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A STOCHASTIC EVALUATION FOR NON-STATIONARY RANDOM NOISE OVER A LONG TIME PERIOD BASED ON LOCAL STATIONARITY AND ITS APPLICATION TO ACTUAL ACOUSTIC ENVIRONMENT

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A non-stationary environmental noise fluctuations can be treated as some composition of local state strips having arbitrary fluctuating pattern in a locally stationary time period. Its non-stationary characteristics can be seen in the form of its changing stage from a locally stationary state to another one by means of its temporal change of probability distribution. For a non-stationarity with continuously slow temporal change, the noise evaluation method is derived by considering the temporal change of moment statistics or distribution parameters. On the other hand, for a non-stationarity with stepwise rapid variation like an ON/OFF operation of machine, the noise evaluation method is derived by considering in principle the occurring probability of each of locally stationary state based on the mutually exclusive property. In the experimental consideration, the above contrastive two evaluation methods have been applied to actual cases with slowly changing environmental noise and rapidly changing machine noise.

1. Introduction

Generally speaking, when we pay our attention to a stochastic noise environmental system, almost all of sound noises show a diversified random fluctuation pattern due to a physical, social and human psychological causes, and furthermore show very often non-stationary properties. For the purpose of evaluating such a noise environmental system, some statistical treatment may need to give a unified and reasonable evaluation reflecting the above factors in it for these non-stationary random process.

In general, from the engineering point of view for evaluating and regulating a stochastic noise environmental system, not only the usual lower order statistics like average, variance and L_{eq} but also any of statistical quantities like higher order moments and $(100 - x)$ percentile L_x sound level directly combined with a whole probability distribution form of noise fluctuation may become very important in connection with a human response to the noise.

In the noise evaluation over a long time period, the noise indexes are calculated by using and summing up a fairly short time observation without considering whether the phenomenon is stationary or non-stationary. For example, the noise indexes are represented in some cases by the first ten minutes observation of sound levels as those of one hour in Japan [1]. For representing more stable evaluation indexes matched to the actual situation, it seems fundamental to find more reasonable evaluation method based on a continuous and long range observation.

In previous studies of non-stationary random processes, it seems to be very often divided into two categories [2]: one is when system parameters vary with a lapse of time (i.e., time-variant system); and the other is when a random excitation of the system exhibits non-stationary behaviour. As is well known, M.B. PRIESTLEY developed an approach to the spectral analysis of non-stationary processes based on the theory of "evolutionary spectra" [3]. On the other hand, the well-known Fokker-Planck equation and/or its generalization describe a temporal change of the probability distribution of a random wave [4, 5]. In our previous studies, some evaluation methods on the probability distribution were derived for the above mentioned non-stationary random noise by reflecting a temporal change of the distribution parameters [6, 7] such as mean and variance under an assumption of Gaussian property on the basis of stability on lower order moment statistics supported by some long time average operation. Furthermore, as a practical study of noise environmental systems, a unified statistical treatment of the probability density function (abr. *pdf*) for non-stationary random processes with a slow temporal change of mean value, variance, and/or arbitrary n-th order moment statistics has been considered from both the theoretical and experimental points of view [8]. Then, the non-stationary property has been reflected into the changing stage from a specific local stationarity to the other ones by considering the temporal fluctuation of distribution parameters and/or moment statistics. However, for actual environmental cases like an inner city situation, the noise level distribution shapes are far from Gaussian type and so the assumption of a Gaussian distribution may be invalid [9].

From the above practical point of view, in this report, without paying attention to the problem on whether the cause of the non-stationary behaviour is either in the temporal change of system parameters or in that of the input characteristic, the statistical estimation methods for the sound level distribution is theoretically proposed on the basis of observations of distribution parameters. Especially, the theoretical methods are derived from two points of view on the stochastic mechanism by dividing the mechanism of non-stationarity into two contrastive situations. The first one is derived for a phenomenon with a gentle non-stationarity caused by further fluctuation of distribution parameters by employing an evaluation method based on an averaging operation with respect to distribution parameters as shown in the previous study [10]. Temporal fluctuations of distribution parameters caused at changing stage from a locally stationary period to the other one is reflected as the averaged information of them in the stochastic evaluation method.

