Islamic Acquisition of the Foreign Sciences: A Cultural Perspective

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The study of the transmission and transformation of ancient science is more than a study of which texts were translated, when, and by whom. It was a complex process, better seen as beginning rather than ending with the translation of relevant books, for the heart of the process is the assimilation rather than the simple reception of the material. Scientific ideas move because people study books, compute with tables, and use instruments, not simply because they translate books, transcribe tables, or buy pretty artifacts. It suffices to recall that the scholars of the Byzantine Empire, despite their status as the direct heirs of the classical Greek scientific tradition and their direct access to whatever classical Greek manuscripts the Islamic world eventually came to possess—indeed to more of them and from an earlier date—were largely uninterested in this knowleldge. Hence no account of the transmission of scientific knowledge can be complete if it does not recognize that it is, at root, an account of the activities of what Dupree has called "*homo sapiens* in a social context."[1]

Two Caveats

At the outset of this paper, two points must be taken into consideration. First, although we may wish to study the whole process of the Islamic acquisition of the foreign sciences as it took place over several centuries and over an area extending from Spain to Afghanistan, it must be realized that the examples given refer to specific events that took place at specific times and in specific places. As a result, eminent Islamic thinkers and writers are quoted without any accompanying claim that each one is representative of all Islamic thinkers at all times and in all places. It is sufficient that when a person such
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[1]A. Hunter Dupree, "The History of American Science: A Field Finds Itself," *American Historical Review*, no. 71 (1969): 869. Quoted from Paul Forman, "Sarton Medal Citation," *Isis* 82:2:213 (1991): 282.

thinkers at all times and in all places. It is sufficient that when a person such as Ibn Khaldūn is quoted that his statement be accepted as indicative of an important attitude held by some of the intelligentsia in North Africa and Muslim Spain in the latter half of the fourteenth century C.E. In order to show the importance of various cultural factors, it is not necessary to demonstrate what is false, namely that each of them operated at all times and in all places.

Secondly, when cultural factors in the transmission and transformation of ancient mathematics in medieval Islamic civilization are studied, a problem arises: there are cultural factors that condition *our* thought. This can be seen most clearly when it is realized that contemporary Western scholars are members of a civilization whose mathematical development was largely dependent on the contributions of medieval Islamic civilization. As Shlomo Pines has put it: "The history of science, in contradistinction for instance to many approaches to the history of philosophy, has ... a built-in principle of valuation. In assessing scientific achievements of the past, it tends to judge their value by the similarity or opposition to the conceptions of modern science."[2] This creates historiographic problems which would hardly arise were we discussing the transmission of Chinese mathematics to Japan, for our undoubted debt to medieval Islamic civilization may be responsible for a tendency to slip unconsciously from this historical fact and to approach any study of the transmission process as one whose end was the creation of our mathematics.

This approach has had two quite opposite but equally regrettable results. The first is to treat medieval Islam as a civilization deserving our gratitude for being a channel through which the great works of the Greeks were carried safely to the eager minds of the European Renaissance. The emphasis falls on the two great periods of translation, the ninth century C.E. (into Arabic) and the twelfth and thirteenth centuries C.E. (into Latin), while the developments of the intervening centuries offer little more than a series of anecdotes about one curious result or another that was proved by an occasional great figure.

The second result of this attitude is to stress those elements of Islamic science closest to our own, for the goal now is to show how far Islamic science prefigured that of modern times. To cite only mathematical examples, rings of polynomials, non-Euclidean geometry, mathematical induction, parts of differential calculus, and other mathematical concepts can be found in Arabic texts, according to the adherents of this approach. As it would be invidious to cite contemporary examples of either of these approaches, and of little interest to cite earlier examples, such matters will not be dealt with here. However, the observation that both of these results, which on the surface seem to place such different values on Islamic civilization, should concur in

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[2] S. Pines, "What Was Original in Arabic Science," *Scientific Change*, ed. A. C. Crombie (New York, 1963).

valuing it only insofar as it served ends not its own is hardly surprising, as both are motivated by a basic interest not in the past but in the present.

