

Selection principles: s -Menger and s -Rothberger-bounded groups

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ABSTRACT

In this paper, selection principles are defined and studied in the realm of irresolute topological groups. Especially, s -Menger-bounded and s -Rothberger-bounded type covering properties are introduced and studied.

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KEYWORDS: *irresolute topological group; s -Menger-bounded group; s -Rothberger-bounded group; selection principle.*

1. INTRODUCTION

Many topological properties are defined or characterized in terms of the following two classical selection principles.

Let \mathcal{P} and \mathcal{Q} be sets consisting of families of subsets of an infinite set X . Then:

$\mathcal{S}_{fin}(\mathcal{P}, \mathcal{Q})$ denotes the selection hypothesis: for each sequence $(P_n)_{n \in \mathbb{N}}$ of elements of \mathcal{P} there is a sequence $(Q_n)_{n \in \mathbb{N}}$ of finite sets such that for each n , $Q_n \subset P_n$, and $\bigcup_{n \in \mathbb{N}} Q_n \in \mathcal{Q}$.

$\mathcal{S}_1(\mathcal{P}, \mathcal{Q})$ is the selection hypothesis: for each sequence $(P_n)_{n \in \mathbb{N}}$ of elements of \mathcal{P} there is a sequence $(p_n)_{n \in \mathbb{N}}$ such that for each n , $p_n \in P_n$, and $\{p_n : n \in \mathbb{N}\}$ is an element of \mathcal{Q} (see [28]).

Let \mathcal{O} denote the family of all open covers of a space X . The property $\mathcal{S}_{fin}(\mathcal{O}, \mathcal{O})$ (resp. $\mathcal{S}_1(\mathcal{O}, \mathcal{O})$) is called the Menger (resp. Rothberger) covering property. For more information about selection principles theory and its relations with other fields of mathematics we refer the reader to see [16, 27, 29, 30].

A topological group is a group with a topology, such that the group operations are continuous. If the group operations are irresolute mappings instead of continuous mappings then we obtain the irresolute topological groups (ITG).

In the recent years many papers about selection principles and topological groups have appeared in the literature. o -bounded topological groups were introduced by O.Okunev. (This notion was also given by Kocinac by the same definition but under the name of Menger-bounded in an unpublished work of Kocinac in [15]. Let us now recall [10].

Definition 1.1. A topological group $(\mathcal{G}, *, \tau)$ is M-bounded (R-bounded) if there is for every sequence $(P_n)_{n \in \mathbb{N}}$ of neighborhoods (nbd) of $1_{\mathcal{G}}$, a sequence $(Q_n)_{n \in \mathbb{N}}$ of finite subsets of \mathcal{G} (a sequence $(P_n)_{n \in \mathbb{N}}$ of elements of \mathcal{G}) such that $\mathcal{G} = \bigcup_{n \in \mathbb{N}} Q_n * P_n$ (resp. $\mathcal{G} = \bigcup_{n \in \mathbb{N}} p_n * P_n$).

ITGs was first studied by Khan, Siab and Kocinac in [13] where their properties were investigated and their differences from topological groups were established. Although many papers on topological groups were published there are very few papers which deal with ITGs.

Our main aim in considering selection principles is to link this with earlier work on irresolute topological groups. Hence, Section 2 contains several definitions and results which will be needed later on. In Section 3 s-Menger-bounded, s-Rothberger-bounded and s-Hurewicz-bounded type covering properties are introduced.

2. PRELIMINARIES

In this section we recall some basic definitions and results that will enable the casual reader to follow the general ideas presented here.

If $(\mathcal{G}, *)$ is a group, and τ a topology on \mathcal{G} , then we say that $(\mathcal{G}, *, \tau)$ is a topologized group with multiplication mapping $\mu : \mathcal{G} \times \mathcal{G} \rightarrow \mathcal{G}, (p, q) \mapsto p * q$ and the inverse mapping $i : \mathcal{G} \rightarrow \mathcal{G}, p \mapsto p^{-1}$. The identity element of \mathcal{G} is denoted by e , or $e_{\mathcal{G}}$ when it is necessary,

Throughout the paper X and Y denote topological spaces. For a subset P of X , $\text{Cl}(P)$ and $\text{Int}(P)$ will denote the closure and interior of P . We denote $f^{\leftarrow}(Q)$ to define the preimage of a subset $Q \subset Y$ for a mapping $f : X \mapsto Y$. The reader is referred to [7] for undefined topological terminology and notations. A subset P of a topological space X is said to be semi-open [20] if there is an open set R in X such that $R \subset P \subset \text{Cl}(R)$. If a semi-open set P contains a point $p \in X$ we say that P is a semi-open nbd of p . If X satisfies $\mathcal{S}_{fin}(s\mathcal{O}, s\mathcal{O})$ (resp. $\mathcal{S}_1(s\mathcal{O}, s\mathcal{O})$), then we say that X has the s-Menger (resp. s-Rothberger) covering property [18, 26], where $s\mathcal{O}$ denotes the family of all semi-open covers

of X . Throughout $SO(X)$ represents the collection of all semi-open sets in X . For terms not defined here we refer the reader to see [26].

