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IDENTIFICATION OF ZONES OF INCREASED VIBROACOUSTIC EMISSION BY MEANS OF THE INVERSE METHOD

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The basic formulas concerning the inversion are presented in the paper. Examples of the author's studies on the possibility of the localization of zones of increased vibroactivity by means of the inverse method are given. The inverse method is very sensitive to the position of substitute sources. This feature can be used for the determination of the position of actual sources. The accuracy of the method depends also on the distribution of the observation points. The dependence of the position of substitute sources on their acoustic power level is discussed in the paper as well.

The distribution of the acoustic fields around the air compressor is given as an example. The measurements were performed at the points distributed on a semi-sphere. Both the amplitude and phase of the acoustic pressure were determined at each measuring point. On the basis of the results obtained, the zones of increased acoustic emission were searched for.

Key words: acoustic modeling, inverse method, sound source.

1. Introduction

Increasing possibilities of parallel obtaining of data and their effective processing as well as improved calculating tools are the reasons that inverse methods are gaining significance. Inverse methods are applicable in various kinds of acoustics: oceanic acoustics, vibroacoustics, aero acoustics, structural acoustics and also in geophysics.

Inverse methods allow to make an acoustic assessment of machines on the bases of the analysis of acoustic field parameters. Modeling of the vibroacoustic energy radiation from the sound source to the receiver and the knowledge of the true value of the acoustic pressure at the measuring points allow to reverse the propagation path and thus to determine the parameters of the source itself.

Inverse methods can be applicable in the identification of vibroacoustic energy sources and in the sound radiation assessment. Reciprocity methods are often used together with inverse methods.

Studies of the possibility of localization of zones of increased vibroactivity by means of the inverse method are presented in this paper. Basic definitions and formulas concerning the inversion itself are also given.

2. Inverse method

The schematic presentation of the determination of parameters of models by the inverse method is shown in Fig. 1. The acoustic pressure at the observation points, marked M in Fig. 1, can be determined on the basis of the inverse method [5].

$$\mathbf{p} = \mathbf{G}\boldsymbol{\alpha} + \mathbf{e} \quad [\mathrm{Pa}], \tag{1}$$

where:

- $\mathbf{p}$ – m-dimensional vector of the measured complex amplitudes of the acoustic pressure at the observation points [Pa],
- m – number of observation points,
- $\boldsymbol{\alpha}$ – n-dimensional vector of complex parameter values of the model source [Pa m],
- n – number of searched source parameters,
- $\mathbf{G}$ – matrix $m \times n$ defining the complex value of the acoustic pressure at the observation points. Parameters of substitute sources were determined on the bases of the assumed emission model [m^{-1}],
- $\mathbf{e}$ – m-dimensional unknown vector of error [Pa].

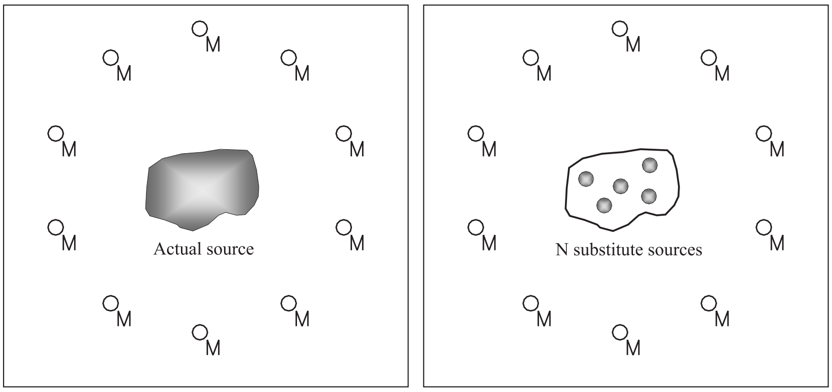

Fig. 1. Schematic presentation of the determination of model parameters by means of the inverse method.

In order to limit as much as possible the error vector, we are fitting the parameters of individual sources of the model. One of the most frequently applied criteria (method of least squares) is the minimization of the expression:

$$K = \mathbf{e}^H \mathbf{e}, \tag{2}$$

where $(\ )^H$ – means conjugate and transposed matrix.

The optimal model parameters can be determined by the method of least squares from formula [5]:

$$\mathbf{a} = \left(\mathbf{G}^H \mathbf{G}\right)^{-1} \mathbf{G}^H \mathbf{p} \quad [\mathrm{Pa\,m}], \tag{3}$$

where **a** – n-dimensional vector of the estimated complex values of parameters of the model sources.

