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Research Paper

Evaluation of the Effect of Uncertainties on the Acoustic Behavior of a Porous Material Located in a Duct Element Using the Monte Carlo Method

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When studying porous materials, most acoustical and geometrical parameters can be affected by the presence of uncertainties, which can reduce the robustness of models and techniques using these parameters. Hence, there is a need to evaluate the effect of these uncertainties in the case of modeling acoustic problems. Among these evaluation methods, the Monte Carlo simulation is considered a benchmark for studying the propagation of uncertainties in theoretical models. In the present study, this method is applied to a theoretical model predicting the acoustic behavior of a porous material located in a duct element to evaluate the impact of each input error on the computation of the acoustic proprieties such as the reflection and transmission coefficients as well as the acoustic power attenuation and the transmission loss of the studied element. Two analyses are conducted; the first one leads to the evaluation of the impacts of error propagation of each acoustic parameter (resistivity, porosity, tortuosity, and viscous and thermal length) through the model using a Monte Carlo simulation. The second analysis presents the effect of propagating the uncertainties of all parameters together. After the simulation of the uncertainties, the 95% confidence intervals and the maximum and minimum errors of each parameter are computed. The obtained results showed that the resistivity and length of the porous material have a great influence on the acoustic outputs of the studied model (transmission and reflection coefficients, transmission loss, and acoustic power attenuation). At the same time, the other physical parameters have a small impact. In addition, the acoustic power attenuation is the acoustic quantity least impacted by the input uncertainties.

Keywords: porous material; physical parameters; transmission loss; acoustic power attenuation; Monte Carlo method.

1. Introduction

Studying acoustic propagation in duct systems containing a porous material is still today important in acoustic research and industrial communities. In fact, this kind of duct element is used in many industrial applications such as transport and building domains. The objective of these studies is to predict and better understand the involved physical phenomena related to the acoustic propagation of these duct elements (reflection, transmission, attenuation, absorption, convection, diffraction, refraction, etc.). The use of porous materials is justified by the fact that they possess good absorption properties, and are easy to manufacture and install. The theoretical description of acoustic propagation in porous media has been constantly progressing since the 1940s. It is now relatively well-known thanks to the contributions of many models which start with Zwikker and Kosten (1949), who are the first to model sound propagation in porous media. Then Delany and Bazley (1970) established an empirical model according to which the acoustic characteristics (the characteristic impedance and the propagation coefficient) depend only on the ratio of the frequency f to the air flow resistivity. Then, the porosity φ and the tortuosity α_∞ were introduced as presented in the models (Attenborough, 1982; 1983) to take into account the complexity of the pore geometry in high frequencies. Johnson *et al.* (1987) introduced the physical concept of viscous characteristic length Λ. This model was next completed by Champoux and Allard (1991) by adding the description of thermal characteristic length Λ' effects. Later on, Lafarge *et al.* (1997) refined the Champoux and Allard model by introducing a new parameter called the thermal permeability k'_0, which describes the damping of sound waves due to the thermal exchanges between the fluid and the structure at the pore surface.

To study the acoustic behavior of duct elements containing porous materials, some matrices can be used coupled with the previous porous acoustic models. Among these matrices, two present a great interest: the first is the transfer matrix as presented in (Peat, 1988; Tanaka *et al.*, 1985; Othmani *et al.*, 2016; 2017; Kani *et al.*, 2019; 2021) from which the acoustic transmission loss (TL) can be computed. The second is the scattering matrix (Bi *et al.*, 2006; Sitel *et al.*, 2006; Taktak *et al.*, 2010; 2013; Jdidia *et al.*, 2014; Othmani *et al.*, 2015; Kessentini *et al.*, 2016; Masmoudi *et al.*, 2017; Ben Souf *et al.*, 2017; Dhief *et al.*, 2020; Tounsi *et al.*, 2022) that contains important information about the transmission and reflection phenomena and is used for the acoustic power attenuation computation. Consequently, these two matrices give complete information about the acoustic behavior of a duct element.

In general, most of the parameters used in the theoretical modeling are characterized by the presence of some uncertainties that affect the robustness of such modeling. In order to avoid any errors, it is necessary to evaluate the effect of these errors in the final results and determine the more influential parameters. To achieve this objective, uncertainty analyses are used. A widely used stochastic technique, called the Monte Carlo method, can be integrated into the theoretical models in order to evaluate the propagation of errors and their degree of influence (Taktak *et al.*, 2009; Trabelsi *et al.*, 2017; Bouazizi *et al.*, 2019).

The aim of the work presented in this paper is to evaluate the impact of uncertainties affecting the physical and geometrical parameters of a duct element containing a porous material on its acoustic behavior. To reach this goal, the Monte Carlo technique is coupled with theoretical modeling to compute the transfer and scattering matrices of the studied duct element. The used acoustic porous model is the Johnson–Champoux–Allard–Lafarge (JCAL) model (Johnson *et al.*, 1987; Champoux, Allard, 1991; Lafarge *et al.*, 1997), which incorporates the maximum of parameters.

In the present study, the effects of uncertainties of model parameters on the porous material acoustic properties such as reflection and transmission coefficients, as well as the transmission loss and the acoustic power attenuation, are evaluated and investigated. This is obtained using the Monte Carlo method allowing the computation of the 95% confidence intervals of the model outputs as well as the corresponding errors.

