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A Differential Evolution Optimization Approach for Parameters Estimation of Truncated and Censored Failure Time Data

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Abstract

Most practically collected datasets are plagued with issues of incompleteness and inaccuracies which will cause erroneous reliability modeling and poor maintenance decisions. This work outlines an investigation into the use of Heuristic techniques to estimate the parameters of a stochastic life distribution by using failure data with Truncation and Censoring. A Maximum Likelihood Estimation (MLE) approach was utilized in which the Log-Likelihood function is modified to account for the Truncation and Censoring factors. A Differential Evolution (DE) algorithm developed in MATLAB R2013a minimizes the Negative Log Likelihood (NLL) function and obtains optimum parameters for the 2 Parameters (2-P) Weibull distribution. Results obtained from a series of designed experimental tests generalized the relationship between the increasing levels of Truncation and Censoring individually on the β and η parameters. The impact of the modified NLL technique was examined under cases of Left Truncated and Right Censored (LTRC) data through evaluations of the MSE metric which were compared to estimations made under the normal NLL equation. Truncation and Censoring percentages were increased from 0% to 50% for testing the modified NLL approach. It is clear from the low MSE values (error) that this approach is successful at estimating the parameters closer to the true values. This approach was applied to failure data of a Gas Engine Power Generator utilized in Offshore Gas Production. The results were compared with those obtained from traditional Weibull Analysis in the ReliaSoft Weibull/Alta package.

Keywords: heuristics, truncation, censoring, maximum likelihood estimation, reliability analysis

1. Introduction

Maintenance Optimization algorithms have now become an indispensable tool that asset intensive organizations utilize to reduce costs and increase profit margins. A significant aspect of this process entails the collection and analysis of lifetime data. Weibull Analysis and other data fitting methods compose the basis of interpretations for equipment degradation over time. However, a drawback of these analyses is the dependency on accurately collected and complete datasets [1]. In many cases, the quality of failure time data has been a major cause for concern. Truncation, Censoring or even missing events are a few of the problems with most practically collected data [2]. In many analyses, these factors are unaccounted and risk impacting failure predictions. This further impacts the accuracy of maintenance planning in the optimization stages.

The popularity of such data in Engineering and other fields such as Medical Sciences has prompted several areas of research to address the above mentioned issues [3-4]. Maximum Likelihood Estimation (MLE) has been ubiquitous in such cases. Its adaptability towards treating with Truncated and Censored data is noted in a number of past publications [5-6].

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Traditionally adopted gradient based methods have been utilized to maximize the likelihood function established. In this regard, the predominance of issues such as trapping at local maxima or minima is noted, leading to failure of convergence in many cases [7]. Heuristics have been proposed for a number of years to estimate function parameters with much success [8]. A novel aspect of the work reported in this paper investigates the applicability and accuracy of extending Heuristic global optimization techniques towards the likelihood function. This would alleviate the issues that have been mentioned while providing simplicity in dealing with the complex function which traditional techniques are not able to address.

Section 2 of this paper details the findings of past literature; section 3 defines the methodology which was undertaken in the current study; section 4 outlines the results illustrating the Truncation and Censoring impact and application to a real case study of an Offshore Gas Power Generator. Section 5 draws conclusions to date from the results and presents future research work.

2. Literature Research

2.1. Left truncation and right censoring

Although there are various categories of incomplete datasets, the current work targets three specific classifications namely Left Truncated, Right Censored, and Left Truncated and Right Censored (LTRC) data. For defining these three (3) classifications, consider failure time data collected from 100 identical Power Transformers between the period 1980 and 2008. The classification of failure cases can be completely defined by four (4) categories as illustrated in Fig. 1. Failure case 1 represents the class of Right Censored data which remains in operation at the end of the observation period of 2008. Case 2 is a Left Truncated observation occurring between installation year and prior to the end of the observation period. Case 3 illustrates a combined class of Left Truncation and Right Censoring. Case 4 represents a Transformer which was installed and failed prior to observation and as a result no data is available. Case 4 is termed as missing data.

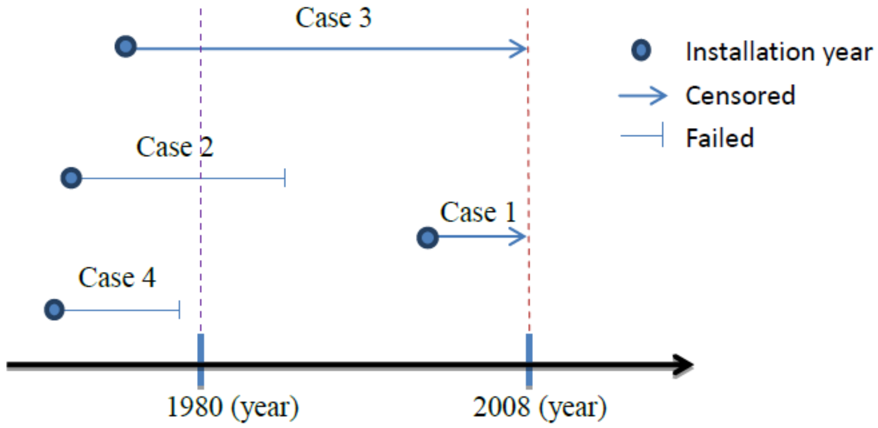


Fig. 1 Classification of Truncated and Censored data [9]

