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GENERAL CONDITIONS OF PHASE CANCELLATION IN AN ACOUSTIC FIELD

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The dependence of the degree of cancellation of an acoustic field, expressed by the field cancellation factor, on the correlation parameters of the signals: cancelling and cancelled is derived. Using this dependence, the necessary conditions – in terms of phase and amplitude – for the occurrence of the cancellation phenomenon at a field point are determined.

The possibility of the cancellation of larger regions of acoustic fields is investigated using such parameters as spectral characteristics of various signal classes and mutual distances of the cancelling and cancelled sources.

A classification into natural and forced cancellation is introduced and the classes of signals which can be cancelled in a natural way are determined.

The conditions for the occurrence of cancellation throughout all space are determined.

1. Introduction

For years now the phenomenon of phase cancellation has constituted a promising chance to workes in the field, to solve the problem of noise control. The literature concerned with acoustics rather infrequently reveals publications reporting trials to utilize this phenomenon for reducing noise [1-5]. At present there does not exist any general theory permitting a quantitative description for all classes of signals, i.e. both deterministic and stochastic. This may have been caused by too large discrepancy between anticipated and obtained results which, in turn, in most cases leads to the abandonment of more thorough investigations. The absence of such a theory for the quantitative conception of the phenomenon can surely not contribute to the realization of positive results as, indeed, it does not favour a proper utilization of this phenomenon in practice.

An attempt has been made in this paper to define the conditions necessary for the creation of the phenomenon of phase cancellation at a field point and to give the dependence between the degree of field cancellation and the correlation parameters of signals. The correlative approach permits to a uniform analysis of all classes of signals.

On the basis of dependence derived between the degree of field cancellation and the correlation parameters of the signals, the range of regions in which the phenomenon of cancellation occurs for various typical classes of signals has been calculated, and the classes of signals subject to natural cancellation defined. The relationship between the mutual spacing of sources, necessary for the occurrence of cancellation in a given region, and the spectral nature of the sound emitted by them has also been determined.

Consideration is merely given in the paper to the cancellation in a field, neglecting the mutual interaction of sources.

2. The field cancellation factor

Let a given source S (Fig. 1) be placed in a propagation medium and emit a signal $x(t)$. It produces in the medium a field $x(r,t)$. At any point A in this field, with the distance $|r_o|$ from the source, the wave $x_A(t)$ is only a function of time. The mean power $\langle P_{xA}\rangle$ of this wave at the point A is

$$\langle P_{xA}\rangle = \frac{1}{T}\int_0^T x_A^2(t)\,dt, \tag{1}$$

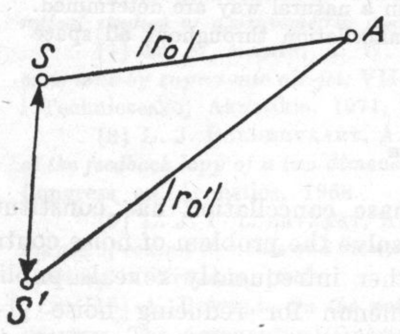

Fig. 1. Spacing of sources – cancelled S, cancelling S' – and the point A at which cancellation is required to take place

where T denotes the duration of signal.

Let another source S', which is placed in this medium at a distance d from the source S, emit a signal $y(t)$.

It produces at point A the wave $y_A(t')$ having a mean power

$$\langle P_{yA}\rangle = \frac{1}{T}\int_0^T y_A^2(t')\,dt, \tag{2}$$

where t' denotes the time calculated with regard to the coordinate system related to the source S' and is equal to

$$t' = t+\tau, \tag{3}$$

where τ is the difference of times at which signals from the sources reach the point A.

If the medium is linear, then the resulting signal $z_A(t)$ at the point A is the total of the component signals from sources S and S':

$$z_A(t) = x_A(t) + y_A(t'). \tag{4}$$

The mean power of this signal is

$$\langle P_{xyA} \rangle = \frac{1}{T} \int_0^T z_A^2(t)\, dt. \tag{5}$$

Cancellation at the point A occurs when

$$\langle P_{xyA} \rangle < \langle P_{xA} \rangle \quad \text{and/or} \quad \langle P_{xyA} \rangle < \langle P_{yA} \rangle, \tag{6}$$

depending on which field is to be cancelled.

Definition. *Phase cancellation* is a phenomenon resulting from such superposition of fields as to bring about a reduction of the signal power measured at a given point or region, compared to the signal power which would have been measured at this point or region as a result of the individual action of each component field.