On the other hand, for the latter case of non-stationarity due to stepwise rapid change of state, it seems difficult to consider any averaging operation because of its rapid discontinuity. In this case, the probability distribution itself shown in a local stationarity and a ratio of its local occurrence time interval to a total time interval seem important information for describing a whole distribution shape of the phenomenon over a long time period. Here, the non-stationary property can be described by this temporal change of distribution parameters due to stepwise temporal change from a locally stationary state to the other one. That is, though the random fluctuation in a local period shows an arbitrary distribution characteristic, the occurrence probability of each local stationarity in which the time ratio of its occurrence is reflected becomes very important information to grasp the mechanism of the global non-stationarity as minutely as possible. Once after this stochastic mechanism of non-stationarity can be grasped globally, the noise level distribution over a long time period can be evaluated under an assumption of simplified expression on noise level distribution (like a Gaussian type distribution within each locally stationary time period) in a local period, since at any rate some smoothing effect can be positively utilized.

Finally, the effectiveness of the proposed method is confirmed experimentally too by applying it to the stochastic evaluation of the non-stationary sound and vibration environmental noises over a long time period.

2. Probabilistic evaluation method for non-stationary noise fluctuation

In general, though the environmental noise shows a stationary property within a local time period, it may often show a non-stationary property over a long time period due to a temporal change of several factors. Here, consider a non-stationary random sound level x. As some probabilistic evaluation method of x over a long time period, there seems to be the following two viewpoints.

1. **Descriptive method:** probability evaluation method over a long time supported by a universal expression like an Hermite type series expansion expression [11] without considering the internal mechanism of non-stationarity.

2. **Stochastic analysis of non-stationary mechanism:** An evaluation method based on globally stochastic structure of non-stationary behavior of phenomenon. There seems to be two contrastive methods.

(a) A method based on the continuous and slow temporal fluctuation of moment statistics and/or distribution parameters (Gentle non-stationarity).

(b) A method based on the rapid discontinuous change of the moment statistics and/or the distribution parameters illustrated by the occurrence probability of each locally stationary state (Stepwise changing non-stationarity).

2.1. Descriptive method

It is already-known that the instantaneous sound level x can be described by the statistical Hermite series expansion type expression [11] as a general explicit expression on the *pdf*, $P(x)$, as:

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \times \left\{ 1 + \sum_{n=3}^{\infty} \frac{A_n}{\sqrt{n!}} H_n\left(\frac{x-\mu}{\sigma}\right) \right\} \tag{1}$$

without concretely considering the stochastic mechanism on the global non-stationarity of phenomenon. Here, we have

$$\left. \begin{aligned} \mu &\triangleq \langle x \rangle, \qquad \sigma^2 \triangleq \langle (x-\mu)^2 \rangle, \\ A_n &\triangleq \left\langle \frac{1}{\sqrt{n!}} H_n\left(\frac{x-\mu}{\sigma}\right) \right\rangle. \end{aligned} \right\} \tag{2}$$

Since this is originally one of general type *pfd* expressions, this can be employed to illustrate descriptively the non-Gaussian property of the phenomenon (the meaning and role of higher order statistics in non-Gaussian distribution are described in Appendix I). Accordingly, the arbitrary distribution characteristic can be evaluated by this expression if passing over the concrete internal mechanism of non-stationarity.

2.2. A stochastic mechanism of non-stationary fluctuation

In general, though the random sound level shows usually a non-stationary property over a long time period, it may very often show locally a stationary property in a short time interval.

Let x be a non-stationary random sound level of arbitrary distribution type over a long time interval. Here, let us introduce a stochastic model of the non-stationary process that x can be composed by N strips of each fluctuation in a locally stationary time period. When x can be described by Eq. (1) in the ith locally stationary period, the probabilistic characteristics of random fluctuation on x can be illustrated by the distribution parameter vector with mean value, variance and higher order nth statistics A_n, as:

$$\mathbf{a}_i \triangleq [\mu_i, \sigma_i^2, A_{3i}, A_{4i}, \cdots] \qquad (\text{for } i = 1, 2, \cdots, N), \tag{3}$$

whose temporal fluctuation produces the non-stationary characteristics of x over a long time interval. Here, the *pdf* of x occuring in a locally stationary period can be described by a conditional *pdf* as $P(x|\mathbf{a}_i)$. Describing the *pdf* of occurrence on $\mathbf{a}$ as $P(\mathbf{a})$, the *pdf* of x over a long time period can be evaluated by

$$P(x) = \int P(x|\mathbf{a}) P(\mathbf{a})\, d\mathbf{a}. \tag{4}$$

Here, $\int \cdot P(\mathbf{a}) d\mathbf{a}$ denotes the averaging operation with respect to $\mathbf{a}$ and $P(\mathbf{a})$ can be evaluated by the occurrence time ratio of the local period to whole ones.

Then, two kinds of non-stationary evaluation method on probability distribution expression of x over a long time interval can be derived from Eq. (4).