Definition 2.1. A mapping $f : X \rightarrow Y$ between spaces X and Y is called irresolute [6] (resp. pre-semi-open) if for each semi open set $Q \subset Y$ (resp. $P \subseteq X$), the set $f^{-1}(Q)$ is semi open in X (resp. $f(P)$ is semi open in Y).

Definition 2.2. A triplet $(\mathcal{G}, *, \tau)$ is called an ITG [13] if for each $p, q \in \mathcal{G}$ and each semi-open nbd R of $p * q^{-1}$ in \mathcal{G} there exist semi-open nbds P of p and Q of q such that $P * Q^{-1} \subset R$.

We note that the union of any family of semi-open sets is semi-open whereas the intersection of two semi-open sets need not be semi-open, thus the family of semi-open sets in a topological space need not be a topology. However in [13] the authors pointed out that if $(\mathcal{G}, *, \tau)$ is an ITG such that the family $SO(\mathcal{G})$ is a topology on \mathcal{G} with $SO(\mathcal{G}) \neq \tau$, then $(\mathcal{G}, *, SO(\mathcal{G}))$ is a topological group. (see, Observation [13]).

Lemma 2.3 ([13]). *If $(\mathcal{G}, *, \tau)$ is an ITG, then*

- (1) $P \in SO(\mathcal{G})$ if and only if $P^{-1} \in SO(\mathcal{G})$.
- (2) If $P \in SO(\mathcal{G})$ and $Q \subset \mathcal{G}$, then $P * Q$ and $Q * P$ are both in $SO(\mathcal{G})$.

Lemma 2.4 ([12]). *A space X is extremely disconnected if and only if the intersection of any two semi-open subsets of X is semi-open.*

Lemma 2.5 ([23]). *Let $P \subset X_0 \in SO(X)$, then $P \in SO(X)$ if and only if $P \in SO(X_0)$.*

Lemma 2.6 ([25]). *Let X_0 be a subspace of X and $P \in SO(X_0)$, then $P = Q \cap X_0$ for some $Q \in SO(X)$.*

Recall the following notations for collection of covers of a space X .

- sw -cover : A semi open cover \mathcal{P} of X is semi- ω -cover (sw -cover) [26] if for each finite subset Q of X there exists $P \in \mathcal{P}$ such that $Q \subset P$ and X is not the member of \mathcal{P} . The symbol $s\Omega$ denotes the family of sw -covers of X .
- $s\gamma$ -cover : A semi open cover \mathcal{P} of X is a $s\gamma$ -cover [26] if it is infinite and for every $p \in X$ the set $\{P \in \mathcal{P} = p \notin P\}$ is finite. The collection of $s\gamma$ -covers of X will be denoted by $s\Gamma$.

We are particularly interested here in the case where \mathcal{P} and \mathcal{Q} are open covers of topological spaces or topological groups. Specifically, let \mathcal{H} and \mathcal{G} be topological spaces with \mathcal{G} a subspace of \mathcal{H} .

- $s\mathcal{O}_{\mathcal{H}}$: The collection of semi-open covers of \mathcal{H} .
- $s\mathcal{O}_{\mathcal{HG}}$: The collection of covers of \mathcal{G} by sets semi-open in \mathcal{H} .
- $s\Omega_{\mathcal{H}}$: The collection of sw -covers of \mathcal{H} .
- $s\Omega_{\mathcal{HG}}$: The collection of sw -covers of \mathcal{G} by sets semi-open in \mathcal{H} .
- $s\mathcal{O}_{\mathcal{H}}(P)$: Let $(\mathcal{H}, *, \tau)$ be an ITG with neutral element $e_{\mathcal{H}}$, if P is a semi-open nbd of $e_{\mathcal{H}}$, then $p * P =: \{p * q : q \in P\}$ is a semi-open nbd

- of p . Thus, $\{p * P : p \in \mathcal{H}\}$ is semi-open cover of \mathcal{H} and will be denoted by $s\mathcal{O}_{\mathcal{H}}(P)$.
- $s\mathcal{O}_{nb\mathcal{d}}(\mathcal{H}) := \{\mathcal{P} \in s\mathcal{O} : (\exists \text{ semi-open nbd } P \text{ of } e_{\mathcal{H}}) (\mathcal{P} = \{p * P : p \in \mathcal{H}\})\}$.
 - $s\Omega_{\mathcal{H}}(P) : \text{For each semi open nbd } P \text{ of } e_{\mathcal{H}}, s\Omega_{\mathcal{H}}(P) = \{Q * P : Q \subset \mathcal{H} \text{ finite}\}$ is an $s\omega$ -cover of \mathcal{H} , where $Q * P := \{q \in Q \text{ and } p \in P\}$ when \mathcal{H} is not an element of this set.
 - $s\Omega_{nb\mathcal{d}}(\mathcal{H}) := \{\mathcal{P} \in s\Omega : (\exists \text{ semi-open nbd } P \text{ of } e_{\mathcal{H}})(\mathcal{U} = \{Q * P : Q \subset \mathcal{H} \text{ finite}\})\}$.
 - $s\mathcal{O}_{\mathcal{H}}^n(P) : \text{For each semi open nbd } P \text{ of } e_{\mathcal{H}}, s\mathcal{O}_{\mathcal{H}}^n(P) = \{Q * P : Q \subset \mathcal{H} \text{ and } 1 \leq |Q| \leq n\}$ is an n -cover of group \mathcal{H} .