The outline of the paper is as follows: in Sec. 2, the theoretical basis of modeling the acoustic behavior of porous element using the JCAL model and the computation of transfer and scattering matrices of the studied duct element are presented in detail. These matrices are then used to calculate the transmission loss and the acoustic power attenuation of the porous material. Section 3 presents the details of the uncertainty analysis based on the Monte Carlo method. Finally, the numerical results are presented and discussed in Sec. 4.

2. Theoretical basis

2.1. Transfer matrix and transmission loss computation

The studied duct element is located between the two axial coordinates z_1 and z_2, as shown in Fig. 1. It contains a porous material with a length equal to L. According to the duct element dimensions used in the present study, only the propagation of the acoustic plane wave through the duct element in the z-direction is assumed. This propagation is modeled using the transfer matrix $[T]$, which provides the relationship between the acoustic pressure P_1 and the particle velocity U_1 in duct I at $z_1 = 0$ and the acoustic pressure P_2 and the particle velocity U_2 in duct II at $z_2 = L$ (Fig. 1).

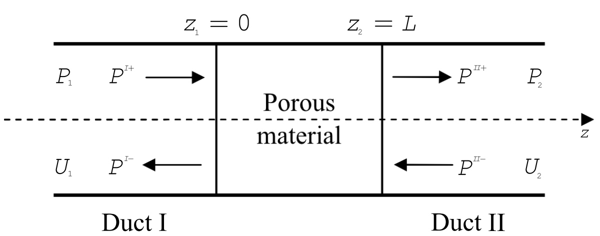

Fig. 1. Description of the studied duct.

The general formulation of the transfer matrix is given as follows (Atalla, Allard, 2009):

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = [T] \begin{pmatrix} P_2 \\ U_2 \end{pmatrix}. \tag{1}$$

For a single fluid layer, the transfer matrix is constructed as:

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos(k(w)L) & jZ(w)\sin(k(w)L) \\ \dfrac{j}{Z(w)}\sin(k(w)L) & \cos(k(w)L) \end{bmatrix}, \tag{2}$$

where $j = \sqrt{-1}$, w is the angular frequency, $Z(w)$ is the characteristic impedance, and $k(w)$ is the acoustic wavenumber of the porous material. These latter two intrinsic quantities are linked to the dynamic mass density $\rho(w)$ and the bulk modulus $K(w)$ of the porous material by the following relations:

$$Z(w) = \sqrt{\rho(w)K(w)}, \tag{3}$$

$$k(w) = w\sqrt{\frac{\rho(w)}{K(w)}}. \tag{4}$$

According to the JCAL model (Johnson *et al.*, 1987; Champoux, Allard, 1991; Lafarge *et al.*, 1997), the expressions of $\rho(w)$ and $K(w)$ are given as follows:

$$\rho(w) = \frac{\alpha_\infty \rho_0}{\phi}\left(1 - j\frac{\sigma\phi}{w\rho_0\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2 \eta\rho_0 w}{\sigma^2 \Lambda^2 \phi^2}}\right), \tag{5}$$

$$K(w) = \frac{\gamma P_0}{\phi}\left(\gamma - \frac{\gamma - 1}{1 - j\dfrac{8\eta}{\Lambda'^2 N_{\mathrm{Pr}}\sqrt{1+j\frac{\Lambda'^2 N_{\mathrm{Pr}}}{16\eta}}}}^{-1}\right), \tag{6}$$

where ϕ is the open porosity, σ is the static air-flow resistivity, α_∞ is the high-frequency limit of the dynamic tortuosity, Λ is the characteristic viscous length, Λ' is the characteristic thermal length, ρ_0 is the density at rest of the fluid saturating the pores, η is its dynamic viscosity, N_{Pr} is its Prandtl number, γ is its specific heat ratio, and P_0 is the atmospheric pressure.

The power transmission factor τ of the porous material is defined as the ratio of the transmitted power W_t and the power incident on the porous material W_i:

$$\tau = \frac{W_t}{W_i}. \tag{7}$$

The transmission loss (TL) is defined in dB as:

$$\mathrm{TL} = 10\log\left(\frac{1}{\tau}\right). \tag{8}$$

Using the four elements T_{11}, T_{12}, T_{21}, and T_{22} of the transfer matrix $[T]$, the transmission loss can be calculated as follows:

$$\mathrm{TL} = 20\log\left(\frac{1}{2}\left|T_{11} + \frac{T_{12}}{Z_0} + Z_0 T_{21} + T_{22}\right|\right), \tag{9}$$

where $Z_0 = \rho_0 c_0$ is the characteristic impedance of the surrounding medium, and c_0 is its speed of sound.