2.2.1. Non-stationarity based on the slow change of distribution parameter

When $\mathbf{a}$ fluctuates continuously and slowly, the boundaries among each locally stationary period can not be clearly decided. Thus, the *pdf* over a long time interval can be evaluated by reflecting the non-stationary fluctuation of distribution parameters given in each local stationarity. Here, the averaging operation $\int \cdot P(\mathbf{a}) d\mathbf{a}$ in Eq. (4) play an important role to smoothing the fluctuation of $\mathbf{a}$.

A moment generating function $m_x(\theta)$ for x can be written as

$$\begin{aligned} m_x(\theta) &\triangleq \int_{-\infty}^{\infty} \exp\{\theta x\} P(x)\, dx \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \exp\{\theta x\} P(x|\mathbf{a})\, dx \right] P(\mathbf{a})\, d\mathbf{a}. \end{aligned} \tag{5}$$

Let the conditional moment generating function $g_x(\theta|\mathbf{a})$ as

$$g_x(\theta|\mathbf{a}) \triangleq \int_{-\infty}^{\infty} \exp\{\theta x\} P(x|\mathbf{a})\, dx. \tag{6}$$

By expanding $g_x(\theta|\mathbf{a})$ around the conditional moment generating function $g_0(\theta|\mathbf{a}_0)$ of an arbitrarily introduced stationary process conditioned by parameter vector $\mathbf{a}_0$ with mean value μ_0, variance σ_0^2 and the mth order moment statistics μ_{0m}, we have

$$g_x(\theta|\mathbf{a}) = g_0(\theta|\mathbf{a}_0) \cdot g_x(\theta|\mathbf{a}) / g_0(\theta|\mathbf{a}_0). \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (5), we have

$$\begin{aligned} m_x(\theta) &= \sum_{r=0}^{\infty} \sum_{t=0}^{\infty} \sum_{n=0}^{\infty} A_{0n} \frac{\langle B_r M_t \rangle}{r!} \sigma_0^n \int_{-\infty}^{\infty} e^{\theta x} \\ &\quad \times \frac{d^{r+t+n}}{dx^{r+t+n}} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(x-\mu_0)^2/2\sigma_0^2}\, dx \end{aligned} \tag{8}$$

with

$$B_r \triangleq \sum_{k=0}^{\left[\frac{r}{2}\right]} \frac{(\mu - \mu_0)^{r-2k} (\sigma^2 - \sigma_0^2)^k}{(r-2k)! k! 2^k} \tag{9}$$

$$\begin{aligned} M_t &\triangleq \frac{d^t}{d\theta^t} \left[1 + \sum_{m=3}^{\infty} \frac{\theta^m}{m!} \left(A_m \sigma^m - A_{0m} \sigma_0^m \right) \right. \\ &\quad \left. \Big/ \left(1 + \sum_{m=3}^{\infty} \frac{A_{0m} \sigma_0^m \theta^m}{m!} \right) \right] \Bigg|_{\theta \to 0}. \end{aligned} \tag{10}$$

Then, we have the *pdf* of x reflecting the non-stationary property of distribution parameters as follows [10]:

$$\begin{aligned} P(x) &= \sum_{r=0}^{\infty}\sum_{t=0}^{\infty}\frac{\langle B_r M_t\rangle_{\mathbf{a}}}{t!}(-1)^{r+t} \\ &\times\left(\frac{d}{dx}\right)^{r+t}\left[\frac{1}{\sqrt{2\pi}\sigma_0}e^{\frac{(x-\mu_0)^2}{2\sigma_0^2}}\right. \\ &\times\left.\left\{1+\sum_{n=3}^{\infty}\frac{A_{0n}}{n!}H_n\left(\frac{x-\mu_0}{\sigma_0}\right)\right\}\right]. \end{aligned} \tag{11}$$

2.2.2. Non-stationarity based on the rapid change of distribution parameter

Next, let us consider a non-stationarity due to a rapid and/or stepwise non-stationary change (e.g., based on ON/OFF operation of a machine). In this case, it is reasonable to introduce a stochastic mechanism of exclusive property (e.g., when a certain local stationarity occurs, other one never occurs) among many of each locally stationary period. In this stochastic mechanism, the contributions from each local stationarity to the whole period can be reflected well rather than averaged distribution parameters for this rapid and stepwise non-stationarity. So, the non-stationary property can be mainly described by a probability of occurrence $P(\mathbf{a})$ on each of locally stationary state in Eq. (4). Not to say, since the occurrence of each locally stationary period is mutually exclusive in principle, $P(\mathbf{a})$ can be evaluated as