Lemma 2.7. *If $(\mathcal{H}, *)$ is an extremely disconnected ITG with the neutral element e , then for each semi-open neighborhood P of e , there exists a symmetric semi-open nbd W of e such that: $Q = Q^{-1} \subset P$.*

3. S-MENGER-BOUNDED, S-ROTHBERGER-BOUNDED AND S-HUREWICZ-BOUNDED GROUPS

Babinkostova, Kocinac and Scheepers in [4] investigated Menger-bounded (o-bounded [9]) and Rothberger-bounded groups in the area of selection principles. On analogues to the Menger-bounded (o-bounded) and Rothberger-bounded groups we examine s-Menger-bounded and s-Rothberger-bounded groups. We also investigate the internal characterizations of groups having these properties in all finite powers (Theorem 3.8, Theorem 3.9, and Theorem 3.13). To introduce this new concept we use covering properties by semi open sets instead of open sets and the ITG properties. Semi-Menger spaces have been investigated in [26]. We recall that a space X is said to have the semi-Menger property (or s-Menger property) if it satisfies $\mathcal{S}_{fin}(s\mathcal{O}, s\mathcal{O})$. Specifically from [26, Theorem 3.8] X is s-Menger if and only if X satisfies $\mathcal{S}_{fin}(s\Omega, s\mathcal{O})$.

Definition 3.1. An ITG $(\mathcal{G}, *, \tau)$ is:

- (1) s-Menger-bounded if for each sequence $(P_n)_{n \in \mathbb{N}}$ of semi-open nbds of the neutral element $e \in \mathcal{G}$, there exists a sequence $(Q_n)_{n \in \mathbb{N}}$ of finite subsets of \mathcal{G} such that $\mathcal{G} = \bigcup_{n \in \mathbb{N}} Q_n * P_n$.
- (2) s-Rothberger-bounded if for each sequence $(P_n)_{n \in \mathbb{N}}$ of semi-open nbds of the neutral element $e \in \mathcal{G}$, there exists a sequence $(p_n)_{n \in \mathbb{N}}$ of elements of \mathcal{G} such that $\mathcal{G} = \bigcup_{n \in \mathbb{N}} p_n * P_n$.
- (3) s-Hurewicz-bounded if there is for each sequence $(P_n)_{n \in \mathbb{N}}$ of semi open nbds of neutral element $e \in \mathcal{G}$, there exists a sequence $(Q_n)_{n \in \mathbb{N}}$ of finite subsets of \mathcal{G} such that each $x \in \mathcal{G}$ belongs to all but finitely many $Q_n * P_n$.

Let $(\mathcal{G}, *)$ be a subgroup of group $(\mathcal{H}, *)$. Then \mathcal{G} is s-Menger-bounded if the selection principle $\mathbb{S}_{fin}(\mathcal{O}_{nb\mathcal{d}}(\mathcal{H}), \mathcal{O}_{\mathcal{H}\mathcal{G}})$ holds, s-Rothberger-bounded if the selection principle $\mathbb{S}_1(\mathcal{O}_{nb\mathcal{d}}(\mathcal{H}), \mathcal{O}_{\mathcal{H}\mathcal{G}})$ holds and s-Hurewicz-bounded if the selection principle $\mathbb{S}_1(\Omega_{nb\mathcal{d}}(\mathcal{H}), \mathcal{O}_{\mathcal{H}\mathcal{G}}^{gp})$ holds.

In subsections 3.1, 3.2 and 3.3 we have verified various properties of each selection principle by taking three types of covering semi open, s-gamma and s-omega and using relation with one another.

3.1. s-Menger-bounded groups. In this subsection we have verified some results on s-Menger-bounded groups.

Theorem 3.2. *Let $(\mathcal{H}, *, \tau)$ be an ITG and $\mathcal{G} \leq \mathcal{H}$. Then $\mathcal{S}_{fin}(s\mathcal{O}_{\mathcal{H}}, s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ implies $\mathcal{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.*

Proof. Since $s\Omega_{nbd}(\mathcal{H})$ is a subclass of $s\mathcal{O}_{\mathcal{H}}$. Therefore, the proof follows immediately. \square

Remark 3.3. Converse of the Theorem 3.2 is not true in general.

