi), \tag{10}$$

where $A_n(r,\varphi)$ and $A_z(r,\varphi)$ denote components of the displacement amplitude in the normal and z-axis directions, respectively.

Using this equation in formula (9), we finally obtain

$$S_c = \frac{1}{\cos\alpha(r_0,\varphi_0)} \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{A_z(r,\varphi)}{A_z(r_0,\varphi_0)} \cos\left\{\frac{\omega r}{c}\sin\delta\cos(\varphi-\gamma) + \frac{\omega f(r,\varphi)}{c}\cos\delta + \theta(r,\varphi)\right\} r\,dr. \tag{11}$$

In this formula $f(r,\varphi)$ determines the distance of each point of the membrane at rest from the plane (r,φ) (Fig. 1); $R(\varphi)$ is the function describing the shape of the border of the membrane on the plane (r,φ) (Fig. 2). The angles δ and γ determine the direction of the incident wave (Fig. 4), and $A_z(r,\varphi)$ denotes the amplitude of deflection of the membrane in the direction of z-axis, measured in the neighbourhood of any point of the membrane (r,φ). $A_z(r_0,\varphi_0)$ and $\alpha(r_0,\varphi_0)$ denote, respectively, the deflection amplitude in the direction of z-axis and the angle of inclination of the element δS (Fig. 3) in the proximity of the point which is vibrating with maximum amplitude in the normal direction A_m. Thus, assuming that $A_z(r,\varphi)$ is measurable, in order to determine the coordinates of the point (r_0,φ_0) and, consequently, $A_z(r_0,\varphi_0)$ and $\alpha(r_0,\varphi_0)$, it is necessary to use equation (10), since only in this way we can find the maximum value of the normal component A_m. Equation (11) contains the quantity $\theta(r,\varphi)$ which denotes the phase of vibration of individual points of the membrane.

In order to define the equivalent area of the membrane, induced to vibrate by a plane wave of angular frequency ω, which falls onto the membrane from a direction determined by the angles (γ,δ) (Fig. 4), we must know the distribution of vibration amplitudes $A_z(r,\varphi)$ and phases $\theta(r,\varphi)$ at each point of the membrane, whose shape is described by the functions $f(r,\varphi)$ and $R(\varphi)$ (the angle $\alpha(r,\varphi)$ can be determined from $f(r,\varphi)$).

In the particular case of the membrane having axial symmetry or its being excited by a plane wave propagating along the axis of symmetry, knowledge of the phases is superfluous. It can be assumed that $\theta = 0$ for each point, i.e. that the instant value of the displacement is

$$\xi = A(r)e^{i\omega t}.$$

If the axis of symmetry of the membrane coincides with z-axis (Fig. 6), then $\delta = 0$ and from (11) we obtain

$$S_c = \frac{2\pi}{\cos\alpha(r_0)}\int_0^R \frac{A_z(r)}{A_z(r_0)}\cos\left\{\frac{\omega f(r)}{c}\right\} r\,dr, \tag{12}$$

where R is the radius of the circle constituting the border of the membrane, $f(r)$ – the function describing its shape in the plane (r, z), and r_0 – the point at which the values of the normal component $A_n(r_0) = A_z(r_0)\cos\alpha(r_0)$ is the greatest ($\alpha(r_0)$ is the angle of inclination of the tangent at the point r_0).

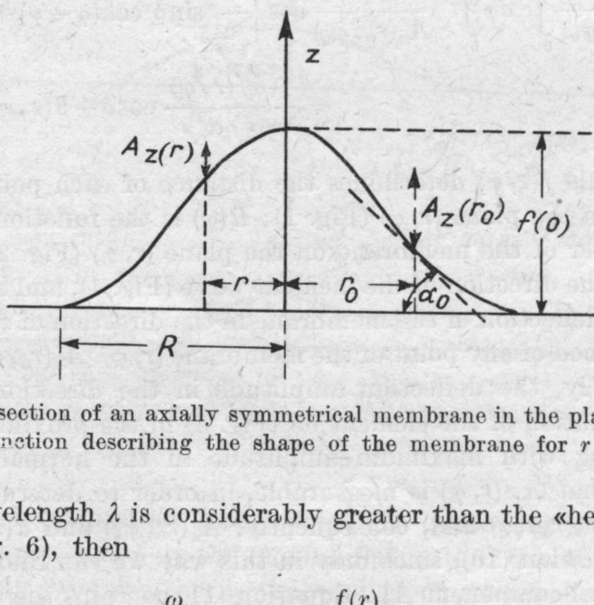

Fig. 6. The cross-section of an axially symmetrical membrane in the plane (r, z); $f(0)$ is the function describing the shape of the membrane for $r = 0$

