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Constraining absolute chronologies with the application of Bayesian analysis

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ABSTRACT
In this work the application of Bayesian statistics to archaeological problems will be discussed. In particular, three case studies will be analyzed, each presenting complex interpretative scenarios, and the most suitable way to solve them. It will be shown that the Bayesian approach allows to refine a dating when in presence of multiple data, even from different dating techniques. The Bayesian approach is presented as the common language between physicists, archaeologists and statisticians to perform more accurate evaluations on stratigraphies and chronologies.

Section: RESEARCH PAPER

Keywords: dating; statistics; Bayes; chronology

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1. INTRODUCTION

Absolute dating has become a powerful tool in archaeology. However, the interpretation of the archaeometric data can be sometimes difficult, especially in the case of non-Gaussian probability distributions (e.g. radiocarbon dates). In fact, sometimes scientific analysis results are not univocal and must be interpreted. In absence of an impartial instrument to discriminate data, the degrees of freedom in the choice of the right results is often left to archaeologists and historians. Moreover, prior information about analyzed samples and provenance sites are not usually taken into account in real synergy with experimental data and historical studies, giving space to subjective interpretations, sometimes in conflict.

In order to overcome this misunderstanding it is important to think about the scientific results as a higher concept than "numbers", and treat the historical data and clues as mathematical terms of an equation.

A great help comes from the Bayesian statistical approach, a model that combines in a single formal analysis the experimental results coming from scientific analyses together with the present knowledge of an archaeological problem, in order to make inferences that can contextualize the problem in a coherent interpretation [1]-[4]. After a brief remind of the theory related to the method, three case studies are reported and discussed. Finally, the potential and advantages of this method will be underlined.

2. THEORY

The Bayes' theorem states that the posterior probability of an event is proportional to the likelihood times the prior probability, or formally:

$$\boldsymbol{P(E_1|E_2) = \frac{P(E_2|E_1)}{P(E_2)} P(E_1)}\,, \tag{1}$$

where $\boldsymbol{P(E_n)}$ are the probabilities of the single events, and $\boldsymbol{P(E_m|E_n)}$ are the likelihood of the two events to be linked.

Bayesian statistics uses probability as a means of measuring one's strength of belief in a particular hypothesis being true [5]. In other words, the application of Bayesian statistics allows the selection of the most significant data in the experimental set, rejecting the ones not supported by historical evidences or by a relevant likelihood.

The application of this method can bring to some strange and unconventional results, which can be understood only after the comprehension of the Bayesian "way of thinking". A special case may help: in Figure 1 a prior probability of an event A (blue curve, labelled R_Date standing for "radiocarbon date") is depicted, which has a maximum probability spanned over 200 years; the likelihood graph (red curve, labelled C_Date standing for "calendar date", normally distributed) states that the probability that event A occurs in conjunction with an event B (such as the use of a coin discovered in the same layer of the material associated to event A) has a normal probability distribution peaked at 920AD with an uncertainty of 20 years. Combining this data produces a posterior probability curve extremely different from the prior one, but closer to the real probability distribution, taken into account all the available data (black curve).

The powerfulness of this method is well represented by the calibration of radiocarbon dates to obtain the probability density curve a scientist is accustomed to (Figure 2). In radiocarbon dating the isotopic concentration of carbon-14 is not univocally related to a single date, but it has to be "calibrated" using a reference curve that links every concentration to a set of calendar ages. In this case the prior information is the existence of the radiocarbon date itself (the result of the isotopic analysis guarantees that this result is associated to at least a date on the calendar timescale); its numerical value is given in the graph title (R_Date, expressed in percent of modern radiocarbon pMC). The likelihood parameter is represented by a mix of the uncertainty associated to the isotopic analysis (red curve, Figure 2) and the probability distribution given by the calibration curve INTCAL (blue curve, Figure 2). The latter is represented in the graph with its uncertainty, which varies with time depending on the precision

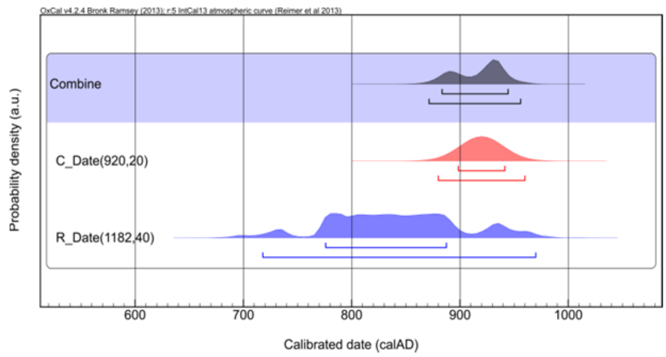

Figure 1. Representation of a Bayesian analysis.