Example 3.4. Real line $(\mathcal{R}, +, \tau)$ with usual topology τ is an ITG under the binary operation of addition. It is known [26], that $(\mathcal{R}, +)$ is a Menger space but not the s-Menger space. Therefore, there is at least one collection of semi open covers say $\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_n, \dots$ for which there exists no collection \mathcal{V}_n of finite subsets of \mathcal{P}_n which satisfy $\bigcup Q_n = \mathcal{H}$. Thus $\mathcal{S}_{fin}(s\mathcal{O}_{\mathcal{H}}, s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ fails to hold. In order to show that $\mathcal{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ holds, we follow as under: Let $(\mathcal{P}_n)_{n \in \mathbb{N}}$ be a sequence from $s\Omega_{nbd}(\mathcal{H})$. Then for each $n, \mathcal{P}_n = \{Q * P_n : Q \subset \mathcal{H} \text{ finite}\}$ and $P_n \in SO(\mathcal{H}, e_{\mathcal{H}})$. Since Q is finite set therefore each \mathcal{P}_n is finite. Then for each $n \in \mathbb{N}$ we can choose \mathcal{R}_n of finite subsets of \mathcal{P}_n such that $\bigcup \mathcal{R}_n = \mathcal{G}$. This proves that, $\mathcal{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ holds.

Theorem 3.5. *Let $(\mathcal{H}, *, \tau)$ be an ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.
- (2) $\mathcal{S}_{fin}(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.
- (3) $\mathcal{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.

Proof. (1) \Rightarrow (2) is straightforward.

(2) \Rightarrow (3) : Since $s\Omega_{nbd}(\mathcal{H})$ is a subclass of $s\mathcal{O}_{nbd}(\mathcal{H})$. Therefore, the proof follows immediately.

(3) \Rightarrow (1) : Let $(\mathcal{P}_n)_{n \in \mathbb{N}} \in s\Omega_{nbd}(\mathcal{H})$. Select a semi-open nbd P_n of $e_{\mathcal{H}}$ for each n such that $\mathcal{P}_n = s\Omega_{\mathcal{H}}(P_n)$. Now, Apply $\mathcal{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ to $(\mathcal{P}_n)_{n \in \mathbb{N}}$: For each n let a finite set $\mathcal{R}_n \subset \mathcal{P}_n$ such that $\bigcup_{n \in \mathbb{N}} \mathcal{R}_n$ is semi-open cover of \mathcal{G} . Then each Q_n is a finite subset of \mathcal{H} . Put $R_n = Q_n * P_n$. Then for each n we have $R_n \in \mathcal{P}_n$, and, thus $\{R_n\}_{n \in \mathbb{N}}$ is a semi-open cover of \mathcal{G} .

Indeed, by writing $N = \bigcup_{n \in \mathbb{N}} Y_n$ here union is disjoint, and applying $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ to each sequence $(s\Omega(P_k) : k \in Y_n)$ independently, one finds a sequence $(Q_n : n \in \mathbb{N})$ of subsets of \mathcal{H} also finite such that for each $p \in \mathcal{G}$ there are infinitely many with $p \in Q_n * P_n$. \square

Theorem 3.6. *Let $(\mathcal{H}, *, \tau)$ be an ITG and a semi open set $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.

$$(2) \mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}}).$$

Proof. (1) \Rightarrow (2) : Let $(s\Omega(P_n) : n \in N) \in s\Omega_{nbd}(\mathcal{G})$, where every P_n is a semi-open nbd in \mathcal{G} containing the group neutral element. Then by Lemma 2.6, select $Q_n \in SO(e, \mathcal{H})$ for each n such that $P_n = Q_n \cap \mathcal{G}$. Now, select $R_n \in SO(e, \mathcal{H})$ for each n such that $R_n^{-1} * R_n \subseteq Q_n$. Apply $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}})$ to $(s\Omega(R_n) : n \in N) \in s\Omega(\mathcal{H})$: We find for each n a set $S_n \subset \mathcal{H}$ which is finite such that $\mathcal{G} \subseteq \bigcup_{n \in N} S_n * R_n$. Since R_n is semi open therefore $S_n * R_n$ is semi open. For each n , and for each $m \in S_n$, choose a $p_m \in \mathcal{G}$ as follows:

$$p_m \begin{cases} \in \mathcal{G} \cap m * R_n & \text{if } \text{nonempty,} \\ = e & \text{if } \text{otherwise.} \end{cases}$$

Then put a finite set $T_n = \{p_m : m \in S_n\} \subset \mathcal{G}$. For each n we have $T_n * P_n \in s\Omega(P_n) \in s\Omega_{nbd}(\mathcal{G})$. Now only remaining to show that $\mathcal{G} = \bigcup_{n \in N} T_n * P_n$. For let $l \in \mathcal{G}$ be given. Choose n so that $g \in S_n * R_n$, and choose $m \in S_n$ so that $l \in m * S_n$. Then obviously $\mathcal{G} \cap m * R_n \neq \phi$, and so $p_m \in \mathcal{G}$ is belonging to $\mathcal{G} \cap m * R_n$. Since $p_m \in m * R_n$, We have $m \in p_m * R_n^{-1}$, and so $l \in p_m * R_n^{-1} * R_n \subseteq p_m * Q_n$. Now $p_m^{-1} * l \in \mathcal{G} \cap Q_n = P_n$, and so we have that $l \in p_m * P_n \subset T_n * P_n$.