2.2. Scattering matrix and acoustic power attenuation computation

The scattering matrix $[S]$ of the studied duct element located between z_1 and z_2 (Fig. 1) is a linear relationship between the incoming pressures vector $\{P^{\mathrm{in}}\}$ and the outgoing pressures vector $\{P^{\mathrm{out}}\}$ and can be expressed as:

$$\{P^{\mathrm{out}}\} = [S]_{2\times2}\{P^{\mathrm{in}}\}, \tag{10}$$

where

$$\{P^{\mathrm{in}}\} = \begin{Bmatrix} P^{I+}(z_1) \\ P^{II-}(z_2) \end{Bmatrix} \quad \text{and} \quad \{P^{\mathrm{out}}\} = \begin{Bmatrix} P^{I-}(z_1) \\ P^{II+}(z_2) \end{Bmatrix}.$$

$P^{I+}(z_1)$, $P^{I-}(z_1)$, $P^{II+}(z_2)$, and $P^{II-}(z_2)$ represent the incident, the reflected, the transmitted, and the retrograde pressures, respectively (Fig. 1).

This matrix

$$[S]_{2\times2} = \begin{bmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{bmatrix}$$

depicts only the studied duct element and is independent of the upstream and downstream acoustic conditions. The physical meaning of each coefficient is as follows:

- S^{11} is the reflection coefficient of the wave coming into the element from the left side,
- S^{22} is the reflection coefficient of the wave coming into the element from the right side,
- S^{12} is the transmission coefficient of the wave coming into the element from the right side,
- S^{21} is the transmission coefficient of the wave coming into the element from the left side.

For a symmetric studied duct element and the same mediums on both sides of the element, the following can be written:

$$\begin{Bmatrix} S^{11} = S^{22} \\ S^{12} = S^{21} \end{Bmatrix}. \tag{11}$$

The four scattering matrix coefficients can be expressed in terms of the transfer matrix coefficients as follows (Hu, 2010):

$$S^{11} = \frac{X^+ - W^+}{X^+ + W^+}, \tag{12}$$

$$S^{22} = -\frac{X^- + W^-}{X^+ + W^+}, \tag{13}$$

$$S^{12} = \frac{X^+W^- - W^+X^-}{X^+ + W^+}, \tag{14}$$

$$S^{21} = \frac{2}{X^+ + W^+}, \tag{15}$$

where

$$\left\{ \begin{array}{l} X^{\pm} = T_{11} \pm \dfrac{T_{12}}{Z_0} \\ W^{\pm} = Z_0 T_{21} \pm T_{22} \end{array} \right\}. \tag{16}$$

The acoustical power attenuation W_{att} of the studied duct element is defined in decibel [dB] as follows:

$$W_{\text{att}} = 10 \log \left(\frac{W^{\text{in}}}{W^{\text{out}}} \right), \tag{17}$$

where W^{in} is the total acoustic power of incoming waves and W^{out} is the total acoustic power of outgoing waves. This acoustic power attenuation can be computed using the scattering matrix $[S]$ (Taktak *et al.*, 2010; Kani *et al.*, 2021):

$$W_{\text{att}} = 10 \log \left(\frac{|d_1|^2 + |d_2|^2}{\lambda_1 |d_1|^2 + \lambda_2 |d_2|^2} \right) \tag{18}$$

with:

- λ_1 and λ_2 are the eigenvalues of the matrix $[H]$ defined as follows:

$$[H] = [S]^H \cdot [S] = [V] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [V]^H, \tag{19}$$

  where the subscript H denotes the conjugate transpose, and $[V]$ is the matrix of the eigenvectors of the matrix $[H]$.
- d_1 and d_2 are the components of the vector $\{d\}$ calculated by the following equation:

$$\{d\} = \sqrt{\frac{1}{2Z_0}} \cdot [V]^H \cdot \left\{ \begin{array}{l} P^{I+} \\ P^{II-} \end{array} \right\}. \tag{20}$$

3. Uncertainty analysis: Monte Carlo simulation

The JCAL model (this model was detailed in Sec. 2) adopted for the sound propagation in the porous material study uses the five material physical parameters: porosity, static air-flow resistivity, high-frequency limit of the dynamic tortuosity, characteristic viscous and thermal lengths.

In order to study the propagation of uncertainties through the used model, uncertainty analysis is conducted (Taktak *et al.*, 2009; Trabelsi *et al.*, 2017; Bouazizi *et al.*, 2019). A probability distribution is first applied to each model's physical parameters input, and then an investigation of their effect on model outputs (transmission and reflection coefficients, transmission loss, and acoustic power attenuation) is performed.

The computing steps for the Monte Carlo simulation used for this analysis are regrouped in the algorithm illustrated in Fig. 2. In the first step, a Gaussian distribution for each input is defined. Then, M-samples are created for each studied model input. After that, the algorithm is executed N-times to generate a set of outputs. Finally, the obtained outputs are regrouped, and the 95% confidence interval with the corresponding maximum and minimum errors are estimated.

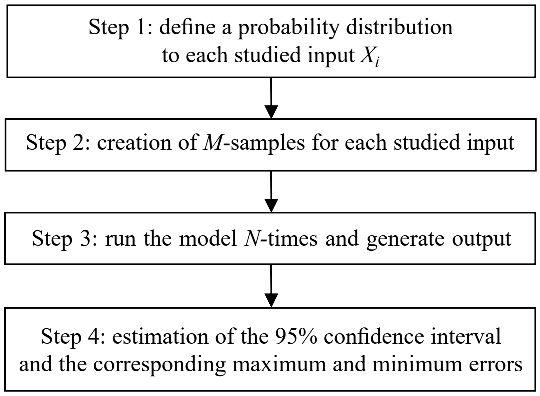

Fig. 2. Monte Carlo algorithm for uncertainty analysis.

