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AN ECONOMETRIC MODEL OF BEEF PRODUCTION FOR OPTIMIZATION PURPOSES

Selostus: Naudanlihan tuotantoprosessia kuvaava taloudellinen malli tuotannon optimointia varten

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Preface

This study has been conducted mainly during the years I was employed at the Agricultural Economics Research Institute. First of all, I wish to express my warmest thanks to professor MATIAS TORVELA, Head of the Institute, for including this study in the research programme of the Institute and for his encouragement and advice throughout the work.

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The basic knowledge which has enabled me to carry out the study stems from my teacher, professor ANTTI MÄKI, and, later, from professor VILJO RYYNÄNEN who also has made valuable comments on the manuscript. Dr. LIISA SYRJÄLÄ has helped in the parts of the study which concern animal husbandry and feeding. I am also grateful to professor MARTTI LAMPILA and to the Department of Animal Husbandry of the Agricultural Research Centre for providing the empirical material needed in this study.

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Helsinki, November 1978 *Jouko Sirén*

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SIRÉN, J. 1978. **An Econometric Model of Beef Production for Optimization Purposes**. J. Scient. Agric. Soc. Finl. 50: 399—000.

Abstract. The aim of this study was to create a model depicting the growth process of a beef animal to provide means for economic optimization of the slaughter weight and of the length of the rearing period. Using data from Finnish beef rearing experiments, a model with two functions, one for weight gain and the other for feed intake, was estimated. In weight gain functions total energy and digestible crude protein were the main explanatory variables. Intake capacity of the animal was estimated as time needed to consume the dry matter quantity of a given feed input using also protein, crude fibre content and energy content of feed as explanatory variables.

The model was estimated separately for Ayrshire bulls and for Friesian-Ayrshire and Charolais-Ayrshire cross-breeds. Functions of Cobb-Douglas or transcendental forms gave the best fits and turned out to be logical. Information given by the regression coefficient estimates was analyzed in detail as background for applications of the model.

When dealing with the optimization problem the dynamics of production was discussed and a maximum daily profit was selected as the optimization criterion. An example was then presented to illustrate the profit maximization. Since they were of essential importance in the production economy, the feed composition and its productivity were analyzed and an example was derived to minimize the price of feed mix with maximum productivity.

I Introduction and aim of the study

Maximizing the profit is normally seen as the object of an economic activity. In agriculture, on the farm level, this object can be reached through optimal organization of the production process. With this aim in mind, the farmer has to choose between different lines and volumes of production, production techniques, feeds for production, intensity levels, etc. The lines of protion must be adjusted to each other, and organized in such a way that an optimal relationship between yields and inputs can be found.

Some inputs in agriculture show a proportionally decreasing return in productivity. An increase of fertilization may result in a progressive increase in the yield, but after a certain level the yield increase becomes smaller and may finally lead to a decrease in the yield. A corresponding phenomenon in animal production can be found in relation to the use of concentrate or protein feeds. In order to obtain the best economic result, recognition of both price and cost levels and the relationship between inputs and output is necessary.

The aim of this study is to illustrate the means for economic optimization of beef production. For this purpose it is necessary to find out the relationship between feed input and the growth of the animal so as to enable the producer to decide:

1. the optimal slaughter weight and age of the animal, and
2. the optimal feed composition for production.

The problem is the same as in the earlier study published by the author (SIRÉN 1974). The present study, however, deals mainly with problems that were not incorporated in the earlier one. The data used available at the time was not adequate for testing the theoretical model, or for an application of the estimated results. The present data derives from experiments conducted in the early 1970's at the Department of Animal Husbandry of the Agricultural Research Centre. This material provides a better starting point for an estimation of the growth process of the beef animal and for an application of the estimates for economic optimization purposes.

Special attention is here paid to problems connected with optimization technique and the application of the estimated growth process model. The use of the new data makes a detailed analysis of the estimated model possible.

II The econometric model depicting the growth process of beef

A. Construction of the model

When a theoretical model is used to describe reality, it is important to make clear the nature of the real phenomenon, the factors that affect each other and the directions of the influence of the factors. NIITAMO (1966), when dealing with the econometric model theory, defines the general principle of model building as clarifying the factors which are essential to the problem under consideration. In this connection, the model can be seen as a simplified and rational illustration of the reality.

In this study the model will be developed to describe the biological growth process of a beef animal, taking the economic aspects into consideration which is why the model is seen as an econometric model. When discussing the economics of production, the most important factors in the growth process are the weight gain resulting from the growth, and the inputs used for this purpose. Even if the economic aspect is of central importance in the model, the biological growth process places limits and a framework to the building of the model.

The literature concerning animal nutrition, animal physiology and utilization of energy and protein is abundant. Among others PALOHEIMO (1956), ANDERSEN (1969, p. 18—30 and 1971), HOMB (1970), KELLNER and BECKER (1971), COLE et al. (1974 and 1976) should be mentioned here. Due to the extensiveness of their work, only a short description will be given below of the central features in the growth process; to serve as backround for the model building.

The growth and maintenance of an animal requires sufficient quantities of energy, nitrogen containing materials, minerals and vitamins; in other words, nutrients. Parts of the nutrients are wasted in faeces, urine and con-

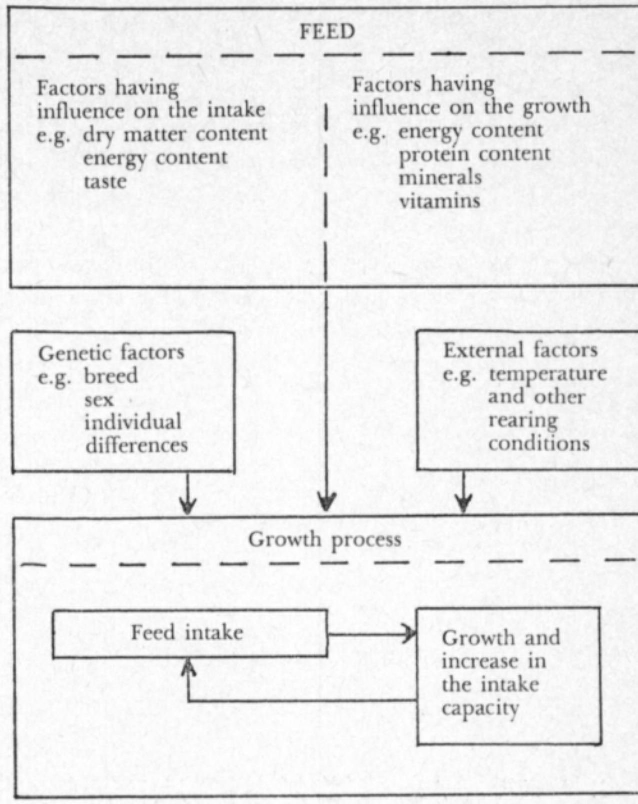

Figure 1. Simplified presentation of the growth process.

version losses, while the rest remains for production. When the size and weight of the animal increase as a result of growth, the feed intake also increases. This is essential for satisfying the growing animal's increased need for feed. In addition, other factors affect the need for nutrients such as individual, breed or sexual differences. Some external factors may also influence the need for nutrients.

The growth process is shown in Figure 1. The lower part of the figure shows the growth process brought about by the proportion of nutrients in the feed which remains for production. As a result of growth, the intake capacity and the need for feed increase, thus increasing the total feed use per weight gain unit.

Taking the problem as a whole, the intake capacity must be considered simultaneously with the nutrients of the feed available for growth. Intake capacity is affected by the size of the animal as well as many other factors. Consequently there are difficulties in finding detailed norms for feed intake (e.g. Lampila 1970 and 1971 and Hyppölä and Hasunen 1970). The daily dry matter intake is usually used as an estimate of the feed intake capacity.

Feed intake capacity plays an important role, as a limiting factor, in the nutrients used in production. If the nutritional value of the feed, in relation to that of dry matter, is low, the feed is bulky; or in the opposite case, concentrated. The energy content of feed, usually measured in feed units per kilogram of dry matter, has an influence on the rate of growth. The weight gain is negative with very low energy contents of feed, and the highest possible, when the energy content is sufficiently rich. (The growth rate is naturally limited by the growth ability of the animal).

As stated above, the growth process is dependent on the relationship between growth and the nutritional value of the feed, the intake capacity of the animal, its changes, and factors affecting it. Growth and intake capacity are linked together by the quantity and quality of the feed nutrients. The growth rate, defined by the limits of the physiological growth ability of the animal, is affected by the supply of nutrients in the feed consumed. This explains the dynamic nature of the growth process.

Feed input in Figure 1 is divided into two parts. One group is formed by factors having an influence on the growth, and the other by factors having an influence on the feed intake of the animal.

Having

X_i = factors of feed affecting growth
X_j = » » » » feed intake
G = genetic factors
U = external factors
Y = weight gain (growth)
T = time needed to consume a given quantity of feed
u = disturbance term,

the model depicting the growth process can be stated as follows:

1. Weight gain

$$Y = f(X_i, G, U) + u_Y$$

2. Intake capacity

$$T = f(X_j, G, U) + u_T$$

The weight gain function of the model specifies the growth of the animal as a function of nutrients of feed, and genetic and external factors. The intake capacity is expressed as the time needed to consume a given quantity of feed, when genetic and external factors are taken into account.

A simultaneous and recursive dependence between weight gain (Y) and intake capacity (T) was expected in the ordinary, basic model (SIRÉN 1974). When testing the different model hypothesis with empirical data it was found that neither simultaneous nor recursive dependence was significant. The analysis, however, was disturbed by multicollinearity between the explanatory endogenous variables and exogenous variables (e.g. JOHNSTON 1972, p. 160, KLEIN 1965, p. 64 and 101). For estimation purposes of the growth process, a model, with separate weight gain and intake capacity functions, was selected. However, the hypotheses on a simultaneous or recursive dependence between the two endogenous variables can not be rejected.

B. Variables of the model

Some references to the choice of variables may be derived from the dependencies under consideration. However, some details must be overlooked or expressed in inaccurate terms because of quantification problems.

Output is measured as liveweight gain, withouth taking the quality of beef into consideration. Here, difficulties arise in the time series data because, without slaughtering results, it can not account for quality differences or the percentage of beef in the liveweight.

The choice of variables depicting the feed input is based on principles of animal nutrition examined in Chapter II A. Suitability of different variable alternatives was tested by SIRÉN (1974). Thus, the basic variables in the growth process are the gross energy, measured in feed units (f.u.), and the digestible crude protein (DCP) of the total feed mixture.

The appearance of protein in energy, and thus, the multicollinearity of these variables provides biased estimates of the regression coefficients. However, the sufficiency of protein needed for growth is not expressed by the energy in the feed. That is why the protein in this study is measured as protein content of feed unit (DCP/f.u.) and will be handled as a factor depicting the quality of the feed energy.

The main factor regulating the time needed to consume a given quantity of feed is the dry matter content of the composition (DM). Because of the differences in the digestibility of different feeds (MÄKELÄ 1956, p. 108), converted dry matter content (suggested, e.g., by HYPPÖLÄ and HASUNEN 1970) would provide a more exact standard for this purpose. For practical reasons, total dry matter of the ration has been chosen as a variable for this study.

The quality of feed affecting the intake of dry matter is measured by the energy content of the dry matter (f.u./kg DM) and the digestible crude protein content of the feed unit (DCP g/f.u.). The crude fibre content of dry matter was also chosen as an indicator of the dry matter intake.

Qualitative factors, such as the taste of the feed, could not be taken into account. In addition, minerals and vitamins were omitted, assuming that the feed in the empirical data was so composed that it satisfied the animal's needs of these ingredients.

Variation in the weight gain and in the feed intake between animals is also caused by sex, breed and individual genetic factors. Some of the quantification problems connected with these variables are avoided when using a model with homogenous groups, or when using dummy variables for these factors. In this study, only bulls will be included and the model will be estimated for some breeds only, partly separately and partly using dummies.

Individual differences of animals within homogenous breed groups are measured by the live weights of the animals at the start of rearing.

Variables of the model are as follows:

Exogenous variables

FU = total energy of the feed composition in feed units (f.u.)
DCP = digestible crude protein, grams/f.u.
EC = energy content of feed, f.u./100 kg dry matter
DM = total dry matter of the feed composition, kg
CF = crude fibre as a percentage of the dry matter
W = live weight of the animal at the start of rearing, kg
D = dummy, representing the breed

Endogenous variables

Y = live weight gain, kg

T = time needed to consume a given quantity of feed (estimate of the cumulative intake capacity of the animal), days

Putting the variables into the structural form of the model the functions can be stated as follows:

1. Weight gain

$Y = f(FU, DCP, W, D) + u_Y$

2. Intake capacity

$T = f(DM, EC, DCP, CF, W, D) + u_T$

III Data

The basic material derived from four beef rearing trials, introduced by the Agricultural Research Centre, forms the data which is used in the estimation of the theoretical model. The experimental data is shown in Table 1.