(2) \Rightarrow (1) : By Lemma 2.5 the proof is evident. □

Corollary 3.7. *Let $(\mathcal{H}, *, \tau)$ be an ITG and $\mathcal{G} \leq \mathcal{H}$. If $\mathbb{S}_{fin}(s\mathcal{O}_{\mathcal{H}}, s\mathcal{O}_{\mathcal{H}\mathcal{G}})$ holds then $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}})$.*

Proof. By Theorem 3.2, Theorem 3.5 and Theorem 3.6, we have $\mathbb{S}_{fin}(s\mathcal{O}_{\mathcal{H}}, s\mathcal{O}_{\mathcal{H}\mathcal{G}}) \Rightarrow \mathbb{S}_{fin}(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}})$. □

Theorem 3.8. *Let $(\mathcal{H}, *, \tau)$ be an extremely disconnected ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$.
- (2) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}})$.

Proof. (1) \Rightarrow (2) : Let $(\mathcal{P}_n : n \in N) \in s\Omega_{nbd}(\mathcal{H})$, and select $P_n \in SO(e_{\mathcal{H}}, \mathcal{H})$ for each n with $\mathcal{P}_n = s\Omega_{\mathcal{H}}(P_n)$. Then define, $Q_n = \bigcap_{j \leq n} P_j$. Each $Q_n = s\Omega_{\mathcal{H}}(Q_n) \in s\Omega_{nbd}(\mathcal{H})$. Apply $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$ to $(Q_n : n \in N)$, then there is a sequence $R_n \in Q_n$ for each n , such that $\{Q_n : n \in N\}$ is a cover of \mathcal{G} and cover is weakly groupable. Suppose an increasing sequence $p_1 < p_2 < p_3 < \dots < p_k < \dots$ such that there is for each finite $S \subset \mathcal{G}$, a k with $S \subset \bigcup_{p_k \leq j \leq p_{k+1}} R_j$. Further, select a finite $S_n \subset \mathcal{H}$ with $R_n = S_n * Q_n$. Since Q_n is semi open so is R_n . For $i < p_1$ set $T_i = \bigcup_{j < p_1} S_j$, and for $p_k \leq i < p_{k+1}$, set $T_i = \bigcup_{p_k \leq j < p_{k+1}} S_j$. So each finite $T_i \subset \mathcal{H}$, and for each i if we put $U_i = T_i * P_i$, then $P_i \in \mathcal{P}_i$ for each i , and $\{P_i\}_{i \in N}$ in $s\Omega_{\mathcal{H}\mathcal{G}}$.

(2) \Rightarrow (1) : This is evident. □

Theorem 3.9. *Let $(\mathcal{H}, *, \tau)$ be an ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) For each n the selection principle $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n})$.
- (2) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}})$.

Proof. (1) \Rightarrow (2) : Suppose $(s\Omega(P_n))_{n \in N} \in s\Omega_{nbd}(\mathcal{H})$. Let P_n be a semi open nbd e_H . Let natural number $N = \bigcup_{n \in N} R_n$, and R_n is infinite for each n , and for m is not equal to n we have $R_m \cap R_n$ is empty. For each k , $(s\Omega(P_n^k) : n \in R_k)$ is in $s\Omega_{nbd}(\mathcal{H}^k)$. By (1), for each k , and for each $n \in R_k$, choose a finite $Q_n \subset \mathcal{H}$ such that $\{Q_n^k * P_n^k : n \in R_k\}$ is a cover of \mathcal{G}^k by sets semi open in \mathcal{H}^k . Now we show that $\{Q_n * P_n : n \in N\}$ is in $s\Omega_{\mathcal{H}\mathcal{G}}$. For let a finite set $Q = \{q_1, q_2, \dots, q_j\} \subset \mathcal{G}$. Then $q = (q_1, q_2, \dots, q_j) \in \mathcal{G}^j$. Select an $n \in R_j$ with $h \in Q_n^j * P_n^j$. Then $Q \subseteq Q_n * P_n \in s\Omega_{\mathcal{H}}(P_n)$.