$$P(\mathbf{a}) = \sum_{i=1}^{N} P_i\cdot\delta(\mathbf{a}-\mathbf{a}_i) \tag{12}$$

with each probability of occurrence of the i-th locally stationary period P_i. As the result, $P(x)$ can be rewritten as

$$P(x) = \sum_{i=1}^{N} P(x|\mathbf{a}_i)P_i. \tag{13}$$

Next, let us consider the random fluctuation of $x(t)$ within each locally stationary time period. By considering the actual situation of environmental noise measurement, the *pdf* expression of x within the i-th local stationarity can be given by the statistical Hermite type expansion expression [11] conditioned by $\mathbf{a}_i$ as:

$$P(x|\mathbf{a}_i) = \frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{x-\mu_i}{2\sigma_i^2}}\times\left\{1+\sum_{n=3}^{\infty}A_{ni}\frac{1}{\sqrt{n!}}H_n\left(\frac{x-\mu_i}{\sigma_i}\right)\right\} \tag{14}$$

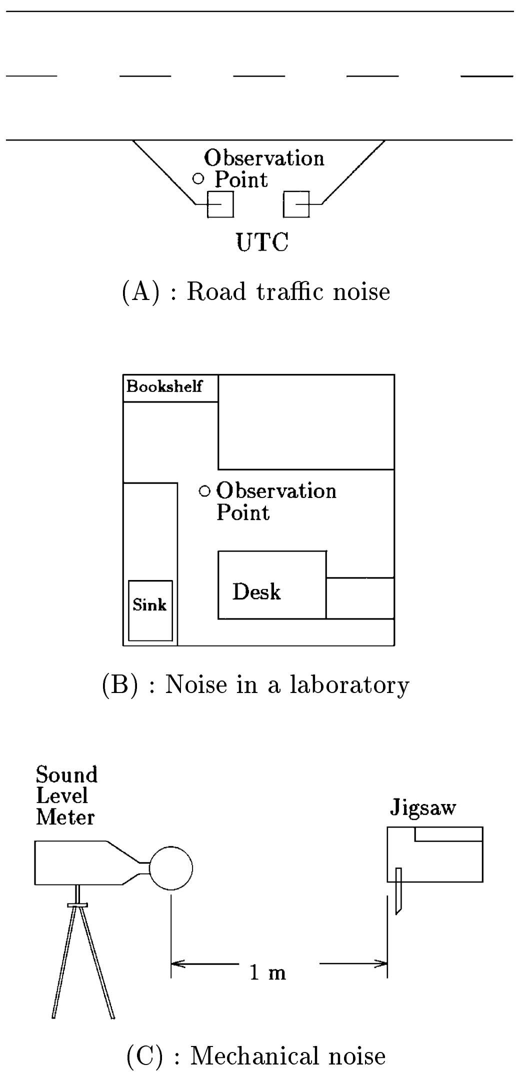
with

$$A_{ni}\triangleq\left\langle\frac{1}{\sqrt{n!}}H_n\left(\frac{x-\mu_i}{\sigma_i}\right)\middle|\mathbf{a}_i\right\rangle, \tag{15}$$

where, μ_i, σ_i^2 and A_{ni} are respectively mean value, variance and higher order distribution parameters of the i-th local stationarity. Here, $H_n(\cdot)$ denotes the n-th order Hermite polynomial.

3. Experimental consideration

The proposed contrastive theoretical methods have been experimentally confirmed by applying them to the non-stationary environmental noise levels based on working machine, road traffic and inside a laboratory. Figure 1 shows there situations of noise measurement — Case A: a street noise with a light traffic flow (about 500 to 600 vehicles per hour), Case B: a sound noise in a quiet laboratory room with rare ring of phone or chattering voices, Case C: a mechanical noise of a jigsaw at a distance of 1 m. The total observation periods were set to 3600 seconds for Cases A and B and 200 seconds for Case C. Characteristics of observed noises are as follows:



(A) : Road traffic noise

(B) : Noise in a laboratory

(C) : Mechanical noise

Fig. 1. Experimental situations of noise measurement.

• Fig. 1(A),(B): Environmental noises whose fluctuation factors contribute to the non-stationarity over a long time period could not be clear and they are not observed in connection with the causes of noise generation. Furthermore, for these noise fluctuations with a gentle non-stationary change of distribution parameters, it is difficult in principle to find accurately a distinguished part of locally stationary period [12]. Accordingly, the locally stationary period has been decided artificially by recognizing each local period as stationary without considering the internal mechanism of non-stationarity of the phenomenon. Measurement has been carried out continuously over a long time period (one hour) with 36 equal local periods of 100 seconds. Distribution parameters have been observed in each locally stationary period. The probability of occurrence on each local stationarity, P_i $(i = 1, 2, \cdots, N)$, has been evaluated as the time ratio of the time length of a locally stationary period to a whole observation period.