When collecting the data, the purpose was to find a large enough collection of animals with individual information on weight gains, age and feed use during as long a rearing period as possible. Likewise, attention was paid to the variation in feeding intensity. However, because of norm feeding only a narrow variation was found, which mainly resulted from different feed compositions and differences in the properties and quality of the feeds used.

All the animals in the experiments were observed on an individual basis from the beginning of the rearing period, which made it possible to discern the total feed consumed during the rearing. During the pasture period, the animals were kept in the barn or in the pen (all feeds were supplied). Experiments were divided into periods of 14 or 28 days. For this study the data was worked out by calculating the following variables as cumulative sums of successive rearing periods:

— age
— live weight gain
— use of different feeds in kilograms and their
— summed feed units
— » digestible crude protein
— » dry matter
— » crude fibre
— in addition the weight and age of animals
at the beginning of the experiment, as well as carcass grading at the end of rearing.

On an average, 16 cumulative observations on every animal were made totalling 1 826 observations on all 113 animals. Because of autocorrelation problems in the time series, which might have caused difficulties in interpretation of the results (e.g. JOHNSTON 1972, p. 246, HEADY et al. 1963, p. 887), only 7 successive observations of each individual animal, at intervals of about 2 months, were selected. The final data, totalling 113 animals and 792 observations, was then divided into two groups, one for Ayrshires and one

Table 1. Facts from the used data.

Name of experiment	Number of animals (bulls)	Breed or cross-breed	Beginning date and duration of experiment	Main feeds used
Ruukki 06	15	Ayrshire	June 1972 12 months	Whole milk, skim milk powder,
	16	Friesian-Ayrshire		barley, silage, hay and pasture
Rehtijärvi 02	28	Ayrshire	December 1973 15–16 months	Whole milk, milk powder, silage, barley, bridgets and pasture
Ruukki 08	28	Ayrshire	December 1973 10–11 months	Skim milk powder, barley and silage
Ruukki 12	15 11	Friesian-Ayrshire Charolais-Ayrshire	December 1974 12–13 months	Skim milk powder, barley and silage
Total	113, of which 71 Ayrshires (Ay), 31 Friesian-Ayrshires (FrAy) and 11 Charolais-Ayrshires (ChAy).			

for Friesian-Ayrshire and Charolais-Ayrshire cross-breeds. Thus, the Ayrshire data consisted of 71 animal time series and the cross-breed data of 42. The final average number of observations per animal in the time series was 7.[1])

The ages of the animals at the beginning of the experiment varied between 0 and 30 days. Because the birth weight of some animals was unknown, the live weight at the age of 30 days was selected to show possible individual differences. That weight was calculated by linear interpolation of the weights at the end of the 14 day periods at the beginning of the experiments. Even if animals with a small age variation were included in the data, an assumption of homogenous data was made.

At its longest, the duration of the Ayrshire rearing experiments was about 16 months and that of the cross-breed experiments 12 months. For the model estimation, a longer rearing period would have been desirable especially for

[1]) The data, as a whole, obtainable from the author.

the cross-breeds. The highest total feed units, dry matter quantities and live weight gains are also presented below. The variation limits of digestible crude protein content, energy content and crude fibre content of feed refer to the whole rearing period.

The variation limits of the variables in the data are shown by the following figures:

Variable	Ayrshire			FrAy and ChAy cross-breeds		
	Average	Variation		Average	Variation	
		lowest	highest		lowest	highest
Live weight at age of 30 days, kg	47.9	38	61	52	34	68
Rearing time, days			476			365
Live weight gain, kg			460			425
Feed units, total	793		2 245	848		1 914
Dry matter, total, kg ...	843		2 419	832		1 847
Digestible crude protein g/f.u.	143.0	108	221	141.8	95	252
Energy content f.u./100 kg DM	94.0	76	168	101.9	92	121
Crude fibre percentage of dry matter	14.3	3.0	23.7	11.7	1.8	19.8

IV Estimation of the model

A. Estimation method, forms of the functions and testing the results

The method of ordinary least squares was applied in the estimation of the model. As stated before, and by SIRÉN (1974), the model was assumed to be a multiequation model with two equations without simultaneous or recursive dependencies in endogenous variables.

The best linear, unbiased estimates of the coefficients are found when the residuals are normally distributed with an expected value of 0, if they are not autocorrelated and if they have a constant variance; or

$$E\ (u_i) = 0; \quad i = 1, 2, \ldots n$$

$$E\ (u_i u_j) = \begin{cases} 0, \text{ when } i \neq j & i, j = 1, 2, \ldots n \\ \delta^2, \text{ when } i = j & i, j = 1, 2, \ldots n \end{cases}$$

In addition we must assume that the residuals are uncorrelated with the exogenous variables in the model.

The consequences of the residual autocorrelation are the unbiased estimates of regression coefficients, however, the sampling variances of the estimates may be unduly large. This is why, e.g., the t-test cannot be used when testing the significance of estimates (JOHNSTON 1972, p. 246). Different methods are available for testing the residual autocorrelation (e.g. Durbin-Watson 1950 and 1951). They are not used in this study, however, because of combined cross section and time series data. To minimize the possible autocorrelation of the time series, only a few observations on each animal were used with intervals of 2 or 3 months; even if in the original data the length of the observation periods was 14 or 28 days. For this reason, no residual autocorrelation is expected and the t-test will be used in the usual way for testing the significance of estimates.

References derived from the results of the model, estimated with some older experimental data, form the basis for the choice of the forms of the functions (Sirén 1974). A function of the Cobb-Douglas type turned out to be suitable for describing the model. Likewise, the transcendental function was tried out, and it provided regression coefficients which were in accordance with the theory on the impact of some feed input factors. Because of a narrow variation in the data, and a very short rearing time, the results obtained in that study were not essentially better than those derived from the Cobb-Douglas type. In the present study the data cover a rearing period of over twelve months, providing the opportunity for a more exact description of the function course. That is why functions of both the transcendental and the Cobb-Douglas forms were also chosen as the basic forms in this study. Selection of the final form is based on the significance level of estimates and R^2-values.

The general form of the *transcendental* function can be stated in logarithmic terms as follows:

$$\log Y = \log a + \sum_{i=1}^{n} b_i \log X_i + \sum_{i=1}^{n} c_i X_i$$

The *Cobb-Douglas* function is a form of the transcendental function when the regression coefficients of linear terms have a value $= 0$.

Properties of the transcendental function type are discussed, e.g., by Halter et al. (1957). Also, Kettunen (1966) and Kettunen and Torvela (1970) have given detailed presentations.

The main properties of the functions are as follows:

	Transcendental type	Cobb-Douglas type
Marginal productivity	$\left(\frac{b_i}{X_i} + c_i\right) \cdot Y$	$\frac{b_i}{X_i} \cdot Y$
Elasticity	$b_i + c_i X_i$	b_i
Minimum or maximum value	$X_i = -\frac{b_i}{c_i}$ minimum, if $b_i < 0$ and $c_i > 0$ maximum, if $b_i > 0$ and $c_i < 0$	does not exist
Inflexion point	$X_i = \frac{-b_i + \sqrt{b_i}}{c_i}$ if $b_i > 1$ and $c_i < 0$, or $0 < b_i < 1$ and $c_i > 0$	does not exist

Also other function types may be used to describe the beef production process. In 1945, input-output relationships were estimated by NELSON (ref. HEADY 1952, p. 72). In that study a function of the Spillman-type

$$Y = m - Ae^{-rX} \text{ was used.}$$

The second degree polynomial in used, among others, by PLAXICO et al. (1959, ref. ANON. 1968, p. 81), HEADY and DILLON (1961, p. 452—475), HEADY et al. (1963) and VOGEL (1965). Properties, suitability and adaptations of the different function types in describing the beef production process are dealt with, e.g. in HJELM (1954) and SIRÉN (1974).

B. Estimates of the model

Estimates of the weight gain and intake capacity functions of the model are presented in the following. Chapter C deals with the analysis of the estimates in greater detail.

1. Estimates of weight gain functions

In the gain functions, total number of feed units (FU), digestible crude protein content (DCP) and weight of the animal at the age of 30 days (W) were included as explanatory variables for live weight gains. When estimating the weight gain function for the two cross-breeds, dummy variable D_{fr} with the value 1 for FrAy and 0 for ChAy was used. Both the transcendental and the Cobb-Douglas functions were estimated. Weight gain was estimated with one of the function's explanatory variables in logarithmic terms only, and the other explanators in both logarithmic and linear terms. The estimation was repeated so that each explanator by turns was placed in the function in logarithmic terms. Comparisons made showed that the estimates derived from transcendental functions were logical and, as a rule, more significant than those derived from other functions. The highest correlation was likewise found with this function form.

Estimates, their standard deviations, and t-values are given in Table 2.

Regression coefficient estimates of energy (FU) became significant at 99.9 % level, giving the theoretical maximum to the function at the energy level of 4 164 f.u. (Ay breed) and 5 516 f.u. (cross-breeds). The maximum live weight gain is located outside the observed region of the data playing no essential role in the actual beef rearing.

The digestible crude protein content of feed unit (DCP) proved to be significant at 99.9 % level as well. Using the regression coefficients it is possible to show that by increasing the protein content of energy, the live weight gain increases first at increasing rates and, beyond the inflection point, at decreasing rates. It reaches the maximum at the level of 135.6 (Ayrshire) and 115.3 grams DCP/f.u. (cross-breeds). The increase in the protein content above this level has a decreasing effect on the productivity of feed energy. The course of the function corresponds, e.g., with results of some recent Danish and Swedish experiments which gave maximum daily gains at protein levels of 120—125 g DCP/f.u. (WIKTORSSON et al. 1975, p. 575).

Table 2. Regression coefficient estimates, their standard deviations and t-values of the weight gain functions. Endogenous variable log Y (live weight gain).

Variable	Regression coefficient	Standard deviation	t-value	
Ayrshire				
log FU	0.8646	0.0158	54.7	
FU	− 0.0001	0.0000	− 6.4	
log DCP	6.4761	0.5878	11.0	
DCP	− 0.0207	0.0017	−12.5	
log W	1.4879	0.8357	1.8	
W	− 0.0130	0.0077	− 1.7	
intercept	−12.9682	1.4179	− 9.1	
				$R^2 = 0.958$
Friesian-Ayrshire and Charolais-Ayrshire cross-breeds				
log FU	0.9368	0.0201	46.6	
FU	− 0.0001	0.0000	− 4.5	
log DCP	3.2938	0.2499	13.2	
DCP	− 0.0124	0.0007	−17.5	
log W	0.7214	0.4591	1.6	
W	− 0.0068	0.0039	− 1.7	
D_{fr}	− 0.0211	0.0069	− 3.1	
intercept	− 6.5071	0.7285	− 8.9	
				$R^2 = 0.983$

The estimated regression coefficients of the animal's weight (W) seem to be logical as well, showing that the growth ability of heavier calves is better than that of light ones. Maximum gains result from animals weighing 49.7 kg (Ayrshire) and 45.7 kg (cross-breeds) at the age of 30 days. However, definite conclusions cannot be drawn because of the low significance of the estimates. This low level is caused by the fact that not only genetic factors but also the effects of different feeding intensities experienced by the animals during their first 30 days are included in the variable W.

When comparing the estimates derived from the cross-breeds data, ChAy has an advantage over FrAy as measured by energy utilization.

2. Estimates of intake capacity functions

Intake capacity was estimated as time (T) needed to consume a given dry matter quantity of feed (DM), also including the energy content (EC), the digestible crude protein content (DCP) and the crude fibre content (CF) as explanatory variables in the function. Time (T), which was expressed in number of days, was characteristic of the cumulative intake capacity of the animal. Here, as well as when the weight gain functions were estimated, individual differences of the calves were described by their live weights at the age of 30 days (W). The cross-breeds were separated from each other by the dummy variable D_{fr}, with a value of 1 for FrAy and 0 for ChAy.

The choice between functions was accomplished by estimating different alternatives between the transcendental and the Cobb-Douglas forms, as was done when estimating the weight gain functions. All the estimated functions gave a high correlation which supported the choice of the Cobb-Douglas form because of its easy adaptation to the model. In the Cobb-Douglas function, correlation coefficients of energy content, the protein content, crude fibre content, and animal weight were partly incongruent with the assumptions and theory, and less significant than those estimated from the functions where the linear terms were also included. That is why, in the final form, only dry matter (DM) is expressed in logarithmic terms and the other variables are in both logarithmic and linear terms.

Estimates are presented in Table 3.

Table 3. Regression coefficient estimates, their standard deviations and t-values of the intake capacity functions. Endogenous variable log T (number of days).