(2) \Rightarrow (1) : Set $m \in N$, and consider $(s\Omega_{\mathcal{H}}(P_n))_{n \in N} \in s\Omega_{nbd}(\mathcal{H}^m)$. For each k choose a semi open nbd S^k of $e_{\mathcal{H}}$ such that $S_k^m \subseteq P_k$. Then $(s\Omega_{\mathcal{H}}(S_k))_{k \in N} \in s\Omega_{nbd}(\mathcal{H})$. Apply $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}})$ and for each k , a finite $Q_k \subset \mathcal{H}$ with $\{Q_k * S_k\}_{k \in N} \in s\Omega_{\mathcal{H}\mathcal{G}}$. Then $\{Q_k^m * S_k^m\}_{k \in N}$ is cover of \mathcal{G}^m by semi open sets. For each k choose $T_k \in s\Omega_H(P_k)$ with $Q_k^m * S_k^m \subset T_k$. Then $\{T_k : k \in N\}$ is a cover of \mathcal{G}^m by semi open sets. \square

Corollary 3.10. *Let $(\mathcal{H}, *, \tau)$ be an ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$.
- (2) For each n , $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}^n), s\mathcal{O}_{\mathcal{G}^n})$.
- (3) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\Omega_{\mathcal{G}})$.
- (4) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}}^{wgp})$.

Proof. (1) \Rightarrow (2) : From Theorem 3.8, we have $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}})$ and from Theorem 3.9, for each n , we have $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n})$. Finally, from Theorem 3.6, $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}^n), s\mathcal{O}_{\mathcal{G}^n})$.

(2) \Rightarrow (1) : If we take $\mathcal{H} = \mathcal{G}$ then from Theorem 3.9, for each n , $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}^n), s\mathcal{O}_{\mathcal{G}^n}) \Rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}})$.

(3) \Rightarrow (4) : This is obvious because $s\omega$ -cover is always weakly groupable cover.

(4) \Rightarrow (1) : This is obvious. \square

3.2. s-Rothberger-bounded groups. In this subsection we have verified some results on s-Rothberger-bounded groups.

Theorem 3.11. *Let $(\mathcal{H}, *, \tau)$ be an ITG and a semi open set $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}})$.
- (2) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}})$.

Proof. The proof is similar to the proof of Theorem 3.6. \square

Theorem 3.12. *Let $(\mathcal{H}, *, \tau)$ be an extremely disconnected ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$.
- (2) $\mathbb{S}_1(\{s\mathcal{O}_{nbd}^n(\mathcal{H})\}_{n \in N}, s\Omega_{\mathcal{H}\mathcal{G}})$.

(3) For each $(T_n)_{n \in N} \in N$ diverging to ∞ , $\mathbb{S}_1(\{s\mathcal{O}_{nbd}^{T_n}(\mathcal{H}) : n \in N\}, s\Omega_{\mathcal{H}\mathcal{G}})$.

Proof. (1) \Rightarrow (2) : Let $s\mathcal{O}_{nbd}^n(\mathcal{H})$ be a collection of all semi open covers of the form $s\mathcal{O}_{\mathcal{H}}^n(P)$. For each n , put $\mathcal{P}_n = s\mathcal{O}^n(P_n)$. Here, P_n is a semi open nbd of the neutral element and $\mathcal{P}_n = s\mathcal{O}^n(P_n) = \{Q * P_n : Q \subset \mathcal{H}, \text{ and } 1 \leq |Q_n| \leq n\}$. For each n put $V_n = \bigcap_{j \leq n} U_j$ and for each n put $\mathcal{R}_n = s\mathcal{O}(R_n)$. Apply $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$ to the sequence $(\mathcal{R}_n : n \in N)$. For each n choose an p_n such that $\{p_n * R_n : n \in N\}$ is in $s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp}$. Choose a sequence $q_1 < q_2 < q_3 < \dots < q_k < q_{k+1} < \dots$ such that: For each finite set $Q \subset \mathcal{G}$ there is an n with $Q \subseteq \bigcup_{m_n \leq j \leq q_{n+1}} p_j * R_j$. For each such i define:

$$Q_i = \begin{cases} \{p_j : j \leq i\} & \text{if } i \leq q_1 \\ \{p_j : q_n \leq j \leq i\} & \text{if } q_n \leq i \leq q_{n+1} \end{cases}$$

Then put $S_n = Q_n * P_n$. For each n we have $S_n \in s\mathcal{O}_{\mathcal{H}}^n(P_n)$, and $\{S_n : n \in N\}$ is an $s\omega$ -cover of \mathcal{G} .