The issue of LTRC data has been explored widely in the fields of medicine and is becoming increasingly popular in engineering applications. Perhaps the most popular technique was applied by Balkrishnan and Mitra [5] and Liu [10] which involved parameter estimation through a modified likelihood function for LTRC data. An Expectation Maximization (EM) and Newton Raphson (NR) algorithm was used for comparison purposes to execute the optimization of the estimated function. Mitra [11] used this approach to estimate the parameters of a Weibull, Gamma and Log-Normal distribution for simulated LTRC failure times with characteristics of that as defined by Hong [2] for Power Transformers. Furthermore, Emura and Shiu [9] adopted and transformed the maximization approach through the use of a one-step NR algorithm for simplicity. Although significant developments have been made drawbacks are apparent in terms of complexity, Non-Convergence and Local Maxima or Minima Trapping when dealing with gradient decent based optimization techniques [7].

2.2. Heuristics optimization techniques differential evolution

Heuristic techniques are a branch of Evolutionary algorithms which perform global optimization searches based on stochastic evolution rather than gradient based decent [12]. For this reason, such techniques remain unaffected by the drawbacks of traditional search techniques. Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Simulated Annealing (SA) are amongst some of the more popularly applied algorithms for parameter estimation purposes. The Differential Evolution (DE) algorithm was developed by Storn and Price in the nineties as a recent advancement in this area [13-14]. DE offers the advantage of determining global minima/maxima points despite the selection of initial values, fast convergence and fewer limitations of control parameters [15]. Das et al. [16] presented an exhaustive survey on DEs and their adaptations over the years. Furthermore, Wang and Huang [17] applied DE to estimate stochastic parameters illustrating the change in population Probability Density Function (PDF) upon Crossover, Mutation and Selection operations. Lobato et al. [18] compared DE to SA for the estimation of Radiative Properties through an inverse problem approach.

2.3. Maximum Likelihood Estimation

The MLE method as defined by Scholz [19] is as follows:

Consider a random vector of observations X = (X_1, X_2,…,X_n) defined by the joint probability distribution function over the n dimensional Euclidean space R^n. The likelihood function is defined as given in Eq. (1).

$$L_x(\theta) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2, \ldots, \theta_r) \quad \text{where } \theta_1, \theta_2, \theta_3, \ldots, \theta_r \text{ are the parameters of the distribution} \tag{1}$$

The PDF for 2-P Weibull with θ= (β, η) is given in Eq. (2) below.

$$f(x; \beta, \eta) = \frac{\beta}{\eta}\left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{x_i}{\eta}\right)^{\beta}} \tag{2}$$

Substitution of Eq. (2) into Eq. (1) and taking logs we obtain Eq. (3) as shown below.

$$LL(x_i{:}\,\beta, \eta) = n\left(log\frac{\beta}{\eta}\right) + \sum_{i=1}^{n}\left\{(\beta-1)\, log\left(\frac{x_i}{\eta}\right) - \left(\frac{x_i}{\eta}\right)^{\beta}\right\} \tag{3}$$

The log-likelihood function is extended to include Left Truncated and Right Censored Data with Truncation indicator, ν, and Censoring indicator, δ, as shown in Eq. (4) [11]. Both indicators are binary in nature [1,0] with $\nu_i \text{ or } \delta_i = 0$ if the sample element is Truncated or Censored and $\nu_i \text{ or } \delta_i = 1$ if it is not. Mitra [11] tested the approach using multiple unit systems where in some cases the dataset belonging to a single unit may or may not be Truncated or Censored. For a Truncated unit, the Truncation time is given by τ_i.

$$L(\beta, \eta) = \sum_{i=1}^{n} \delta\left\{log\beta - log\eta + (\beta-1)log\frac{x_i}{\eta}\right\} - \left(\frac{x_i}{\eta}\right)^{\beta} + \sum_{i=1}^{n}(1-\nu)\left(\frac{\tau}{\eta}\right)^{\beta} \tag{4}$$

Although DE has been applied extensively towards parameter estimation its extension in the context of dealing with incomplete datasets is yet to be investigated. Thus, the main focus of the current work is to analyze the application of the DE heuristic global optimization technique for parameter determination purposes as applied to Truncated and Censored datasets. The modified MLE technique will be implemented within the DE algorithm to account for the incomplete data.