• Fig. 1(C): A random noise radiated from a jigsaw under stepwise changed speed operation (e.g., three steps of operating speed with low, middle and high) was employed as an example of mechanical noise. At each stage of constant speed, the radiated noise fluctuation was regarded as stationary and its time length and the distribution parameter vector have been observed. The operating time table is shown in Table 1. The time ratio of operating time length of a locally stationary period to a whole one has been adopted as the probability of occurrence on each stage of operation, P_i $(i = 1, 2, \cdots, N)$.

Table 1. Non-stationary operation pattern of jigsaw and P_i's.

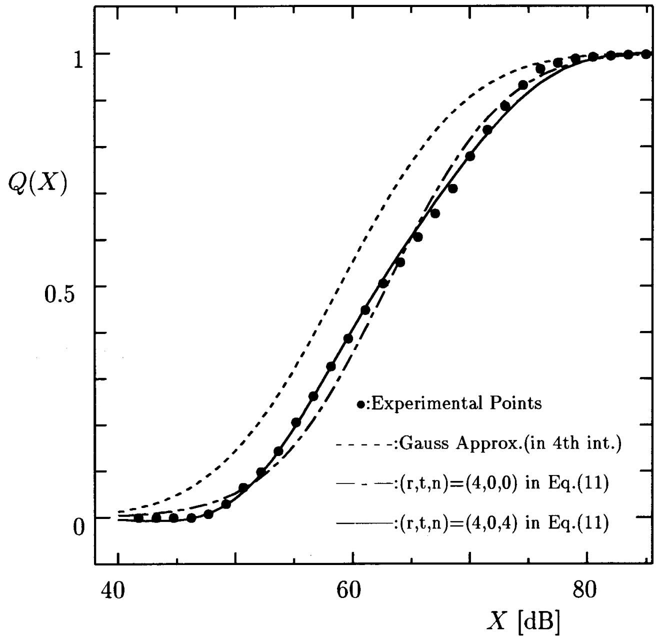
i operation	1 Low	2 Mid.	3 High	4 Mid.	5 Low
period [sec] P_i	15 0.075	30 0.15	125 0.625	15 0.075	15 0.075

Considering that the sound noise level fluctuation is usually evaluated by noise index L_x ($(100 - x)\%$ sound level), Eqs. (11) and (13) are evaluated in the form of cumulative distribution function of sound noise level fluctuation as:

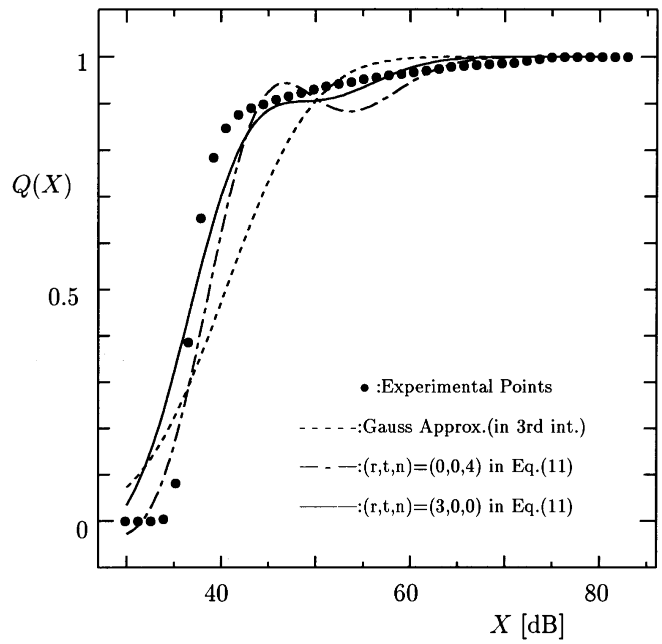
$$Q(X) \triangleq \int_{-\infty}^{X} P(x)\,dx. \tag{16}$$

3.1. Evaluation of noise level distribution with slow temporal change of distribution parameter

The non-stationary fluctuation of distribution parameters obtained in each locally stationary period has been evaluated by Eqs. (9) and (10) as expansion coefficients B_r and M_t. The noise level distribution has been evaluated by Eq. (11). The degree of approximation for non-stationary property was employed up to 4 for r and 0 for t (see Appendix II). Theoretically estimated results of the cumulative noise level distribution are shown in Fig. 2. In the Case (B), as its circumstance was mainly quiet with occasional

