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Standing Waves and Acoustic Heating (or Cooling) in Resonators Filled with Chemically Reacting Gas

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Standing waves and acoustic heating in a one-dimensional resonator filled with chemically reacting gas, is the subject of investigation. The chemical reaction of $A \to B$ type, which takes place in a gas, may be reversible or not. Governing equations for the sound and entropy mode which is generated in the field of sound are derived by use of a special mathematical method. Under some conditions, sound waves propagating in opposite directions do not interact. The character of nonlinear dynamics of the sound and relative acoustic heating or cooling depends on reversibility of a chemical reaction. Some examples of acoustic heating in a resonator are illustrated and discussed.

Keywords: standing wave, acoustic resonator, non-equilibrium media.

1. Introduction

Investigation of acoustic oscillations in resonators is of great importance because sound waves always propagate in bounded volumes. Recent interest in this subject is caused by engineering and technical application of resonators. Acoustic field in resonators filled with a Newtonian fluid has been studied thoroughly theoretically and analytically, for example in the papers (Biwa, Yazaki, 2010; Mortell, Mulchrone, Seymour, 2009). It has been established that nonlinearity may lead to periodic shock waves for intense perturbations in a resonator (Keller, 1977; Chester, 1964; Ockendon *et al.*, 1993). Kaner *et al.* (1977) were the first to introduce the analytical method of different scales to describe the acoustic field in closed volumes. This method uses the slow dependence of the shape of progressive wave on nonlinearity and attenuation, and its fast dependence on the retarded time. However, most of previous analytical considerations require the sound to be periodic in time (Kaner, Rudenko, Khokholov, 1977; Ochmann, 1985).

In this study, sound perturbations and acoustic heating in a resonator filled with chemically reacting gas (where the chemical reaction of $A \to B$ type may make a gas acoustically active) is the subject of investigation. Chemically reacting gas, under some conditions, is a non-equilibrium medium. The interest in the non-equilibrium hydrodynamics constantly grows because of a wide application of non-equilibrium media in lasers, physics of atmosphere, and physics of plasma (Osipov, Uvarov, 1992). There are a lot of papers concerning the sound propagation in unbounded volumes of such media (Demidov, Rytenkov, Skrebov, 1988; Blinov, 1989). Bauer and Bass (1973) have investigated the possibility of the sound amplification in a gas maintained in ambient vibrational and radiative non-equilibrium by thermal radiation. Srinivasan and Vincenti (1975) introduced different ways to excite the vibrational mode of a gas, for example, chemical reaction. Kogan and Molevich (1985) have taken into account the possibility of a non-equilibrium state, not only of the vibrational but also of the rotational degrees of a molecule's freedom. Molevich (2002; 2003) was the first to mention some special mechanisms of nonlinear self-action of the acoustic beam which cause the self-focusing of sound in an acoustically active medium, cooling of gas by sound, and the excitation of acoustic streaming in opposite direction as compared to the one of sound propagation. The acoustic cooling in a gas with the chemical reaction of $A \to B$ type in unbounded volumes was also studied by the authors for the high and low-frequency sound in (Perelomova, Pelc-Garska, 2011).

In this paper, the special mathematical method based on projecting was used to separate the equa-

tions governing the sound and non-acoustic modes in a resonator. The method has been worked out and applied previously by one of the authors for some problems of weakly nonlinear flows (Perelomova, 2003; 2006; 2010). The projecting method leads to governing equations of acoustics modes which in general do not require the sound be periodic in time. To describe the sound itself in a resonator, the method of projection is complemented by the method of different scales (Kaner, Rudenko, Khokholov, 1977). The equations governing the sound and non-acoustic modes are discussed in Sec. 5 and the numerical examples of acoustic perturbations and acoustic heating are presented in Sec. 6.

2. Basic equations

The momentum, energy, and continuity equations in a gas where a simple chemical reaction of $A \to B$ type takes place, read:

$$\begin{aligned} \rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= -\boldsymbol{\nabla}P, \\ \frac{C_{V,\infty}}{R}\frac{\mathrm{d}T}{\mathrm{d}t} - \frac{T}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} &= Q, \\ \frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\boldsymbol{\nabla}\cdot\mathbf{v} &= 0. \end{aligned} \tag{1}$$

The system (1) should be complemented by the dynamic equation of the mass fraction Y of reagent A and the thermal equation of state:

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -\frac{Q}{Hm}, \qquad P = \frac{\rho T}{m}. \tag{2}$$

In the systems of Eq. (1), (2), $\mathbf{v}$, ρ, P denote the velocity, density, and pressure of a gas, respectively; T is the temperature measured in Joules per molecule, $C_{V,\infty}$ is the molar "frozen" heat capacity at a constant volume (i.e., correspondent processes take place at infinitely high frequencies), $R = C_{P,\infty} - C_{V,\infty}$ is the universal gas constant, $C_{P,\infty}$ is the "frozen" heat capacity at a constant pressure, Q is the heat produced in the medium per one molecule due to a chemical reaction, H denotes the reaction enthalpy per unit mass of reagent A, and m denotes the averaged molecular mass of a gas.