The main analytical tool for solving the inversion problems is the matrix distribution versus singular value decomposition (SVD). The suitability of this distribution results from the fact that the matrix of a transfer function **G** can be expressed as [6]:

$$\mathbf{G} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^H, \tag{4}$$

where **U**, **V** – unitary matrices, $\mathbf{\Sigma}$ – diagonal matrix $m \times n$, where individual elements of a diagonal satisfy the condition.

When v and u are the i-th columns of the matrices **U** and **V**, the following equations can be written:

$$\mathbf{G}\,\mathbf{v}_i = \sigma_i\,\mathbf{u}_i\,; \qquad \mathbf{G}^H\mathbf{u}_i = \sigma_i\mathbf{v}_i\,. \tag{5}$$

The following matrix is called the pseudo inverse matrix:

$$\mathbf{G}^+ = \left(\mathbf{G}^H\mathbf{G}\right)^{-1}\mathbf{G}^H. \tag{6}$$

Thus, the vector of the estimated complex values of the parameters of the model sources can be determined from:

$$\mathbf{a} = \mathbf{G}^+\mathbf{p}. \tag{7}$$

The matrix **G** can be determined from the distribution SVD:

$$\mathbf{G}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\mathbf{U}^H, \tag{8}$$

where:

$$\begin{aligned} \mathbf{\Sigma}^+ &= [S^{-1}|0], \\ S^{-1} &= \mathrm{diag}\,(1/\sigma_1\ ...\ 1/\sigma_N). \end{aligned}$$

A measure of the accuracy of the performed simulations is the value of the expression:

$$\kappa(\mathbf{G}) = \frac{\sigma_{\max}}{\sigma_{\min}}\,. \tag{9}$$

Thanks to this value we can determine the rational error of the estimation of source parameter $\dfrac{\delta\alpha}{\alpha}$ based on the estimation of the rational error during estimating the amplitudes of the acoustic pressure $\dfrac{\delta\pi}{\pi}$ during determining the model parameters:

$$\left|\frac{\delta a}{a}\right| \le \kappa(\mathbf{G})\left|\frac{\delta\pi}{\pi}\right|, \tag{10}$$

where π means the true value of the complex amplitudes of the acoustic pressure.

Searching for noise sources of the model parameters was carried out only for stationary processes and noise sources located on the surface of the hard screen. Single omnidirectional sources were assumed as replacement noise sources. When the angles of phase shift are known, the model of the source was assumed as follows:

$$\pi = \sqrt{\frac{N\rho_0 c}{3.14...}}\,\frac{\exp(ikr)}{r}\,, \tag{11}$$

where k [m^{-1}] means wave number.

When the angles were unknown, it was assumed that:

$$\pi^2 = \frac{A^2}{r^2}\,. \tag{12}$$

The assumptions and the bases of the inverse method presented in this part of the paper will be utilized later.

The knowledge of the actual distribution of the acoustic pressure around the sound sources is essential for the calculation of the model parameters. To achieve this, it is necessary to determine the distribution of the acoustic pressure amplitudes as well as the distribution of the phase shift angles between the acoustic signals at the selected measuring points (e.g. on the surface of a semi-sphere). The knowledge of angles of the phase shift is a crucial factor since the value $\exp(ikr)$ is very sensitive to changes of the distance from the source (independent of the distance of the measurement location from the source).

The determination of the phase shift angle is possible only when the individual sound sources (e.g. elements of machines) are kinematically coupled with each other. Otherwise the measurement of the average phase shift angle is impossible (since the angle changes in time). In such cases, the inverse method can also be applied but one must realize that the accuracy of the calculations will be significantly lower. The errors in parameters of the model $\kappa(\mathbf{G})$ can be several thousand times larger. Then $\mathbf{p}$ (in Eq. (1)) will mean the vector of the measured acoustic pressure squares at the observation points, while $\boldsymbol{\alpha}$ will mean the vector of the acoustic power of individual sources. The matrix $\mathbf{G}$ will be the actual matrix.

3. Method's sensitivity to the source position

To be able to assess properly the parameters of the substitute sources, those sources should be located exactly at the places of the actual sources. The inverse method (especially when the measurements involve the phase shift angle) is highly sensitive to the localization of the substitute sources. This feature can be utilized for the determination of the position of the actual sources. The accuracy of the method depends also on the arrangement of the observation points. Some examples are presented below. These examples with the equations and pictures are made by the author.

3.1. Observation points arranged on the surface of the sphere

In the first part of the study, the influence of the substitute source localization on the value of its acoustic power was investigated. Observation points were arranged in a continuous way on the surface of the semi-sphere. The omni-directional sound source model of 20 dB power (imitating the actual source) was situated in the centre of the coordinate system. During the tests the substitute source was shifted inside a square of side equal to the radius of the sphere (Fig. 2). Then the parameters of the substitute source (acoustic power) were calculated on the basis of Eq. (7).