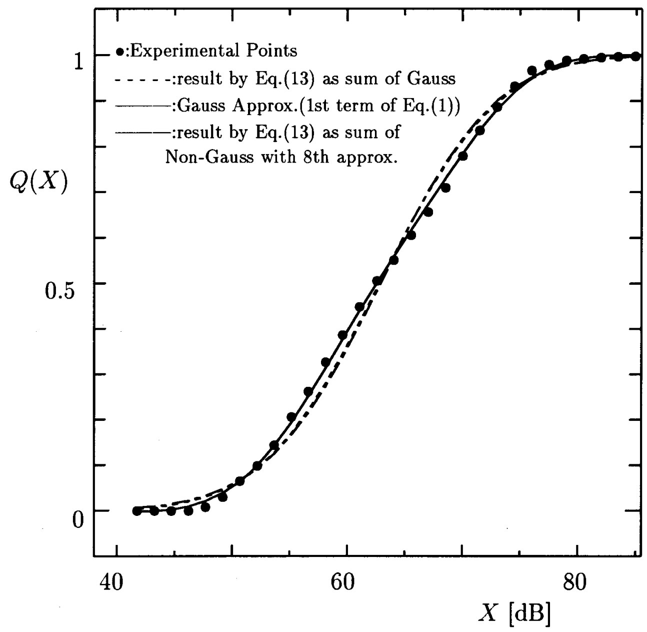


(a) Case (A) : Traffic noise

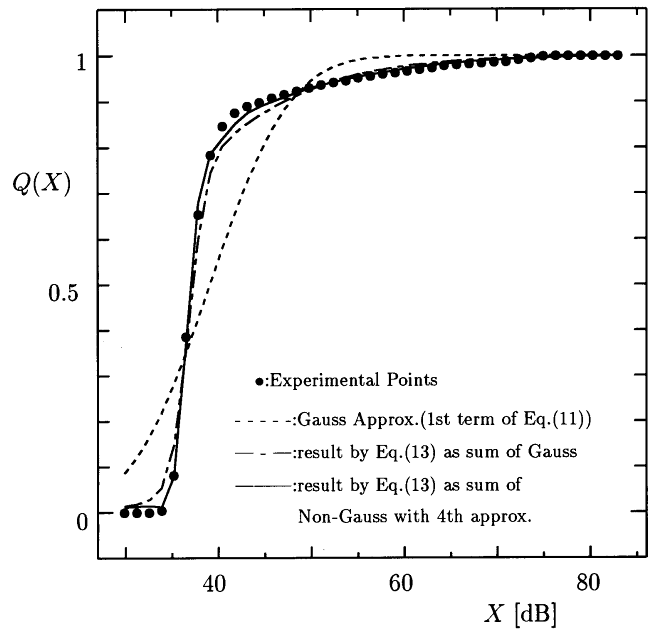


(b) Case (B) : Laboratory noise

Fig. 2. Comparisons between experimentally sampled points and theoretically estimated curves for cumulative distribution of environmental noise levels as gentle non-stationary phenomenon. The estimation curves are shown with 1st term in Eq. (1) and with the degree of approximation (r, t, n) in Eq. (11).



(a) Case (A) : Traffic noise



(b) Case (B) : Laboratory noise

Fig. 3. Comparisons between experimentally sampled points and theoretically estimated curves for cumulative distribution of environmental noise levels as stepwise changing non-stationary phenomenon. The estimation curves are shown with 1st term in Eq. (1) and with the degree of approximation $n = 0$, 4 and 8 in Eqs. (13) and (14).

ring of phone and chattering voices, the fluctuation pattern was rather a rapid changing non-stationarity case. So, the theoretical result could not illustrate well the experimental points.

3.2. Evaluation of noise level distribution with probability of occurrence on local stationarity

3.2.1. Application to environmental noises

The noise level distribution within each local period was evaluated with distribution parameters obtained in each period by Eq. (14). Furthermore, the noise level distribution over a long time period has been evaluated by Eq. (4) by setting P_i to 1/36. The estimation results are shown in Fig. 3. Though the theoretical result in Fig. 2(b) Case (B) could not illustrate well the experimental points, it can illustrate well them within a permissible error less than 1 dB in Fig. 3(b) Case (B).

3.2.2. Application to a mechanical noise (Case C)

For a stepwise operated case of non-stationary type, let us apply the proposed method of Eq. (13). P_i's in Eq. (12) and the operating time pattern are shown in Table 1.