In order to describe quantitatively the phenomenon of cancellation, we shall define the field cancellation factor k as the ratio of the cancelled power $\langle P_{cA} \rangle$ to the primary wave power $\langle P_{xA} \rangle$ which existed at a given point A prior to the superposition of the cancelling field:

$$k \overset{\text{df}}{=} \frac{\langle P_{cA} \rangle}{\langle P_{xA} \rangle}. \tag{7}$$

But

$$\langle P_{cA} \rangle = \langle P_{xA} \rangle - \langle P_{xyA} \rangle, \tag{8}$$

therefore

$$k = 1 - \frac{\langle P_{xyA} \rangle}{\langle P_{xA} \rangle}. \tag{9}$$

Since the mean cancelled power $\langle P_c \rangle$ cannot exceed the mean primary power $\langle P_x \rangle$, the field cancellation factor must lie between zero and unity:

$$0 < k \leqslant 1. \tag{10}$$

3. Dependence of the field cancellation factor on the correlation factor of the signals

The correlation functions for both of the signals $x(t)$ and $y(t)$ are calculated as the limit of the following integral for various values of the variable τ [6]:

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\,y(t+\tau)\,dt. \tag{11}$$

In general, the integration should extend over the whole range of $x(t)$ and $y(t)$. If the processes $x(t)$ and $y(t)$ are stationary, then their statistical properties will be the same for various samples of the processes. Thus the correlation functions of various samples will also be the same. It is then possible to calculate correlation functions of a given process for finite time intervals of width T:

$$R_{Txy}(\tau) = \frac{1}{T}\int_0^T x(t)\,y(t+\tau)\,dt. \tag{12}$$

Similarly, autocorrelation functions of component signals can be defined by

$$R_{Txx}(\tau) = \frac{1}{T}\int_0^T x(t)\,x(t+\tau)\,dt, \tag{13}$$

$$R_{Tyy}(\tau) = \frac{1}{T}\int_0^T y(t)\,y(t+\tau)\,dt. \tag{14}$$

For $\tau = 0$ the autocorrelation functions represent the mean power of the signal in the interval T:

$$R_{Txx}(0) = \frac{1}{T}\int_0^T x^2(t)\,dt = \langle P_x\rangle, \tag{15}$$

$$R_{Tyy}(0) = \frac{1}{T}\int_0^T y^2(t)\,dt = \langle P_y\rangle. \tag{16}$$

The ratio of the cross-correlation function, determined at a point of the field, to the square root of the product of values of the auto-correlation functions for the component signals at zero, is termed the *cross-correlation factor* b_τ for a time difference τ:

$$b_\tau = \frac{R_{Txy}(\tau)}{\sqrt{R_{Txx}(0)R_{Tyy}(0)}}. \tag{17}$$

The mean power of the total signal at the point A over a time T is

$$\langle P_{xyA}\rangle = \frac{1}{T}\int_0^T z_A^2(t)\,dt = \frac{1}{T}\int_0^T x_A^2(t)\,dt + \frac{1}{T}\int_0^T y_A^2(t+\tau)\,dt + \frac{2}{T}\int_0^T x(t)\,y(t+\tau)\,dt$$

$$= R_{Txx}(0) + R_{Tyy}(0) + 2R_{Txy}(\tau). \quad (18)$$

Substituting this value into formula (9) and considering condition (10), with a denoting the ratio of the cancelling signal power to the cancelled signal power,

$$a = \frac{R_{Tyy}(0)}{R_{Txx}(0)}, \quad (19)$$

we obtain the following condition for cancellation:

$$\frac{1}{2}\sqrt{a} < -b_\tau \leqslant \frac{1}{2}\,\frac{1+a}{\sqrt{a}}. \quad (20)$$

We also obtain the following relationship between the field cancellation factor k, the cross-correlation factor b_τ of component signals, and the ratio a of the powers of these signals:

$$k = -2\,b_\tau\sqrt{a} - a. \quad (21)$$

Consider first condition (20). Since a is positive, because it is the ratio of the powers of signals, both the right- and left-hand sides of inequality (20) are positive. Thus condition (20) is satisfied only if the correlation factor b_τ assumes a negative value, i.e.,

$$b_\tau < 0. \quad (22)$$

This relation is called the *phase condition*.

Let us now examine for what power ratios a the cancellation is possible and how strongly the component signals should be correlated with one another for a given power ratio a.

The right-hand side of inequality (20) represents a concave function having a minimum equal to 1 for the value $a = 1$ (Fig. 2).

