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Review of the book of Vladimir Kovalevsky "Geometry of Locally Finite Spaces"

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The title of the book may create an impression that it is for geometers and topologists. That is right, but I think that the main purpose of the book is to construct a broad road for specialists in digital geometry, image recognition, and other branches of computer and applied mathematics. Where to? Into the world of computational and algorithmic topology. Topological notions such as surface, manifold, connectedness, boundary, orientation, dimension unavoidably appear when you start to think about basic theoretical problems in the above-mentioned branches of science. However, existing topological books are devoted to the purely theoretical interior problems of topology and are very far from practical applications. The theory of locally finite spaces presented in the 330-page monograph of Professor Vladimir Kovalevsky fills this gap. The theoretic part of the book contains numerous new definitions and theorems which express in a form understandable for non-specialists applications of those notions of set topology which are needed for computer imagery. The part devoted to applications presents some algorithms for investigating topological and geometrical properties of digitized two- and three-dimensional images. The book assumes a certain level of preparation, but some elementary knowledge in topology and in image processing would be sufficient. One of the important features of this book is that it provides a means which gives the possibility to overcome the existing discrepancy between theory and applications: The traditional way of research consists in making theory in Euclidean space with real coordinates while applications deal only with finite discrete sets and rational numbers. The reason is that even the smallest part of the Euclidean space cannot be explicitly represented in a computer and computations with irrational numbers are impossible since there exists no arithmetic of irrational numbers. The author demonstrates that locally finite spaces are explicitly representable in a computer, only rational coordinates are used and the theory of those spaces is in accordance with classical topology. The book consists of 14 chapters: an introductory section followed by thirteen main sections. The introductory section presents a short retrospect to the origin of the book followed by an overview of the contents and of the aims of the monograph. Section 2 presents a new set of axioms and a proof that classical axioms of a topological space follow from the new axioms as theorems. This means that a locally finite space satisfying the new axioms (ALF spaces) is a particular case of a classical space: it is a T0 Alexandroff space which is not a T1 space. I would like to stress that the axioms are actually designed to applications. In particular, they are very convenient while working with computer presentations of geometric objects. Section 3 is devoted to the theory of the spaces under consideration. It contains numerous theorems about the properties of ALF spaces and definitions of balls, spheres and of the dimension of the space elements. These definitions are important for describing combinatorial homeomorphisms between spaces. This section also contains a new generalization of the orientation of simplicial complexes to the general case of abstract complexes and a generalization of the classical notion of a boundary. Section 4 considers mappings among locally finite spaces. The author has demonstrated that classical homeomorphisms based on continuous maps being applied to locally finite spaces degrade to isomorphisms. He suggests to replace them by a much more convenient notion of connectedness preserving correspondences (CPMs), which can map one space element to many. He also has demonstrated that a combinatorial homeomorphism based on elementary subdivisions of space elements uniquely defines a continuous CPM whose inverse is also continuous. Sections 6 to 9 are devoted to a new concept of digital geometry which reflects Euclidean geometry numerically. There is among others a new and complete theory of digital straight segments being considered as one-dimensional complexes rather than sequences of pixels. A digital straight segment is considered here as a subset of the boundary of a digital half-plane rather than as digitization of a Euclidean straight line. This theory leads to interesting efficient applications to image analysis (Section 11.3). Sections 10 to 13 are devoted to applications. They contain descriptions of numerous algorithms based on the theory of locally finite spaces. There are among them algorithms for tracing and encoding boundaries in two- and three-dimensional digital images, exactly reconstructing images from their boundary codes, labeling connected components, computing skeletons, constructing convex hulls and others. Some sections of the book contain problems to be solved which will stimulate further research. Particularly interesting is Section 14 "Topics for Discussion". The author discusses here the possibility to avoid irrational numbers and to use finite differences instead of derivatives. He demonstrates the possibilities of his approach while presenting an inference of the Taylor formula based on finite differences. The book is unique: it is the first one in its own way presenting a self-contained theory of locally finite spaces that is independent of the Hausdorff topology. It also contains a concept of digital geometry that is independent of Euclidean geometry. I do not know any similar books. The book is carefully and clearly written and contains numerous well-made excellent illustrations. It is understandable for those whose interests are close to digital geometry / topology as well as for specialists in topology craving for applications. This book would be very useful and guiding for students and researchers, or for anyone interested in digital topology, digital geometry and computer imagery. It also may be of interest for physicists working on quantum gravity since it presents a well founded theory of spaces that could serve as the base of this branch of physics. My main conclusion is that the book should certainly find its way into university libraries and onto many private book shelves.

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