3. Dispersion relations in a one-dimensional flow

The one-dimensional gas flow along axis OX is considered. Every quantity in Eq. (1) represents a sum of unperturbed value and its variation, for example: $\rho = \rho_0 + \rho'$ (where in a weakly nonlinear flow $|\rho'| \ll \rho_0$, and so on). Following (Molevich, 2002; 2003), we assume that the stationary quantities Y_0, T_0, P_0, ρ_0 are maintained by a transverse pumping, so that in the longitudinal direction pointed by axis OX the unperturbed medium is homogeneous and $v_0 = 0$. The dimensionless quantities which characterise a chemical reaction Q_T, Q_ρ, Q_Y depend on the type of heat production:

$$\begin{aligned} Q_T &= \frac{T_0}{Q_0}\left(\frac{\partial Q}{\partial T}\right)_{T_0,\rho_0,Y_0}, \\ Q_\rho &= \frac{\rho_0}{Q_0}\left(\frac{\partial Q}{\partial \rho}\right)_{T_0,\rho_0,Y_0}, \\ Q_Y &= \frac{Y_0}{Q_0}\left(\frac{\partial Q}{\partial Y}\right)_{T_0,\rho_0,Y_0}. \end{aligned} \tag{3}$$

The characteristic duration of a chemical reaction is:

$$\tau_c = \frac{HmY_0}{Q_0 Q_Y}. \tag{4}$$

Equations (1), with account for Eq. (2) within accuracy up to quadratic nonlinear terms, take the forms as follows:

$$\begin{aligned} &\frac{\partial v'}{\partial t} + \frac{T_0}{m\rho_0}\frac{\partial \rho'}{\partial x} + \frac{1}{m}\frac{\partial T'}{\partial x} \\ &\qquad = -v'\frac{\partial v'}{\partial x} + \frac{T_0\rho'}{m\rho_0^2}\frac{\partial \rho'}{\partial x} - \frac{T'}{m\rho_0}\frac{\partial \rho'}{\partial x}, \\ &\frac{\partial T'}{\partial t} + (\gamma-1)\left(T_0\frac{\partial v'}{\partial x} - Q_T\frac{Q_0}{T_0}T' - Q_\rho\frac{Q_0}{\rho_0}\rho' - Q_Y\frac{Q_0}{Y_0}Y'\right) \\ &\qquad = -v'\frac{\partial T'}{\partial x} - (\gamma-1)T'\frac{\partial v'}{\partial x}, \\ &\frac{\partial Y'}{\partial t} + \frac{1}{Hm}\left(Q_T\frac{Q_0}{T_0}T' + Q_\rho\frac{Q_0}{\rho_0}\rho' + Q_Y\frac{Q_0}{Y_0}Y'\right) \\ &\qquad = -v'\frac{\partial Y'}{\partial x}, \\ &\frac{\partial \rho'}{\partial t} + \rho_0\frac{\partial v'}{\partial x} = -v'\frac{\partial \rho'}{\partial x} - \rho'\frac{\partial v'}{\partial x}, \end{aligned} \tag{5}$$

where $\gamma = \dfrac{C_{P,\infty}}{C_{V,\infty}}$ denotes the frozen adiabatic exponent. The linearized version of Eq. (5) describes a flow of infinitely-small magnitude:

$$\begin{aligned} &\frac{\partial v'}{\partial t} + \frac{T_0}{m\rho_0}\frac{\partial \rho'}{\partial x} + \frac{1}{m}\frac{\partial T'}{\partial x} = 0, \\ &\frac{\partial T'}{\partial t} + (\gamma-1)\left(T_0\frac{\partial v'}{\partial x} - Q_T\frac{Q_0}{T_0}T' - Q_\rho\frac{Q_0}{\rho_0}\rho' - Q_Y\frac{Q_0}{Y_0}Y'\right) = 0, \\ &\frac{\partial Y'}{\partial t} + \frac{1}{Hm}\left(Q_T\frac{Q_0}{T_0}T' + Q_\rho\frac{Q_0}{\rho_0}\rho' + Q_Y\frac{Q_0}{Y_0}Y'\right) = 0, \\ &\frac{\partial \rho'}{\partial t} + \rho_0\frac{\partial v'}{\partial x} = 0. \end{aligned} \tag{6}$$

All perturbations may be represented as sums of planar waves:

$$f(x,t) = \int \widetilde{f}(k,t)\exp(-ikx)\,\mathrm{d}k = \int \widetilde{f}(k)\exp(i\omega t - ikx)\,\mathrm{d}k, \tag{7}$$

($\widetilde{f}(k,t)$ denotes the Fourier transform of $f(x,t)$, $\widetilde{f}(k,t) = \frac{1}{2\pi}\int f(x,t)e^{ikx}\,\mathrm{d}x$, where k is the wave number). Every type of motion is determined by one of the four roots of dispersion equation of the linear flow, $\omega(k)$. The dispersion relations for three non-thermal modes (two acoustic and one non-wave) and the thermal one, which is not progressive, are described by the equalities:

$$\omega^2 = k^2\frac{T_0}{m}\frac{C_P}{C_V} = k^2\frac{T_0}{m}\left(\frac{C_{P,\infty} + mH\left(\frac{\partial Y}{\partial T}\right)_P}{C_{V,\infty} + mH\left(\frac{\partial Y}{\partial T}\right)_V}\right) = k^2\frac{T_0}{m}\left(\frac{C_{P,\infty} + \frac{(Q_\rho - Q_T)\tau_c}{1+i\omega\tau_c}\frac{Q_0}{T_0}}{C_{V,\infty} - \frac{Q_T\tau_c}{1+i\omega\tau_c}\frac{Q_0}{T_0}}\right), \quad \omega = 0. \tag{8}$$