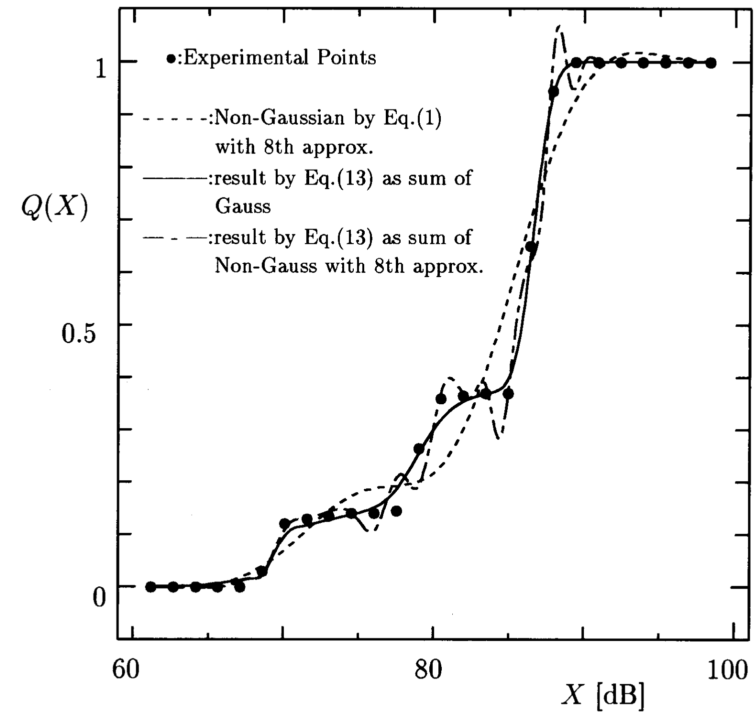


Fig. 4. Comparisons between experimentally sampled points and theoretically estimated curves for cumulative distribution of noise level fluctuation radiated from a jigsaw under known non-stationary operation. The estimation curves are shown with the degree of approximation $n = 8$ in Eq. (1) and with the degree of approximation $n = 0$ and 8 in Eqs. (13) and (14).

The mean value and the variance of x have been observed in each locally stationary period. A conditional *pdf* within the i-th locally stationary period, $P(x|\mathbf{a}_i)$ is given by Eq. (14) especially under a simplified assumption of standard type Gaussian *pdf.* The estimation result is shown in Fig. 4. In spite of stepwise fluctuating pattern of sound noise, the theoretical curve illustrates well the experimental points within a practically permissible error (about 1 dB).

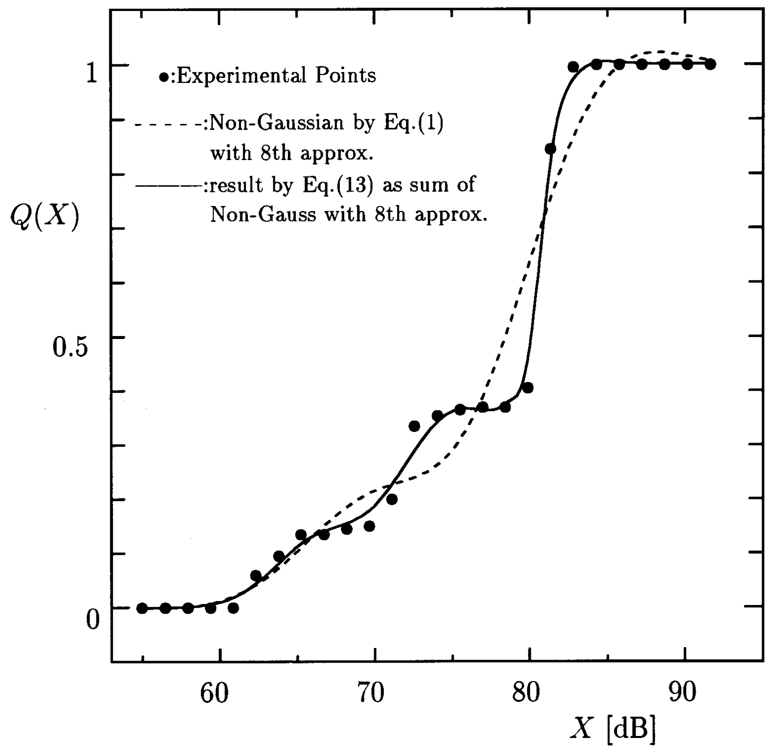


Fig. 5. Comparisons between experimentally sampled points and theoretically estimated curves for cumulative distribution of noise level fluctuation radiated from a jigsaw under unknown non-stationary operation pattern. The estimation curves are shown with the degree of approximation $n = 8$ in Eq. (1) and with the degree of approximation $n = 8$ in Eqs. (13) and (14).