From the Schwartz inequality we get a stronger condition, namely

$$|b_\tau| \leqslant 1. \quad (23)$$

Combining conditions (20) and (23), we get

$$\tfrac{1}{2}\sqrt{a} < 1, \quad (24)$$

hence the range of permissible powers for cancelling waves becomes

$$0 < R_{Tyy}(0) < 4\,R_{Txx}(0). \quad (25)$$

Thus the phenomenon of cancellation may occur for ratios of the power of the cancelling signal to the cancelled one within the range

$$0 < a < 4, \tag{26}$$

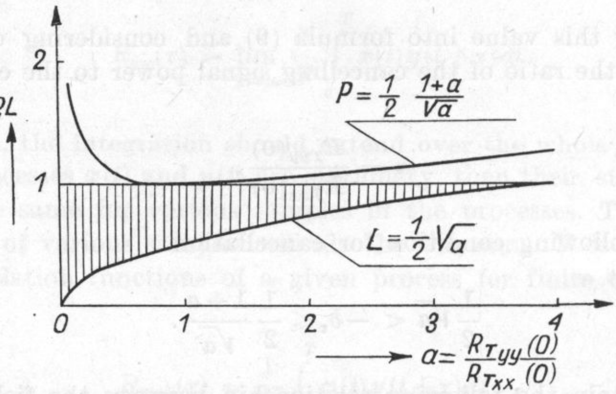

Fig. 2. A graph of the function P [which represents the right side of inequality (20)] and the function L (which represents the left-hand side of this inequality) versus the power ratio of the component signals. The shaded region determines the range of the value of the modulus of the cross-correlation factor $|b_\tau|$ necessary to obtain cancellation at a certain ratio a

while the required cross-correlation factor of both signals for given power ratio a should vary within the limits

$$\tfrac{1}{2}\sqrt{a} < -b_\tau \leqslant 1. \tag{27}$$

This condition is called the *amplitude condition.*

These relationships are shown in Fig. 2. The region of permissible values for $-b_\tau$, determined by condition (27), is shaded in Fig. 2.

Let us now analyze the mutual dependence of the degree of cancellation of a field and the degree of correlation of its component signals. To this end we present relation (21) in the form of a family of curves $k = f(a)$ for different values of the parameter b_τ(Fig. 3).

The maximum cancellation occurs for a value of the power ratio given by

$$a = b_\tau^2, \tag{28}$$

with the field cancellation factor attaining the value

$$k_{\max} = b_\tau^2. \tag{29}$$

Thus the maxima of the family of curves represented by (21) lie on the straight line

$$k_{\max} = a \quad \text{for } a \leqslant 1. \tag{30}$$

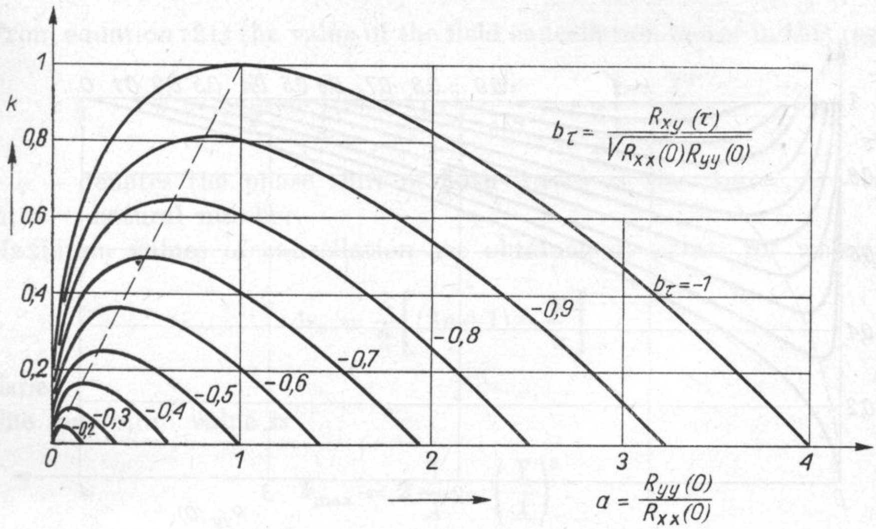

Fig. 3. Dependence of the field cancellation factor k on the power ratio a of both signals for various values of the cross-correlation factor b

Cancellation larger than any k_i is possible for signals for which the power ratio a is within the limits

$$(1-\sqrt{1-k_i})^2 < a < (1+\sqrt{1-k_i})^2, \tag{31}$$

while the modulus of the correlation factor is higher than $|b_\tau|_{\min}$, where

$$|b_\tau|_{\min} = \sqrt{k_i}. \tag{32}$$

The relations (31) and (32) are more apparent in the diagram of the required degree of correlation of the signals versus their power ratio at an assumed field cancellation factor (Fig. 4).