The approximate roots of dispersion relations for both acoustic branches were firstly derived in (Molevich, 2003). A weak dependence of heat release in a chemical reaction on the temperature and density ($|Q_\rho| \ll 1$, $|Q_T| \ll 1$) is assumed. The dispersion relations have been evaluated in the high-frequency case, i.e. when acoustic frequency is large as compared with the inverse characteristic duration of a chemical reaction, τ_c: $\omega\tau_c \approx |k|u\tau_c \gg 1$:

$$\begin{aligned}
&\omega_1 = u(k - iB),\\
&\omega_2 = u(-k - iB),\\
&\omega_3 = i\left(\frac{1}{\tau_c} + \frac{(\gamma-1)Q_0(Q_\rho - Q_T)}{u^2 m}\right),\\
&\omega_4 = 0,
\end{aligned} \tag{9}$$

where

$$B = \frac{Q_0(\gamma-1)(Q_\rho + (\gamma-1)Q_T)}{2u^3 m} \tag{10}$$

and $u = \sqrt{\gamma\frac{T_0}{m}}$ is the frozen sound velocity. The first two roots in (9), ω_1, ω_2, are acoustic and ω_4 corresponds to the thermal (entropy) mode. The third non-acoustic root ω_3 is responsible for the non-wave variation in mass fraction of reagent A. For the sound to be a wave process, attenuation (when B is negative), or amplification, (if B is positive) should be small as compared with the characteristic acoustic wavenumber, $|B| \ll k$. That in fact determines the smallness of $|Q_\rho|$, $|Q_T|$ more precisely in dependence with the characteristic domain of sound wavenumbers.

4. Definitions of modes in linear flow

In general, every perturbation of the field variables contains contributions from each of the four modes, for example, $\rho' = \rho'_1 + \rho'_2 + \rho'_3 + \rho'_4$. That allows decomposition of equations which govern every mode in their linear parts using specific properties of the modes. Keeping in mind that

$$\int f(x,t)\,\mathrm{d}x = \int(-ik)^{-1}\widetilde{f}(k)\exp(i\omega t - ikx)\,\mathrm{d}k, \tag{11}$$

one may determine the modes as specific relations of perturbations.

$$\begin{aligned}
\psi_1 &= \begin{pmatrix} v'_1 \\ T'_1 \\ Y'_1 \\ \rho'_1 \end{pmatrix} = \begin{pmatrix} \frac{u}{\rho_0} - \frac{uB}{\rho_0}\int \mathrm{d}x \\ \frac{(\gamma-1)T_0}{\rho_0} - \frac{2\gamma T_0 B}{\rho_0}\int \mathrm{d}x \\ \frac{2Bu^2}{(\gamma-1)H\rho_0}\int \mathrm{d}x \\ 1 \end{pmatrix}\rho'_1,\\
\psi_2 &= \begin{pmatrix} -\frac{u}{\rho_0} - \frac{uB}{\rho_0}\int \mathrm{d}x \\ \frac{(\gamma-1)T_0}{\rho_0} + \frac{2\gamma T_0 B}{\rho_0}\int \mathrm{d}x \\ -\frac{2Bu^2}{(\gamma-1)H\rho_0}\int \mathrm{d}x \\ 1 \end{pmatrix}\rho'_2,\\
\psi_3 &= \begin{pmatrix} \left(\frac{(\gamma-1)Q_0\,(Q_\rho - Q_T)}{\rho_0 T_0 \gamma} + \frac{1}{\tau_c\rho_0}\right)\int \mathrm{d}x \\ -\frac{T_0}{\rho_0} \\ \frac{u^2}{H(\gamma-1)\rho_0} \\ 1 \end{pmatrix}\rho'_3,\\
\psi_4 &= \begin{pmatrix} 0 \\ -\frac{T_0}{\rho_0} \\ -\frac{\tau_c Q_0\,(Q_\rho - Q_T)}{Hm\rho_0} \\ 1 \end{pmatrix}\rho'_4.
\end{aligned} \tag{12}$$

The linear equations governing any mode may be decomposed directly from the system (6) by use of relations (12). The dynamic equations governing the excess density in acoustic wave progressive in the positive and negative direction of axis OX, take the forms:

$$\begin{aligned}
&\frac{\partial\rho'_1}{\partial t} + u\frac{\partial\rho'_1}{\partial x} - uB\rho'_1 = 0,\\
&\frac{\partial\rho'_2}{\partial t} - u\frac{\partial\rho'_2}{\partial x} - uB\rho'_2 = 0.
\end{aligned} \tag{13}$$

The equations for excess densities of the entropy and the chemical modes are as follows:

$$\frac{\partial \rho_4'}{\partial t} = 0, \qquad \frac{\partial \rho_3'}{\partial t} + \left(\frac{Q_0(\gamma-1)(Q_\rho - Q_T)}{T_0 \gamma} + \frac{1}{\tau_c} \right) \rho_3' = 0. \tag{14}$$

The Eqs. (13), (14) correspond to the dispersion relations (9).