4. Results and discussion

The presented uncertainty analysis is applied to a porous material located in a duct element in the frequency band (0–4000 Hz). The properties of this studied porous material are presented in Table 1. Only the plane wave is propagating in the duct element. This table also presents the mean value and the standard derivation used for each parameter to obtain the 10 000 values of studied acoustic parameters, which are then used in the proposed Monte Carlo method.

Table 1. Properties of the studied porous material.

Parameter	Mean value	Standard deviation
Flow resistivity σ $[\text{N} \cdot \text{s} \cdot \text{m}^{-4}]$	25000	306
Porosity ϕ	0.95	0.012
Tortuosity α_∞	1	0.013
Viscous length Λ [μm]	170	5.10^{-6}
Thermal length Λ' [μm]	510	5.10^{-6}
Length of the porous material L [m]	0.05	7.10^{-4}
Density $[\text{kg} \cdot \text{m}^{-3}]$	60	0.9

This section presents the results of the developed uncertainty analysis when considering the variation of one of the used parameters separately. The uncertainty analysis through the Monte Carlo method is performed as described in Fig. 2 through MATLAB software.

Initially, a ±5% variation is applied to each mean input value (shown in Table 1) according to the Gaussian distribution and by performing $N = 10\,000$ calculations. After randomly selecting a set of each input (resistivity, porosity, tortuosity, viscous and thermal lengths), the corresponding model outputs (transmission and reflection coefficients, transmission loss, and acoustic power attenuation) are computed. Finally, these steps are repeated N-times to obtain, for each input parameter, the 95% confidence intervals limited by the minimum and the maximum values of each model output of the studied porous material and the corresponding mean value.

Uncertainty analysis results considering the flow resistivity uncertainty are presented in Fig. 3. This figure illustrates that the flow resistivity has an important effect on the studied acoustic outputs, with errors varying from 0.5% to 6%. This observation is confirmed by the 95% interval of each output parameter: these intervals present a thickness, which means that the influence of this parameter is significant. It is important