4. Spatial conditions for the existence of phase cancellation for monochromatic (sinusoidal) signals

Let the source S (Fig. 1) emit the wave

$$x(t) = X\sin\omega_1 t, \tag{33}$$

and the source S' the wave

$$y(t) = Y\sin(\omega_2 t+\varphi). \tag{34}$$

Let the difference of the distances from both sources to the reception point A be

$$\Delta r = \tau c, \tag{35}$$

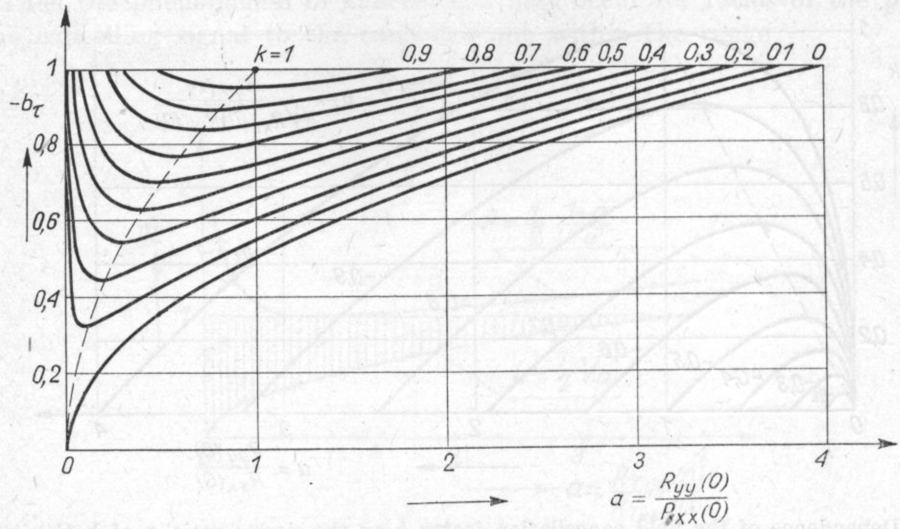

Fig. 4. Dependence of the required correlation factor b_τ on the power ratio a of both signals for an assumed field cancellation factor k

where c is the velocity of sound.

The autocorrelation functions of signals for $\tau = 0$ are:

$$R_{Txx}(0) = \frac{X^2}{2}, \quad R_{Tyy}(0) = \frac{Y^2}{2}. \tag{36}$$

The cross-correlation function for $\omega_1 \neq \omega_2$ is equal to zero only if T is sufficiently long,

$$R_{Txy}(\tau) = 0, \tag{37}$$

so that stable cancellation cannot occur at the point A.

On the other hand, if frequencies of both signals are identical, i.e. $\omega_1 = \omega_2 = \omega$, then

$$R_{Txy}(\tau) = \frac{XY}{2}\cos(\omega\tau + \varphi), \tag{38}$$

and substituting (36) and (37) in equation (27) and considering (35) we obtain a condition for the range of the differences of distances covered by the wave and thus determine the range of the area of cancellation:

$$\left[(2n+1)\frac{\varphi}{\pi} - \frac{1}{\pi}\arccos\frac{1}{2}\frac{Y}{X}\right]\frac{\lambda}{2} < \Delta r < \left[(2n+1)\frac{\varphi}{\pi} + \frac{1}{\pi}\arccos\frac{1}{2}\frac{Y}{X}\right]\frac{\lambda}{2}. \tag{39}$$

From equation (21) the value of the field cancellation factor in this region is

$$k = 2\frac{Y}{X}\cos\left[(2n+1)\pi + \frac{\Delta r}{\lambda/2}\pi + \varphi\right] - \frac{Y^2}{X^2}, \quad (40)$$

where φ — denotes the phase shift of both waves at the source, λ — wavelength, n — natural number.

Maximum values of cancellation are obtained at points for which

$$\Delta r_0 = \frac{\lambda}{2}\left[(2n+1) - \frac{\varphi}{\pi}\right] \quad (41)$$

is satisfied.

The maximum value is

$$k_{\max} = 2\frac{Y}{X} - \left(\frac{Y}{X}\right)^2. \quad (42)$$

This value can be obtained for two ratios of signal amplitudes, namely

$$\frac{Y}{X} = 1 \pm \sqrt{1 - k_{\max}}. \quad (43)$$

The dependence between these amplitude ratios is the following:

$$\left(\frac{Y}{X}\right)_1 \left(\frac{Y}{X}\right)_2 = k_{\max}. \quad (44)$$

Thus cancellation occurs for the amplitude ratios of signals in the interval $0 < Y/X < 2$, while the maximum possible value of cancellation $k_{\max} = 1$ is obtained for $Y/X = 1$.