2.4. Mean Squared Error performance metric

Past authors have often implemented the Mean Squared Error (MSE) metric for determining the levels of success attained through their approaches [2, 9, 11]. MSE estimates the error between the predicted and true values as represented in the undermentioned Eq. 5. For n estimates, if $\widehat{X_i}$is the vector of estimates and $X_i$is the vector of True values then the MSE is calculated as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{X_i} - X_i\right)^2 \tag{5}$$

3. Methodology

MATLAB R2013a software was utilized for coding the DE algorithm in the current study. The algorithm would refer to the normal and modified NLL equations with the aim being to minimize the function and follows the standard flow diagram proposed by Storn and Price [13]. To effectively test the capability of estimating the parameters with minimum error, data were simulated using Reliasoft's Weibull/Alta Monte Carlo Simulation (MCS) package. This approach also allowed for comparative testing with both complete and incomplete datasets. Complete datasets were firstly simulated with the choice of Beta values selected to cover cases of Reliability Growth (β<1) and Reliability Degradation (β>1). For many industrial systems a combination of static and rotational equipment typically coexists. Thus to properly represent the failure nature for both types, beta values of 0.6, 2.5 and 4.5 were randomly selected. Table 1 shows the full composition of the data simulation program.

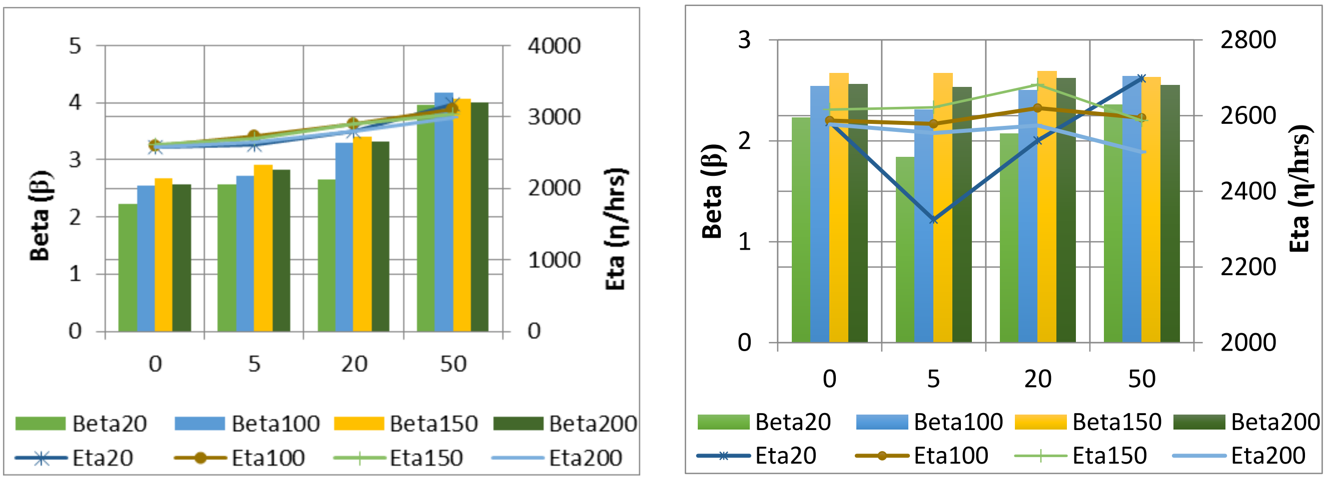
Additionally, four sample sizes of 20, 100, 150 and 200 were selected in the current study. In comparison to previous work, this range of sample sizes is smaller. However, this was the choice in the current work since much of the practical data collected from the industry conform to sample sizes within the range of 20-200. Thus, to allow for conformity in the practical application section similar sample sizes were selected. Since the MCS allowed for different levels of Censoring to be included, Censored samples corresponding to 5%, 20% and 50% for each Weibull parameter vector set and sample size were also simulated. All data generations were then exported, chronologically ordered and then artificially Left Truncated by omitting the initial number of failures corresponding to the Truncation percentage as a function of sample size. A total of 384 tests were performed, and the MSE values were calculated for each dataset tested. Fig. 3 and Fig. 5 illustrate a comparison of the PDF plots for the true and estimated Weibull parameters.