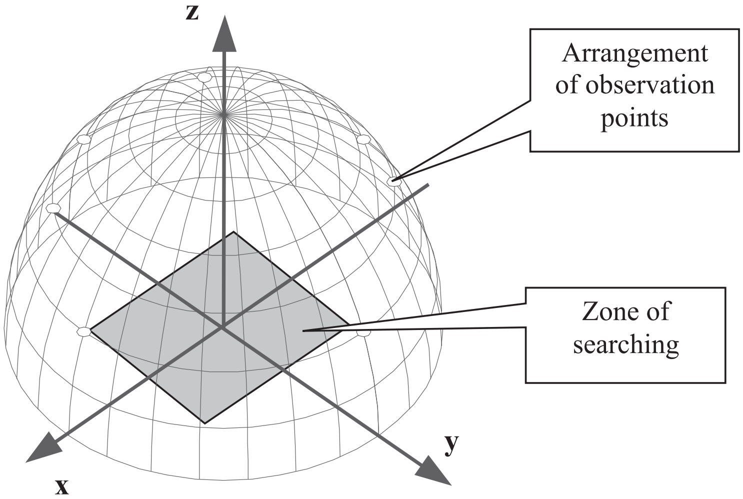

Fig. 2. Arrangement of sound sources and observation points on the surface of the semi-sphere.

Then, the acoustic power value of the substitute source, determined by the inverse method (with phase), can be calculated by the formula:

$$N_z = N \frac{\sin(c\,k\,R)^2}{(c\,k\,R)^2} \quad [\mathrm{W}], \tag{13}$$

where N – actual source power; N_z – substitute source power [W]; R – radius of the semi-sphere [m]; cR – distance between the substitute and the actual source [m]; c – rational shift of replacement the source with respect to the real source which is located in the middle of the coordination system.

If the phase shift angles are not known, we can estimate the substitute source power from the following equation:

$$N_z = N \frac{(1-c^2)^2}{2c} \log\left(\frac{1-c}{1+c}\right) \quad [\mathrm{W}]. \tag{14}$$

Examples of the calculation results – assuming $R = 2m$ – are presented graphically in Fig. 3.