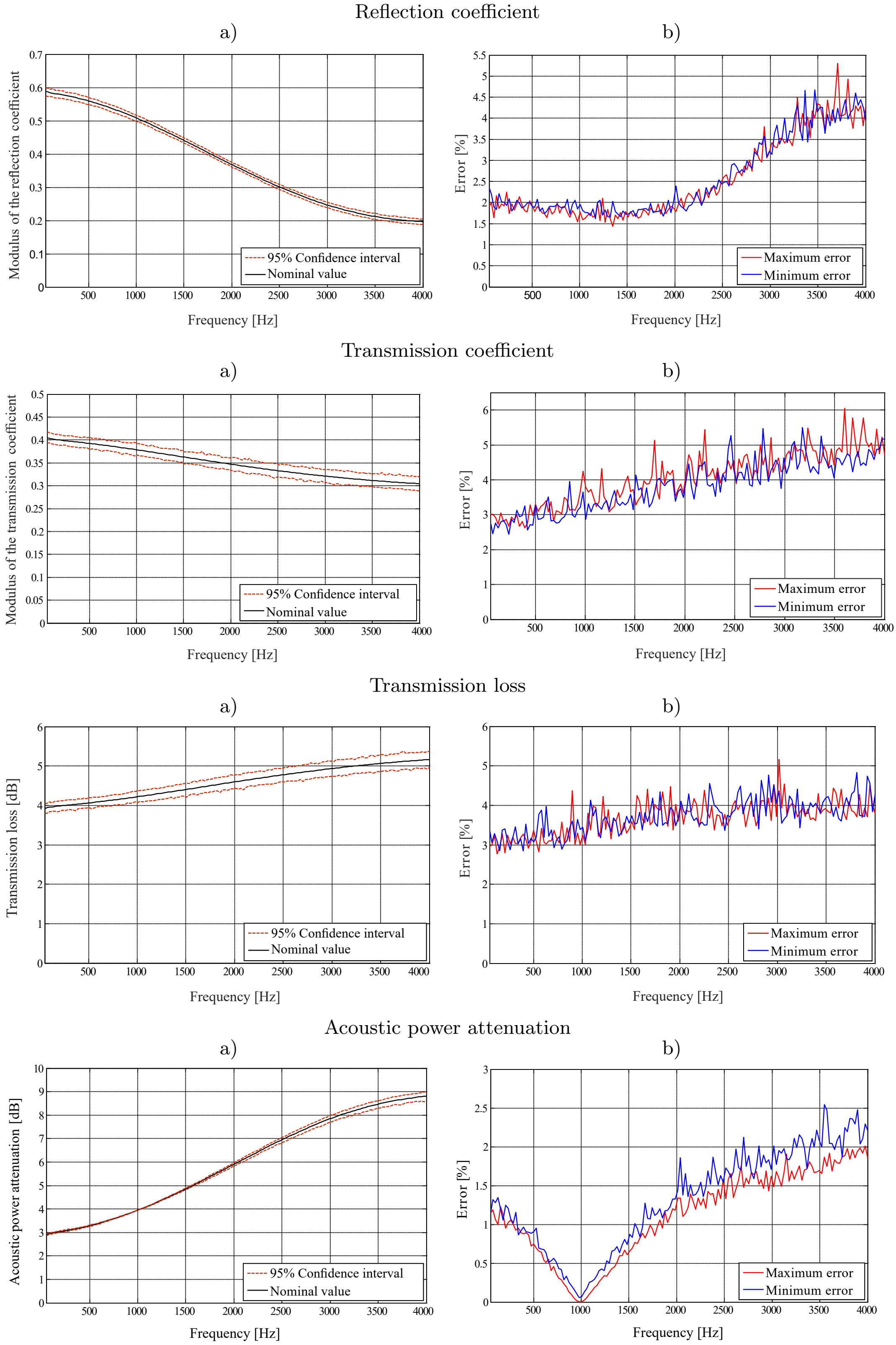

Fig. 3. Flow resistivity uncertainties' effect on the acoustic outputs: 95% confidence interval (a) and the corresponding error (b).

to indicate also that the uncertainty of the flow resistivity has a minimum influence on the acoustic power attenuation, as indicated in Fig. 3 (with an error not exceeding 2.5%). It is also observed that errors on all the output parameters dues to uncertainties increase when the frequency increases.

Figure 4 shows that the porosity has the same effect as the flow resistivity with a significant influence on the

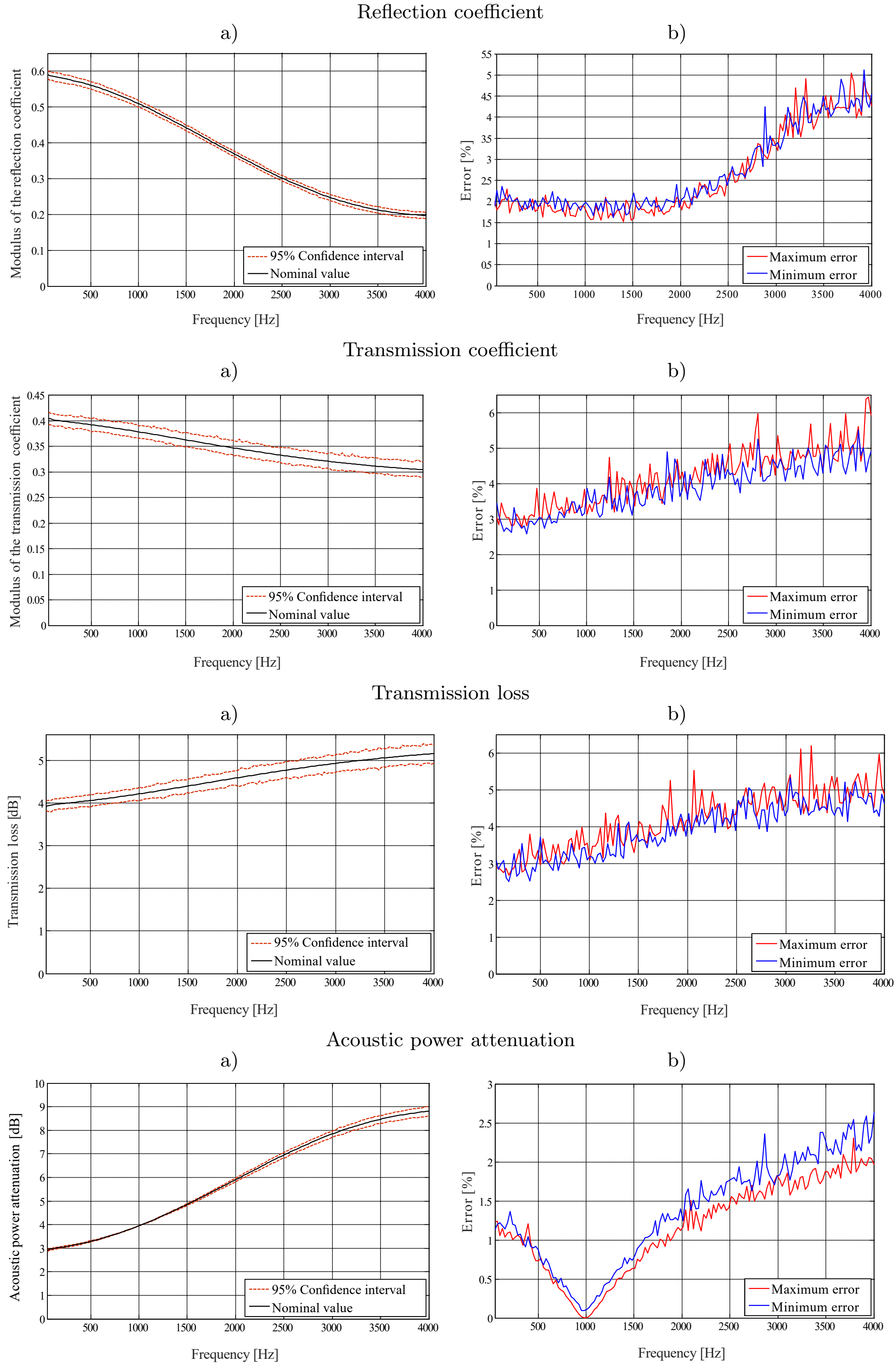

Fig. 4. Porosity uncertainties' effect on the acoustic behaviors:
95% confidence interval (a) and the corresponding error (b).