Table 1 Dynamics of the simulated datasets used for testing the DE algorithm

Parameter Varied	# of Variations	Details	References
Weibull distribution parameter vector	3	Beta, β=0.6 , Eta, η=500 Beta, β=2.5, Eta, η=2500 Beta, β=4.5, Eta, η=4000	[20]
Sample Size	4	20, 100,150 and 200	N/A
Percentage Censoring	4	0%,5%, 20% and 50%	[5], [7], [11]
Percentage Truncation	4	0%,5%, 20% and 50%	[5], [7], [11]
Total # of Datasets	192		
Total # of Tests	384 (Normal and Modified NLL)		

4. Results

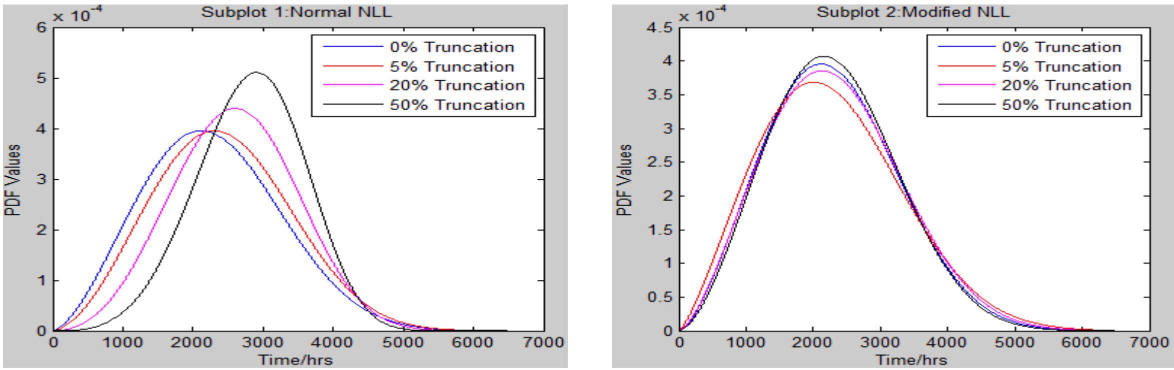
4.1. Truncation effect



(a) Effect of Truncation Percentages on Beta and Eta Estimates with normal NLL

(b) Effect of Truncation Percentages on Beta and Eta Estimates with modified NLL

Fig. 2 Effect of truncation percentages of Beta and Eta Values



(a) Effect of increased truncation percentages on the PDF curve with normal NLL L

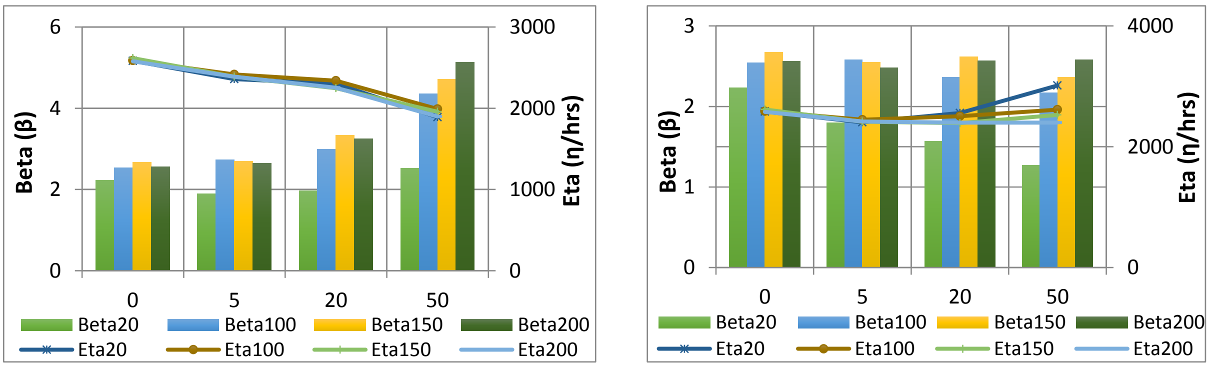
(b) Effect of increased truncation percentages on the PDF curve with modified NLL

Fig. 3 Effect of increased truncation percentages on the PDF curve

The individual effect of Truncation and Censoring percentages was investigated initially. Fig. 2 shows the impact when using (a) the normal NLL and (b) the modified NLL functions for varying percentages of Truncation. The True Beta and Eta values for the data were 2.5 and 2500 hours respectively and the results are displayed for all sample sizes. The estimated values of β and η have surpassed the true values when utilizing the normal NLL function. Significant improvement can be noted with the use of the modified NLL equation in Fig. 2(b). The values for Beta and Eta are estimated much closer to the true values even at increased levels of Truncation. The accuracy of the estimates is, however, dependent on the sample size. At larger sample sizes of 150 and 200 the accuracy of estimation was higher as expected. Fig. 3 further illustrates the overall effect of Left Truncation on the Weibull PDF, where it can be noted that the Truncation percentage increases, the distribution is shifted to the right indicating increased characteristic life. Therefore, as the Truncation percentage increases, the errors in characteristic life would also increase. Larger Truncation percentages cause inaccurate magnification in β values resulting in the higher failure rate than the true value, again introducing errors in reliability of the asset.