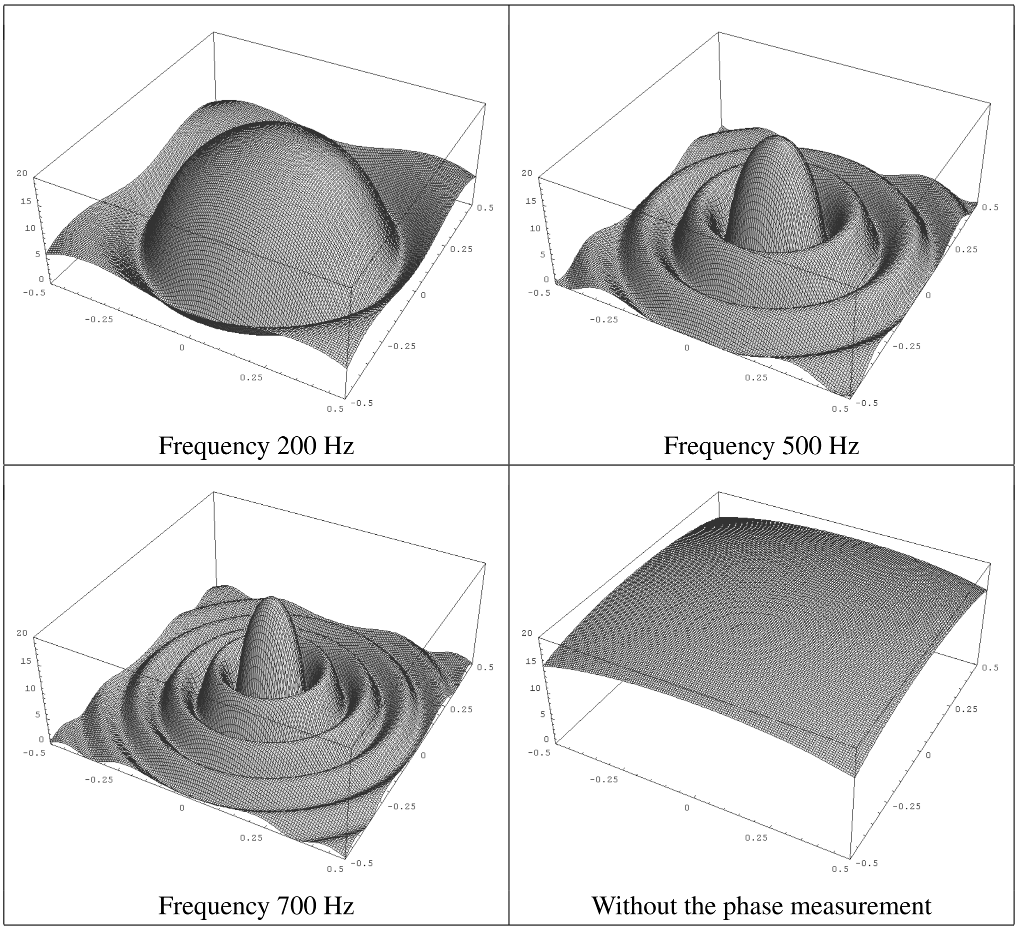

Fig. 3. Sound power level of the substitute source depending on its position – observation points arranged on the surface of the semi-sphere.

3.2. Observation points arranged on the circle

In the second part of the study, the observation points were arranged in a continuous way on the circle of radius R. The omni-directional sound source was placed in the centre of the coordinate system. During the tests, the substitute source was shifted inside the square of a side equal to the radius of the circle (Fig. 4). Acoustic power values were calculated on the basis of Eq. (7).

The acoustic power value of the substitute source determined by the inverse method (with phase) can be calculated then from the formula:

$$N_z = \frac{N}{4\pi^2} \left(\int_0^{2\pi} \cos \left[kR \left(1 - \sqrt{1 + c^2 - 2c\cos(\varphi)} \right) \right] d\varphi \right)^2 \quad \text{[W]}. \tag{15}$$

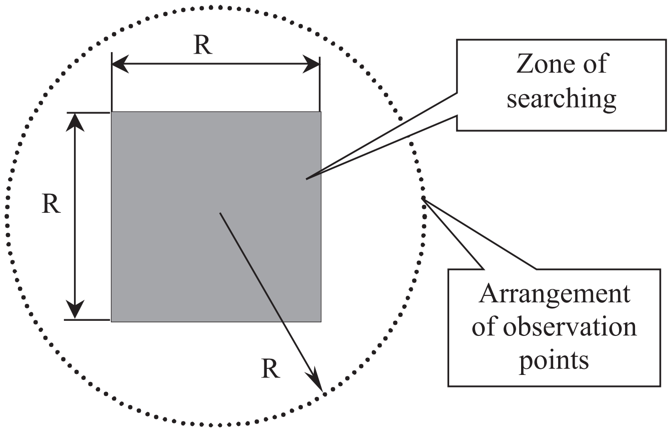

Fig. 4. Arrangement of sound sources and observation points on the circle.