Variable	Regression coefficient	Standard deviation	t-value	
Ayrshire				
log DM	0.6063	0.0059	102.6	
log EC	−0.6192	0.3705	− 1.7	
EC	0.0039	0.0014	2.7	
log DCP	2.4383	0.3773	6.5	
DCP	−0.0084	0.0011	− 7.9	
log CF	−0.1194	0.0599	− 2.0	
CF	0.0100	0.0023	4.4	
log W	2.3988	0.4766	5.0	
W	−0.0236	0.0044	− 5.4	
intercept	−5.4787	1.1764	− 4.7	
				$R^2 = 0.983$
Friesian-Ayrshire and Charolais-Ayrshire cross-breeds				
log DM	0.6177	0.0074	83.6	
log EC	0.9056	3.2457	0.3	
EC	0.0049	0.0132	0.4	
log DCP	1.0674	0.2286	4.7	
DCP	−0.0054	0.0007	− 8.1	
log CF	0.1174	0.0657	1.8	
CF	0.0150	0.0047	3.2	
log W	0.9525	0.2940	3.2	
W	−0.0089	0.0025	− 3.5	
D_{fr}	−0.0142	0.0045	− 3.2	
intercept	−4.7873	5.1757	− 0.9	
				$R^2 = 0.989$

According to the results, the time needed to consume a given feed quantity depends, at 99.9 % significance level, on the total of the dry matter of the feed. Moreover, energy, protein and crude fibre contents have a significant influence on the feed intake. The increase in the weight of the animal has a negative correlation to the time which the animal needs to consume a constant dry matter quantity. This signifies a positive correlation between the daily dry matter intake and the size of the animal.

When comparing the cross-breeds with each other, it can be seen that the ChAy needs more time than the FrAy to consume a constant feed input. This proves that ChAy have a smaller intake capacity than FrAy. In spite of a smaller feed intake, the ChAy turned out to have a better growth ability, as stated in Chapter B 1.

According to the regression coefficient estimates, increasing the energy content of feed retards the feed intake. Up to a certain limit this is apparent from the protein content of the feed. This is due to the fact that a proportionately smaller quantity of concentrated feed with a high protein content satisfies the nutrient needs of animals better than a feed with a lower nutritional value. An increase in the crude fibre content retards the feed intake as well.

Information derived from the estimates will be examined in greater detail in Chapter C.

C. *Evaluation of the estimation results*

Estimates derived from the model offer abundant information concerning the influence of feed quantity and properties on the growth, and on the feed intake of the animal. Because they are of essential importance to the weight gain and feed utilization, and to profits, factors in the feed are examined below as they appear from the model estimates. The main features of the influence of energy, protein, energy content and crude fibre content of the feed in the growth process are discussed. The results refer to the rearing period from the animal's birth to the age of 16 months or 460 kg live weight (Ayrshire), and to 12 months or 425 kg (cross-breeds).

1. Energy and digestible crude protein content of feed

Variations in live weight gains were explained by the total feed unit quantities of the feed ration, and by the average digestible crude protein content of feed units. Increases in energy (number of feed units) produce an increase in weight at a decreasing rate. The gain reaches its maximum at the energy level shown by Figure 2.

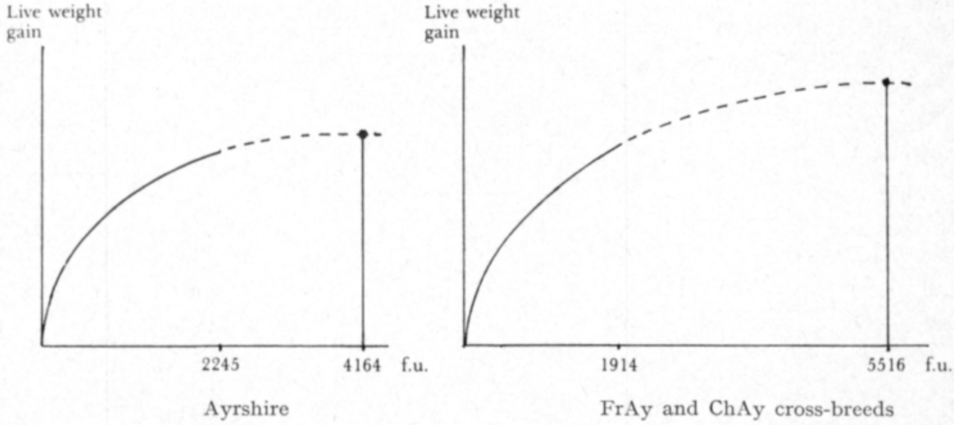

Figure 2. Live weight gain as a function of the number of feed units. The dotted line shows the area outside the data observations.

Weight gain curves of the examined breeds, as a function of feed unit quantity when other variables are fixed at their mean level, are shown in Figure 3. Also, the curves of marginal and average feed uses are presented. Corresponding figures are shown in Table 4.

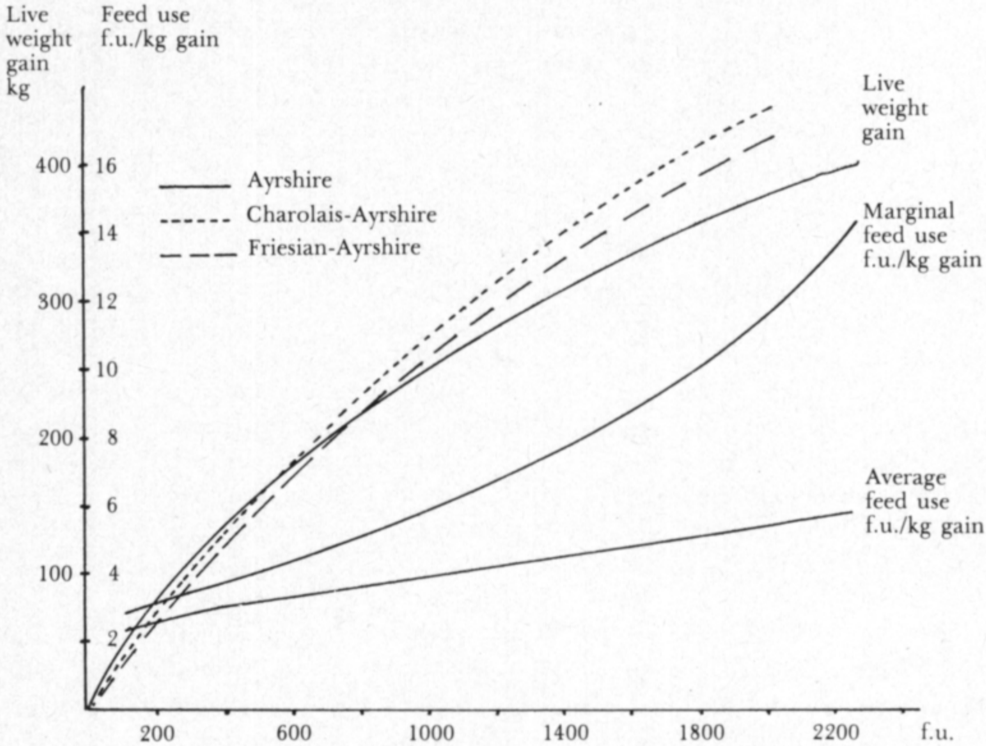

Figure 3. Live weight gain of the breeds examined as a function of the number feed units and marginal and average feed use of the Ayrshire breed Other variables are fixed at mean level.

Table 4. Live weight gain estimates and average and marginal feed use given by the model as a function of feed unit quantity. Other variables are fixed at mean level of Ay-data.

Number of feed units	Ayrshire			Friesian-Ayrshire			Charolais-Ayrshire		
	Gain	f.u./kg gain		Gain	f.u./kg gain		Gain	f.u./kg gain	
	kg	Aver.	Marg.	kg	Aver.	Marg.	kg	Aver.	Marg.
100	42	2.4	2.8	35	2.8	3.1	37	2.7	2.9
300	105	2.9	3.6	96	3.1	3.5	100	3.0	3.4
500	157	3.2	4.2	149	3.4	3.9	156	3.2	3.8
750	212	3.5	5.0	209	3.6	4.4	219	3.4	4.2
1 000	258	3.9	5.9	262	3.8	5.0	275	3.6	4.7
1 250	297	4.2	7.0	310	4.0	5.6	325	3.9	5.3
1 500	330	4.6	8.2	352	4.3	6.3	370	4.1	6.0
1 750	358	4.9	9.7	390	4.5	7.0	409	4.3	6.7
2 000	381	5.2	11.7	423	4.7	7.9	444	4.5	7.6
2 250	401	5.6	14.1	—	—	—	—	—	—

Comparing the animal groups one can see that the feed utilization of ChAy cross-breeds is obviously better than that of FrAy. Excluding the earliest rearing phases, the Ayrshire has the lowest rate of feed utilization.

An increase in the protein content of the feed shows, up to a certain limit, an increasing productivity of feed unit, as is seen in Figure 4.

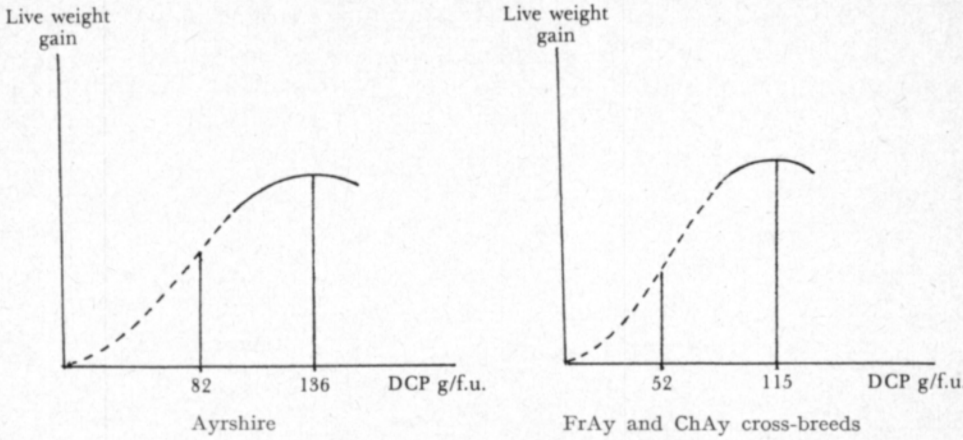

Figure 4. Live weight gain as a function of digestible crude protein content of feed unit. Other variables are fixed at mean level. The dotted line shows the area outside the data observations.

The course of the function has a form similar to traditional agricultural production functions. Thus, increasing the protein content, the productivity of the feed unit increases at first at an increasing rate; after the inflexion point at a decreasing rate; and after reaching the maximum, it actually decreases.

The Ayrshire has a maximum at the level of 136 and the cross-breeds at 115 g DCP/f.u. These levels can be seen as the technical optimum of the digestible crude protein content of energy.

In Table 5 the productivity of digestible crude protein at two different energy levels is presented. The levels correspond with the average and highest observations in Ay-data.

Table 5. Live weight gain, and average and marginal productivity of feed unit at two fixed energy levels (average and highest observation in Ayrshire-data).

Protein content of feed DCP g/f.u.	Input level of 793 f.u.			Input level of 2 200 f.u.		
	Gain kg	Average productivity g/f.u.	Marginal productivity g/f.u.	Gain kg	Average productivity g/f.u.	Marginal productivity g/f.u.
110	184	236	165	350	159	65
115	197	248	174	368	167	68
120	204	257	180	382	174	71
125	210	265	185	392	178	73
130	213	268	188	398	181	74
135	214	271	189	401	182	74
140	214	269	188	399	181	74

One can see quite a high increase in the weight gain and feed productivity when the protein content of the feed unit is increased up to 135 g DCP/f.u. Substituting, e.g., feed of 120 g DCP for that of 135 g DCP the average productivity of one feed unit increases with 14 g at an input level of 793 f.u. and with 8 g at a level of 2 200 f.u. Respective increases in total live weight gains are 10 and 19 kg.

To show the importance of protein in beef rearing, the value of weight gain increase can be compared with cost increase resulting from more protein containing feed. The following figures show how much one feed unit can increase in price in relation to the value of weight gain increase when a feed unit of 120 g DCP/f.u. is substituted for that of 135 g DCP/f.u.

	Input level, f.u.	
	793	2 200
Live weight gain with 120 g DCP/f.u., kg	204	382
» » » » 135 » » »	214	401
Increase in live weight gain »	10	19
Estim. increase in carcass weight gain »	5	9,5
Value of weight gain increase		
at beef price of 13 Fmk/kg, p/f.u.	8,2	5,6
» » » » 14 » »	8,8	6,1
» » » » 15 » »	9,5	6,5

Protein content has an influence on the intake capacity as well. Estimates derived from intake capacity functions indicate an increase in the time needed to consume a given dry matter quantity up to the maximum; first at increasing and after the inflexion point at decreasing rates as shown in Figure 5. The maximums are, for the Ayrshire, at a level of 126, and for cross-breeds, at 86 g DCP/f.u. Conversely expressed, the daily dry matter intake at first decreases and, after the minimum, increases with an increasing protein content of the feed.

The conclusion is reached that the animal needs more feed of a low nutritive value than feed with high protein to statisfy its nutritional needs. Beyond the maximum (minimum daily intake), given by the function, the time needed to consume a given dry matter quantity decreases, thus indicating an increase in the feed intake. This is probably due to the fact that the data used in this study consisted of experiments with plenty of silage and grass feeds which, if harvested young, have more taste and a higher protein content.