If in equation (41) it is assumed that the phase shift of signals at the source is equal to zero, then maximum cancellation will occur at such points in space that the difference of distances from the sources is exactly an odd multiple of half wavelengths. In the proximity of these points there will also be cancellation, which will decrease with the cosine of the ratio $\Delta r/\frac{1}{2}\lambda$; symmetrically for distance differences Δr smaller and higher than Δr_o.

The limiting values of the difference Δr_g at which the cancellation finishes is determined by

$$\Delta r_g = \Delta r_0 \pm \frac{\lambda}{2\pi}\arccos\frac{1}{2}\frac{Y}{X}. \quad (45)$$

Hence, it can be concluded that the cancellation region extends for a phase difference $\Delta\psi$ between the signals given by

$$\Delta\psi = 2\arccos\frac{1}{2}\frac{Y}{X}. \quad (46)$$

The relationships described in equations (42) and (46) are shown in Fig. 5, and that of equation (40) in Fig. 6. From these figures it can be seen that the question as to whether the ratio Y/X is larger or smaller than unity is not trivial. In both cases, the same values of k_{max} can be obtained, but for $Y/X < 1$

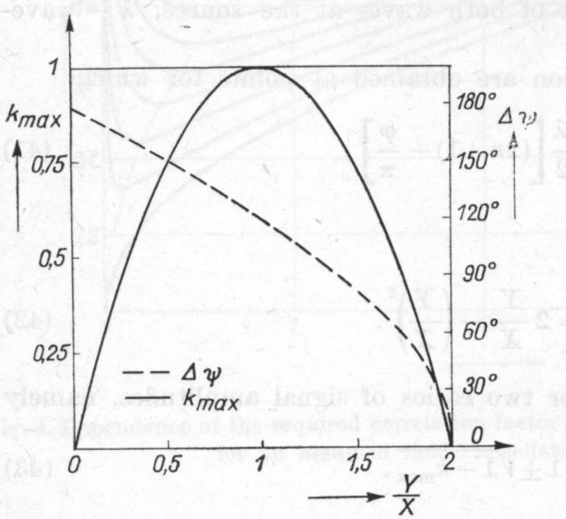

Fig. 5. Dependence of the maximum field cancellation factor k_{max} and the maximum permissible phase difference $\Delta\psi$ between signals on the amplitude ratio of the signals

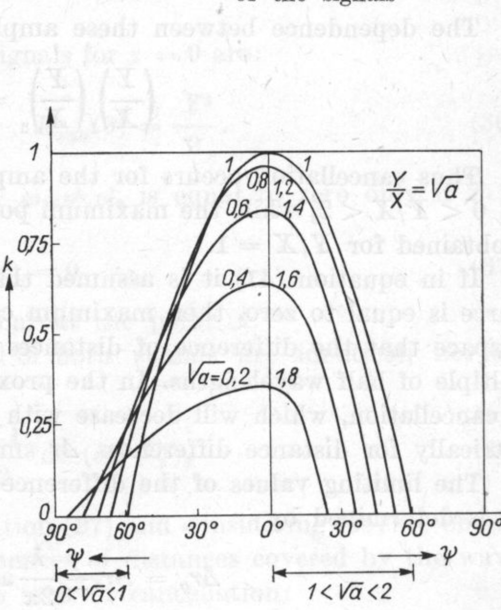

Fig. 6. Dependence of the field cancellation factor k on the phase

$$\psi = (2n+1)\pi + \frac{2\Delta r}{\lambda}\pi + \varphi$$

at various amplitude ratios of the signals

the range of phase difference $\Delta\psi$, at which the cancellation occurs, and hence the range of cancellation, is larger than in the case $Y/X > 1$. Another consequence of equation (45) is that the range of the cancellation region is proportional to the wavelength, and thus higher for lower frequencies.

5. Phase cancellation of periodic signals

Let the sources S and S' (Fig. 1) emit suitable periodic signals of the forms

$$x(t)=\sum_{n}^{\infty}X_n\sin(n\omega t), \tag{47}$$

$$y(t)=\sum_{n}^{\infty}Y_n\sin(n\omega t+\varphi_n). \tag{48}$$

We shall limit ourselves to signals of the same fundamental and harmonic frequencies, since in the case of different frequencies the cross-correlation factor is equal to zero and it is not possible to achieve stable cancellation.

The correlation functions of singals (47) and (48) for a point whose distances from the sources differ by $\Delta r = c\tau$ are

$$R_{Txx}(0)=\sum_{n}^{\infty}\frac{X_n^2}{2}, \quad R_{Tyy}(0)=\sum_{n}^{\infty}\frac{Y_n^2}{2}, \tag{49}$$

$$R_{Txy}(\tau)=\sum_{n}^{\infty}\frac{X_nY_n}{2}\cos(n\omega\tau+\varphi_n). \tag{50}$$

A general analysis for any values of Y_n and X_n is too complicated and we will consider the simple case $X_n = Y_n$.