Furthermore, as a trial for application to a case of unknown operation time pattern, the whole observation period was divided into 5 equal local periods and the distribution parameters were observed precisely up to higher expansion terms of non-Gaussian property in each local period. The estimation result is shown in Fig. 5. When the operating time pattern is unknown, the non-Gaussian property will increase since some stepwise change of operation would be contained in a local period. Accordingly, it seems effective to evaluate accurately the distribution information in each local period in this case.

4. Conclusion

In this study, two contrastive evaluation methods for non-stationary random noise have been theoretically proposed from a viewpoint of stochastic mechanism of non-stationarity in connection with relationship between the locally stationary period and a whole long time period. The non-stationary property was divided into two types: one is caused by continuous and gentle change of distribution parameters and another is caused by rapid discontinuous change of them. Especially, in the latter case, when the stochastic mechanism of non-stationarity has been globally grasped, a probability distribution of sound level over a long time period can be precisely estimated by using positively the exclusive property among locally stationary periods. In the experimental consideration, the above contrastive two kinds of evaluation method have been applied to the actual cases when slowly changing environmental noise and rapidly changing mechanical noise. Thier noise level distributions have been evaluated theoretically by concretely employing the information on parameter $\mathbf{a}$ in good agreement with experimental ones.

Appendix I. Meaning and role of higher order statistics in non-Gaussian distribution

The meaning and the role of higher order statistics in non-Gaussian distribution (of not only single-variate but also multi-variate phenomenon) seems to be itemized as follows:

1. In the study on one stochastic variate of non Gaussianly fluctuating wave, many higher order statistics like skewness, kurtosis and so on seem to correspond to the scale on an existence style (fluctuation pattern, behaviour or probability distribution form) of physical quantity, differing from the lower order statistics like mean and variance directly connected to the scale of physical existence itself. For example, in the similar way as the well-known study on a distorted periodical wave in an elementary electric circuit theory (deviated from a pure sinusoidal wave), the degree of distortion is hierarchically reflected in many higher harmonics and is measured sometimes in a scale of distortion factor as an existence style (behaviour or wave pattern) of physical quantity differing from the direct connection to physical existence itself on an energy scale (like an effective value). Furthermore, for a study on more than 2 stochastic variates (necessary to the special study on mutual correlation among them), it is inevitable to introduce step by step the idea of higher order mutual correlation as a scale of mutual stochastic relationship, especially among respective remote levels apart from mean value, in addition to the well-known idea of the 1st order (linear) correlation (directly connected the mutual relationship mainly focussed on the neighborhood of the mean value).

2. As seen in Figs. 4 and 5, it is obvious that our experimental results can be theoretically understood by taking some higher order moment statistics into consideration in order.

3. In the well-known signal processing techniques like AR, MA, ARMA and ARIMA models, the above mentioned higher order statistics is usually and artificially reflected

only in the latent error style from an opperational viewpoint. But, since the human being is not only a biological but also high-ranking valuable existence (connected with truth, good and beauty) beyond only an energetic physical existence, in our living nature and human, it is sure that many of the meaningful error, accident, exception, great genius, secret essence *etc.* certainly exist (might be partly related to the above higher order mutual correlation quantities?) and should not be excluded artificially in our eagerness to hurry up our artificial realization (only from workman's opperational standpoint forgetting our original quality of humanity).

4. Especially, in the extremum statistics with more than 2 random variates, if we want to study hierarchically step by step based on the bottom-up way viewpoint (from the lower order to the higher order statistics) beyond only an apparently descriptive style study, it seems inevitable to introduce such an idea of higher order mutual correlation, even if it is not well-known.

Appendix II. The Order of non-stationarity

In general, statistical expansion series type expression is convenient to illustrate a diversified stochastics phenomenon with a higher precision by taking step by step more and more expansion coefficients of higher order from a theoretical viewpoint as mentioned in the previous studies [13, 14]. This concept seems fundamental in our study of non-stationarity treated in this paper. However, in this expansion expression, it becomes necessary to investigate up to what order the expansion coefficients should be reasonably determined, especially from problem-oriented viewpoint. So, in the previous papers [13, 14], a method to determine the model order corresponding to an optimal number of expansion terms was theoretically proposed in connection with the experimentally sampled data. In this paper, by considering the previously proposed method and the past experience on the order decision, the order of non-stationarity was selected as shown in Fig. 2. That is, the order r in Eq. (11) was set 3 or 4, the higher order t of non-Gaussian property did not give a good estimation result because of the astable propoerty of higher order statistics over long time interval.

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