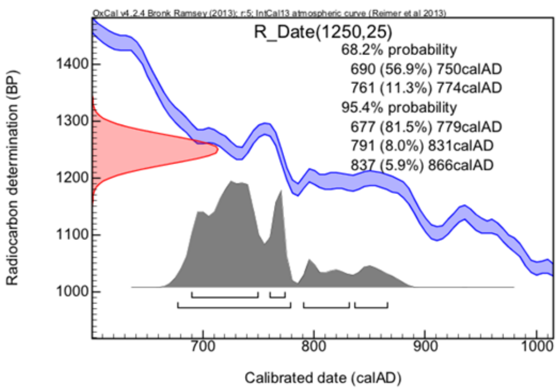

Figure 2. Example of radiocarbon date calibration.

and the number of measures used to build the curve. The projection of the Gaussian variable on the calibration curve is a density curve (black curve) from which the most credible date intervals (HPD, highest posterior density regions) can be extracted. The table in the graph describes the age ranges with 1-sigma and 2-sigma probability: note that the probability shape changes radically from the Gaussian initial one [6], and the probability densities can be splitted in multiple peaks characterized by a fraction of the initial 68 % and 95 %.

This approach is extremely useful when there is the need to match multiple dates and prior knowledge about a site or an artifact.

In this case, the same formal procedure is used, putting one of the collected dates as a prior information, and using the known connections between the stratigraphy layers or building phases as likelihood parameters.

3. MATERIALS AND METHODS

In this work three different examples of Bayesian statistics applications will be shown, each of them introducing a more complex approach to archaeological questions that can be solved with the help of prior information. Each posterior probability curve is evaluated numerically through a Markov Monte Carlo Chain analysis implemented in OxCal 4.2 [6]. This is a software package developed to calibrate radiocarbon concentrations and to calculate the most probable dates associated to a single concentration. It is also possible to apply the most used Bayesian models to a set of data distributions, both before and after the calibration process. The Monte Carlo analysis is performed during the calibration to evaluate the best probability density distribution. More precisely, the Metropolis-Hastings algorithm is used, since it only requires relative probability information [7]; in addition, it uses a set of proposal moves which can both result in changes to single elements of the model or changes to the duration and timing of whole groups. This provides much faster convergence for complex models.

4. RESULTS

A first example of this approach is the dating of the site of My-Son, a cluster of abandoned and partially ruined Hindu temples constructed between the 4th and the 14th century AD by the kings of Champa in Quang Nam Province (Central Vietnam) [8]. The available materials included 3 sets of bricks from one of the latest construction phases, and some charcoal pieces embedded in a brick (Figure 3A). A memorial stele commemorating the dedication of the temple (1155 AD, C_Date T) gives an historical boundary. Both thermoluminescence (TL) and radiocarbon were used when possible.

The selected samples come from three different sections of the structure walls, and the archaeological question was if all the masonries were contemporary or not. The ages obtained for the bricks allowed to identify three main groups: one (60 %) consisted in reused material, whose dates were well before 1155 AD, the dedication year; another surely posterior to that date; the last (G3 phase) with dates in between. The components of the first group are characterized by a density distribution completely shifted on the left of the stele boundary and their date is univocally assessed before the stele foundation, suggesting a probable reuse from preexisting structures. The components of the second group are completely shifted on its