Substitution of functions (49) and (50) in equation (27) gives

$$\tfrac{1}{2}\sum_{n}^{\infty}X_n^2<\sum_{n}^{\infty}X_n^2\cos[(2m+1)]\pi+n\omega\tau+\varphi_n]\leqslant\sum_{n}^{\infty}X_n^2, \tag{51}$$

where $m = 0,1, \ldots$

The right-hand side of (51) determines complete cancellation. It occurs if

$$(2m+1)\pi+n\omega\tau+\varphi_n=0, \tag{52}$$

i.e. if the phases φ_n are such that, for all n,

$$\varphi_n=\pi\left(2m+1-2n\frac{\tau}{T}\right) \tag{53}$$

is satisfied, where T denotes the period of the periodic signal. Let us assume that both sources emit the same signals, i.e. $\varphi_n = 0$ for all n.

To bring about a complete cancellation, all the cosines must be equal to unity, thus we obtain the condition

$$\tau=\frac{(2m+1)\pi}{2n\pi f}=\frac{\Delta r}{\lambda f}, \tag{54}$$

and hence

$$\Delta r = \frac{\lambda}{2}\frac{2m+1}{n}, \tag{55}$$

where λ is the wavelenght of the periodic signal, n — the n-th component, m — the m-th multiple of the half wavelength.

Equation (55) must be satisfied for all components, i.e.

$$\Delta r_k = \Delta r_l, \tag{56}$$

where k, l denote any two components of the periodic signal. Substituting in (55), we have

$$\frac{2m_k+1}{k} = \frac{2m_l+1}{l}, \tag{57}$$

so that

$$m_k = \frac{k}{l} m_l + \frac{k-l}{2l}. \tag{58}$$

In order to satisfy this equality, m_k should be integral for all m_l. This is the case if, assuming $k > l$,

$$\frac{k-l}{2l} = \text{integer}. \tag{59}$$

In this case the ratio k/l is also an integer, and equation (58) will be valid when this is an odd number.

From this discussion it can be concluded that complete cancellation of periodic signals is possible only for signals with only odd harmonics.

Periodic signals with only odd harmonics possess half-wave symmetry, i.e.

$$y(\theta+\pi) = -y(\theta). \tag{60}$$

This means that the part of the wave in the interval $(\pi, 2\pi)$, after being rotated about the θ axis and shifted by π, coincides exactly with that part of the wave in the interval $(0, \pi)$ (Fig. 7). Thus at points which the waves from

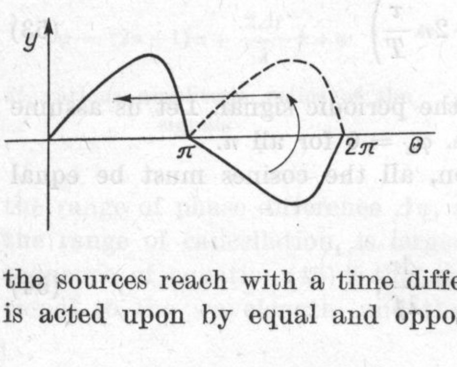

Fig. 7. A signal $y(\theta)$ exhibiting half-wave symmetry

the sources reach with a time difference $\tau = T/2$, the element of the medium is acted upon by equal and opposite forces which are completely balanced.

6. Phase cancellation for other classes of signals

Other signals do not all exhibit a natural half-wave symmetry, and for this reason it is not possible to achieve complete cancellation with them.

However, any signal (e.g. noise) may be taken, reversed artificially, and superimposed on the original signal. If the primary signal is $x(t)$, then the reversed and delayed signal is $y(t+\tau) = -x(t)$, and the cross-correlation function of these signals will be

$$R_{Txy}(\tau) = \frac{1}{T}\int_0^T x(t)y(t+\tau)\,dt$$

$$= \frac{1}{T}\int_0^T x(t)[-x(t)]\,dt = -R_{Txx}(0) = -R_{Tyy}(0). \qquad (61)$$

When substituting from equation (61) into equation (27), one can see that the condition for the complete cancellation is satisfied. Thus, a signal having no symmetry whatsoever can be completely cancelled by the use of its artificial reverse with respect to phase. This operation gives any signal half-wave symmetry with regard to the primary signal, since the relation between the secondary and primary signals is the following:

$$y(t+\tau) = -y(t) \qquad (62)$$

(cf. equation (60)).