5. Dynamic equations in a weakly nonlinear flow

5.1. Dynamic equations for the sound in a weakly nonlinear flow

To obtain Eqs. (13), (14), we may also use a special mathematical method based on projectors. Projecting is in fact a linear combination of Eqs. (6) in order to hold in the linear part of resulting equation only terms corresponding to the chosen mode. For example, by multiplying the first and second equations of the system (6) by the following factors, correspondingly,

$$\frac{\rho_0}{2u} + \frac{(\gamma-1)Q_0\left(Q_\rho + (\gamma-1)Q_T\right)\rho_0}{2mu^4} \int \mathrm{d}x,$$

$$\frac{\rho_0}{2mu^2} + \frac{(\gamma-1)Q_0\left(3Q_\rho + (\gamma-3)Q_T\right)\rho_0}{4m^2u_\infty^5} \int \mathrm{d}x,$$

and the third and fourth equations of the system (6) by the following factors, respectively:

$$-\frac{H(\gamma-1)\rho_0}{2\tau_c u^3} \int \mathrm{d}x,$$

$$\frac{1}{2\gamma} - \frac{(\gamma-1)Q_0\left(-3Q_\rho + \gamma\left(2Q_\rho - 3Q_T\right) + 3Q_T\right)}{4m\gamma u^3} \int \mathrm{d}x,$$

and taking their sum after that, one gets an equation governing the excess density in acoustic wave progressive in the positive direction of axis OX (it coincides with the first equation from the system (13)). The terms belonging to all other modes become reduced in the final equation. In a similar way it is possible to obtain an equation for the second acoustic mode and for the non-acoustic modes in an unbounded space. To describe the sound in a resonator, the method of different scales will be used (Kaner, Rudenko, Khokholov, 1977; Rudenko, Soluyan, 1977). This method exploits the idea of a slow dependence of the shape of progressive modes caused by the nonlinearity and attenuation, and a fast dependence on the retarded time. The acoustic field is considered in new variables: $\eta = x - ut$, $\xi = x + ut$, corresponding to the progressive in the positive and negative direction of axis OX waves, and μt, where $\mu = Max(B/k, M)$ (M denotes the acoustic Mach number). In the linear non-viscous flow, acoustic perturbations are functions exclusively of the retarded variables, $\rho_1'(\eta)$, $\rho_2'(\xi)$. We fix relations (12) in a weakly nonlinear flow and combine Eqs. (5) in the way described at the beginning of this subsection. We will also consider the magnitudes of non-acoustic perturbations much smaller than that of the sound, so that every perturbation in the non-linear part of equations is in the leading order a sum of specific acoustic quantities, $\rho' = \rho_1' + \rho_2'$ and so on. Dynamic equations for acoustic branches take the forms:

$$\begin{aligned} \frac{\partial \rho_1'}{\partial t} - uB\rho_1' &= -\frac{u(\gamma+1)}{2\rho_0}\rho_1'\frac{\partial \rho_1'}{\partial \eta} - \frac{u(\gamma-3)}{2\rho_0}\rho_2'\frac{\partial \rho_1'}{\partial \eta} \\ &\quad + \frac{u(\gamma^2-3\gamma+4)}{2\gamma\rho_0}\left(\rho_1'\frac{\partial \rho_2'}{\partial \xi} + \rho_2'\frac{\partial \rho_2'}{\partial \xi}\right), \\ \frac{\partial \rho_2'}{\partial t} - uB\rho_2' &= \frac{u(\gamma+1)}{2\rho_0}\rho_2'\frac{\partial \rho_2'}{\partial \xi} + \frac{u(\gamma-3)}{2\rho_0}\rho_1'\frac{\partial \rho_2'}{\partial \xi} \\ &\quad - \frac{u(\gamma^2-3\gamma+4)}{2\gamma\rho_0}\left(\rho_2'\frac{\partial \rho_1'}{\partial \eta} + \rho_1'\frac{\partial \rho_1'}{\partial \eta}\right). \end{aligned} \tag{15}$$

Let ρ_1' and ρ_2' be periodic functions of η and ξ, and their averaged over periods values be zero: $\overline{\rho_1'(\eta)} = 0$, $\overline{\rho_2'(\xi)} = 0$. It can easily be concluded that averaging the first equation over period in ξ, and the second over period in η leads to equations for non-interacting acoustic modes progressive in different directions,

$$\begin{aligned} \frac{\partial \rho_1'}{\partial t} - uB\rho_1' + \frac{u(\gamma+1)}{2\rho_0}\rho_1'\frac{\partial \rho_1'}{\partial \eta} &= 0, \\ \frac{\partial \rho_2'}{\partial t} - uB\rho_2' - \frac{u(\gamma+1)}{2\rho_0}\rho_2'\frac{\partial \rho_2'}{\partial \xi} &= 0. \end{aligned} \tag{16}$$

5.2. Acoustic heating

Acoustic heating is an increase in the ambient temperature associated with nonlinear losses in the acoustic energy. It associates with the entropy mode. To obtain an equation describing interaction between the acoustic and entropy modes in a resonator we multiply the first equation of the system (5) by 0 and the second, third, and fourth equations by following factors:

$$-\frac{\rho_0}{mu^2} + \frac{\rho_0 Q_0(Q_\rho - Q_T)(\gamma-1)\tau_c}{m^2u^4},$$

$$\frac{\rho_0 H(\gamma-1)(-mu^2 + Q_0(Q_\rho - Q_T)(\gamma-1)\tau_c)}{mu^4},$$

$$1 - \frac{1}{\gamma} - \frac{(\gamma-1)^2 Q_0(Q_\rho - Q_T)\tau_c}{mu^2\gamma}.$$