However, as is seen in Table 6, variations in the protein content have only a slight influence on the daily dry matter intake.

An essential difference between the Ayrshire and the two cross-breeds may be observed where protein is concerned. Both maximum feed productivity and minimum daily dry matter intake are located at the protein levels which in the Ayrshire are higher than with the cross-breeds examined. The explanation may lie in the better protein utilization of the cross-breeds in comparison to the Ayrshire. This cannot be proved, however, because of the lack of experimental data and research results. One of the reasons might be, as seen in Table 4 and Figure 2, that the cross-breeds need more energy at the

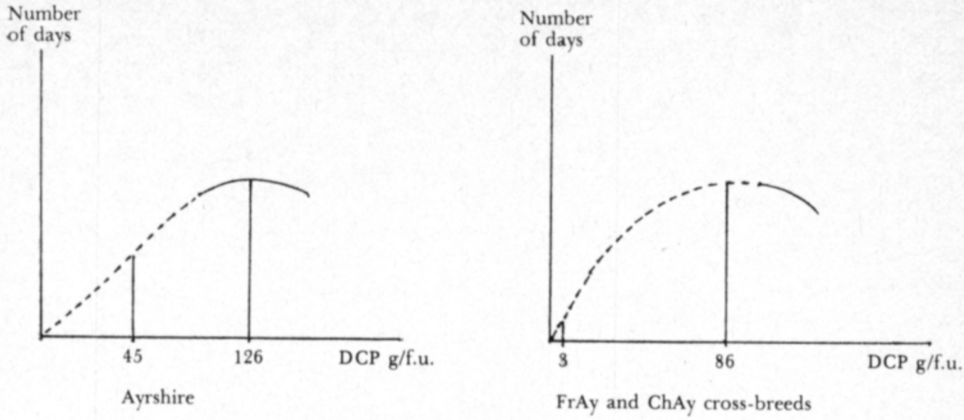

Figure 5. Time needed to intake a given quantity of dry matter as a function of protein content of feed. Other variables are fixed at mean level. The dotted line shows the area outside the data observations.

Table 6. Average and marginal daily dry matter intake as a function of protein content of feed at two dry matter levels (average and highest observation in Ayrshire-data).

Protein content of feed DCP g/f.u.	Daily dry matter intake, kg			
	843 kg of DM		2 400 kg of DM	
	Average	Marginal	Average	Marginal
110	3.36	5.55	5.08	8.38
115	3.32	5.49	5.02	8.28
120	3.30	5.43	4.98	8.22
126	3.29	5.43	4.97	8.20
130	3.30	5.43	4.98	8.21
135	3.31	5.46	5.00	8.25
140	3.34	5.49	5.04	8.31
143	3.36	5.52	5.07	8.36

beginning of rearing than the Ayrshires. Thus, their need for protein can be satisfied by a relatively lower digestible crude protein content of the feed than that of the Ayrshire. In addition, differences between the Ay- and cross-breed data may explain a phenomenon of this kind.

2. Crude fibre content of the dry matter

Increase in the crude fibre content of dry matter affects negatively the daily dry matter intake. This is shown by Figure 6 in which the time needed to intake a given dry matter quantity as a function of the crude fibre percentage, is presented.

According to the estimates of the model, the time which the Ayrshire needs to consume a given dry matter quantity is at its shortest (daily dry matter intake is at its greatest) when the crude fibre content is 5.2 per cent of the dry matter. Increases in the crude fibre content decrease the daily dry matter intake. Below the 5.2 per cent level, the daily dry matter intake

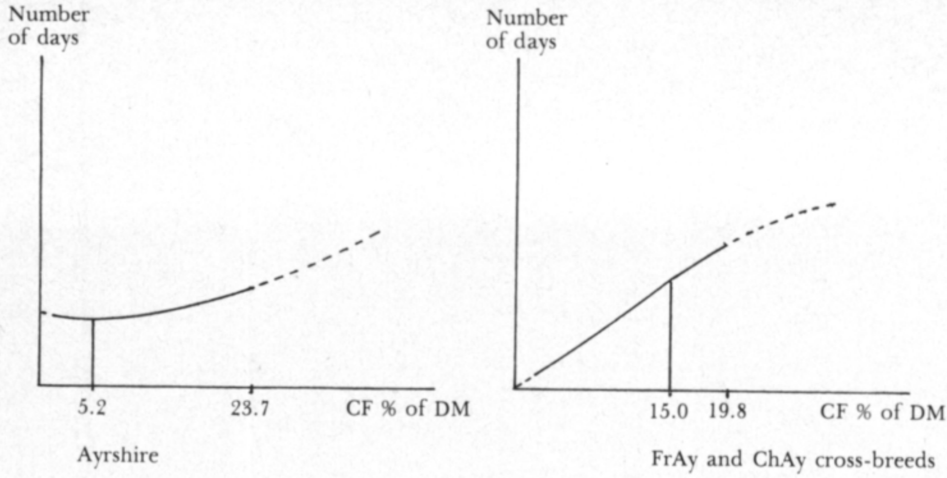

Figure 6. Time needed to intake a given quantity of dry matter as a function of crude fibre content of feed. Other variables are fixed at mean level. The dotted line shows the area outside the data observations.

decreases as well. However, only some feed compositions, such as those of animal origin, or of very concentrated feeds, can attain so low a crude fibre content.

As for the cross-breeds, differences in the course of the function appear. A maximum daily intake of feed for crude fibre percentage does not exist. Instead, the time needed to intake a given dry matter quantity increases at an increasing rate, and after 15 per cent at a decreasing rate. Compared with the Ayrshire the curve is similar between 5.2—15 per cent of crude fibre in dry matter.

The time needed to intake two different quantities of dry matter, depending on the crude fibre content, is shown in Figure 7 a. Other variables are fixed at their mean level. In Figure 7 b, marginal and average daily dry matter intakes are presented. E.g., intake of 843 kg dry matter requires 237 to 275 days when the crude fibre content varies from 10 to 20 per cent of total dry matter. The corresponding average daily intake is 3.55 and 3.07 kg DM. A dry matter quantity of 2 400 kg, including 10 or 20 per cent of crude fibre, takes 447 and 518 days, respectively, yielding corresponding average daily intake of 5.37 and 4.63 kg DM.

As stated above, a high crude fibre in dry matter tends to slow down the feed intake. Thus, depending on the nutritive value of the feed, an excess of crude fibre can act as a growth limiting factor.

3. Energy content of feed

On the assumption that the energy content of the feed (f.u./100 kg DM) has an influence on the feed intake, it was included as an explanatory variable in the intake capacity function of the model. An increase in the energy content was expected to decrease the dry matter intake because an animal's need for energy will be satisfied by a smaller dry matter quantity of highly concentrated feed than of lower concentrated feed.

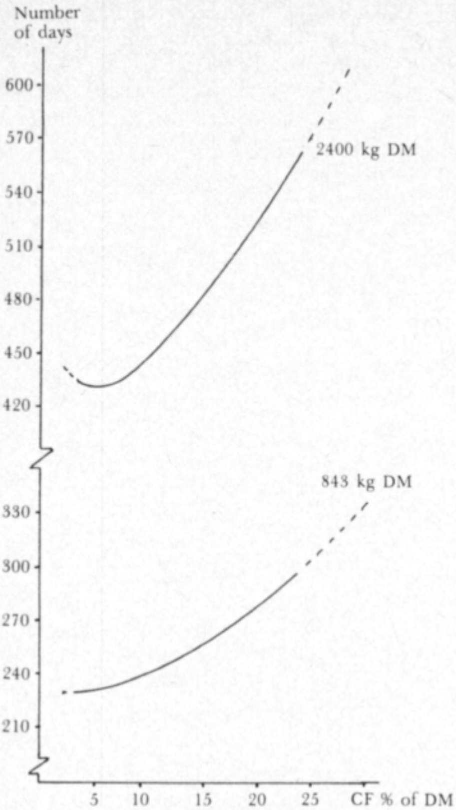

Figure 7 a. Time needed to intake 843 kg and 2 400 kg of dry matter as a function of crude fibre content, Ayrshire breed. Other variables are fixed at mean level.

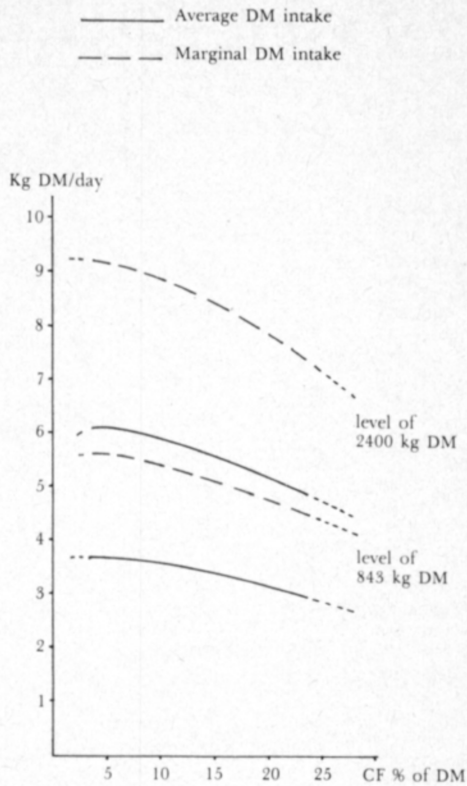

Figure 7 b. Marginal and average daily dry matter intake of dry matter quantities in figure 7 a.

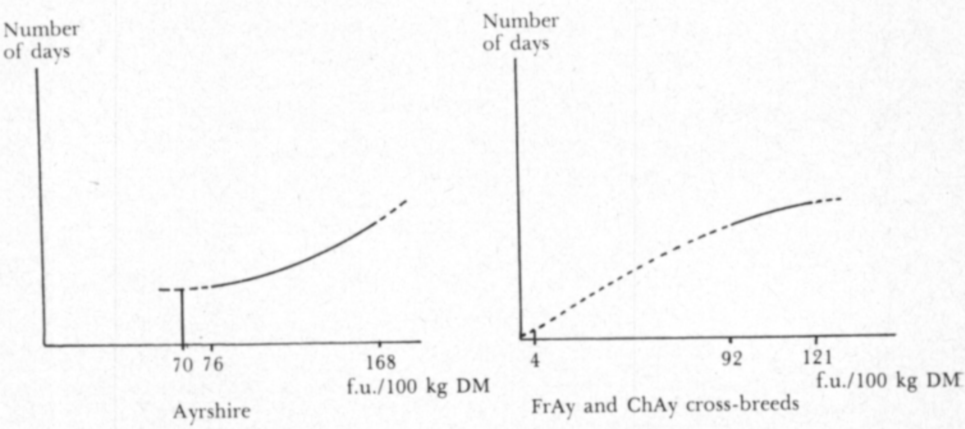

Figure 8. Time needed to intake a given quantity of dry matter as a function of energy content of feed. Other variables are fixed at mean level. The dotted line shows the area outside the data observations.

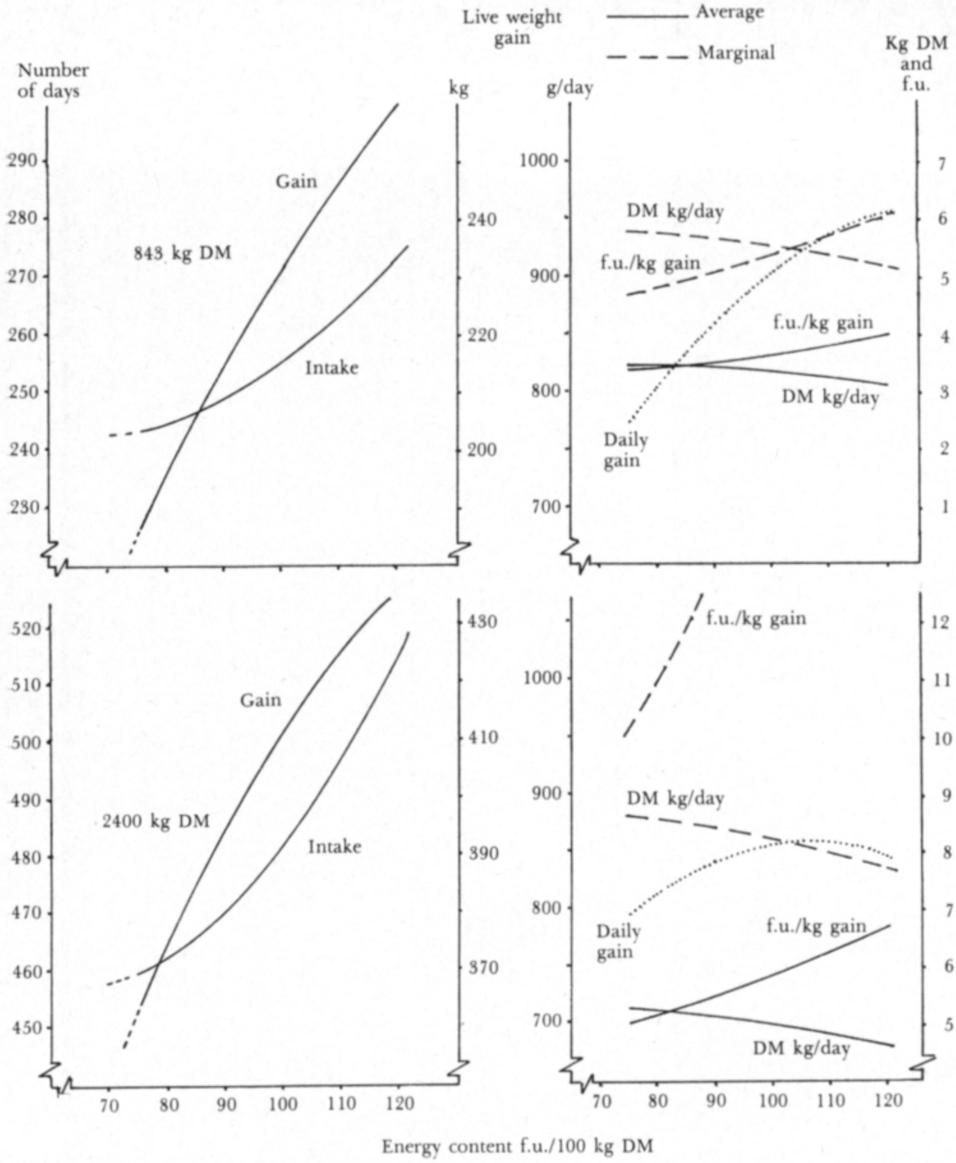

Figure 9. Time needed to intake 843 kg and 2 400 kg of dry matter and live weight gain curves (left side figures), and average daily gain, average and marginal dry matter intake and energy need (right side figures) as a function of energy content of feed, Ayrshire breed. Other variables are fixed at mean level.