If the wavelength λ is considerably greater than the «height» of the mem brane $f(0)$ (Fig. 6), then

$$\frac{\omega}{c}f(r) = 2\pi\frac{f(r)}{\lambda} \ll 1.$$

In this case, after a series expansion of the function $\cos\{\omega f(r)/c\}$, it follows from (12) that

$$S_c \approx \frac{2\pi}{\cos\alpha(r_0)}\int_0^R \frac{A_z(r)}{A_z(r_0)} r\,dr - \frac{\pi}{\cos\alpha(r_0)}\int_0^R \left\{\frac{\omega f(r)}{c}\right\}^2 \frac{A_z(r)}{A_z(r_0)} r\,dr, \tag{13}$$

he second term being considerably smaller than the first.

If the maximum value of the amplitude occurs at the centre of the membrane ($r_0 = 0$), then $\cos\alpha(r_0) = 1$ and we can make the approximation

$$S_c \approx 2\pi \int_0^R \frac{A_z(r)}{A_z(0)}\, r\,dr. \tag{14}$$

As was stated at the beginning, it is very often assummed that the equivalent area of the membrane S_c is equal to the area of its projection onto the plane (r, φ), i.e. $S_c = \pi R^2$ (Fig. 6). From the relations which have been derived from the definition of the equivalent area, on the assumption of the equivalence of the work performed by the real membrane and a flat piston, it can be seen that the equivalent surface cannot be equal to the projected one. For instance, in considering equation (13) we see that $S_c = \pi R^2$ only when the membrane is flat ($f(r) = 0$), and each point is vibrating with the same amplitude $A(r) =$ const. It is obvious that the last condition cannot be satisfied because of the necessity of mounting the membrane by its edges.

3. The active mass of the membrane

It is assumed that the active mass m_c of the membrane is that mass which, when vibrating with a velocity amplitude equal to the maximum velocity amplitude of the membrane v_m, has a kinetic energy equal to the total kinetic energy of the membrane.

Similarly as in the case of the equivalent area, this definition differs from the definition stated in the monograph of ŻYSZKOWSKI [4] in that it does not give the centre of the membrane the value v_m because in the general case the maximum velocity amplitude can be recorded at any point.

If the instantaneous value of the velocity of vibration of the element δS is

$$v = v(r, \varphi)\, e^{i\{\omega t + \Psi(r,\varphi)\}},$$

then the time average of the kinetic energy is equal to

$$\delta E_k = \frac{1}{4}\, \varrho(r, \varphi)\, \delta S\, v^2(r, \varphi),$$

where $\varrho(r, \varphi)$ is the surface density of the membrane.

Summing over the whole surface S, we obtain the total kinetic energy of the membrane:

$$E_k = \frac{1}{4} \iint_S \varrho(r, \varphi)\, v^2(r, \varphi)\, \delta S.$$

Using equation (3) we subsequently obtain

$$E_k = \frac{1}{4} \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{\varrho(r, \varphi)\, v^2(r, \varphi)}{\cos\alpha(r, \varphi)}\, r\,dr. \tag{15}$$

Let us assume that the velocity amplitude $v(r_0, \varphi_0)$ at the point $(r_0, {}_0\varphi)$ is the maximum. According to the definition of the active mass m_c, for the mean kinetic energy we have

$$E_k = \frac{1}{4} m_c v^2(r_0, \varphi_0). \tag{16}$$

From equations (15) and (16) we get

$$m_c = \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{\varrho(r, \varphi) v^2(r, \varphi)}{\cos\alpha(r, \varphi) v^2(r_0, \varphi_0)} r dr. \tag{17}$$

In this expression $\varrho(r, \varphi)$ denotes the surface density of the membrane which, in general, may change from point to point; $R(\varphi)$ is the function describing the shape of the border of the membrane (Fig. 2), while $v(r_0, \varphi_0)$ and $v(r, \varphi)$ denote the maximum velocity amplitude and the velocity amplitude of the membrane vibrations, measured in the proximity of any point (r, φ). The methods of measuring these quantities and the displacement amplitude will be presented in a subsequent paper.