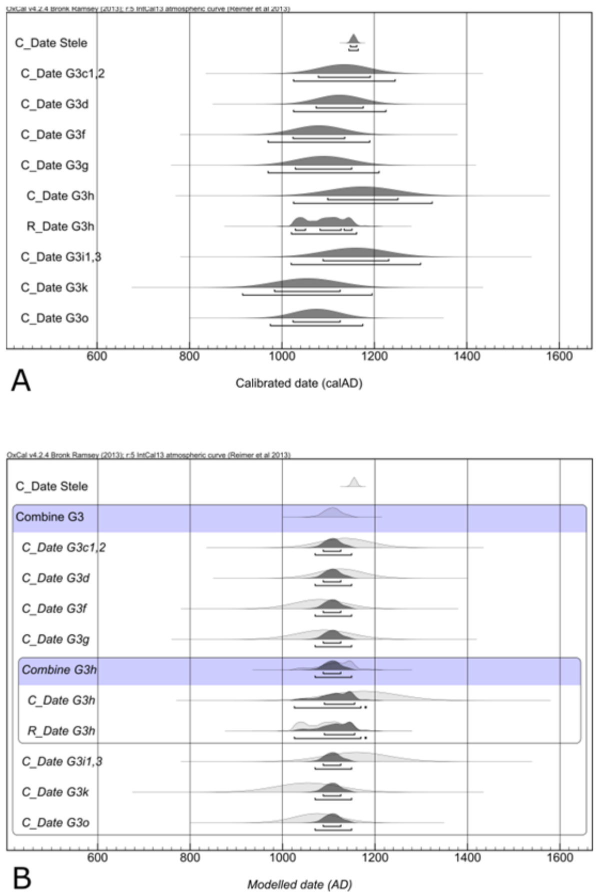

Figure 3. A: unmodelled representation of radiocarbon and thermoluminescence dates on My-Son samples; B: Bayesian analysis of the same dates.

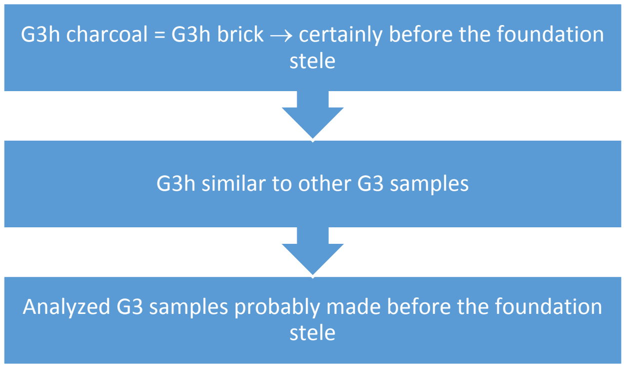

Figure 4. Logical deduction path for G3 phase dating.

right (G4 phase, Figure 3A), stressing the possibility of restoration performed in later times. The main question regarding the samples of the third group was if they were reused or purposely made for the edification of the site.

Unlike the calibration example, here the prior information is represented by a supposed *terminus post quem* given by the dated stele; this information is formalized by the equation:

$$\boldsymbol{p(t, t_{stele})} = \begin{cases} \boldsymbol{1, if\ t > t_{stele}} \\ \boldsymbol{0, otherwise} \end{cases} . \tag{2}$$

This equation and the following don't take into account the uncertainty associated to the boundary events. This is done to explain the model function, in the algorithm the whole range of probabilities is used, projecting the tales of the boundary distribution and smoothing the step slope of the resulting probability.

In Figure 3B the same results as in Figure 3A are represented, but after the application of the Bayesian statistics. The dark grey curves represent the posterior probability density distribution: it is clear that G3h has a large probability to be dated just before 1155 AD (stele), supported by a non-zero probability that comes from the radiocarbon dating of a piece of burned wood embedded in the brick paste. The "Combine" distribution is an operation on probability distributions that combines any number of probability distribution functions which give independent information on a parameter. The similarity in the probability distribution allows to extend the production of all G3 bricks before 1155 AD (Figure 4). A further modelling takes into account the probable contemporaneity of production of the G3 bricks, adding a prior function: if we assume that the production period is constrained by a start unknown event (t_a) and a finish unknown event (t_b), the formalization of the model is as follows:

$$\boldsymbol{p(t) \propto \frac{\prod_i p(t_a, t_i, t_b)}{(t_b - t_a)^n}} . \tag{3}$$