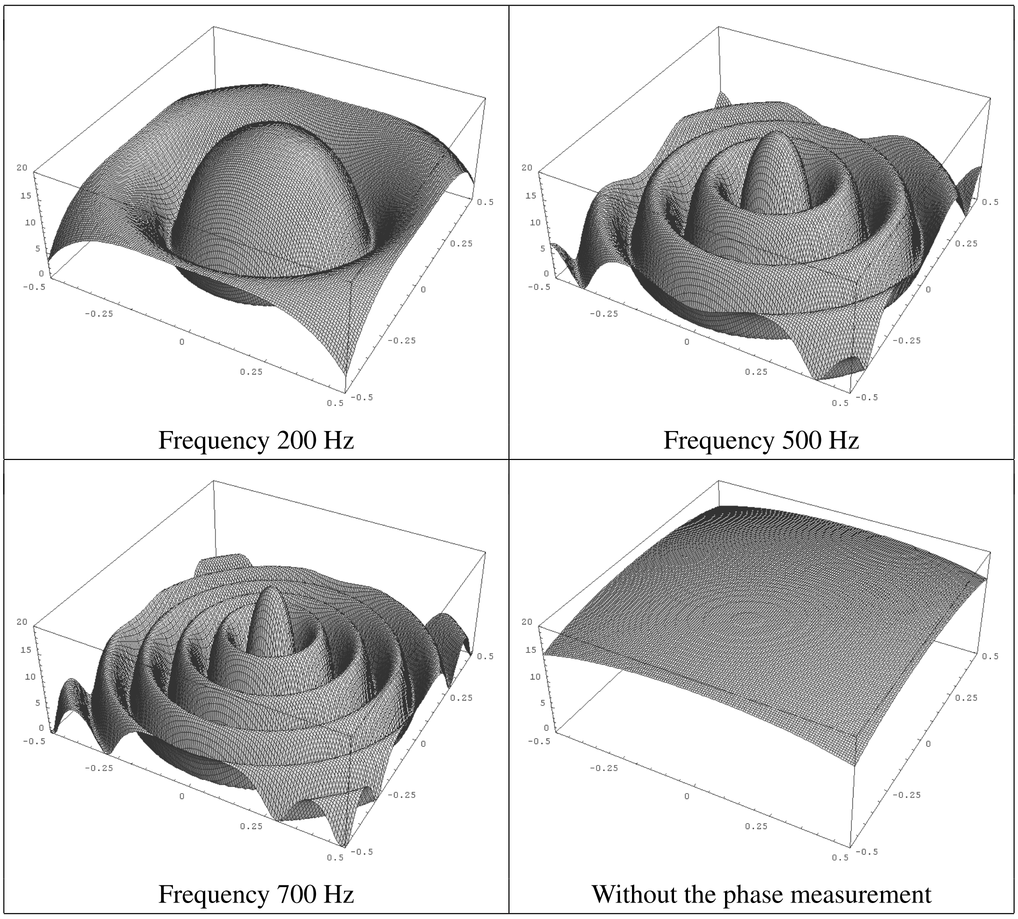

Fig. 5. Sound power level of the substitute source depending on its position – observation points arranged on the circle.

If the phase shift angles are not known, we can estimate the substitute source power from the equation:

$$N_z = N\frac{(1-c^2)^2}{1+c^2} \quad \text{[W]}. \tag{16}$$

Examples of the calculation results – assuming $R = 2m$ – are presented graphically in Fig. 5.

3.3. Observation points arranged on straight line

Consecutively the observation points were arranged in a continuous way on a straight line, which distance to the model source is equal to d. The omni-directional sound source was placed in the centre of the coordination system. During the tests the substitute source was shifted inside the square of a side equal to d (Fig. 6.). The acoustic power values were calculated on the basis of Eq. (7).

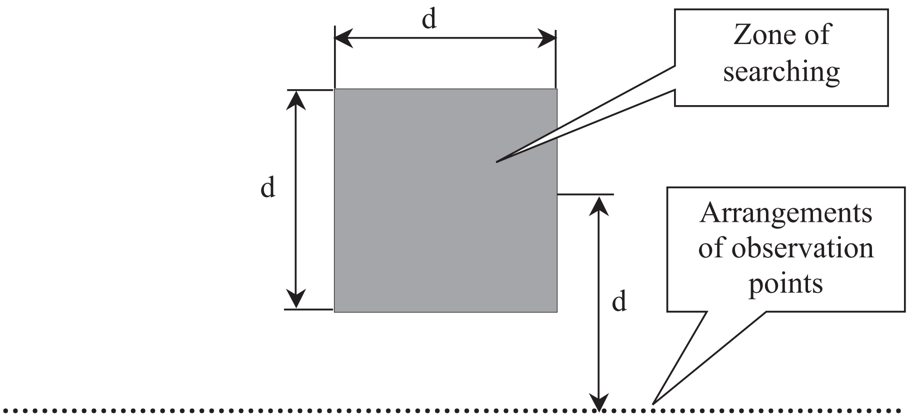

Fig. 6. Arrangements of sound sources and observation points on the straight line.

In this case the acoustic power value of the substitute source determined by the inverse method (with phase) can be calculated from the equation:

$$\begin{aligned} N_z \; = \; & \frac{N(1+2c_y+c_y^2)d^2}{4\pi^2} \\ & \cdot \left(\int_0^{2\pi} \frac{\cos\left[k\sqrt{d^2+x^2} - k\sqrt{d(1+c_y)^2+(x-c_xd)^2}\right]}{\sqrt{d^2+x^2}\,\sqrt{d(1+c_y)^2+(x-c_xd)^2}}\,dx \right)^2 \quad \text{[W]}, \end{aligned} \tag{17}$$

where c_xd – distance from the substitute to the model source versus x-axis, c_yd – distance from the substitute to the model source versus y-axis.

If the phase shift angles are not known, we can estimate the substitute source power from the equation:

$$N_z = 2N\frac{(1+c_y)^2(2+c_y)}{c_x^2+(2+c_y)^2} \quad \text{[W]}. \tag{18}$$

Examples of the calculation results – assuming $R = 2m$ – are presented graphically in Fig. 7.