For the particular case of axial symmetry and a homogeneous membrane (Fig. 6) we have from equation (17),

$$m_c = 2\pi\varrho \int_0^R \frac{v^2(r)}{\cos\alpha(r) v^2(r_0)} r dr. \tag{18}$$

If the angle of inclination of the tangent to the surface of the membrane $\alpha(r)$ is not too great (i.e. membrane is almost flat), then using the series expansion

$$\frac{1}{\cos\alpha(r)} = 1 + \frac{1}{2} \alpha^2(r) + \ldots$$

the right-hand side of equation (18) can be expanded as a series

$$m_c = 2\pi\varrho \int_0^R \frac{v^2(r)}{v^2(r_0)} r dr + \pi\varrho \int_0^R \alpha^2(r) \frac{v^2(r)}{v^2(r_0)} r dr + \ldots$$

It can be seen that in this case it is not necessary to know the geometry of the membrane (the function $f(r)$) in order to define the active mass m_c, since the succeeding terms of the series are very small in comparison with the first.

Similarly as in the case of the equivalent area, we can see from the foregoing equations that the active mass m_c can be equal to the mass of the membrane only when the whole surface of the membrane is vibrating with the same amplitude ($v(r, \varphi) = \text{const}$). Such a case is excluded because of the necessity of mounting the membrane by its border.

4. The equivalent area in a diffuse field

Formula (11), defining the equivalent area of a membrane in a plane wave field, is based on the assumption that the wave falls at an angle δ (Fig. 4) which is smaller than the boundary angle δ_g(Fig. 5). This formula is in agreement with the definition of the equivalent area S_g. In this section a formula describing the active surface of a membrane in a diffuse field will be derived. It differs somewhat from the definition of S_c accepted above as regards both its form and its generality (compared with the formula (11)), and is therefore derived separately.

From the assumed diffusivity of the field, i.e. the probability that a plane wave is incident on any element of the membrane δS in a direction determined by the angles (γ, δ) (Fig. 4), remaining at the same time within the solid angle

$$d\Omega = \sin\delta\, d\delta\, d\gamma,$$

is equal to

$$dp(\gamma, \delta) = \frac{d\Omega}{2\pi} = \frac{\sin\delta\, d\delta\, d\gamma}{2\pi}. \tag{19}$$

In this case 2π is the value of the solid angle which fills the half-space $z > 0$.

It can be assumed that the equivalent area of the membrane S_c within the field of the plane wave whose direction of propagation is determined by the angles (γ, δ) is a random variable of these two quantities. This means that the mean value S_c (the expected value) at any moment is equal to

$$\bar{S}_c = \iint\limits_{2\pi} dp(\gamma, \delta) S_c(\gamma, \delta).$$

Substituting for S_c and dp from formulae (11) and (19), and subsequently integrating with respect to γ and δ so as to cover the whole of the solid angle 2π, we obtain

$$S_c = \frac{1}{\cos\alpha(r_0, \varphi_0)} \int_0^{2\pi} d\varphi \int_0^{R(\varphi)} \frac{A_z(r, \varphi)}{A_z(r_0, \varphi_0)} H(r, \varphi) r\, dr, \tag{20}$$

where

$$H(r, \varphi) = \int_0^{\pi/2} \sin\delta\, d\delta \int_0^{2\pi} \cos\left\{\frac{\omega r}{c} \sin\delta \cos(\varphi - \gamma) + \frac{\omega f(r, \varphi)}{c} \cos\delta + \theta(r, \varphi)\right\} d\gamma. \tag{21}$$

In assuming the limits of integration for $\delta(0$, to $\pi/2)$, and for $\gamma(0$ to $2\pi)$, we introduce some error since waves from all possible directions can reach only the elements δS located on the top of the membrane. For any other location there exist values of the angles γ and δ for which the corresponding plane waves do not reach the element δS. For example, points in the «geometrical shade» (Fig. 5) have waves corresponding to values of the angle δ greater than δ_g.

In order to have a convenient form of the formula determining the mean value of the active surface S_c, we assume the integration limits of equation (21). It is, however, to note that this formula is valid only if the membrane is not too convex, i.e. if $\delta_g \to \pi/2$ (Fig. 5).

It we subsequently assume that the membrane has the axial symmetry and that its border is a circle of radius R lying in the plane (r, φ) (Fig. 6), we may expect that in a diffuse field the phase of vibration θ of all points of the membrane will be equal (assuming, as in section 2, that $\theta = 0$).