(2) \Rightarrow (3) : Choose $1 < r_1 < r_2 < \dots < r_n < \dots$ such that $(\forall j \geq r_n)(T_j \geq n)$. Let $\mathcal{P}_n = s\mathcal{O}_{\mathcal{H}}^{T_n}(P_n)$, $n \in N$ be given. For each sequence $(s\mathcal{O}_{nbd}^{T_n}(\mathcal{H}) : n \in N)$, where each P_n is a semi open nbd of $e_{\mathcal{H}}$, there exist a sequence $(R_n : n \in N)$ such that for each n , $R_n \in s\mathcal{O}^{T_n}(P_n)$ and $\{R_n : n \in N\} \in s\Omega_{\mathcal{H}\mathcal{G}}$. Define $R_1 = \bigcap_{j \leq r_1} P_j$, and for each n put $R_{n+1} = \bigcap_{r_n \leq j \leq r_{n+1}} P_j$. To apply (2) to $(s\mathcal{O}_{\mathcal{H}}^n(R_n) : n \in N)$. For each n , choose $Q_n \subset \mathcal{G}$ with $|Q_n| \leq n$ with $S_n = Q_n * G_n$, $n \in N$, the set $\{S_n : n \in N\} \in s\Omega_{\mathcal{H}\mathcal{G}}$.

(3) \Rightarrow (1) : Let $(T_n : n \in N)$ be given. Choose $1 \leq q_1 < q_2 < \dots < q_k < q_{k+1} < \dots$ such that $T_1 \leq q_1$ and for each k , $T_{k+1} \leq (q_{k+1} - q_k)$. Now, let $\mathcal{P}_n = s\mathcal{O}_{\mathcal{H}}(P_n)$ be given for each n . Define $R_1 = \bigcap_{j \leq n} P_j$ and $R_{k+1} = \bigcap_{q_k \leq j \leq q_{k+1}} P_j$. Then put $\mathcal{R}_n = s\mathcal{O}_{\mathcal{H}}^{T_n}(R_n)$, $n \in N$. Apply (3) to the sequence $(\mathcal{R}_n : n \in N)$ and choose for each n an $S_n \in \mathcal{R}_n$ so that $\{S_n : n \in N\} \in s\Omega_{\mathcal{H}\mathcal{G}}$. For each n choose finite set $Q_n \subseteq \mathcal{H}$ such that $|Q_n| \leq T_n$, and $S_n = Q_n * R_n$. For each m write $Q_m = \{p_{q_m+1}, \dots, p_{q_{m+1}}\}$ with repetitions, if necessary. Then $(p_k * P_k : k < \infty)$ is the sequence with $p_k * P_k \in \mathcal{P}_k$ for each k , the sequence of n_j 's witness the weak groupability $\{p_k * P_k : k < \infty\}$. \square

Theorem 3.13. Let $(\mathcal{H}, *, \tau)$ be an extremely disconnected ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:

- (1) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp})$
- (2) For each n , $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n}^{wgp})$
- (3) For each n , $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n})$

Proof. (1) \Rightarrow (2) : Put $n > 1$ and suppose $\mathcal{H}^n = \mathcal{H} \times \mathcal{H} \times \dots \times \mathcal{H}$ (n copies). Let $\mathcal{P}_p = s\mathcal{O}^{p^p}(P_{p,1} \times P_{p,2} \times \dots \times P_{p,n})$ for each p and define $Q_p = \bigcap_{j \leq n} U_{p,j}$, a semi open nbd of $e_{\mathcal{H}}$. For select a finite set $Q_p \subset \mathcal{H}$ such that $|Q_p| \leq p$, and such that $\{R_p * Q_p : p < \infty\} \in s\Omega_{\mathcal{H}\mathcal{G}}$. Since $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp}) \rightarrow \mathbb{S}_{fin}(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{wgp}) \rightarrow \mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Omega_{\mathcal{H}\mathcal{G}})$, as we saw in Theorem 3.8. Then for each m put $G_p = R_p \times R_p \times \dots \times R_p$ (n copies), $p < \infty$. Then put $S_p = G_p * (P_{p,1} \times P_{p,1} \times \dots \times P_{p,n})$. For each p we have $S_p \in s\mathcal{O}^{p^p}(U_{p,1} \times U_{p,2} \times$

$\dots \times U_{p,n}$) and we have $\{S_p : p < \infty\} \in s\Omega_{\mathcal{HG}}$. By (3) \Rightarrow (1) of Theorem 3.12, $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}^n), s\mathcal{O}_{\mathcal{H}^n\mathcal{G}^n}^{wgp})$ holds.

(2) \Rightarrow (3) : This is obvious.