It would appear advisable to classify cancellation into natural and forced, especially in view of the fact that the phenomenon of natural cancellation is observed without performing any operations on the signal. The possibility of this phenomenon occurring results from the characteristics of the signal itself, namely its half-wave symmetry. Such cancellation can therefore occur only for classes of signals which have a natural halfwave symmetry. They include monochromatic and periodic signals with only odd harmonics. All other signals can, however, be cancelled by performing on them a phase reversal operation, but because of this operation, cancellation obtained in this way will be defined as *forced cancellation*.

7. Dependence of the range of the cancellation region on the signal power density spectrum

In section 4 it has been shown that the range of the cancellation region for monochromatic signals is inversely proportional to their frequency.

Let us now consider how the range of the cancellation region depends on the power density spectrum of any signal. In the case of the superposition of two identical signals only reversed in phase and shifted by τ_A, the cross-correlation function of both signals will be a mirror reflection of the autocorrelation function a_τ of primary signal shifted by τ_A relative to the τ axis. Since — as a result of the uncertainty relation [7] — the width of the autocorrelation function is inversely proportional to the upper frequency limit of the signal,

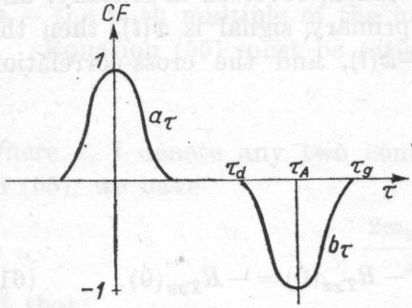

Fig. 8. a_τ — **autocorrelation function of the primary signal,** b_τ — **cross-correlation of the primary signal with the primary signal inversed in phase and shifted by** τ_A

the width of the correlation function will also be inversely proportional to this frequency in agreement with the formula

$$\tau_g - \tau_d = \frac{C}{f_g}, \tag{63}$$

where C is a constant.

Since the width of the interval $\tau_g - \tau_d$, in which the cross-correlation function assumes negative values, is decisive for the range of the cancellation region, signals whose power density spectra are shifted in the direction of low frequencies will produce larger cancellation regions.

8. Dependence of the range of the cancellation region on the source spacing

The time delay τ_A of the signals reaching point A depends on the distance d between the sources and on the position of point A relative to both sources (Fig. 9).

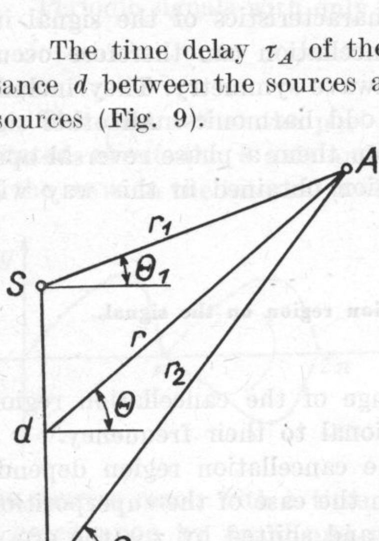

Fig. 9. The mutual spacing of the sources and of the receiving point A **in a polar system**

The smaller the distance d, the smaller the time delay τ_A will be and the cross-correlation curve in Fig. 8 will be shifted towards zero. If the distance d is reduced so that, for given signal, τ_d becomes negative (passes through point $\tau = 0$), the cancellation region will start from the sources themselves.

We shall now investigate the condition which the distance d between sources should satisfy to obtain cancellation throughout the whole region.

The condition for cancellation throughout the whole region is that the cross-correlation factor should be negative at every point of the region.

Such a condition is satisfied if

$$\Delta r = r_2 - r_1 = c\tau \leqslant c\tau_g \tag{64}$$

for all r_1 and r_2, where

$$r_1 = \sqrt{r^2 + \frac{d^2}{4} - rd\sin\theta}, \tag{65}$$

$$r_2 = \sqrt{r^2 + \frac{d^2}{4} + rd\sin\theta}. \tag{66}$$

Substituting (65) and (66) in to (64) we obtain the following condition for the distance d:

$$d \leqslant c\tau_g \sqrt{\frac{4r^2 - c^2\tau_g^2}{4r^2\sin^2\theta - c^2\tau_g^2}} = B. \tag{67}$$

If the cancellation is to extend to infinity, the condition

$$d \leqslant \lim_{r\to\infty} B = \frac{c\tau_g}{\sin\theta} \tag{68}$$

must also be satisfied.

If condition (68) is met, the cancellation region will extend to infinity but only within an angle $\pm\theta$. To obtain cancellation throughout the whole region, condition (68) is further limited to

$$d \leqslant c\tau_g. \tag{69}$$

Considering (63) and assuming $\tau_d = 0$ (the condition of cancellation from the sources themselves) we finally obtain

$$d \leqslant C\frac{c}{f_g}. \tag{70}$$

Thus, the higher the upper frequency limit of a signal, the closer the sources must be placed to obtain cancellation throughout the whole region.