The integration relative to γ will be carried out using the identity $\cos\alpha = \frac{1}{2}(e^{i\alpha}+e^{-i\alpha})$:

$$\int_0^{2\pi} \exp\left\{\pm i\frac{\omega r}{c}\sin\delta\cos(\gamma-\varphi)\right\}d\gamma = 2\int_0^{\pi}\exp\left\{\pm i\frac{\omega r}{c}\cos\gamma\right\}d\gamma.$$

According to WHITTAKER [2]

$$\int_0^{\pi} e^{\pm iz\cos\gamma}d\gamma = \pi J_0(z),$$

where $J_0(z)$ is a zero-order Bessel function. Using these equalities we obtain

$$\int_0^{2\pi}\cos\left\{\frac{\omega r}{c}\sin\delta\cos(\varphi-\gamma)+\frac{\omega f(r)}{c}\cos\delta\right\} = 2\pi\cos\left\{\frac{\omega f(r)}{c}\cos\delta\right\}J_0\left\{\frac{\omega r}{c}\sin\delta\right\}.$$

Thus, the function H in (21) (with $\theta = 0$ and the axial symmetry assumed for the membrane) has the form

$$H(r) = 2\pi\int_0^{\pi/2}\sin\delta\cos\left\{\frac{\omega f(r)}{c}\cos\delta\right\}J_0\left(\frac{\omega r}{c}\sin\delta\right)d\delta.$$

Substituting $\cos\delta = x$, we obtain

$$H(r) = 2\pi\int_0^{1}\cos\left\{\frac{\omega f(r)}{c}x\right\}J_0\left(\frac{\omega r}{c}\sqrt{1-x^2}\right)dx.$$

Using integral transformation tables [1] we finally obtain

$$H(r) = 2\pi\,\frac{\sin\left\{\frac{\omega}{c}\sqrt{r^2+f^2(r)}\right\}}{\frac{\omega}{c}\sqrt{r^2+f^2(r)}}.$$

Substituting this function into equation (20) we obtain an expression for the mean value of the equivalent area of an axially symmetrical membrane in a diffuse field:

$$\bar{S}_c = \frac{4\pi^2}{\cos\alpha(r_0)} \int_0^R \frac{A_z(r)}{A_z(r_0)} \frac{\sin\left\{\frac{\omega}{c}\sqrt{r^2+f^2(r)}\right\}}{\frac{\omega}{c}\sqrt{r^2+f^2(r)}}. \tag{22}$$

The notation in this formula is the same as in formula (12), which describes the equivalent area of an axially symmetrical membrane induced to vibrate by a plane wave.

As has already been stated, formula (22) is valid only if the membrane is not too convex, i.e. if $f(r) \to 0$ ($\delta_g \to \pi/2$). Thus in the conclusion we shall refer only to the formulae derived in sections 2 and 3.

5. Conclusions

The method proposed in this paper for the determination of active parameters, together with the procedure of measuring the vibrations of a membrane in a plane wave field, give a precise determination (according to the definition) of the equivalent area and the active mass of microphone membranes. Using the method of trial and error we can, for instance, construct membranes of various shapes. By measuring their active parameters we can also find membranes which possess such values of the parameters as are required by the designer. An asset of the method of describing the equivalent area (section 2) and the active mass (section 3), as proposed in this paper, is that it is based consistently on the equivalence of work and energy for the real membrane and the equivalent piston. It results from the relations obtained that the active parameters depend on the shape of the membrane and the quantities describing the acoustic field. Some of the previous formulae describing the equivalent area and the active mass do not consider the real shape of the membrane (for instance, by assuming that the equivalent area is equal to the projection of the surface). These formulae also do not contain quantilics describing the acoustic field (e.g. the wavelength and the angle of incidence), which are taken into consideration in the method described in this paper.

In many papers devoted to the problem of equivalent diagrams, attempts have been made to introduce into the definitions of the active parameters the velocity and displacement amplitudes which are the solutions of the correspondingly formulated boundary problems. This trend in the investigations constitutes the next step, after the method of trail and error (to which we have already referred), on the way to the optimal design of microphone membranes. The results of these investigations would be analytical relations between the active parameters and the quantities describing the material properties of the membrane, the manner of mounting, etc. Due to the complexity of the problem, the determination of the distribution of the velocity and displacement amplitudes of a real microphone membrane, excited by an acoustic wave, is possible only with substantial simplification, thus considerably reducing the validity of the results. In this situation there is only one approach to finding the relations between the active parameters and the quantities describing the material properties of the membrane, the manner of mounting, etc.: by experimental investigations. These relations can be found with the aid of the method of trial and error provided that the algorithm defining the active parameters is based consistently on their energetic definition. This condition is satisfied by the method proposed in this paper.

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