To eliminate the acoustic and relaxation terms in the linear part of the final equation we calculated the sum of all four expressions. In the nonlinear part, only quadratic terms correspondent to the acoustics modes are kept, since their magnitudes are large as compared with the non-acoustic ones. The equation governing acoustic heating takes the form:

$$\frac{\partial \rho_4'}{\partial t} = \left(\frac{u(\gamma-1)(\gamma-2)}{\gamma\rho_0} - \frac{\tau_c Q_0 (Q_\rho - Q_T)(\gamma-1)^2(\gamma-2)}{mu\gamma\rho_0}\right) \cdot \left(\rho_1' \frac{\partial \rho_1'}{\partial \eta} - \rho_1' \frac{\partial \rho_2'}{\partial \xi} + \rho_2' \frac{\partial \rho_1'}{\partial \eta} - \rho_2' \frac{\partial \rho_2'}{\partial \xi}\right) - \frac{2Bu(\gamma-1)}{\rho_0}\left(\frac{\partial \rho_1'}{\partial \eta}\int \rho_1' \,\mathrm{d}\eta - \frac{\partial \rho_1'}{\partial \eta}\int \rho_2' \,\mathrm{d}\xi - \frac{\partial \rho_2'}{\partial \xi}\int \rho_1' \,\mathrm{d}\eta + \frac{\partial \rho_2'}{\partial \xi}\int \rho_2' \,\mathrm{d}\xi\right) - \frac{uB(\gamma-1)(\gamma-2)}{\rho_0\gamma}\left(\rho_1'^2 + 2\rho_1'\rho_2' + \rho_2'^2\right). \quad (17)$$

After averaging over the period in η and ξ, Eq. (17) becomes reduced to:

$$\frac{\partial \rho_4'}{\partial t} = -\frac{2Bu(\gamma-1)}{\rho_0}\left(\overline{\frac{\partial \rho_1'}{\partial \eta}\int \rho_1' \,\mathrm{d}\eta} + \overline{\frac{\partial \rho_2'}{\partial \xi}\int \rho_2' \,\mathrm{d}\xi}\right) - \frac{uB(\gamma-1)(\gamma-2)}{\rho_0\gamma}\left(\overline{\rho_1'^2} + \overline{\rho_2'^2}\right). \quad (18)$$

Thus, the periodic waves almost do not interact in the volume of a resonator.

6. Acoustic field and relative nonlinear phenomena

In an acoustic nonlinear resonator, the perturbations of density and velocity in the leading order are sums of specific parts:

$$\rho' = \rho_1' + \rho_2', \quad (19)$$

$$v' = v_1' + v_2' \approx \frac{u}{\rho_0}(\rho_1' - \rho_2'), \quad (20)$$

where ρ_1', ρ_2' are solutions of Eqs. (16). We leave only leading-order relations from Eqs. (12) in view of the fact that the squared acoustic terms are the source of heating. We will consider zero initial perturbation of density $\rho'(x, t=0) = 0$ and initial excess velocity as $v'(x, t=0) = 2Mu\sin(\omega x/u)$ in the sections below. It is convenient to rearrange equations in the dimensionless variables,

$$b = Bu/\omega, \quad X = \omega x/u, \quad \tau = \omega t, \quad \eta = X - \tau, \quad \xi = X + \tau, \quad \theta = \exp(b\tau) - 1, \quad R_1 = \frac{\rho_1'}{M\rho_0}\exp(-b\tau), \quad R_2 = \frac{\rho_2'}{M\rho_0}\exp(-b\tau), \quad K = \frac{(\gamma+1)M}{2b}. \quad (21)$$

In these notations, Eqs. (16) take the forms

$$\frac{\partial R_1}{\partial \theta} + KR_1\frac{\partial R_1}{\partial \eta} = 0, \qquad \frac{\partial R_2}{\partial \theta} - KR_2\frac{\partial R_2}{\partial \xi} = 0. \quad (22)$$

It is clear that if $R_1(\theta, \eta)$ is a solution of the first equation, $R_2 = -R_1(\theta, \xi)$ is also a solution of the second equation in Eqs. (22). Velocity in a flat resonator satisfies the following boundary conditions: $v'|_{X=0} = v'|_{X=L} \equiv 0$, so that the dimensionless length of a resonator L is π-fold. The governing equation for the excess dimensionless temperature associating with the entropy mode follows from Eq. (18):

$$-\frac{\partial \rho_4'}{\partial \tau} = \frac{\partial T_4'/T_0}{\partial \tau} = bM^2(\gamma-1)\exp(2b\tau) \cdot \left(2\left(\overline{\frac{\partial R_1}{\partial \eta}\int R_1 \,\mathrm{d}\eta} + \overline{\frac{\partial R_2}{\partial \xi}\int R_2 \,\mathrm{d}\xi}\right) + \frac{\gamma-2}{\gamma}\left(\overline{R_1^2} + \overline{R_2^2}\right)\right), \quad (23)$$

that yields for the periodic sound

$$\frac{\partial T_4'/T_0}{\partial \tau} = -bM^2\exp(2b\tau) \cdot \left(\frac{(\gamma-1)(\gamma+2)}{\gamma}\left(\overline{R_1^2} + \overline{R_2^2}\right)\right). \quad (24)$$