4.2. The censoring effect

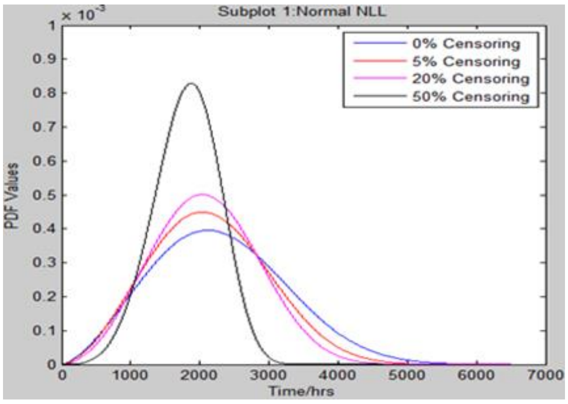
Figs. 4(a) and 4(b) illustrate the impact of Right Censoring on both Beta and Eta parameters for True Beta =2.5 and True Eta=2500 for all sample sizes. An increase in the Beta values and reduction in the Eta values were noted with increasing percentages of Censoring. Increased values of Beta indicate a higher failure rate for the asset than the true value while decreasing Eta implies a reduction in Characteristic Life. The Censoring impact is noted in the PDF curve as illustrated in Fig. 5(a) as it shifts it to the left along the time axis. This reduction in Eta decreases the Characteristic Life and exaggerates the state of unreliability. Utilization of the DE algorithm has proven effective in nullifying the effects of Censoring as demonstrated in Fig. 5(b). There is a minimal impact on the PDF curves even at a high percentage of Censoring. An added benefit noted with the use of the DE algorithm was its ability to enforce repeatability in the results. Under different internal parameters, the DE algorithm shows no variation in the results, indicating that it is robust.



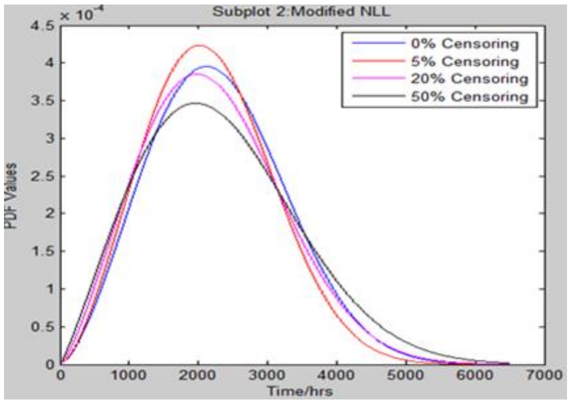
(a) Effect of censoring percentages on Beta and Eta values with normal NLL

(b) Effect of censoring percentages on Beta and Eta values with modified NLL

Fig. 4 Effect of censoring percentages on Beta and Eta values



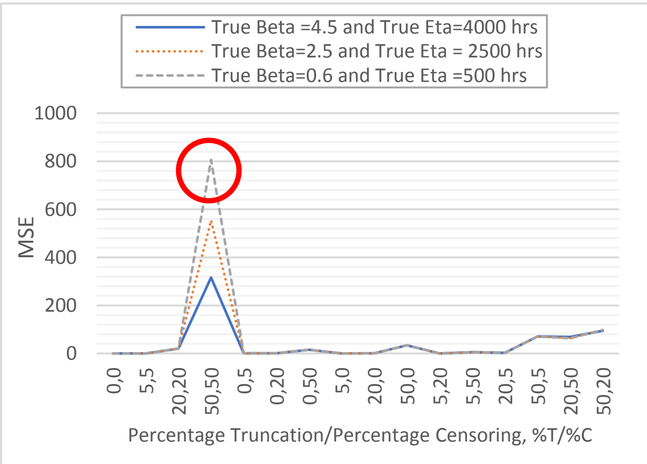
(a) Effect of increased censoring percentages on the PDF curve with normal NLL



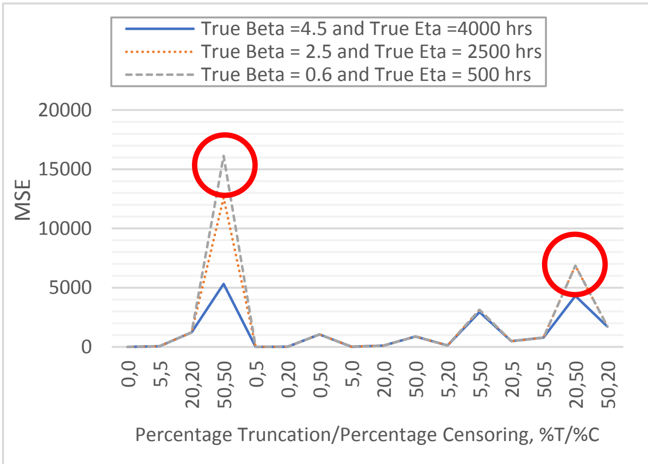
(b) Effect of increased censoring percentages on the PDF curve with normal NLL

Fig. 5 The Effect of increased censoring percentages on the PDF curve.