transmission and reflection coefficients as well as the transmission loss (TL) with errors reaching 6% and a small effect on the acoustic attenuation of the porous material with a maximum error equal to 3%. It is observed that the thickness of the confidence interval in Fig. 4a is small, which indicates that the effect of this

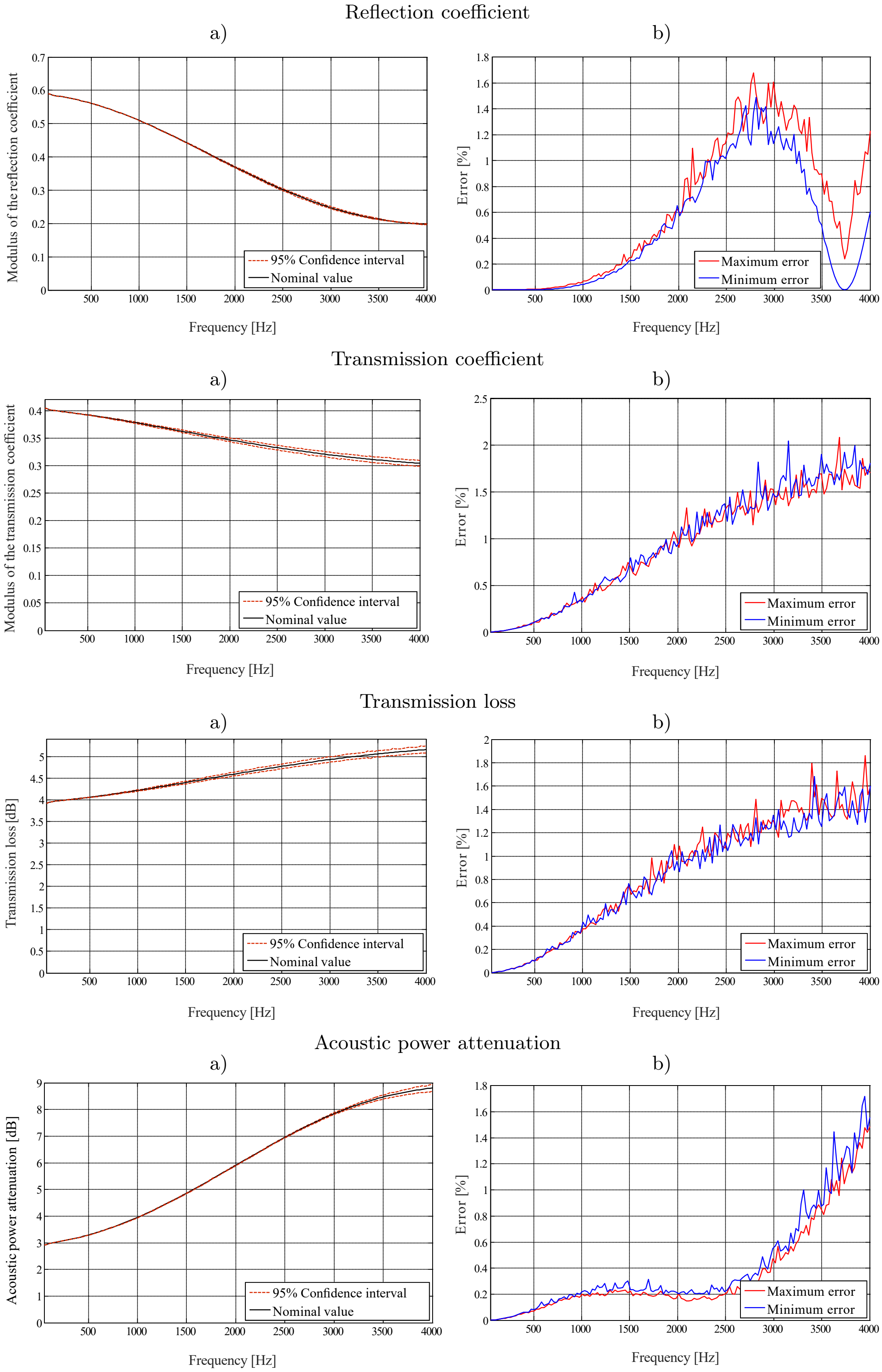

Fig. 5. Tortuosity uncertainties' effect on the acoustic behaviors: 95% confidence interval (a) and the corresponding error (b).

parameter is minor. This result is confirmed by the results presented in Fig. 4b showing the variation of the maximum and minimum errors due to the normal distribution of the porosity variation. Thus, a variation of ±5% of the nominal value of the porosity generates a minimum error equal to 2% (at high frequency) in