6.1. Standing waves and relative heating before forming of waves discontinuity

For times smaller than the characteristic time of formation of the saw-tooth wave, $\widetilde{T}$: $\widetilde{T} = \frac{1}{b}\ln(1 + 1/K)$, the periodic solutions of (22) take the forms (Rudenko, Soluyan, 1977; Riemann, 1953):

$$R_1 = -\sum_{n=1}^{\infty}\frac{2J_n(nK\theta)\sin(n\eta)}{nK\theta}, \qquad R_2 = \sum_{n=1}^{\infty}\frac{2J_n(nK\theta)\sin(n\xi)}{nK\theta}, \quad (25)$$

where J_n is the Bessel function of the n-th order. Note that in a strongly damping resonator with negative b, discontinuities may not form at all. The velocity and density fields in a resonator before formation of the saw-tooth wave for different times τ are presented in Fig. 1. The density of total energy of the sum of the Riemann's waves (22) is constant, its dimensionless quantity equals

$$0.5L^{-1}\int_0^L \left((R_1+R_2)^2 + (V_1+V_2)^2\right)\mathrm{d}X = 0.5L^{-1}\int_0^L \left((R_1+R_2)^2 + (R_1-R_2)^2\right)\mathrm{d}X = L^{-1}\int_0^L \left(R_1^2 + R_2^2\right)\mathrm{d}X = 1,$$

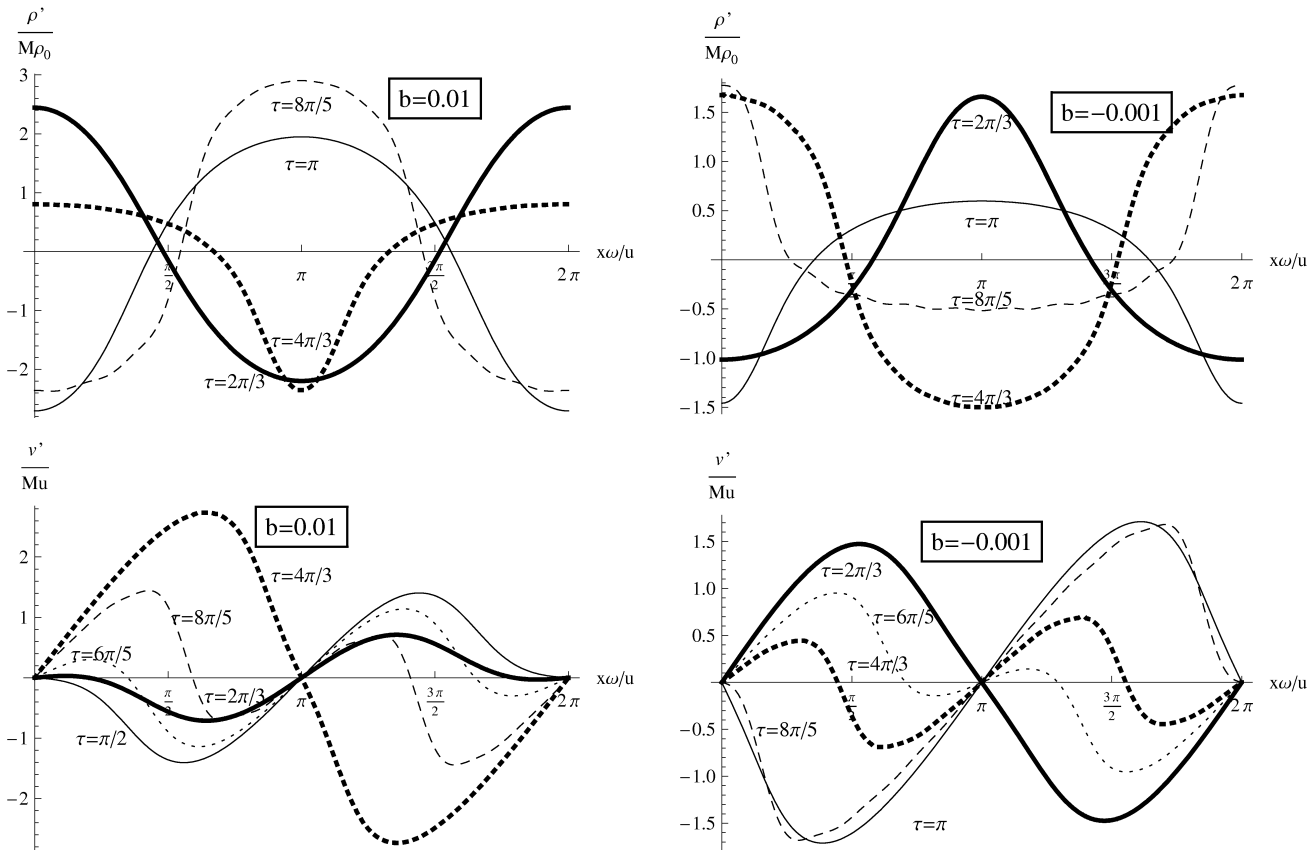

Fig. 1. Velocity and excess density in the standing wave in a resonator filled with chemically reacting gas before formation of the saw-tooth wave, at different fractions of period.