A second case regards the dating campaign of three burials in the archaeological site of Sipan in Northern Peru. It shows that the precision of the chronological boundaries of an event can be enhanced combining the site stratigraphy with the whole set of dating results. During this campaign, three different tombs were dated, using both TL and radiocarbon techniques. While the stratigraphic evidence clearly stated their relative temporal sequence, the archaeological request was the refinement of the absolute dating of the Warrior-Priest tomb (T14). Looking at the raw results, it appears that the age of all the examined materials, given the experimental uncertainty, is practically the same (range of phase T3: 215-435 AD; range of phase T14: 255-775 AD; range of phase T1-T2: 595-775 AD) (Figure 5A). However, using the stratigraphy of the site as the main constraint, the most probable period of T14 construction is severely restricted (Figure 5B). The prior information applied here is similar to the one from the previous example, with the difference that here there are three different groups of events, with two unknown intermediate periods that represent the end of construction of a tomb and the start of construction of the next tomb. The formal condition of this model is:

$$\boldsymbol{p(t) \propto \frac{\prod_i p(t_a, t_i, t_b)}{(t_b - t_a)} \frac{\prod_j p(t_b, t_j, t_c)}{(t_c - t_b)} \frac{\prod_k p(t_c, t_k, t_d)}{(t_d - t_C)}} , \tag{4}$$

where t_a, t_b, t_c and t_d are the boundaries of the grouped events.

As reported in Figure 5B, the prior condition of non-contemporaneity of the three phases reduces the span of T14 use by 80 %, especially involving some particularly spanned radiocarbon dates (RC124).

The last example regards a Neolithic site in Southern Italy. It shows how the Bayesian analysis can extract extended information on a site integrating prior hypothesis and dating. Here the information needed is not about the refining of the site chronology, as the collected samples are common wares used in everyday life during all the occupation period, and cannot represent an *a priori* restrain; instead, it is possible to extrapolate data about the site life span, beside the beginning and end of its occupation. For this settlement, the continuity of site occupation was hypothesized. After a first analysis (Figure 6), it was possible to identify two sub-phases, which were further modeled to find a possible hiatus between the two periods. A further complication in the prior probability model

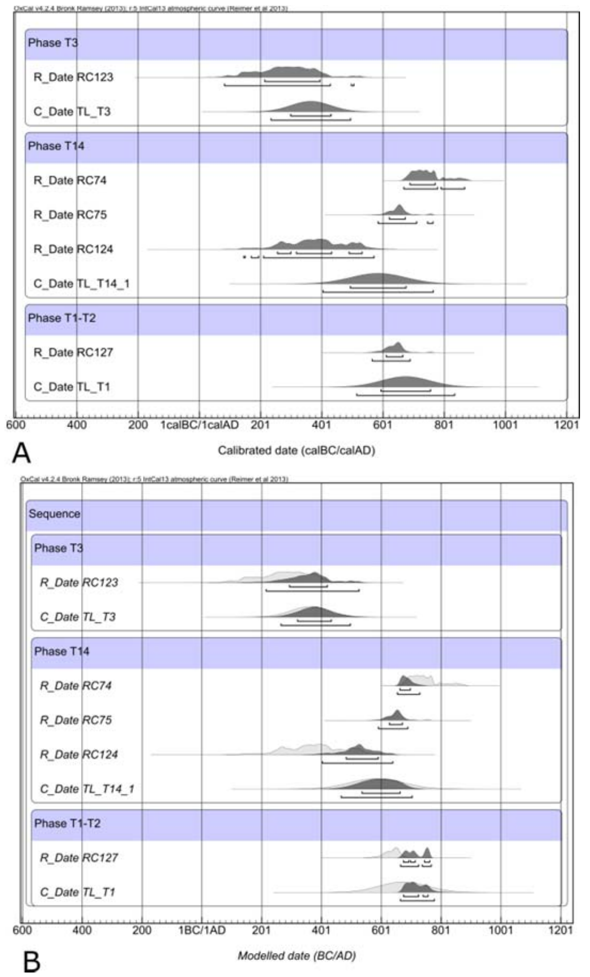

Figure 5. A: unmodelled representation of radiocarbon and thermoluminescence dates on Sipan samples; B: Bayesian analysis of the same dates.

is introduced taking into account the presence of a gap between the end of the first event (t_b) and the start of the second one (t_c):

$$p(t) \propto p(t_b, t_c)\frac{\prod_i p(t_a, t_i, t_b)}{(t_b - t_a)}\frac{\prod_j p(t_c, t_j, t_d)}{(t_d - t_c)}. \quad (5)$$

The resulting data are showed in Figure 7: the end of the first sub-phase and the beginning of the second are not overlapping, but there is a gap of about 100 years in the probability distribution curves. This can be a signal of a temporary site leaving, as well as of a period of crafting decadence. It is clear that such a result could have not been obtained through a rough qualitative interpretation of the results.