Four Factors Affecting Islamic Assimilation

There are several factors affecting the Islamic assimilation of foreign sciences. The first one is the principal unifying factor of Islamic civilization: the religion of Islam, which includes both moral and religious teachings and the Sharī'ah.

The Sharī'ah contributed to Islamic civilization in two ways. On a general level, it provided a set of precepts on which was based a unified attitude towards most of life's activities. One specific requirement—the pilgrimage to Makkah at least once in the lifetime of any Muslim physically and financially able to do so—had such an important effect in its own right that it demands a separate mention here. This requirement legislated the means for the cultural interchange that is such a marked feature of Islamic civilization. The resulting unprecedented scale of travel provided Muslims from Afghanistan, for example, a chance to meet and to share life's great experience with Muslims from Samarqand or Spain. It also affected Muslim science by creating a sacred center of the Muslim world, a development that provided a viable alternative to Greek scientific geography, a sacred geography which resulted in the production of a special literature, special methods for finding the direction of Makkah (the *qiblah*), and the making of maps based on this geography.[3]

The second factor, and another unifying principle in this vast empire, was the Arabic language, a tongue chosen by God as the vehicle for His final revelation to humanity. Since the Qur'an was decreed to be untranslatable, the Arabic language automatically became the first subject studied in Muslim education, and thus was created a *lingua franca* of the written and, for scholars, the spoken word. Fruits of this luxuriant linguistic growth are now preserved in our own contemporary scientific language: alcohol, algebra, algorithm, Betelgeuse (indeed, most star names), azimuth, alkali, zero, zenith, nadir, and cipher.

This supremacy of Arabic in the religious sciences seems to have had an effect on the mathematical sciences where, after an initial period of transliteration of the difficult terms, the foreign sciences took on a generally Arabic garb. It is possible that this represents a response to a strong social bias in favor of a particular language, for the Arabization of foreign sciences (Islamization would come later) must have been an important factor in any judgment made by the Muslim intelligentsia concerning their acceptability. In a nearly

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[3]David King has drawn attention to this tradition in medieval Islam.

contemporary and linguistically similar case, that of Syriac, the situation was otherwise. Rosenthal cites as an example the Syriac translators' failure to coin a Syriac equivalent for the Greek word *oudeteros* (neuter) and remarks: "The number of comparable cases is extremely large. Syriac thus became hospitable to Greek words, some of them long known .. and fully adapted, many others .. very conspicuous in their foreignness."[4] That quite the opposite happened in Arabic was, I suggest, partially the result of cultural factors.

Thus Islam provided a common faith, law, and language that created an intellectual unity which was enriched, rather than destroyed, by geographical, ethnic, and political diversity.

A third factor, one which gave an element of diversity to this unity, was the widespread tolerance of other religious groups within the Islamic world. The fact that Christians, Jews, and Sabians came to share in an "Islamicate" culture had widespread consequences for the spread of the foreign sciences in the Islamic world. What would have been the effect on Islamic science if scholars like Ḥunayn, Qusṭā, Mā Shā' Allāh, or Thābit had been excluded from intellectual participation in the Islamic community? It is a paradox that, although Islam is unthinkable apart from the revelation to the Prophet, Rosenthal is correct when he writes that Islamic civilization as we know it "would simply not have existed without the Greek heritage,"[5] which was communicated in large part by non-Muslims.

A fourth factor, one which encouraged variety within a larger unity, was the tendency of large kingdoms to break up into smaller units. For example, the 'Abbāsid revolt in the mid-eighth century C.E. and the subsequent political separation of Muslim Spain and North Africa were only the earliest of many examples of schisms with important consequences for the development of science. On a general level, Djebbar, in his study of Ibn Bajjā, has noted that the fragmentation of Andalusia into small kingdoms gave rise to a "fruitful scientific competition between such regional centers as Seville, Toledo, Valencia, Cordova and Saragossa."[6] One thinks, too, of the differences in forms of the numerical ciphers and the development of an arithmetic-

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[4]F. Rosenthal, "Near Eastern Continuity in Translation Activity: Syriac and Arabic," unpublished.