where $V_1 = v_1'/u$, $V_2 = v_2'/u$ are dimensionless velocities of a fluid in the rightwards and leftwards propagating waves. Thus, the acoustic energy in a resonator E varies with time as $\exp(2b\tau)$. Equation (24) readily may be rearranged into

$$\frac{\partial T_4'/T_0}{\partial(b\tau)} = -\frac{M^2(\gamma-1)(\gamma+2)}{\gamma}\exp(2b\tau). \tag{26}$$

After integration with the initial condition

$$T_4'(\tau=0)=0,$$

it yields

$$M^{-2}\frac{T_4'}{T_0} = \frac{(\gamma-1)(\gamma+2)}{2\gamma}(\exp(2b\tau)-1). \tag{27}$$

The important characteristics which describes losses of acoustic energy in a resonator, is the quality factor, or q-factor:

$$q = \frac{E}{|\partial E/\partial\tau|}. \tag{28}$$

The quality factor before formation of the discontinuity is simply $q = |2b|^{-1}$.

6.2. Saw-tooth waves and relative heating

For the times greater than the characteristic time of formation of the saw-tooth wave, $K\theta > \pi/2$, R_1 and R_2 take the saw-tooth shapes consisting of straight-line parts,

$$\begin{aligned} R_1 &= \frac{\eta}{1+K\theta}, \quad \text{if} \quad -\pi \le \eta < \pi, \\ R_2 &= -\frac{\xi}{1+K\theta}, \quad \text{if} \quad -\pi \le \xi < \pi, \end{aligned} \tag{29}$$

and are periodic in η or ξ, respectively. The saw-tooth shapes of R_1 and R_2 are shown in Fig. 2. The velocity and density perturbations in a resonator after formation of the saw-tooth wave for different times τ are plotted in Fig. 3. For positive coefficient b, the peak value of waves increases and de-

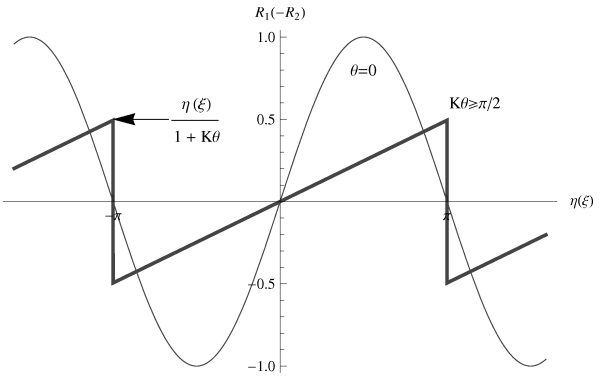

Fig. 2. Saw-tooth waveforms in accordance to Eqs. (29).