5. DISCUSSION AND CONCLUSION

The described examples aim to underline the importance of a correct approach during the statistical elaboration of the results of absolute dating techniques. They are not only numbers, but a source of linked information that can be extracted imposing the right conditions and constraints. The right approach is not to put the obtained data in a supposed model, rejecting what doesn't fit or "doesn't sound well"; all information should be analyzed and criticized, trying to shape the historical model in a feedback process, involving archaeologists, physicists and statisticians in the discussion.

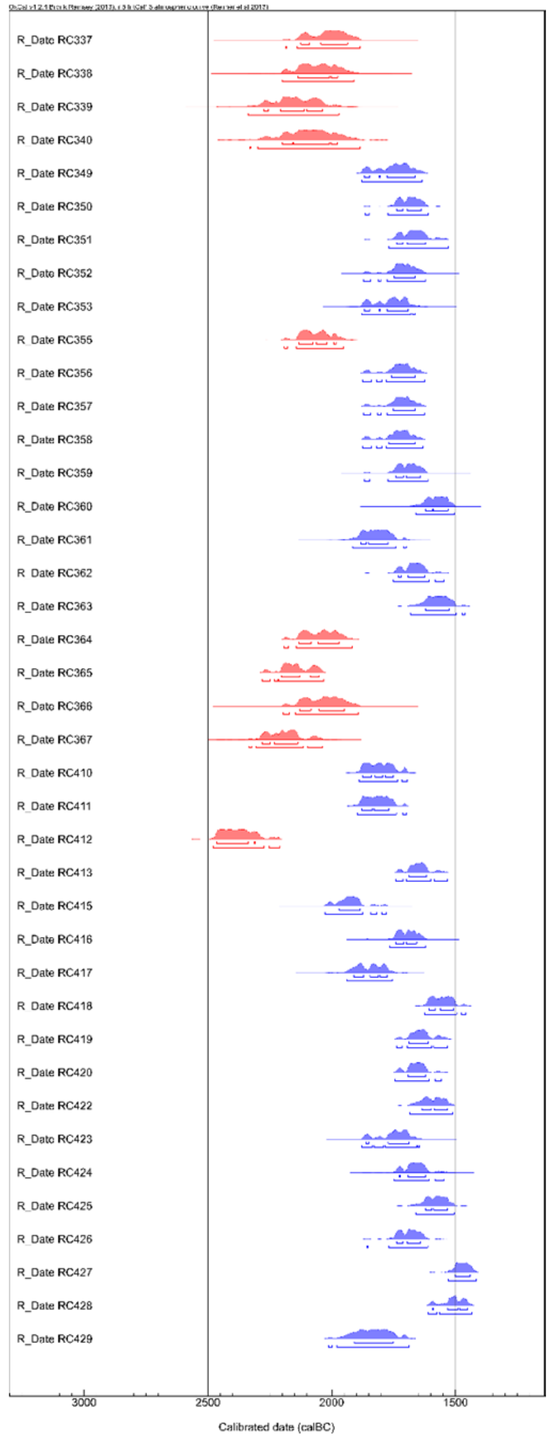

Figure 6. Distribution of the radiocarbon dates on Southern Italy samples.

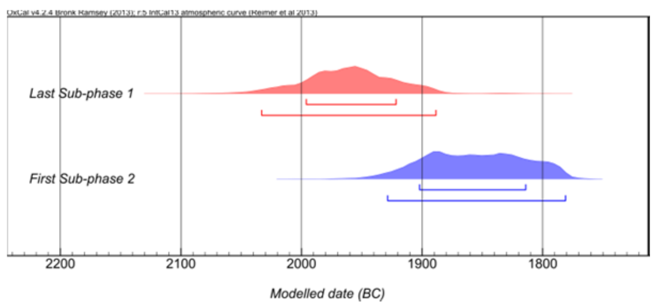

Figure 7. Sub-phases discrimination.

Furthermore, the use of all the available information in the refining process of the statistical data considers the uniqueness of every site, with the great advantage of taking into account the uniqueness of the experimental evidences [9].

The potential of a Bayesian approach in archaeology will reach its maximum when a tight interdisciplinary collaboration between archaeologists and staticians will come true. This requires archaeologists to be comfortable in analyzing a situation and defining the archaeological problem in a realistic but not over-refined way. In wider terms, they require to communicate their ideas to statisticians speaking a "shared" language, and to explain the importance of their work in simple terms.

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