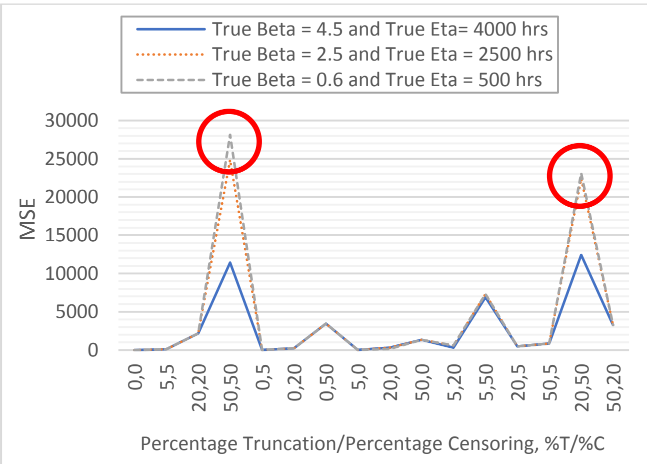
4.3. Left truncation and right censoring



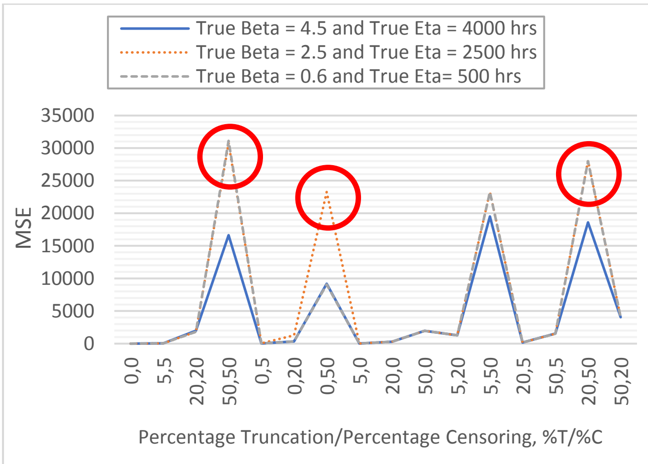
(a) MSE values for sample size 20 with normal NLL equation



(b) MSE values for sample size 100 with normal NLL equation



(c) MSE values for sample size 150 with normal NLL equation



(d) MSE values for sample size 200 with normal NLL equation

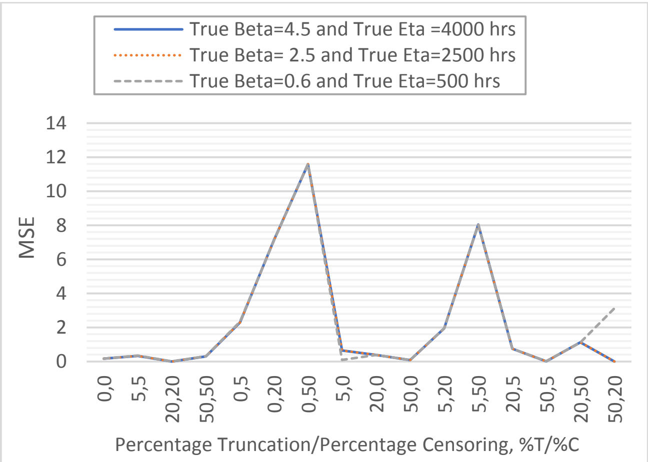
Fig. 6 MSE values for three simulated Weibull parameter vector sets under varying combinations of truncation and censoring

The majority of testing investigated different percentage combinations of LTRC data. Comparison of the estimates illustrated the benefits when utilizing the modified NLL equation and DE algorithm. Table 2 summarizes the results of MSE for different distribution parameters at increasing combinations of Truncation and Censoring. Although the percentages of Truncation were increased from 5% to 50%, the increase in the MSE values remained low, indicating that the estimated values are much closer to the true values. A range of the MSE from 0.89 to 16,133 was obtained in the case of True β=0.6 and True η=500 under the normal NLL while a range from 0.13 to 5.97 was noted under the modified NLL and DE algorithm for Truncation and Censoring percentages increasing from 0% to 50%. Similar ranges were obtained with the other Weibull Parameter vector sets as well. The reduction in MSE levels were noted for all sample sizes tested. Table 2 shows the results for sample size of 100. In the current work, a smaller range of sample sizes was tested compared to past publications. This was done to make the practical and simulated datasets of similar sizes for comparison purposes.