As seen in Figure 8, Ay-estimates show an increase in the time needed to intake a given dry matter quantity when the energy content exceeds 70 f.u./ 100 kg DM., which also represents the minimum time for the intake of given dry matter quantity. This means that the decrease in the daily dry matter intake is caused by the increase in the energy content of feed. With the cross-breeds, some differences in the course of the function exist. However, the variation range of the energy content of feed in the cross-breed data was too narrow for definite conclusions to be drawn, as may be seen in the figure.

Energy content can be regarded as one of the most important factors of feed. This fact is shown by Figure 9, where the two upper figures indicate the feed input with a dry matter of 843 kg (average in Ay-data) with an energy content ranging from 70 to 120 f.u./100 kg DM. Thus, different quantities of energy are included in the feed with a constant dry matter quantity. The figure on the left shows the time needed to consume the feed and the resulting live weight gain, while the average and marginal daily dry matter intakes and energy needs (f.u./kg gain) are presented on the right. Here, also the average daily live weight gain, calculated from the model, is presented. When comparing the feed inputs with varying energy contents, increases in daily gain at decreasing rates are to be found, even at the level of 120 f.u./100 kg DM (1012 f.u.).

The two figures below present a similar case in which 2 400 kg dry matter is included in the feed composition. The figure on the right shows a very rapid increase in the energy needs at higher energy content levels. This is due to the fact that the animal has nearly exhausted its growth ability. The daily gain curve reaches the maximum here, at the energy content level of 105 f.u./100 kg DM or at 2 520 f.u. in total.

It is evident that by changing the energy content grade of a fixed dry matter quantity, different daily gains are obtained. High gains, of course shorten the rearing time, thus reducing fixed production costs. However, extra costs usually result from a more concentrated feed which cancels out the increase in the net return. That is why the maximum daily gain, often defined as a technical optimum of feeding, does not necessarily correspond with the highest economic growth rate or relatioship between feed and gain.

V Adaptation of the model in the economic optimization of production

A. *Aim in beef production and appraising the economic result*

In an economic activity production resources are used in a way which should result in as great a difference between return and costs as possible. In a static production process this aim is achieved by profit maximization or by cost minimization.

In agriculture, the aim is valid on the farm level as a whole, and in reference to different lines of production. Thus, both return and costs, and the physical relationship between inputs and output are of crucial importance when aiming at the best economic result.

Return in beef production depends on the gain and the price obtained. In practice, at least, differentiation according to the carcass quality and weight is practiced, which makes specifying the exact return difficult. However, since costs are the item of greatest importance in the economic analysis of beef production they will be analyzed below in greater detail.

According to the terminology used, for example, in the Scandinavian countries (Anon. 1967), *the production cost* is determined as the total cost of production, thus including ordinary production expenses or operating costs,

costs of labour and interest claimed on total capital. The production costs of beef can be grouped, e.g., as follows:

1. cost of calf
2. cost of feed
3. cost of other supplies (electricity, fuel, etc.)
4. cost of capital (depreciation and repair of buildings and machines, insurance, and interest claimed on capital invested)
5. cost of labour.

From the different concepts of the business result of agriculture, farm family income and profits (net and gross) are examined below.

The amount of gross return remaining after operating costs have been deducted is called *the farm family income*, or the value of the farm family labour plus interest on total capital invested. It is often used as a business result for the whole farm but very seldom for separate lines of production because of the difficulties in separating capital and labour costs from different farm activities.

Net profit is what remains when all the production costs are deducted from the gross return. A positive net profit means a higher labour income or interest on capital than was presupposed. In the opposite case, the farmer has to be content with a lower salary or lower interest on capital investments used in production. Evaluation of the labour input and its value is liable to be erroneous. In addition, the net profit is of little value when measured in monetary units. For these reasons, as stated by MÄKI (1964, p. 76), the errors in, e.g., labour use evaluation, can strongly affect the result. As a business result of beef production, however, net profit can be seen as correct, because statistics on the use of labour are available.

Comparisons between the profitability of different lines of production are often made according to the *gross profit*. This is reached when variable costs are deducted from the gross return. According to MELLEROWITCZ (1950, p. 23—24), MÄKI (1964, p. 272) and RYYNÄNEN and PÖLKKI (1973) costs that depend on production levels are variable. Thus, gross profit is the remainder of gross returns after the subtracting of the fixed costs of production, of which interest on capital, depreciation and repairs are primary items.

An exact definition of gross return cannot be found. This is due to the problem of definition of fixed and variable costs. Whether the cost item is fixed or variable depends on the problem in question. Examining beef production as a dynamic process yields the information that some (apparently) fixed costs are variable in regard to time. Capital costs, for example are fixed in a static production process but variable in a dynamic one. Moreover, they have a linear correlation with time, at least in the short run. Approximately the same applies to the costs of labour and of other supplies for production. In view of the fact that the optimization of beef production can be examined from several standpoints, as will be shown in the following chapter (B), there are arguments which evaluate the economic result according to the net profit so that all costs are taken into account. If only variable costs are deducted, gross profit on labour and capital costs is the result.

B. Optimization criterion

When maximum net or gross profit, or some other business oriented result is chosen to show the results of animal rearing, the following three standpoints may be taken into consideration:

1. maximal profit per animal
2. maximal » » gain unit
3. maximal » » time unit of rearing.

In this chapter adaptations of the model according to the criterion above are examined.

Supposing that daily costs of labour, capital and other supplies (except feed) are fixed; and with:

NPa, NPkg, NPd = net profit per animal, weight gain unit or day
GPa, GPkg, GPd = gross profit per animal, weight gain unit or day
Y = live weight gain, kg
T = time used in production, days
X = feed input, f.u.
py, px = prices of beef (per live weight) and feed (per f.u.)
p_{calf} = price of calf
p_{oc} = other costs (daily costs of labour, capital and other supplies)

equations for profits can be stated. Net and gross profits, and their maximization, are presented in the following.

1. Profit maximization per animal

Profits are written as follows:

Net profit:

(1. 1.) $NPa = pyY - (pxX + p_{calf} + Tp_{oc})$

Gross profit:

(1. 2.) $GPa = pyY - (pxX + p_{calf})$.

The first derivate of the function for X put = 0 presents the maximum[1]) or

(1. 3.) Max NPa: $py\frac{dY}{dX} - px - p_{oc}\frac{dT}{dX} = 0$

and

(1. 4.) Max GPa: $py\frac{dY}{dX} - px = 0$.

---

[1]) The function changes from positive to negative with the increase of X over Xo, which proves that the function has a maximum value at the level of Xo. (e.g. Kivikoski, 1968).

Substituting $\frac{dY}{dX}$ for marginal productivity of X (f.u.), estimated from the gain function and $\frac{dT}{dX}$[1] for marginal time needed to consume a given marginal quantity of feed, the equations can be written as follows:

$$\text{(1. 5.) Max NPa:} \quad py\left(\frac{b}{X} + \ln 10\, c\right)\left(10^{a}X^{b}10^{cX}\right) - px$$

$$-p_{oc}\left(\frac{e \cdot z}{X}\right)\left(10^{d}\left(\frac{X}{z}\right)^{e}\right) = 0$$

$$\text{(1. 6.) Max GPa:} \quad py\left(\frac{b}{X} + \ln 10\, c\right)\left(10^{a}X^{b}10^{cX}\right) - px = 0.$$

In the equations

a = constant term in the gain function resulting from the given feed input (energy not included)

b, c = logarithmic and linear regression coefficients of energy (FU) in the weight gain function

d = constant term in the intake capacity function resulting from the given feed input (dry matter not included)

e = logarithmic regression coefficient of dry matter (DM) in the intake capacity function

z = energy content of the given feed input.

Examining the equations, it can be seen that the term p_{calf} (cost of the calf) disappears when taking the derivate. As a fixed cost item it has no influence on the maximum. Moreover, the dynamic nature of production, or time, is taken into consideration only when net profit is maximized.

2. Profit maximization per weight gain unit

With a fixed beef price level, the maximum profit per gain unit produced, is reached by minimizing the unit costs. BLOHM (1966), e.g., prefers maximum profit per gain unit to that per animal. This is valid when the vantage point is static.

Profit can be stated as follows:

$$\text{(2. 1.) Net profit:} \quad NPkg = py - \left(\frac{pxX}{Y} + \frac{p_{calf}}{Y} + \frac{Tp_{oc}}{Y}\right)$$

$$\text{(2. 2.) Gross profit:} \quad GPkg = py - \left(\frac{pxX}{Y} + \frac{p_{calf}}{Y}\right).$$

---

[1]) The marginal productivity of dry matter, estimated from the intake capacity function must be substituted for the marginal productivity of a feed unit. This can be done when the energy content of feed is known as is assumed by the model.

Deriving the cost function in brackets for X, and setting the derivate = 0, the unit cost minimum is obtained. Making C = cost function we may write:

$$(2.3.)\quad \frac{dC_{NPkg}}{dX}: \quad \frac{dY}{dX}\left(pxX + p_{calf} + Tp_{oc}\right) - Y\left[px + p_{oc}\left(\frac{dT}{dX}\right)\right] = 0$$

and

$$(2.4.)\quad \frac{dC_{GPkg}}{dX}: \quad \frac{dY}{dX}\left(pxX + p_{calf}\right) - px = 0.$$

Substituting Y and T for the weight gain and intake capacity functions of the model, the cost minimum per kilogram of live weight gain can be written:

$$(2.5.)\quad MinC_{NPkg:}{}^{1)} \quad \left(\frac{b}{X} + \ln 10\, c\right)\left[pxX + p_{calf} + p_{oc}\left(10^{d}\left(\frac{X}{z}\right)^{e}\right)\right]$$
$$-p_{oc}\left[\left(\frac{e \cdot z}{X}\right)\left(10^{d}\left(\frac{X}{z}\right)^{e}\right)\right] - px = 0$$

$$(2.6.)\quad MinC_{GPkg:}{}^{1)} \quad \left(\frac{b}{X} + \ln 10\, c\right)\left(pxX + p_{calf}\right) - px = 0.$$

When comparing the cost minimums, or the net and gross profit maximum values with each other, as is seen in equation 2. 5., all cost items are included in the function as variable costs. Gross profit maximization (equation 2.6.) only takes the costs of feed and the calf into consideration, thus omitting the time factor and signifying a static approach.

3. Profit maximization per day of rearing

Beef production on farms is normally an activity of a continuing nature. Because the number of animals on farms is limited by production resources, such as the size of buildings, land area, labour force etc. it is important for the farmer to specify the length of the animal's rearing period which produces the maximal yearly profit per farm. After this, the farmer can decide the optimal circulation of animals per animal place.

The problem can be solved by choosing the rearing time of every individual animal so that the average daily profit during the whole rearing period is of maximum value. The profits can now be stated as follows:

Net profit:

$$(3.1.)\quad NP_d = \frac{pyY}{T} - \left(\frac{pxX + p_{calf}}{T} + p_{oc}\right)$$

Gross profit:

$$(3.2.)\quad GP_d = \frac{pyY}{T} - \left(\frac{pxX + p_{calf}}{T}\right).$$

---

[1]) The function changes from negative to positive when increasing X over Xo. Thus, the function has the minimum value on the input level of Xo.