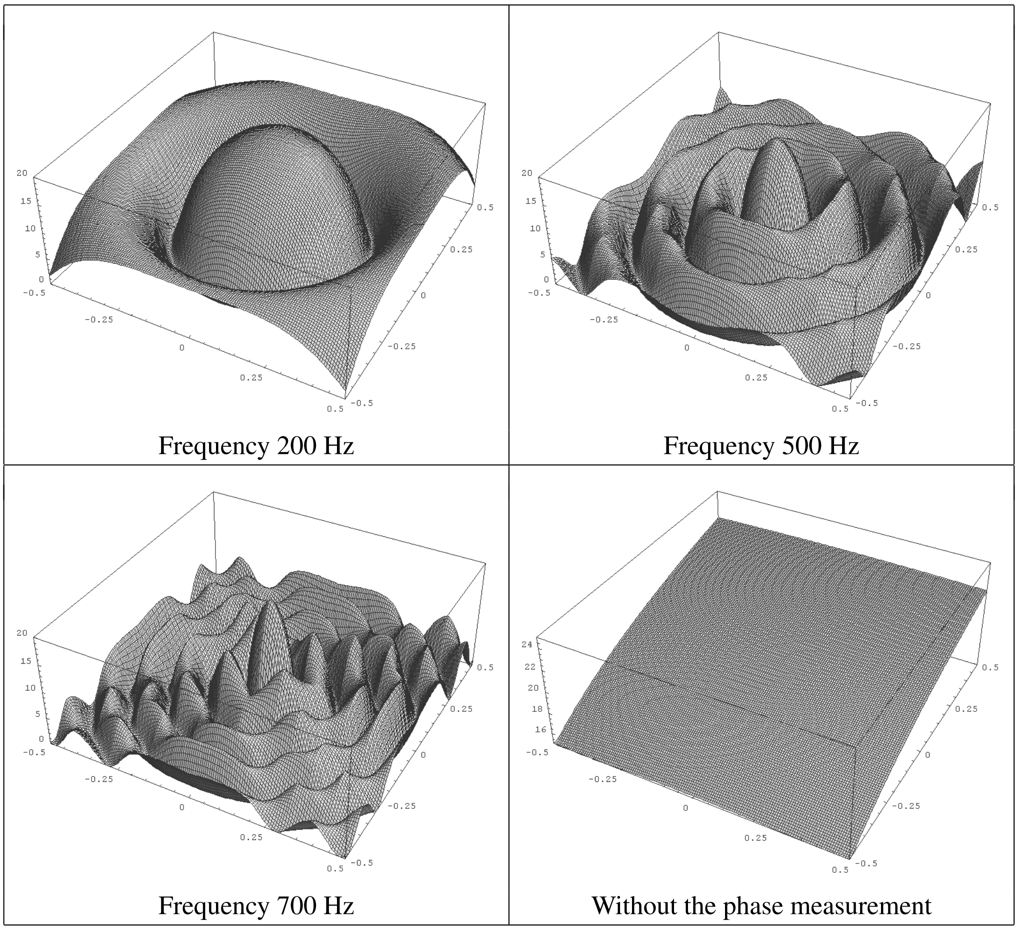

Fig. 7. Sound power levels of the substitute source depending on its position – observation points arranged on the straight line.

Having information on the phase shifts of signals in each of the three various arrangements of the observation points, we can find the power extreme of the substitute source. The global extreme agrees well with the position of the reference source. Apart from the significant global maximum, several local maxima occur. This is an undesirable effect since it may impede finding the global extreme. When the observation points are properly arranged, then we can observe the global power maximum of the substitute source even without phase shift measurements. However, the maximum is not as distinct as that one determined when the phase shift is measured. In the case of small disturbances of the signal it can distort the information on the model source position. If the observation points are not properly arranged (such as their arrangement on the straight line – Fig. 7), the finding of the maximum power level of the substitute source might be impossible.

4. Acoustic measurements

As the next stage of our research we tried to find out whether the sensitivity of the inverse method to the source position can be utilized for the localization of zones of increased acoustic emission.

In order to achieve this aim, measurements of the acoustic pressure distribution around the actual source – the air compressor – were performed. A schematic presentation of the measuring equipment in an anechoic chamber is presented in Fig. 8. To determine the acoustic power pressure distribution, the measurements were done in 197 measuring points for each sound source.