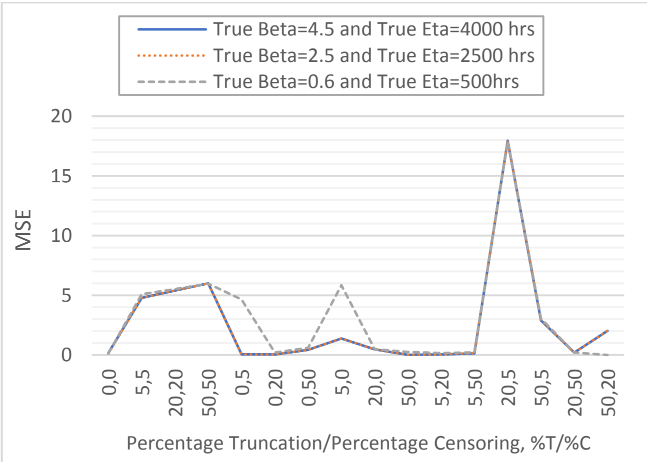
Table 2 MSE values for normal and modified NLL equations at sample size 100 for combinations of percentage truncation and censoring (%T/%C)

Mean Squared Error (MSE)	True β=0.6 and True η=500				True β=2.5 and True η=2500				True β=4.5 and True η=4000			
	Percentage Truncation/Percentage Censoring											
	0/0	5/5	20/20	50/50	0/0	5/5	20/20	50/50	0/0	5/5	20/20	50/50
Normal NLL	0.89	54.0	1220	16133	0.115	53.7	1221	12599	0.17	53.8	1220.6	5313.7
Modified NLL	0.13	5.09	5.51	5.97	0.116	4.78	5.39	6.00	0.17	4.78	5.39	6.01

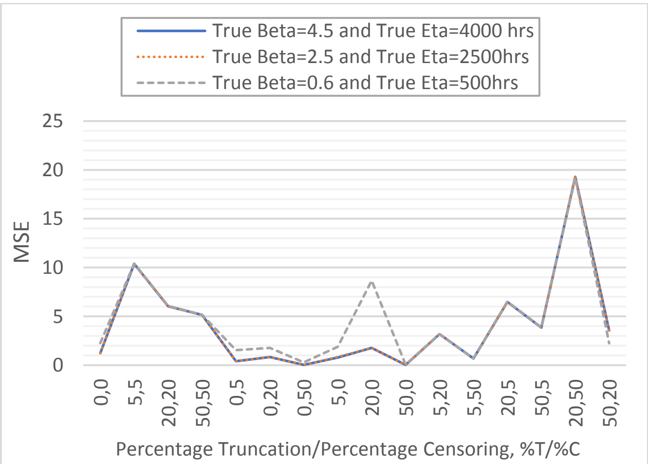
The reduction and stabilization of MSE values are noted in Fig. 7 (a)-(d). The larger differences which were noted across the Weibull distributions at higher percentages of Censoring were also minimized. In addition, contrary to Fig. 6 which shows a steady increase in the MSE for increasing sample sizes, these values have remained consistently lower throughout all sample sizes when the modified NLL and DE algorithm are implemented. This indicates a much more precise and accurate estimation to the true values.



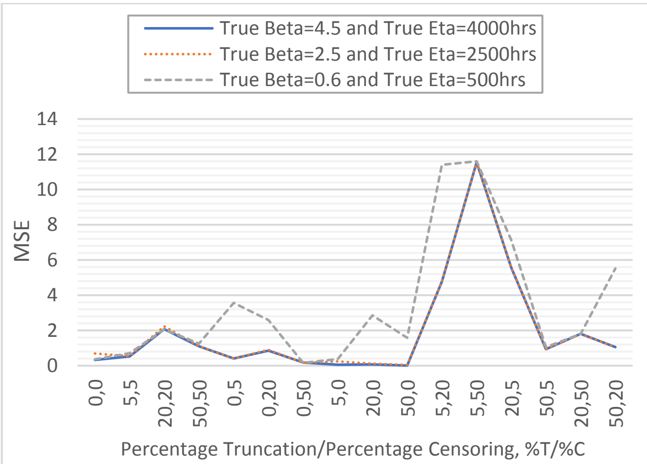
(a) MSE values for sample size 20 with modified NLL equation



(b) MSE values for sample size 100 with modified NLL equation



(c) MSE values for sample size 150 with modified NLL equation



(d) MSE values for sample size 200 with modified NLL equation

Fig. 7 MSE values for three simulated Weibull parameter vector sets under varying combinations of truncation and censoring

4.4. Practical application

The data used for the practical validation was the failure times relating to an Offshore Platform Gas Engine Generator. The Power Generator is a critical asset on the self-sustainable platform since it provides a source of power to many other pieces of equipment. Since the failure time data from the organization's Computer Maintenance Management System (CMMS) is incomplete, modified approaches are required to account for these deficiencies. Traditional Weibull Analysis has been performed on the Generator by using Reliasoft's Weibull/ Alta software package. Subsequently, the dataset with n=40 was, then, entered into the developed algorithm and tested to determine the estimated parameters of Beta and Eta which would account for the LTRC effect.