When deriving the functions for X and setting it = 0, the value of X, or the number of feed units which yields the maximum daily average profit can be discovered. In the short run, cost items included in p_{oc} are supposed to be fixed. Thus, the derivate of both net and gross profit is as follows:

$$(3.3.)\quad \frac{dNP_d}{dX} \text{ and } \frac{dGP_d}{dX} = \frac{dT}{dX}\left(pxX + p_{calf} - pyY\right) - T\left(px - py\frac{dY}{dX}\right).$$

Substituting Y and T in (3.3.) for respective functions of the model and setting the derivate = 0, maximum daily profit is obtained from function 3.4.

Max NP_d and GP_d:

$$(3.4.)\quad \left(\frac{e \cdot z}{X}\right)\left[pxX + p_{calf} - py\left(10^a X^b 10^{cX}\right)\right] - px$$

$$+ py\left(\frac{b}{X} + \ln 10\, c\right)\left(10^a X^b 10^{cX}\right) = 0.$$

Using the value of X, the model estimates and the feed ingredients, estimates for the live weight gain and the length of the rearing period can de calculated. With the daily costs of labour, capital and supplies, other than feed, fixed, the optimum is determined only by the prices of beef, feed and the calf. The value of X is independent of the concept of the business result used, because the time factor, or dynamics of production, is included in the analysis.

4. An example for evaluation of the optimization criterion

In this chapter, application of the model for optimization purposes will be dealt with mainly to evaluate the different optimization criteria discussed above. Therefore, a practical example will be presented, in which a calf is reared up to the stage which gives the maximum net and gross profit — a) per animal, b) per live weight unit produced and c) per day used in rearing.

For the calculation, it is necessary to make some assumptions about the animal, the composition of the feed and the prices of inputs and output.

The animal is assumed to be an Ayrshire bull calf, weighing 47.9 kilograms at the age of 30 days, which corresponds to the average of the empirical data used in the model estimation. In addition, data averages are used for feed, which means that the average energy content will be 0.94 f.u./100 kg DM, the protein content of 143 g DCP/f.u. and the crude fibre content 14.3 % of DM. Because of its essential importance in the growth process, the feed composition will be analyzed in greater detail later in this chapter (V C 2).

The feed price is appraised at 0.85 Fmk/f.u. and other costs such as those of labour, capital and other supplies total 1.50 Fmk/day. The calf is purchased at the price of 400 Fmk.

The price of the beef is based on the actual price of bull beef attainable by the producer. Since 1962, a target price system for beef has been in existence. According to the Agricultural Price Acts, an average target price is confirmed for all beef. Prices for different beef, like those of calves, cows, heifers and bulls are then derived from the average target price according to their share of the total supply. The average beef target price used in the following is 13.65 Fmk/kg and for bulls 14.30 Fmk/kg. Assuming the carcass weight is 50 per cent of the live weight, the price can be converted to 7.15 Fmk/live weight kg.

However, beef prices vary during the course of the rearing primarily for two reasons. Firstly, a price differentiation is made in slaughterhouses according to the quality of beef, and secondly, since 1974, there has been a state subsidy (at the moment 130 p/kg) for beef exceeding a carcass weight of 160 kg. With this incentive to producers it is hoped to increase the average slaughtering weight of beef animals, and to stimulate beef production in a situation where the number of calves is decreasing due to the decreasing number of milk cows. Thus, the return from the rearing is not necessarily based on a linear dependency of the weight gain.

The quality of the beef could not be quantified as a dependent variable in the estimated model for reasons stated earlier, and only state intervention in beef production is taken into account in the following example. That means graded beef pricing, fixed at the level of 14.30 Fmk/kg for animals under 160 kg carcass weight and 15.60 Fmk/kg for animals over 160 kg. Converted to live weight, the prices would be 7.15 Fmk/kg and 7.80 Fmk/kg, respectively. Thus, all the prices used in the example are as follows:

beef py_1 = 7.15 Fmk/kg live weight, under 160 kg of carcass weight
(14.30 » carcass »)
py_2 = 7.80 Fmk/kg live weight, over 160 kg of carcass weight
(15.60 » carcass »)
feed px = 0.85 Fmk/f.u.
calf p_{calf} = 400 Fmk/pcs
other
costs p_{oc} = 1.50 Fmk/day.

When applying the estimated model, constant term values of the gain and intake functions must first be calculated. They become available after plugging the values of the known variables into the functions of the model. Furthermore, by putting the constant terms calculated, and the regression coefficient estimates of the model into the functions 1. 5., 1. 6., 2. 5., 2. 6. and 3. 4. (which define maximum net or gross profits), the functions presented below are obtained. Using the prices and costs of the example, the value of X, or the number of feed units can be solved. The live weight gain of the animal is now obtained by putting the value of X into the weight gain function of the model. The rearing time can then be estimated from the intake capacity function of the model according to the total dry matter quantity of the feed input, expressed by the average energy content of the feed ration.

Maximal profits per animal:

Maximal net profit, NPa

$$(4.\ 1.)\quad py\left(\frac{0.8646}{X}-0.0002\right)\left(10^{-0.0927}X^{0.8646}10^{-0.0001X}\right)-px$$

$$-p_{oc}\left(\frac{0.6063\cdot 0.94}{X}\right)\left(10^{0.6257}\left(\frac{X}{0.94}\right)^{0.6063}\right)=0$$

Maximal gross profit, GPa

$$\text{(4. 2.)} \quad \mathrm{py}\left(\frac{0.8646}{X} - 0.0002\right)\left(10^{-0.0927}X^{0.8646}10^{-0.0001X}\right) - \mathrm{px} = 0.$$

Maximal profits per live weight kg produced:

Maximal net profit, NPkg

$$\text{(4. 3.)} \quad \left(\frac{0.8646}{X} - 0.0002\right)\left[\mathrm{px}X + \mathrm{p_{calf}} + \mathrm{p_{oc}}\left(10^{0.6257}\left(\frac{X}{0.94}\right)^{0.6063}\right)\right]$$
$$-\mathrm{p_{oc}}\left[\left(\frac{0.6063 \cdot 0.94}{X}\right)\left(10^{0.6257}\left(\frac{X}{0.94}\right)^{0.6063}\right)\right] - \mathrm{px} = 0$$

Maximal gross profit, GPkg

$$\text{(4. 4.)} \quad \left(\frac{0.8646}{X} - 0.0002\right)\left(\mathrm{px}X + \mathrm{p_{calf}}\right) - \mathrm{px} = 0.$$

Maximal profits per day of rearing:

Maximal net and gross profit, NPd and GPd

$$\text{(4. 5.)} \quad \left(\frac{0.6063 \cdot 0.94}{X}\right)\left[\mathrm{px}X + \mathrm{p_{calf}} - \mathrm{py}\left(10^{-0.0927}X^{0.8646}10^{-0.0001X}\right)\right]$$
$$-\mathrm{px} + \mathrm{py}\left(\frac{0.8646}{X} - 0.0002\right)\left(10^{-0.0927}X^{0.8646}10^{-0.0001X}\right) = 0.$$

The optimum, expressed by the maximum profits, according to the different optimization criteria, will be analyzed separately below.

a. Maximum profit per animal

A step in the producer's price, at the level of 160 kg of carcass weight, represents a point of discontinuity on the return curve. Assuming the carcass weight of the calf as 15 kg, or as 50 % of the live weight, the net weight gain up to 160 kg would be 145 kg of carcass or 290 kg of live weight. Using the feed composition presented earlier, the gain of 145 kg can be attained with 1204 f.u. (Figure 10).

By using the two producer prices maximum net profits (NPa) are derived from the function 4. 1. at the levels of 1 170 f.u. and 1 320 f.u. They are indicated by A_1 and A in Figure 10. Net profit at the level of 1 170 f.u. is 166.40 Fmk/animal and at the level of 1 320 f.u. 356.48 Fmk/animal. Thus, the optimum, assessed according to net profit, is attained at 1 320 f.u.

Concerning gross profits (GPa) at the two producer price levels, only one solution is valid. Maximum gross profit 906.26 Fmk/animal results from function 4. 2. at input level of 1 686 f.u., which is shown by B in Figure 10.

b. Maximum profit per weight gain unit

With fixed prices for beef, the maximum profit per gain unit may be attained by cost minimization. This is seen in Figure 11. There is the step in the producer's price at the level mentioned earlier (1 204 f.u.). The step, however, has no effect on the minimum cost.

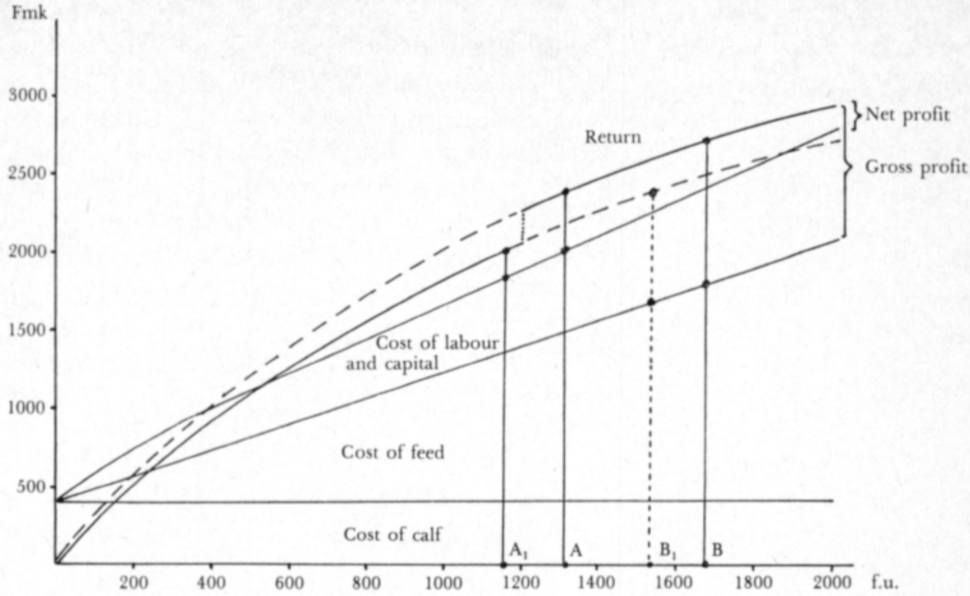

Figure 10. Return, costs and profits as a function of feed unit quantity, Fmk/animal.

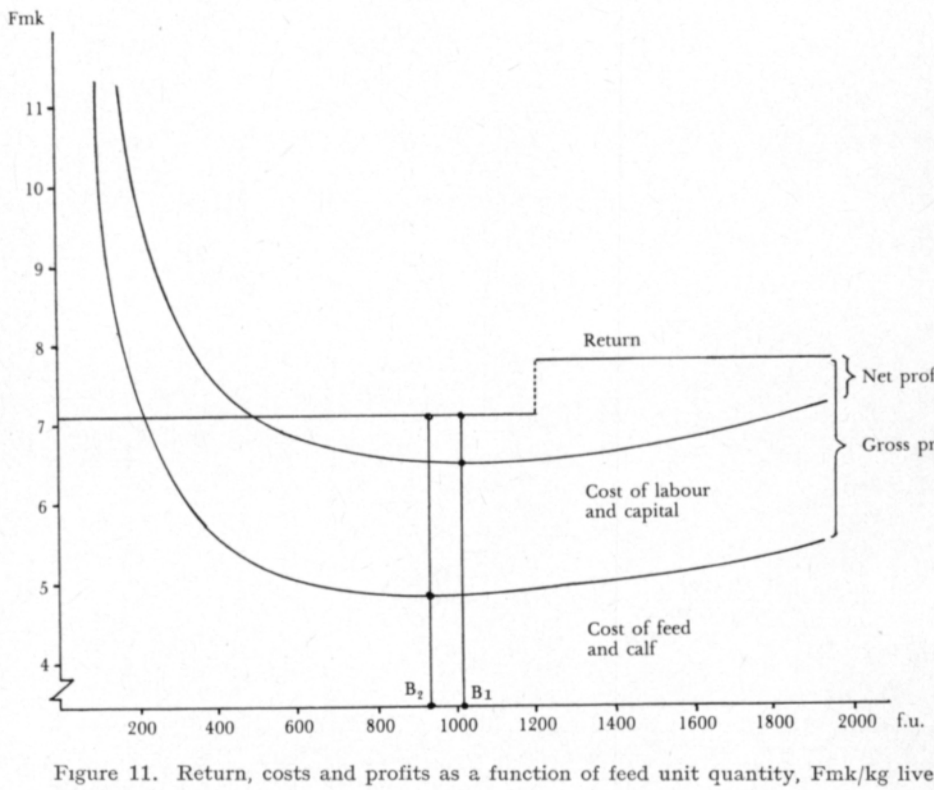

Figure 11. Return, costs and profits as a function of feed unit quantity, Fmk/kg live weight gain.