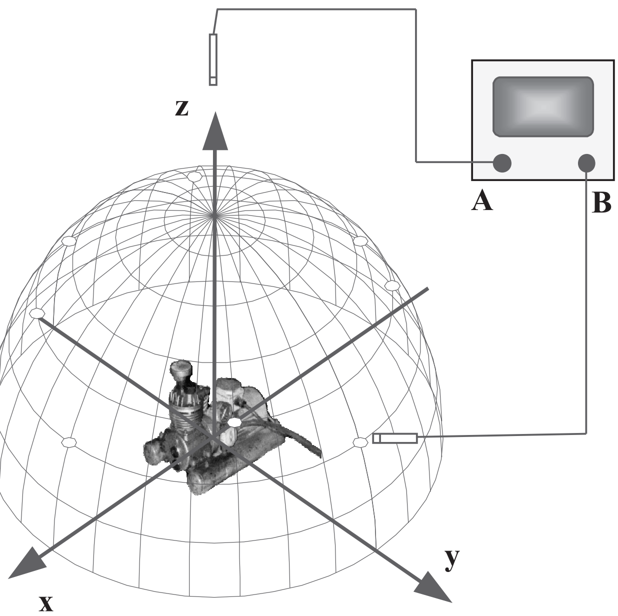

Fig. 8. Schematic diagram of the measuring setup.

The acoustic pressure was measured by means of the microphone B. This microphone was placed subsequently at 197 measuring points, 1.5 m distant from the sound source ($l = 1.5$ m). The microphone A, positioned perpendicularly above the sound source (versus the source in a non-variable direction), was used for the comparisons of the acoustic signals. Due to the simultaneous measuring of the signals from both the microphones, the phase shift angles between the acoustic pressure in the specified direction and the reference direction could be determined.

The measurements were performed in bands of the constant width of 2 [Hz] in the frequency range from 0 to 1600 [Hz]. A two-channel analyzer BK 2133 was used for simultaneous analyzing of signals from both the microphones.

The acoustic pressure amplitude, p, was read from the function of the natural density spectrum of the signal from microphone A.

$$\overline{p^2} = G_{22} \quad [\mathrm{N}^2/\mathrm{m}^4], \tag{19}$$

where

$$G_{22} = \int\limits_{f-\frac{1}{2}\Delta f}^{f+\frac{1}{2}\Delta f} G_{22}(f)\, df \quad [\mathrm{N}^2/\mathrm{m}^4],$$

G_{22} – function of the auto power spectrum, f – investigated frequency [Hz], Δf – bandwidth frequency [Hz].

The function of the mutual density spectrum of signals from both the microphones was applied for the determination of the phase shift angles between the signals from the microphones A and B. The phase shift angle y was determined from the ratio of the imaginary part to the real part of this function:

$$\psi = \arctan\left(\frac{\mathrm{Im}\,(G_{12})}{\mathrm{Re}\,(G_{12})}\right), \tag{20}$$

where

$$G_{12} = \int\limits_{f-\frac{1}{2}\Delta f}^{f+\frac{1}{2}\Delta f} G_{12}(f)\, df \quad \text{– function of the cross power spectrum.}$$

5. Zones of increased vibroacoustic emission

On the basis of the measured acoustic pressure distribution around the actual source, the zones of increased vibroacoustic emission (maximal power level of the substitute source determined from formula (7)) were searched by means of the shifting of the substitute source inside the rectangular prism of dimensions corresponding to the overall dimensions of the machine under test.

In this particular case it was a rectangular prism of dimensions: $0.6 \times 0.4 \times 0.45$ [m]. The cross-section of the zone under testing in the XY plane at the height $z = 0.25$ [m] is shown in Fig. 9.

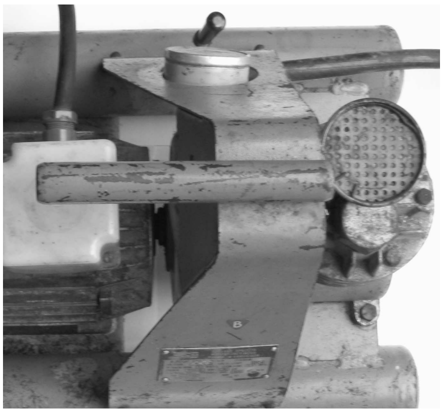

Fig. 9. Zone of the assessment of the substitute source power.

As the result of such searching the substitute source power level in the bandwidth of 2 [Hz] were determined. Some examples of the calculation results are presented in Fig. 10.