For white noise $f_g \to \infty$ while $d \to 0$, thus the primary and secondary sources should be placed at the same point.

9. Conclusions

The investigations carried out lead to the following conclusions:

1. To obtain field cancellation at a point, a suitable value of the coefficient a, the ratio of the cancelling signal power to the cancelled one, is necessary, as it is also for such a correlation of the signals that phase condition (22) (a negative cross-correlation factor for the signals) as well as amplitude condition (27) (that the modulus of the cross-correlation factor should be higher than half square root of the ratio of the cancelling signal power to the cancelled signal power) should be met.

2. Complete cancellation occurs only if signals are entirely correlated, directed in opposition (correlation factor equal to -1), and have equal powers. This means that they are the same signals but reversed in phase.

3. The maximum feasible field cancellation factor is equal to the square of the modulus of the cross-correlation factor of the component signals. It is obtained at a precisely defined ratio of the powers of these signals which is also equal to the square of the modulus of cross-correlation factor. This implies that the maximum degree of field cancellation is obtained when the power of the cancelling signal is smaller than or equal to the power of the cancelled signal.

4. The maximum range of changes of the ratio a of the cancelling signal power to the cancelled signal power, at which cancellation can be obtained, is $0 < a < 4$. As the power ratio tends to the limit values, the degree of cancellation decreases. If the degree of cancellation is to be higher than a given value k_i, then the range of required power ratio of the signals is diminished to

$$(1-\sqrt{1-k_i})^2 < a < (1+\sqrt{1-k_i})^2.$$

5. The value of modulus of the cross-correlation factor necessary to obtain the required degree of field cancellation depends on the ratio of powers of the component signals. It is least for the condition $a = k_i$, i.e. when the power of the cancelling signal is smaller than or equal to the power of the cancelled signal.

6. The range of the cancellation regions in the case of monochromatic signals is inversely proportional to their frequency, while in the case of composite signals it is inversely proportional to their upper frequency limit.

7. To obtain complete cancellation in a certain region, the cancelled and cancelling waves should propagate along parallel tracks. This ensures the conservation of constant phase and amplitude ratios within this region throughout the whole process of cancellation. This condition determines the location of source of the cancelling signal relative to the cancelled one and relative to the region in which the primary signal is to be cancelled.

8. It appears constructive to distinguish between natural and forced cancellation. Natural cancellation occurs because of the superpositions of various phases of the same signal. It is possible only for signals having half-wave symmetry, i.e. for monochromatic and periodic signals of only odd harmonic components. Forced cancellation can involve all classes of signals. For this purpose it is necessary to use a special device for the phase inversion and time delay of a given signal.

9. An interesting class of signals classified under natural cancellation are monochromatic signals. The cancellation regions of these signals are concentrated around points whose distances from the sources differ by an odd number of half wavelengths. At these points the degree of cancellation assumes a maximum value and it gradually decreases when moving away from them. Cancellation is possible for ratios of the signal amplitudes within the limits $0 < Y/X < 2$, the closer this ratio is to unity, the closer to unity the field cancellation factor becomes.

The same degree of cancellation can be obtained for two ratios of the signal amplitudes Y/X — one smaller and one larger than unity. However, this ratio has some effect on the range of the cancellation region. If $Y/X < 1$, the range of the differences of the component signal phases at which the cancellation occurs is higher, and this results in a larger value of the range of the cancellation region.

References

[1] H. F. Olson, M. E. May, *Electronic sound absorbers*, JASA, **25**, 6, 1130-6 (1953).

[2] S. Czarnecki, *Investigations of the sound transmission properties of a medium resulting from sound wave compensation caused by other sources*, JSVib, **11**, 2, 225-233 (1970).

[3] O. Bschorr, *Lärmauslöschung durch gesteuerte Interferenz*, 7 ICA, Budapest 1971, Rep. 21N8.

[4] B. D. Tartakovsky, *Systems of active compensation of sound field for bounded structures*, 7ICA, Budapest 1971, Rep. 21V1.

[5] M. J. M. Jessel, G. A. Mangiante, *Active sound absorbers in an air duct*, JSVib, **23**, 3, 383-390 (1972).

[6] J. S. Bendat, A. G. Piersol, *Measurement and analysis of random data*, John Wiley Sons, Inc. New York 1966, p. 1-35.

[7] R. Bracewell, *Fourier transformation and its applications*, WNT, Warszawa 1968, p. 153-194 (in Polish).

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