When solved from the functions 4. 3. and 4. 4. the following cost minimums are found (per carcass weight): production cost 13.07 Fmk/kg at the level of 1 016 f.u. (B_1) and variable cost of 9.69 Fmk/kg at 948 f.u. (B_2). Thus, profits per kg gain are $NP_{kg} = 1.23$ Fmk and $GP_{kg} = 4.61$ Fmk. They are attained below the weight limit of 160 kg or at the lower producer price. The costs increase after the minimum at an increasing rate which means a decrease in profits. Therefore, only increases in beef prices for heavier animals might change the optimum. A price change occurs at the input level of 1 204 f.u., leading, in the example, to net profit of 2.53 Fmk/kg and a gross profit of 5.91 Fmk/kg. Thus the weight limit of 160 kg represents the optimum as measured by the minimum unit cost.

c. Maximum profit per day of rearing

Solving the function 4. 5. the value of 1 037 feed units is obtained for X, which gives the maximum net and gross profits per day during the entire rearing period.

Figure 12 presents the average daily return and costs as a function of the feed unit quantity. The daily return tends to increase at the beginning of the rearing, caused by the increase in the growth ability of the animal. After an

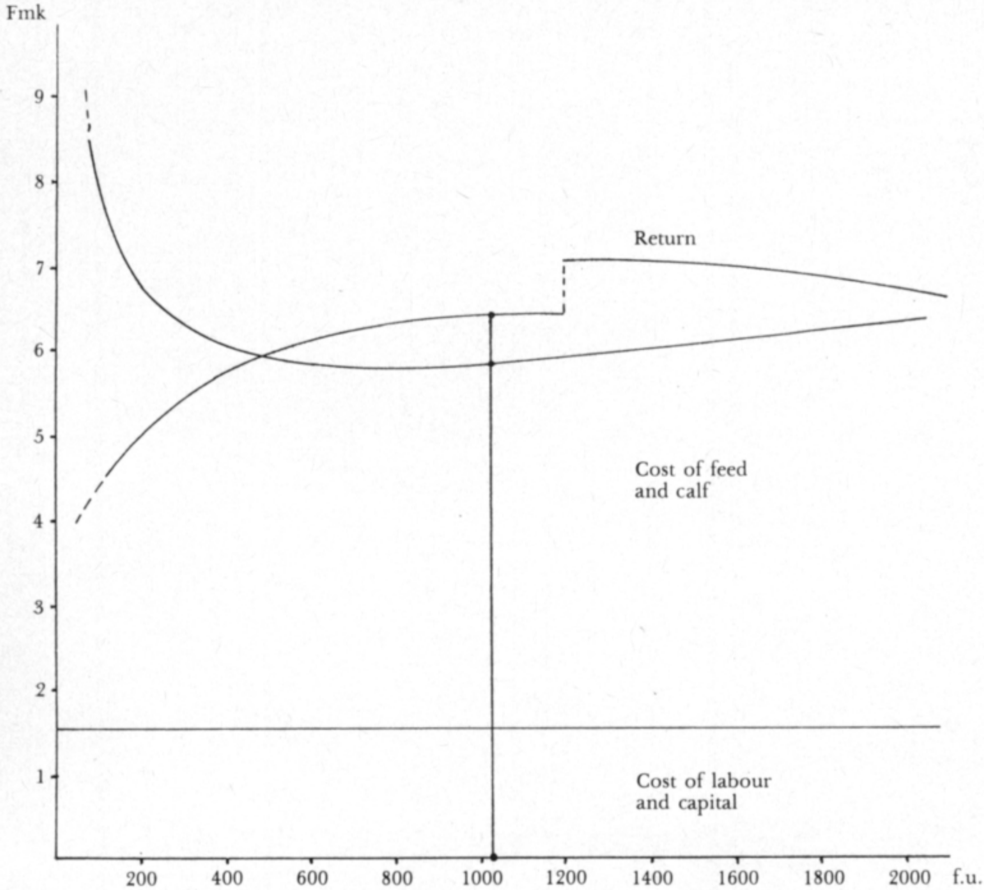

Figure 12. Return and cost as a function of feed unit quantity, Fmk/day of rearing.

input level of about 1 200 f.u., it starts to decrease. At the level of 1 204 f.u. there is a point of discontinuity on the return curve, as stated earlier.

Regarding costs, those of labour and capital are supposed to be fixed per day in the short run. Thus, the course of the production cost curve is determined solely by the costs of the calf and the feed.

Maximum profits, net = 0.55 Fmk/day and gross = 2.05 Fmk/day are obtained at the level of 1 037 feed units. To find the optimum, the effect of the change in the price of beef at the weight of 160 kg must be taken into consideration. When the return curve decreases and the cost curve increases, a decrease in profits is a natural result. Thus, profits at the weight level of 160 kg, or at the level of 1 204 feed units must be compared with those derived from the example. Assessed according to the average daily profits, rearing to the weight of 160 kg gives a net profit = 1.13 Fmk/day and a gross profit = 2.63 Fmk/day.

d. Comparison between the maximums derived from the example

In the present example, the number of the feed unit quantity was specified, giving the maximum profit of the rearing. Profit was maximized a) per animal under rearing, b) per kilogram weight gain and c) per day of the rearing. Moreover, the beef price differentiation was taken into account as a factor affecting the final optimum. Results from the example are summarized in Table 7.

Table 7. Maximal profits as derived from the example.

	Profit per animal		Profit per kg gain		Profit per day	
	Net	Gross	Net	Gross	Net	Gross
Number of feed units	1 320	1 686	1 204	1 204	1 204	1 204
Live weight gain, kg	306,6	351.2	290.1	290.1	290.1	290.1
Rearing time, days	342	396	323	323	323	323
Return, Fmk	2 391.48	2 739.36	7.80	7.80	7.01	7.01
Costs, Fmk	2 035.00	1 833.10	6.59	4.91	5.91	4.41
Profit, Fmk	356.48	906.26	1.22	2.89	1.10	2.60

The maximums can be compared with each other when converting the profits to refer, e.g., to a year and farm. The yearly profits, and the number of animals per place in the cow shed would then be as follows:

	Maximums		
	per animal	per kg gain	per day
Net profit Fmk/year	380.45	401.03	401.03
animals pcs/year/place	1.07	1.13	1.13
Gross profit Fmk/year	833.84	948.53	948.53
animals pcs/year/place	0.92	1.13	1.13

Thus, under the assumptions of the example, the best yearly profit is attained when rearing the calf to a carcass weight of 160 kg which means 1.13 calves per year and place.

5. Specification of the optimum in beef production

In the preceding chapter different optimization criteria and business results were discussed and an example was presented for application of the estimated model. It was found that the optimum is affected by both the criterion used and the business concept under maximization. The reasons for different optimums, measured by maximal profit, depend on

1) whether the standpoint in the analysis is static or dynamic,
2) which of the total costs are taken into account, and
3) which costs are seen as variable.

Relation between weight gain and time, or the growth rate, is of special importance in reference to the profitability of production. A dynamic approach is consequently required in the economic analysis of beef production. Already in the 1950's, Brown and Arscott (1958), among others, had paid attention to the dynamic nature of animal production as an integral part of the production function analysis when they developed the methods for optimization of broiler production.

As already stated, some cost items are integrated with time. If these costs are omitted, the approach tends to be of a static nature. When maximizing net profit, for example, when all costs are taken into account, the time factor is included in the analysis, as is apparent in the following:

Criterion	Business result under maximization	
	Net profit	Gross profit
per animal	dynamic	static
per kg gain	dynamic	static
per day of rearing	dynamic	dynamic

The validity of the approach likewise depends on the optimization criterion. As is apparent below, the character of production cost items varies when different criteria are used.

Production cost item	Optimization criterion		
	per animal	per kg gain	per day
Cost of calf[1])	fixed	variable	variable
Cost of feed[1])	variable	»	»
Cost of other requisites[1])	»	»	fixed
Cost of capital	»	»	»
Cost of labour	»	»	»

The optimum is determined only by variable costs. Thus, omitting some of the variable costs or adopting a static approach, leads to an erroneous optimum. This happens when gross profit per animal or per weight gain unit is used as the business result under maximization.

The conclusion may therefore be drawn that the maximum daily profit, net or gross, most completely specifies the optimum in beef production.

[1]) Cost items usually taken into account when gross profit is calculated.

C. *Application of the model*

1. Use of the model

The model estimated in the present study incorporates the principal activities needed for profit maximization in beef rearing. Ordinarily only facts on feeds available for production, animals and price levels of beef, feeds and other inputs are needed to find out the optimal length of rearing period and weight gain.

It has been explained earlier in this chapter why maximization per day of rearing leads to the highest net and gross profit results. The number of feed units, or the value of X, yielding the optimum levels, is produced by the equations 5. 1. and 5. 2.

Ayrshire

$$(5.1.)\qquad \left(\frac{0.6063 \cdot z}{X}\right)\left[pxX + p_{calf} - py\left(10^{a} \cdot X^{0.8646} \cdot 10^{-0.0001X}\right)\right] - px + py\left(\frac{0.8646}{X} - 0.0002\right)\left(10^{a} \cdot X^{0.8646} \cdot 10^{-0.0001X}\right) = 0$$

Friesian-Ayrshire and
Charolais-Ayrshire

$$(5.2.)\qquad \left(\frac{0.6177 \cdot z}{X}\right)\left[pxX + p_{calf} - py\left(10^{d} \cdot X^{0.9368} \cdot 10^{-0.00007X}\right)\right] - px + py\left(\frac{0.9368}{X} - 0.00017\right)\left(10^{d} \cdot X^{0.9368} \cdot 10^{-0.00007X}\right) = 0$$

In the equations px = price of feed unit in the feed composition
py = beef price per kg live weight
p_{calf} = price of calf
z = energy content of feed composition
a and d = constant term of weight gain function, which results when digestible crude protein content of feed unit and weight of animal are put into the function (a = Ayrshire, d = cross-breeds).

Putting the prices and values of z, a and d into the respective equations 5. 1. or 5. 2., the value of X can be solved.

In the second phase, the live weight gain and rearing time estimates can be calculated from the model. As observed in the example above (V B 4), possible producer price differentiation, according to the weight of the animal, must be taken into account.

2. On the feed composition as an economic factor

Among the different inputs, feed composition is of special importance. Even if the model estimated in the present study gives the optimum by any kind of useful feed composition, the question still remains as to how to choose the feed which maximizes the profit. In other words, how to find the optimum among all the possible optimal feeding alternatives.

The present study does not attempt to answer the question of how to compose the feed input in optimal accordance with its productivity and price. Some quidelines on the requirements of the feed, however, could be derived for maximization of its productivity. The feed having the maximal productivity — we can call it the technical optimum — does not necessarily correspond to the most economic feed, for the prices of feeds with high nutritive value are generally high. If the increase in return is lower than the cost addition of more productive feed, there is no point in aiming at the technical optimum mentioned above.

In practice, the most profitable feeding can be approached by first specifying the most productive feed composition with respect to the protein level, energy content, etc. The feeds available should then be combined to fulfil the requirements at minimum costs. The specification of the technical optimum seems to incorporate the greatest number of weaknesses, even if some trends on relationships between feed ingredients and gain or intake have been derived also here (see Chapter IV C). Feed cost minimization can be solved, e.g., by different mathematical methods.

Applications of linear programming methods have been common in solving feed mix problems. As early as 1951 WAUGH presented solution minimizing the cost of dairy feed by linear programming. Since then numerous studies (e.g. Mc ALEKSANDER and HUTTON, 1957, and RAHMAN and BENDER, 1971) have been carried out and different programming methods have been developed in several countries. At present, numerous computer programmes exist for solving the problems. In Finland linear programming for solving feed mix problems has been used by SUOMELA et al. (1961) and WECKMAN (1972), among others.

In the following a short account will be given of the method of minimizing the price of feed mix.

As stated earlier in this chapter, there are some requirements that feed should fulfil in order to assure maximum productivity. The requirements, even if difficult to specify, are set in the example for Ayrshire breed based on the results from the estimated model as follows:

- The protein content of the feed must be at least 126 g and at most 136 g DCP/f.u. According to the estimates derived, if the protein content of 126 g DCP/f.u. is exceeded the feed intake and with it the energy for production increases, up to the maximum productivity level of 136 g DCP/f.u.
- The crude fibre content should be as low as possible, at least 5.2 per cent of dry matter — anyway, the level which gives the maximum daily dry matter intake. Most home-produced feeds have a crude fibre content of more than 20 per cent.
- The energy content of feed affects the total energy in the feed taken in by the animal. Increase in the energy content, to a certain limit, produces an increase in the daily weight gain. According to the estimates the feed should contain at least 90 f.u./100 kg DM.

Here, the feed mix is made up only for an ordinary production period thus leaving out feeds like milk and skim milk needed during the first few weeks of the animal's life. Feeds available are presented in Table 8.

Table 8. Basic feeds available in the example.