Fig. 8 illustrates the obtained results and compares it to the traditional Weibull analysis. Since there is no information on the failures prior to the observable period, the Truncation percentage was calculated by using time. The number of failures obtained during an eight-year observable period from 2007 to 2014 was used to estimate the numbers of failures missing over a five-year period. However, it should be noted that this Truncation Percentage is calculated on the assumption of a constant failure rate. To validate this assumption, a graphical Cumulative Failures versus Time trend test was performed on the data for which the correlation coefficient (R^2) was found to be 97%. This allows for reasonable justification of the assumption and calculation of the Truncation percentage. The final values noted were 40% Left Truncation and 5% Right Censoring. The estimated values of Beta and Eta were found as 3.32 and 91,058 hours respectively under the DE algorithm and modified NLL as opposed to 4.13 and 95,747 hours under the traditional approach based on the normal NLL equation not considering the Truncation and Censoring impact. Since the percentage of Truncation is 40% whilst the Censoring impact is only 5% the increases in both Beta and Eta values are higher than the values obtained under the developed DE with modified NLL equation. This is similar to the demonstrated cases of Truncation impact in the experimental study in section 4.1, where increases in both Beta and Eta values were noted with higher levels of Truncation.

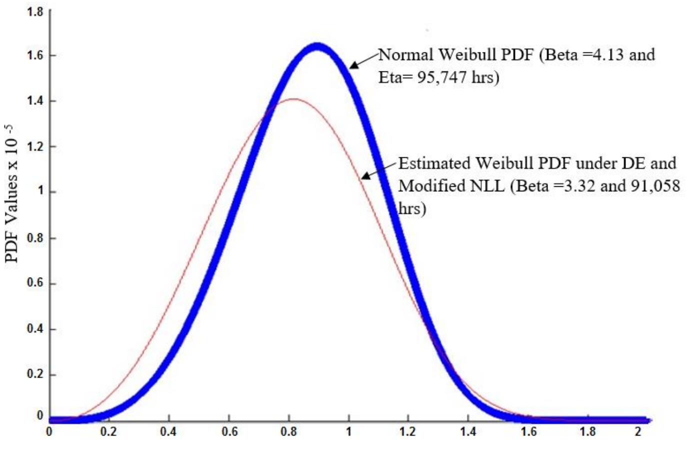


Fig. 8 PDF comparison plot of traditional Weibull testing and the modified approach for a power generator

5. Conclusions

Much of the maintenance databases are plagued with Truncated, Censored, or missing data, thus the accurate prediction is hampered leading to the poor maintenance decisions. Reductions in the MSEs show that the parameter estimations are much closer to the true values when using the Differential Evolution (DE) algorithm approach for comprehensive range of combinations of LTRC. Results from experimentation with the individual effect of increasing percentages of Truncation highlighted a general tendency to progressively overestimate the values of both β and η. In the case of censored data, while the β value is increased, the η value decreased with larger percentages of Censoring. Under the practical application, a difference of 0.81 was noted in the β parameter and 4,689 hours for η. Both values were greater under the normal NLL (traditional technique) as was illustrated in cases with larger values of Truncation (40%) present in the data. Both values remained greater than 1 indicating the asset was in a state of Reliability Degradation. The algorithm has provided a simple and efficient way of correcting such issues. The use of the current approach however requires knowledge of the number of data points missing as illustrated in the practical application which may not always be available. The future work will target further improvement and refinement of the algorithm so that these data points can be estimated more accurately.

Acronyms	
MLE	Maximum Likelihood Estimation
DE	Differential Evolution
NLL	Negative Log-Likelihood
LL	Log-Likelihood
MSE	Mean Squared Error
2-P	2 Parameters
LTRC	Left Truncated and Right Censored
EM	Expectation Maximisation
NR	Newton Raphson
MCS	Monte Carlo Simulation
PDF	Probability Density Function
GA	Genetic Algorithms
PSO	Particle Swarm Optimisation
SA	Simulated Annealing
CMMS	Computer Maintenance Management System
Notations	
β	Beta Parameter
η	Eta Parameter
θ	Parameter vector for 2-P Weibull distribution (β, η)
n	Sample Size of Data Points
x_i	Data Point i from sample
X	Vector of Data Points
v	Truncation Indicator
τ	Censoring Indicator
$L_X(\theta)$	Likelihood Equation
R^2	Correlation Coefficient

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