(3) \Rightarrow (1) : Let $\mathcal{P}_n = s\mathcal{O}^n(P_n)$ for each p . Write $N = \bigcup_{k < \infty} B_k$ where for each k , $k \leq \min(B_k)$ and B_k is infinite, and for $p \neq n$, $B_p \cap B_n = \emptyset$. For each k : For $p \in B_k$ put $\mathcal{Q}_m = s\mathcal{O}(P_m^k)$. Then $(\mathcal{Q}_p : p \in B_k)$ is a sequence from $s\mathcal{O}_{nbd}(\mathcal{H}^k)$. Applying (3) choose for each $p \in B_k$ an $q_p \in \mathcal{H}^k$ such that $\{q_p * P_p^k : p \in B_k\}$ is a semi open cover of \mathcal{G}^k . For each p in B_k write $q_p = (q_p(1), \dots, q_p(k))$, and then set $\phi(q_p) = \{q_p(1), \dots, q_p(k)\}$. Note that for each $p \in B_k$ we have $|\phi(q_p)| \leq k \leq p$, and so $\phi(q_p) * P_p$ is in $s\mathcal{O}^p(P_p)$. Set $Q_p = \phi(q_p) * P_p$ for each p , a member of $s\mathcal{O}^p(P_p) = \mathcal{Q}_p$. Claim that $\{Q_p : p < \infty\}$ is in $s\Omega_{\mathcal{HG}}$. For let $R \subset \mathcal{G}$ be a finite and put $k = |R|$. Write $R = \{r_1, \dots, r_k\}$. Suppose $q = (r_1, \dots, r_k) \in \mathcal{G}^k$. For some $p \in B_k$ we have $q \in q_p * P_p^k$, and so $R \subset \phi(q_p) * P_p = Q_p$. Now (2) \Rightarrow (1) of Theorem 3.12 implies that $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{HG}}^{wgp})$ holds. \square

Theorem 3.14. *Let $(\mathcal{H}, *, \tau)$ be an extremely disconnected ITG and a semi open set $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{HG}}^{wgp})$
- (2) $\mathbb{S}_1(s\mathcal{O}_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}}^{wgp})$

Proof. The proof is similar to the proof of Theorem 3.6. \square

3.3. s-Hurewicz-bounded groups. In this subsection we have verified some results on s-Hurewicz-bounded groups.

Theorem 3.15. *Let $(\mathcal{H}, *, \tau)$ be an extremely disconnected ITG and $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{HG}}^{gp})$.
- (2) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Gamma_{\mathcal{HG}})$.

Proof. (1) \Rightarrow (2) : For each $n \in N$ let $\mathcal{P}_n \in s\Omega_{nbd}(\mathcal{H})$, and select $P_n \in SO(e_H, H)$ with $\mathcal{P}_n = s\Omega(P_n)$. Put $Q_n = \bigcap_{j \leq n} P_j$. For each n put $\mathcal{Q}_n = s\Omega(Q_n)$ is in $s\Omega_{nbd}(\mathcal{H})$. Then apply $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{HG}}^{gp})$ to $(\mathcal{Q}_n : n \in N)$. Choose $R_n \in \mathcal{Q}_n$ such that $\{R_n : n \in N\}$ is a groupable semi open cover of \mathcal{G} . Choose a sequence $p_1 < p_2 < p_3 < \dots < p_k < \dots$ such that x belongs to \mathcal{G} , for all but finitely many n , $x \in \bigcup_{p_n \leq j \leq p_{n+1}} R_j$. Select finite set $S_n \subset \mathcal{H}$ with $R_n = S_n * Q_n$. So R_n is also semi open because of ITG. Now define, for each k , the finite set T_k by,

$$T_k = \begin{cases} \bigcup_{i \leq m_1} S_i & \text{if } k \leq p_1 \\ \bigcup_{p_n \leq i \leq p_{n+1}} S_i & \text{if } p_n \leq k \leq p_{n+1} \end{cases}$$

For each n put $A_n = T_n * P_n$ an element of $s\Omega(P_n)$. Claim that $\{A_n : n \in N\}$ is a $s\gamma$ -cover of \mathcal{G} . For consider g is an element of \mathcal{G} . Select $M \in N$ in such a way for all $n \geq M$ we have $g \in \bigcup_{p_n < i \leq p_{n+1}} R_i$. But for $p_n < i \leq p_{n+1}$ we have

$R_i = S_i * Q_i \subset A_k = T_k * P_k$ for $p_n < i \leq p_{n+1}$. Thus for all $k > p_M$ we have $g \in A_k$. It follows that $\{A_k : k \in N\}$ is $s\gamma$ -cover of \mathcal{G} .

(2) \Rightarrow (1) : This is evident. \square

Theorem 3.16. *Let $(\mathcal{H}, *, \tau)$ be an ITG and a semi open set $\mathcal{G} \leq \mathcal{H}$. Then the following statements are equivalent:*

- (1) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\mathcal{O}_{\mathcal{H}\mathcal{G}}^{gp})$.
- (2) $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\mathcal{O}_{\mathcal{G}}^{gp})$.

Proof. The proof is similar to the proof of Theorem 3.6. \square

Corollary 3.17. *If $(\mathcal{H}, *, \tau)$ has property $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{H}), s\Gamma_{\mathcal{H}})$, then for each $\mathcal{G} \leq \mathcal{H}$, $\mathbb{S}_1(s\Omega_{nbd}(\mathcal{G}), s\Gamma_{\mathcal{G}})$ holds.*

4. CONCLUSIONS

We have introduced three new types of selection principles in the realm of irresolute topological groups. We have also proved that these new notions are well defined, by means of studying their internal characterizations. Kocinac introduced several types of selection principles available in the literature. For future work one can see selection principle in the domain of soft sets.

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