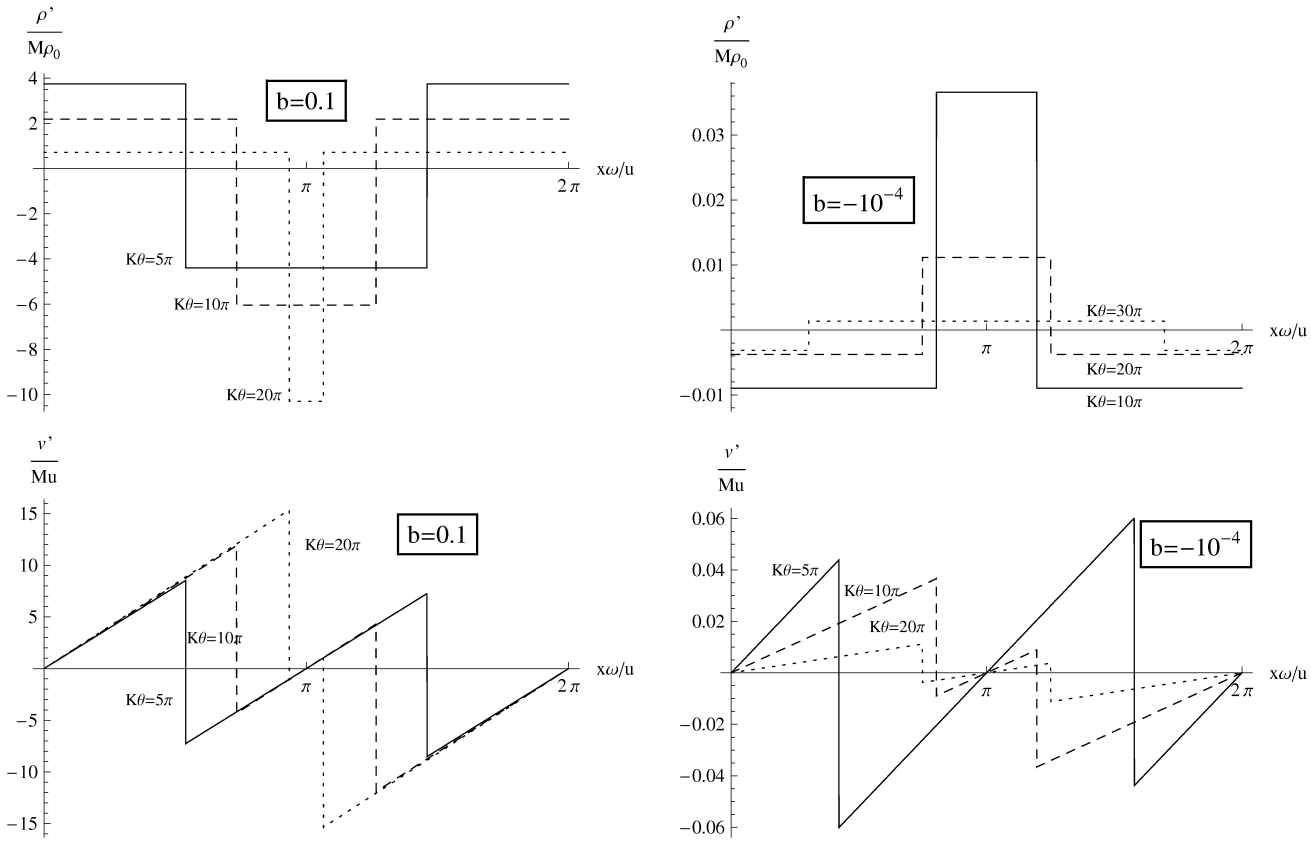

Fig. 3. Velocity and excess density in the standing wave in a resonator filled with chemically reacting gas after formation of the saw-tooth wave, for different values of $K\theta$ and b.

creases for $b < 0$. The quality factor q after formation of the saw-tooth waves is time-dependent, it equals $\left|\frac{1}{2b}\left(1+\frac{K\exp(b\tau)}{1-K}\right)\right|$. The acoustic energy in resonator for times greater then the characteristic time of formation of the saw-tooth wave, takes the following form:

$$\frac{E}{E_0} = \frac{e^{2b\tau}}{\left(1+K\left(e^{b\tau}-1\right)\right)^2}, \tag{30}$$

where

$$E_0 = E(0) = \frac{2}{3}M^2.$$

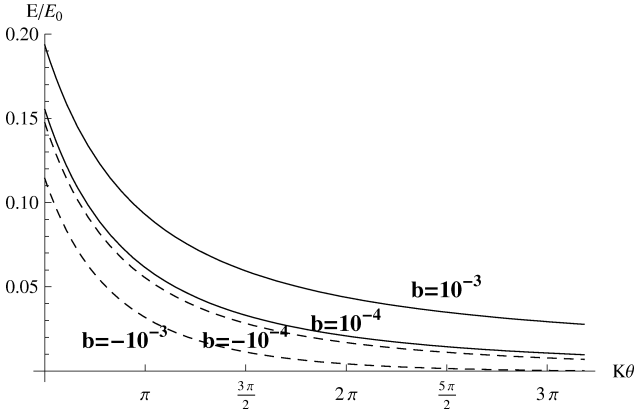

Fig. 4. Evolution of dimensionless energy for different coefficients b.

Figure 4 represents dimensionless energy for different coefficients b. As for acoustic heating, it is described by equation

$$\frac{\partial T_4'/T_0}{\partial(b\tau)} = -\frac{2M^2\pi^2(\gamma-1)(\gamma+2)}{3\gamma(1+K(\exp(b\tau)-1))^2}\exp(2b\tau). \tag{31}$$

7. Conclusions

The nonlinear standing waves in a resonator filled with Newtonian fluid have been well-studied, for example in (Kaner, Rudenko, Khokholov, 1977). As far as the authors know, standing acoustic waves in a non-equilibrium medium like gas with chemical reaction were not investigated. In this study, a flat resonator filled with chemically reacting gas, was considered. A special analytical method based on projectors was used to obtain equations governing the sound and relative heating in a resonator. The method leads to general Eqs. (15) which do not require the sound be periodic in time. The projecting method is complemented by the method of different scales. To exclude interactions between acoustics modes, the periodicity of acoustic perturbations was assumed as well as equal zero on average. In a Newtonian fluid, curves of the velocity in a standing wave have nodal points placed at dimensional distances π from each other.

In a medium considered in this study, the curves of velocity possess additional nodal points which travel between the constant ones (see Fig. 1). The shapes of velocity after formation of discontiniuty look similar to those observed experimentally in Newtonian fluid (Biwa, Yazaki, 2010). Efficiency of acoustic heating (or cooling) in resonators is of a great importance because of their wide physical and technical applications. In this study, Eqs. (26) and (31) which describe acoustic heating (or cooling) before and after formation of a saw-tooth wave, respectively, were obtained. In the case of irreversible chemical reaction $A \to B$ into a gas, the sound amplifies before formation of discontinuity ($b > 0$), and the medium becomes cooler. It never occurs in a resonator filled with a Newtonian fluid. After formation of discontinuity, attenuation at the front of the shock wave makes the peak pressure to decrease independently of the sign of b. The acoustic energy in a resonator, four times greater than the characteristic time of formation of the saw-tooth wave, tends to zero for negative coefficients b. For $b > 0$, the energy tends to value $\dfrac{4b^2}{(\gamma+1)^2M^2}E_0$, in accordance to Eq. (30), as the time increases. That reflects the equilibrium between nonlinear attenuation on the fronts of the saw-tooth wave and anomalous enlargement in their magnitude. The temperature, associated with the entropy mode, increases if $b < 0$ and decreases otherwise in the both cases, before and after formation of discontinuities. q-factor of a resonator filled with chemically reacting gas also depends on the dimensionless coefficient b. It is constant before shock formation, and time-dependent after that. The efficiency of the acoustic heating (or cooling) depends on the Mach number and the coefficient b, Eq. (26), (31).

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