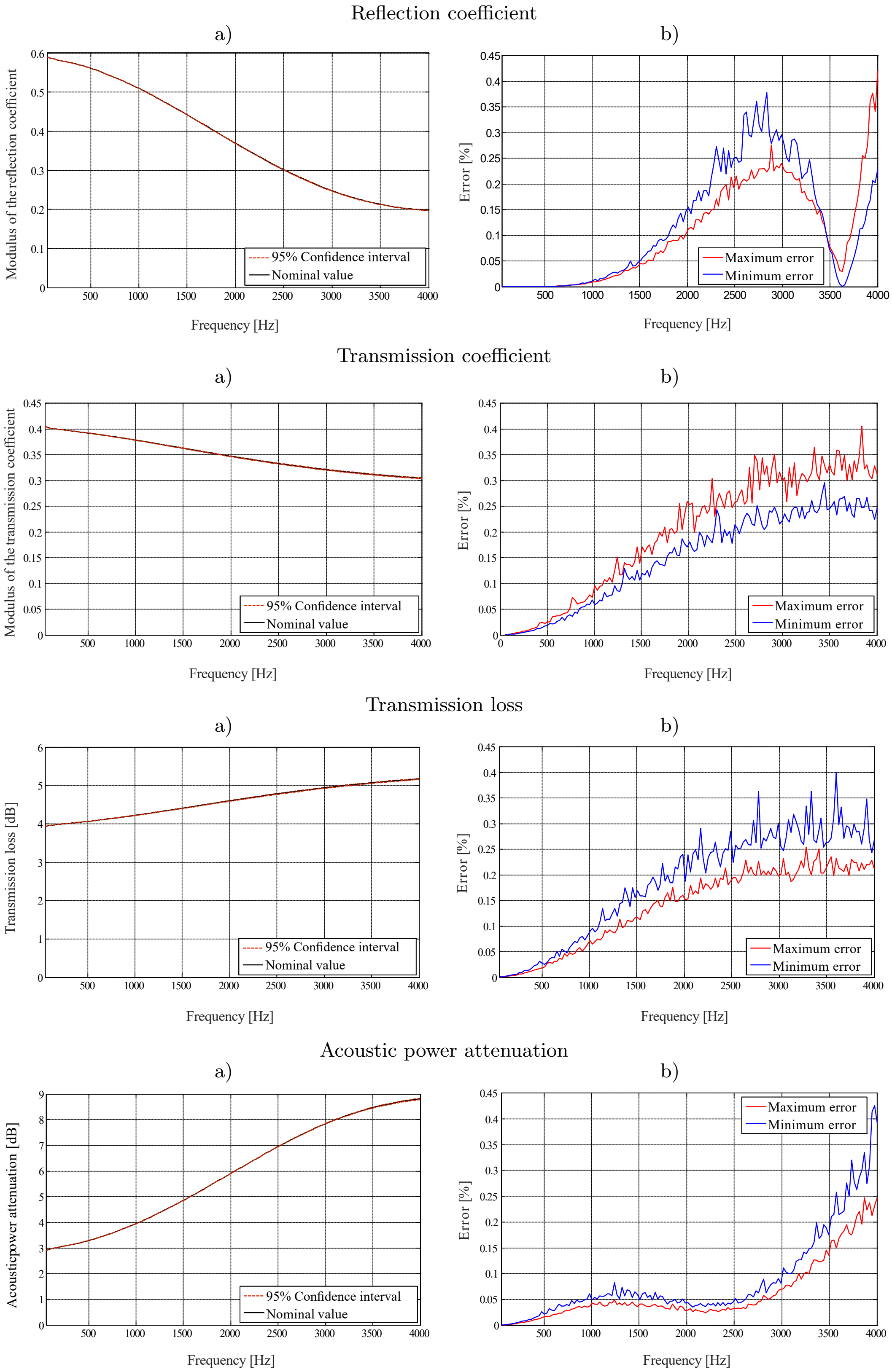

Fig. 6. Viscous characteristic length uncertainties' effect on the acoustic behaviors: 95% confidence interval (a) and the corresponding error (b).

the frequency band between 2000–4000 Hz and a maximum error equal to 2.5% in this band.

Figure 5 shows that the tortuosity has a small effect on the acoustic output parameters with small errors