Feeds		kg/f.u.	Price Fmk/f.u.	DCP g/f.u.	kg DM in 100 f.u.	Crude fibre percentage of DM	of f.u.
Barley	X_1	1.00	0.7609[1]	75.00	87.00	4.0	3.5
Oats	X_2	1.15	0.8034[1]	100.00	100.05	9.2	9.2
Silage	X_3	7.00	0.8705[2]	160.00	140.00	25.0	35.0
Hay	X_4	2.25	1.0508[2]	130.00	180.00	31.3	56.3
Purchased feed	X_5	1.10	1.5290[1]	160.00	99.00	12.2	12.1
Straw	X_6	3.60	0.1800[2]	50.00	306.00	41.2	126.0

[1]) Market price
[2]) Price is based on production costs

As stated above, for construction of the linear model the constraints for the feed mixture have to be specified. Here, they are set as follows:

1. f.u. quantity of the feed mix $= 100$
2. DCP content » » » » ≥ 130 g/f.u.
3. crude fibre content of the feed mix $\leq$ 20 % of DM
4. energy content » » » » $\geq$ 90 f.u./100 kg DM
   or bulkiness » » » » $\leq$ 111 kg DM/100 f.u.
5. barley and oats total $\geq$ 20 % but $\leq$ 40 % of the f.u. quantity

After converting the constraints to correspond with 100 feed units, the object function and constraint equalities may be stated as follows:

Object function

$$76.09\, x_1 + 80.34\, x_2 + 87.05\, x_3 + 105.08\, x_4 + 152.90\, x_5 + 18.00\, x_6 = \text{Min}$$

Constraints

1. f.u. quantity

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 100$$

2. DCP quantity

$$75.00\, x_1 + 100.00\, x_2 + 160.00\, x_3 + 130.00\, x_4 + 160.00\, x_5 + 50.00\, x_6 \geq 13\,000$$

3. Crude fibre quantity

$$3.50\, x_1 + 9.20\, x_2 + 35.00\, x_3 + 56.30\, x_4 + 12.10\, x_5 + 126.00\, x_6 \leq 2\,222^{1)}$$

4. Bulkiness

$$87.00\, x_1 + 100.05\, x_2 + 140.00\, x_3 + 180.00\, x_4 + 99.00\, x_5 + 306.00\, x_6 \leq 1\,111$$

5. Barley and oats

$$x_1 + x_2 \geq 20$$
$$x_1 + x_2 \leq 40$$

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[1]) 20 % in DM as converted per f.u. when energy content is 90 f.u./100 kg DM.

Table 9. Minimum cost feed mixture.

Feeds	Percentage of total f.u.	Price Fmk/f.u.	DCP g/f.u.	kg DM in 100 f.u.	Crude fibre percentage	
					of DM	of f.u.
Barley	24.00	0.7609	75.00	87.00	4.0	3.5
Oats	16.00	0.8034	100.00	100.05	9.2	9.2
Silage	36.20	0.8705	160.00	140.00	25.0	35.0
Purchased feed ...	23.80	1.5290	160.00	99.00	12.2	12.1
Total feed mixture	100.00	0.9905	130.00	111.13[1])	17.0	16.1

[1]) Corresponds to an energy content of 90 f.u./100 kg DM.

For solving the model, the linear programming method ILONA was applied (e.g. SEPPÄNEN, 1973), which gave as a result the feed mixture presented in Table 9.

The minimum cost feed combination applies to an ordinary production period. Thus, milk, milk powder and other feeds which are needed in the first phases of an animal's life have to be taken into consideration outside the model. Their quantities are naturally dependent on the age of the calf when bought to the farm. The cost of these feeds may also be included in the price of the calf, which means that the weight gain produced by these feeds should be handled as a separate item of output.

VI Summary

The aim of this study has been to create a model depicting the growth process of a beef animal to provide means for economic optimization of the slaughter weight and of the length of the rearing period.

The background for the model building has been the biological growth process and factors affecting it. The model consists of two functions, one explaining the weight gain and the other explaining the increase in the intake capacity of an animal. The main explanatory variables in the weight gain function were the total number of feed units and the protein content of feed. The intake capacity was estimated as a number of days needed to consume the total dry matter of a given quantity of feed using also protein, crude fibre and energy contents of the feed as explanatory variables. In this form the model is dynamic in nature taking the growth rate of an animal into consideration.

Dependencies of the model were estimated using data from beef rearing experiments conducted at the Agricultural Research Centre in the early 1970's. The model was estimated for bulls of three breeds; Ayrshire, Friesian-Ayrshire and Charolais-Ayrshire.

A function of transcendental type was selected to describe the weight gain. Also for intake capacity the function of transcendental form turned out to be suitable. R^2-values of the functions were high and the estimates were as a rule significant at high level.

Main points given by the regression coefficient estimates of the model can be summarized as follows:

— maximum live weight is attained at input level of **4 164** f.u. (Ayrshire) and **5 516** f.u. (cross-breeds)
— feed use (f.u./kg gain) is highest in Ayrshire, lower in Friesian-Ayrshire and lowest in Charolais-Ayrshire
— productivity of f.u. is at its highest when the energy content of the feed is **136** g DCP/f.u. (Ayrshire) and **115** g DCP/f.u. (cross-breeds)
— daily intake of dry matter is at its lowest when the protein content of the feed is **126** g DCP/f.u. (Ayrshire) and **86** g DCP/f.u. (cross-breeds)
— increases in the crude fibre content or in the energy content of the feed decrease the daily dry matter intake.

Adaptations of the model in the economic optimization were discussed with net and gross profits of the rearing as the business results under maximization. Equations were derived showing maximum profits a) per animal, b) per weight gain unit produced and c) per day of rearing. It was noted that the best economic result is obtained when the profit, net or gross, is maximized per day of rearing. The equation presented gives the number of feed units which maximizes the profit assuming that the other variables of the model and the prices of beef, feed and other inputs are known. Using the estimated functions of the model, the optimal rearing time and final weight of an animal can be decided. An example on adaptations of the model was presented.

The model gives the economic optimum with any kind of feed ration. The importance of the ration as an economic factor was discussed and an example was presented where, with linear programming a minimum cost ration with maximum productivity was attained.

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SELOSTUS

Naudanlihan tuotantoprosessia kuvaava taloudellinen malli tuotannon optimointia varten

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Naudanlihan tuottajalle olisi suureksi avuksi tieto siitä, minkä ikäisenä ja painoisena eläin olisi kannattavinta myydä. Paras tuloshan saavutetaan silloin, kun kasvatuksen antaman rahallisen tuoton ja tuotantoon käytetyistä panoksista aiheutuvan kustannuksen erotus on suurimmillaan.

Tuotannon kannattavuuteen ehkä ratkaisevimmin vaikuttaa se, että eläin kasvaessaan tarvitsee kutakin lisäkasvukiloa kohti yhä enemmän rehua. Siinä vaiheessa kun lisäkasvukilon hinta ei enää riitä peittämään niitä kustannuksia, jotka sen aikaansaamisesta aiheutuvat, on kasvatuksen jatkaminen kannattamatonta.

Tämän tutkimuksen tarkoituksena on ollut selvittää lähinnä rehun ja kasvun välistä suhdetta maassamme yleisimmin kasvatetuilla lihanaudoilla. Tavoitteena on ollut kehittää rehupanoksen ja lihatuotoksen välistä suhdettä kuvaava matemaattinen malli, joka voisi toimia sinänsä monimutkaisen kasvuprosessin mahdollisimman yksinkertaisena kuvaajana. Malli on pyritty kehittämään sellaiseksi, että sen avulla voidaan määrittää eläimen optimaalinen kasvatusajan pituus ja myyntipaino käytettäessä ruokintaan vapaavalintaista rehuyhdistelmää.

Mallin kehittämisen taustana on ollut eläimen biologinen kasvuprosessi, jossa rehun sisältämä ravinto on keskeisessä asemassa. Malli muodostuu kahdesta yhtälöstä, joista ensimmäinen kuvaa kasvua ja toinen eläimen syöntikyvyn kehitystä. Kasvua mitattiin elopainon lisäyksellä, jonka oletettiin riippuvan annetun rehuyhdistelmän kokonaisrehuyksikkömäärästä (ry) ja valkuaispitoisuudesta (srv g/ry) sekä eläimen perinnöllisistä ominaisuuksista. Syöntikykyä mitattiin päivien lukumääränä, jonka eläin tarvitsee annetun rehuyhdistelmän syömiseen. Selittävinä muuttujina käytettiin rehuannoksen kokonaiskuiva-ainemäärä (ka), väkevyyttä (ry/100 kg ka), valkuaispitoisuutta (srv g/ry) ja kuitupitoisuutta (% ka:sta) sekä eläimen perinnöllisiä ominaisuuksia. Mallin rakenne on edelläolevasta johtuen dynaaminen; se siis ottaa huomioon myös kasvun ja ajan välisen yhteyden, kasvunopeuden.

Mallin riippuvuussuhteiden estimointi suoritettiin kokemusperäisellä aineistolla, joka saatiin Maatalouden tutkimuskeskuksen kotieläinhoidon tutkimuslaitoksen 1970-luvun alkupuolella suorittamista kasvatuskokeista. Tästä materiaalista valittiin yhteensä 113 sonnivasikkaa, jotka oli otettu kokeisiin heti syntymän jälkeen ja joiden yhtenäinen kasvatusaika oli mahdollisimman pitkä. Aineistossa oli ayrshirerotuisia (Ay) sonnivasikoita 71 kpl, friisiläis-ayrshire risteytyksiä (FrAy) 31 kpl ja charolais-ayrshire risteytyksia (ChAy) 11 kpl. Malli estimoitiin kullekin rodulle erikseen.

Kasvua selitti parhaiten transcendenttifunktio, jonka selitysaste oli korkea. Syöntikykyä kuvaavaksi funktioksi valittiin transcendenttifunktio, jossa kuitenkin rehuannoksen kuiva-ainemäärä oli vain logaritmisena. Myös syöntikykyfunktion selitysaste oli korkea. Muuttujien regressiokertoimien estimaatit olivat merkitseviä yleensä varsin suurella luotettavuudella.

Kertoimien estimaattien perusteella voidaan tiivistetysti esittää seuraavat tulokset:

- korkein elopaino saavutetaan ry-määrällä 4 164 ry (Ay) ja 5 516 ry (FrAy- ja ChAy-risteytykset),
- rehuyksikkötarve lisäkasvukiloa kohti on suurin Ay-rodulla, pienempi FrAy- ja pienin ChAy-risteytyksillä,
- rehuyksikön tuottavuus on suurimmillaan, kun se sisältää sulavaa raakavalkuaista 136 g/ry (Ay) ja 115 g/ry (risteytykset),
- rehun kuiva-aineen päivittäinen syöntimäärä on pienimmillään, kun sulavan raakavalkuaisen määrä on 126 g/ry (Ay) ja 86 g/ry (risteytykset),
- rehun raakakuitupitoisuuden lisäys samoin kuin rehun väkevyyden lisääminen vähentävät kuiva-aineen päivittäistä syöntimäärää.

Tulokset ovat samansuuntaiset kuin ne teoriat ja tutkimustulokset, joita on rehun ominaisuuksien vaikutuksesta eläintuotannossa.

Riippuvuussuhteiden estimoinnin jälkeen tarkasteltiin mallin soveltamista tuotannon optimointiin käyttämällä tuloksen indikaattoreina nettovoittoa, joka saadaan vähentämällä tuotosta tuotantokustannukset, sekä katetuottoa, joka tässä oletettiin katteeksi työlle ja pääomille. Taloudellisesti edullisin kasvatusajan pituus ja eläimen paino katsottiin saavutetun silloin, kun käytetyt tulossuureet saavat suurimman mahdollisen arvon.

Nettovoiton ja katetuoton maksimointi suoritettiin a) eläintä kohti, b) eläimen tuottamaa lisäkasvukiloa kohti ja c) eläimen kasvattamiseen käytettyä päivää kohti. Voitiin osoittaa, että parhaaseen taloudelliseen lopputulokseen päästään maksimoimalla tulos kasvatusajan suhteen (vaihtoehto c). Tutkimuksessa esitetyllä yhtälöllä voidaan ratkaista se kokonaisrehuyksikkömäärä, jolla saadaan kasvatuksesta paras rahallinen tulos, kun tiedetään käytetyn rehuyhdistelmän ominaisuudet, vasikan rotu ja syntymäpaino sekä rehujen ja lihan hinnat. Estimoidun mallin avulla voidaan edelleen laskea tarvittava kasvatusajan pituus ja loppupaino. Mallin soveltamista on havainnollistettu esimerkillä.

Tutkimus ei pyri ratkaisemaan taloudellisesti edullisinta rehuyhdistelmää vaan mallin avulla voidaan määrittää tuotannon optimi kaikkien käyttökelpoisten rehuyhdistelmien suhteen. Tutkimuksessa on kuitenkin tarkasteltu rehuyhdistelmän merkitystä taloudellisena tekijänä. Mahdollisimman hyvän tuottavuuden omaavan rehuyhdistelmän hinnan minimoinnista esitettiin myös lineaarisen ohjelmoinnin sovellutusesimerkki.