[5]F. Rosenthal, *The Classical Heritage in Islam* (Berkeley: University of California Press, 1975), 14. Rosenthal relates this phenomenon to the fact that "for them [the Syriac writers] Greek represented a higher civilization," an attitude reflected in the oft-quoted remark of Bishop Sebokht apropos of the Hindu numerals that "if those who believe they have arrived at the limit of science because they speak Greek had known these things they would be convinced, even if it is a bit late, that there are others who know something, not only Greeks but men of a different language." Of course the Syriac writers could not share the Arabic writers' confidence that God had favored their language by making it the vehicle of His final revelation to humanity.

[6]A. Djebbar, "Abu Bakr ibn Bajja et les Mathématiques de son Temps" (preprint, U. de Paris-Sud), 5-6.

algebraic symbolism unique to the western regions of the Muslim world. Indeed, Sabra has pointed out one of the consequences of the "heightened Andalusian self-consciousness" found in many thinkers of that region, beginning with Ibn Ḥazm of Cordova in the eleventh century C.E. and finding its fullest expression with al Bitrūjī: a revolt against Ptolemaic astronomy very different in character from that which arose in the East with Ibn al Haytham and culminated in the work of the Marāghah astronomers.[7]

Consequences for the Assimilation of Foreign Mathematics

The Islamic basis of the civilization described above had profound consequences for mathematics. First of all, a great deal of the fundamental attitude was reflected in the division of knowledge into two categories: 1) the religious, traditional, or Arabic sciences, and 2) the rest, variously labeled "the sciences of the ancients," "the philosophical sciences," or "the foreign sciences."[8] In the words of Ibn Khaldūn, the fourteenth-century North African scholar, the religious sciences are founded on the legal material of the Qur'an and the Sunnah and include the sciences of Qur'an interpretation and reading, the various subdivisions of religious law, speculative theology, and dream interpretation. The other—ancient—sciences (i.e., logic, the four mathematical sciences, physics, and metaphysics) are studied by people of all religions.

Unfortunately, much of the writing on this topic betrays a tendency to take the data cited in Goldziher's classic study,[9] as if they represent the only Islamic attitude toward the ancient sciences. Goldziher himself must share some of the responsibility for this sort of reading of his data when he writes: "As soon as someone displayed an interest in the *'ulūm al awā'il* [ancient sci-

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[7]In a paper delivered at a session on "The Idea of Progress in Different Cultures," Sabra refers to this Eastern critique of Ptolemy's astronomy as an endeavor which, unlike the critique of his *Optics*, failed to unseat the work as the dominant one in the field. Sabra also poses the question, although he says that he is not "overhopeful" of the answer, of whether one might find cultural factors to have been of importance. I would hazard the guess that there is an explanation which, if not cultural, at least relates to the sociology of science. This is that, after Ibn al Haytham, there were very few people working on optics, and hence there was no strong commitment to a received paradigm. I say this on the basis of the following facts mentioned by Sabra in the "Introduction to Part II" of *The Optics of Ibn al-Haytham* (London, 1989). They are: (1) "After the eleventh century Ptolemy's Optics seems to disappear from ... medieval Arabic scholarship," and (2) " ... no mention of [Ibn al-Haytham's] Kitab al Manazir has so far been found in the writings of Islamic mathematicians and philosophers of the eleventh and twelfth centuries." (Nor, indeed, in the ophthalmological literature through the end of the thirteenth century.)

[8]For the prevalence of this division, see Rosenthal, *The Classical Heritage in Islam*, 54ff.