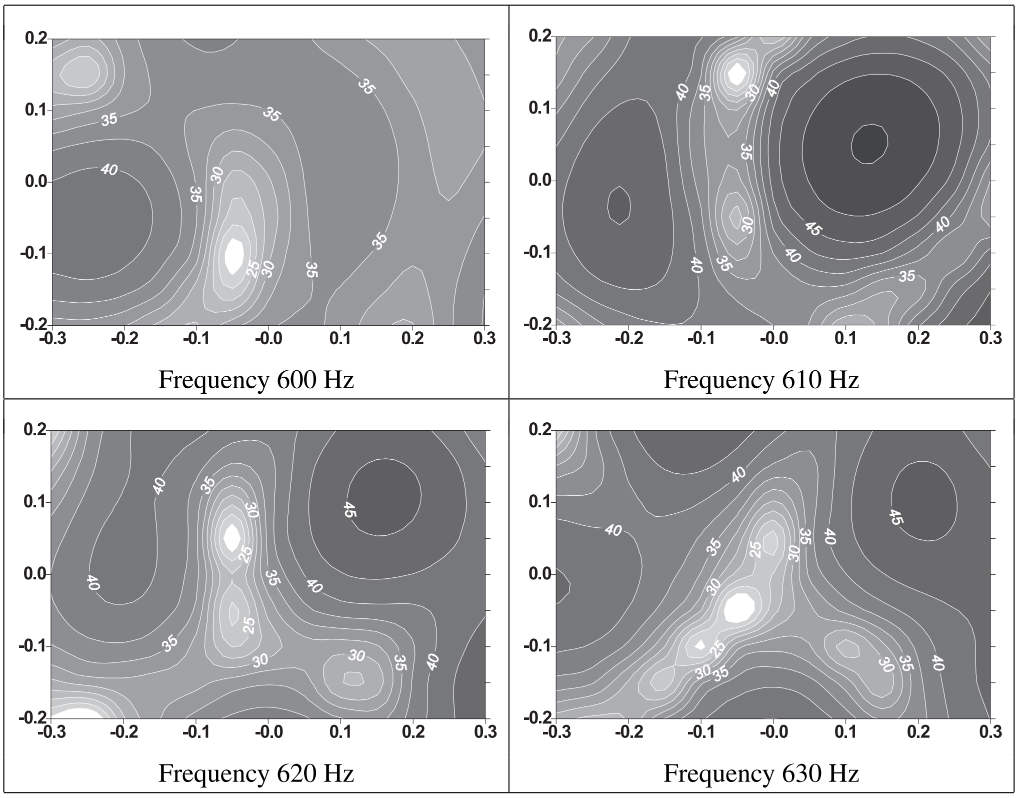

Fig. 10. Power level of the substitute sound source in a bandwidth of 2 Hz.

The position of the global extreme depends on the position of the actual source. Positions of the local extremes depend also on the investigated frequency. Therefore, the averaging of the calculation results obtained at similar frequencies in order to smooth out the local minima seems to be logical.

An example of the result of averaging the power level of the substitute source in the one-third-octave band is given in Fig. 11.

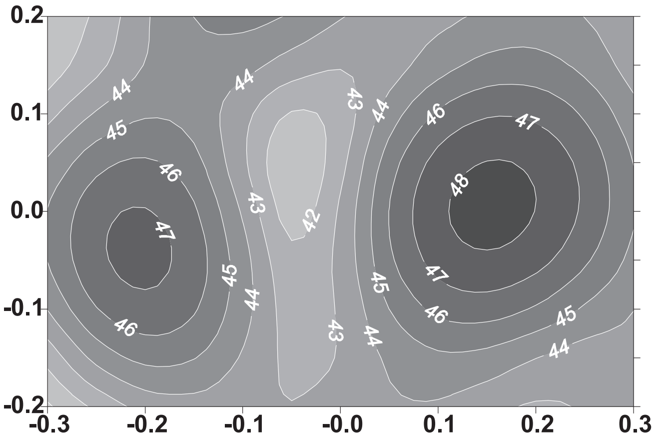

Fig. 11. Averaged level of the substitute source power in the one-third-octave band 630 Hz.

6. Conclusions

The inverse method can be utilized for the identification of vibroacoustic energy sources and the assessment of sound radiation. The examples of the performed investigations mentioned above were aimed at showing how the acoustic field can be investigated and how the models of vibroacoustic emission can be presented – on the bases of the spectrum of substitute sources.

The knowledge of the actual distribution of the acoustic pressure around the sound sources is necessary for the calculations of model parameters. In order to be able to perform those calculations, the determination of the distribution of the acoustic pressure amplitudes as well as the distribution of the phase shift angles between acoustic signals at selected measuring points is necessary. When the phase shift angles are not known, the inverse method can be applied as well, however, one must take into account the worse accuracy of the measurements and the distribution of observation points have to be arranged with an utmost care.

The inverse method (especially when the measurements involve the phase shift angles) is very sensitive to the position of the substitute sources. This feature can be utilized also for the determination of the position of the actual sources.

The accuracy of the method depends also on the distribution of the observation points.

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