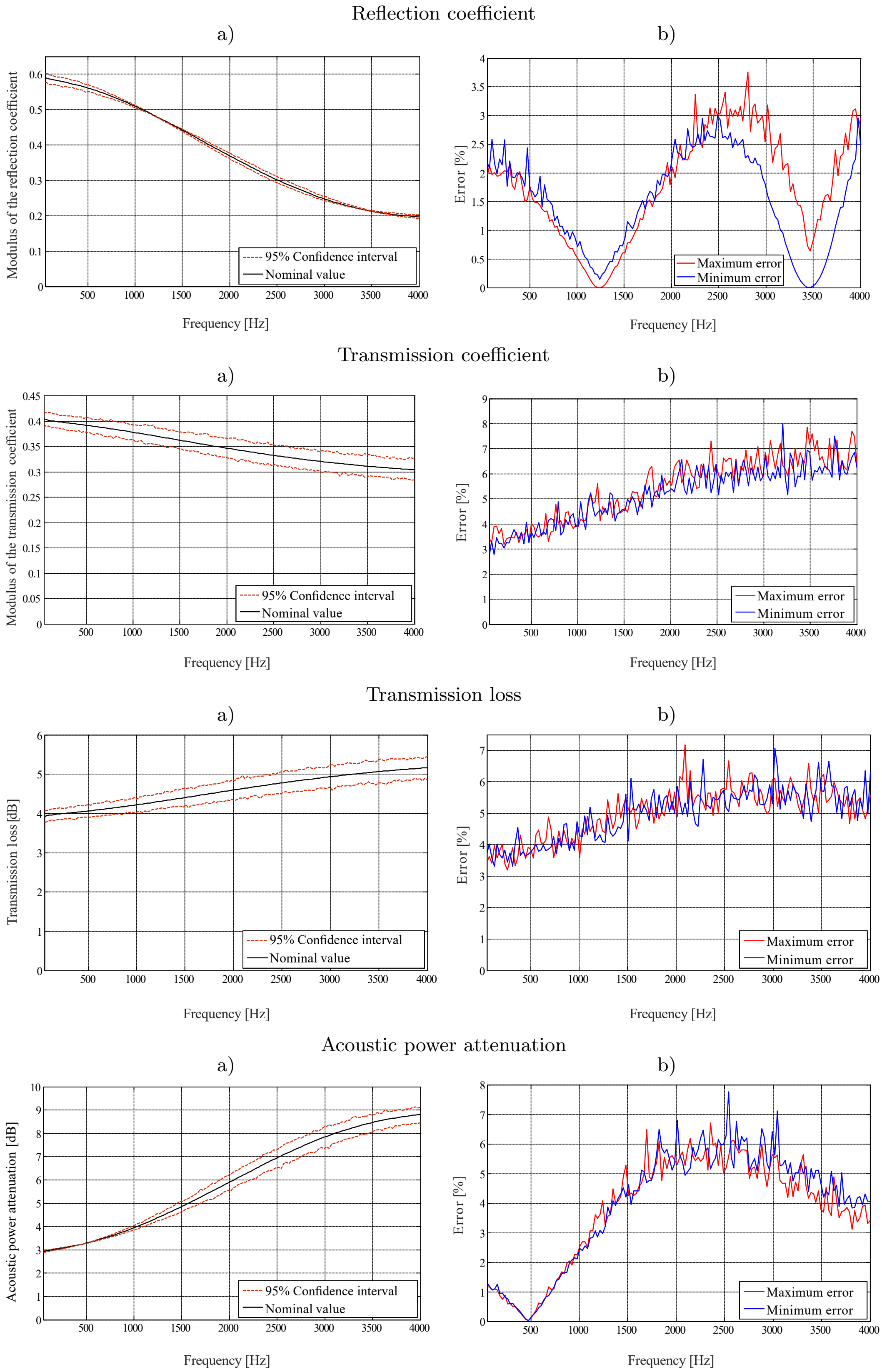

Fig. 7. Porous material length uncertainties' effect on the acoustic behaviors: 95% confidence interval (a) and the corresponding error (b).

and 95% confidence. This result is confirmed by the results presented in Fig. 5b, which shows the variation of the maximum and minimum errors due to the normal distribution of the variation of tortuosity; the maximum of these errors reaches the value of 2.5%.

Thus, a variation of ±5% of the nominal value of the tortuosity generates a minimum error equal to 0.3% (at low frequency) in the frequency band between (0–2500 Hz) and then an increase in value to 1.4% in the frequency band (2500–4000 Hz) and a maximum error equal to 1.5% in the frequency band between (2500–4000 Hz).

Figure 6 demonstrates that the viscous characteristic length Λ (as well as the thermal length Λ') has a negligible effect on the acoustic outputs the studied porous material. It is noticed that the thickness of the confidence interval is very small, which shows that the effect of this parameter is small, as shown in Fig. 6a, and with minimum errors. Thus, a variation of ±5% of the nominal value generates a minimum error equal to 0.23% and a maximum error equal to 0.33% at high frequency.

The effect of a variation of ±5% in the nominal value of the viscous and thermal characteristic lengths are, respectively, presented in Figs. 6 and 7. It is observed that the effect of these two lengths is similar.

When 5% errors are added to the nominal value of the length of porous materials, we observe that the thickness of the confidence interval is high in all the acoustic outputs. The curves show that this parameter has an important influence. A variation of ±5% of the nominal value of the length of porous materials on the attenuation generates a minimal error equal to 7.5% and a maximal variation error equal to 7% at a frequency equal to 2000 Hz.

5. Conclusion

This paper presented the results of the simulation of errors obtained by the Monte Carlo method, which allowed to determine the confidence interval of the coefficients of the scattering matrix transmission loss and acoustic attenuation for each input parameter and then for all parameters together. The Monte Carlo method is interesting for studying the degree of dependence of the model output on the inputs.

The results concluded that the resistivity and length of the porous material have a great influence on the acoustic outputs of the studied model (transmission and reflection coefficients, transmission loss, and acoustic power attenuation). At the same time, the effect of the uncertainty on the other parameters is negligible. It is important to indicate that acoustic power attenuation is the parameter less affected by input errors. The present study justifies the choice of this parameter in a previous work (Kani *et al.*, 2021) in the cost function of an inverse technique to determine porous parameters of a porous material in a duct element. By using this parameter, it is guaranteed to have results less sensitive to parameter uncertainties.

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