[9]Ignatz Goldziher, "Die Stellung der alten Islamischen Orthodoxie zu den antiken Wissenschaften," *Abhand. der Preuss. Akad. d. Wiss.* (Philos.-hist. Kl.) 18 (Berlin, 1916), English translation: "The Attitude of Orthodox Islam toward the 'Ancient Sciences,'" in Studies on Islam, ed. M. L. Swartz (New York: Oxford University Press, 1981), 195-215.

ences] he was regarded as suspect." Indeed, Heinen has even challenged the applicability of this division of knowledge to the intellectual situation in the first centuries of Islamic civilization. He argues that it is a backwards projection of a situation that was valid only from the tenth century C.E. and onwards, and further claims that there is some evidence to suggest that these distinctions did not exist in those early centuries. As Heinen's arguments are persuasive, they deserve some consideration when the cultural norms that were operative in the eighth and ninth centuries C.E., the period of the initial acquisition of the foreign sciences, are being assessed.

However, Heinen's argument does not affect the fact that assimilation is an ongoing process. In assessing the way in which Greek science was assimilated, the distinction between "religious" and "ancient" sciences must be kept in mind. For example, an important segment of Islamic thinkers—al Ghazzālī among them—argued that although a knowledge of the basics of the ancient sciences was important for the Islamic community, excessive study of them could lead to conceit and a falling away from the faith. A remark by Ibn Khaldūn sums up this ambivalent situation very nicely:

> Many scientists *restricted* themselves [italics added] to cultivating the mathematical disciplines and the related sciences of astrology, sorcery and talismans.... The intellectual sciences and their representatives ... seduced many people who were eager to study these sciences and accept the opinions expressed in them. In this respect the sin falls on the person who commits it.

A recognition of widespread hostility is also reflected by al Bīrūnī in his great work on mathematical geography, in which he says that unfortunately men of his time are proud of their ignorance and plot to harm the learned scholar. Among them the extremists "stamp the sciences as atheistic," while the "rude and stubborn critic" only cloaks his hostility with the seemingly wise question: what is the benefit of these sciences?

Although enthusiasts such as al Bīrūnī made it clear that they believed in the maxim "the more the better" as regards the study of the mathematical sciences, Ibn Khaldūn probably reflected the attitude of the ʻulama as a whole when he warned that excessive study in this area could misdirect one's efforts. Abū Bakr al Rāzī, a ninth-century C.E. physician, writes: "As for mathematics I confess that I have only studied this subject to the extent that was absolutely indispensable, not wasting my time on tricks and refinements.... I make (my excuse) boldly on the grounds that what I have done is the right course, rather than the way chosen by those so-called 'philosophers' who devote their whole lives to indulging in geometrical superfluities."

One observes in the above quotations two strands of criticism: identifying the foreign sciences as potentially subversive of the faith and stigmatizing them as, on the whole, superfluous to the needs of life both here and in the hereafter. These criticisms are combined in the thought of al Ghazzālī who, conceding that some geometry, arithmetic, medicine, and logic are necessary to a Muslim community, nevertheless asks rhetorically how many experts in mathematics, medicine, or astronomy a given community needs. He also points out the danger in the study of mathematics as being that persons who are ignorant of its companion sciences, such as astrology, might conclude that they are as certain in their arguments as mathematics is in its arguments.

These comments reflect a basic concern that knowledge advance the well-being of the community[10] and not just provide an intellectual stimulus to the individual. To the extent that Islamic thinkers share this view, they come to value intellectual endeavors less as a means of providing an indivi-dual with satisfaction through his/her solution of intricate problems than as contributing to the better functioning of the community. Even if such utilitarian criticism as was given by al Bīrūnī's "rude and stubborn critic" discouraged only a few of those otherwise willing to endure the hardships of a life devoted to the foreign sciences, its consequences can be proportionately large when the base of scientific workers was as small as it was in ancient and medieval cultures.

Consequences for Education

The division of knowledge referred to above was reflected in the educational system, through which elementary school pupils obtained an introduction to the religious disciplines. Thus the rules of arithmetic were usually taught as a propaedeutic to the laws of inheritance, as the relevant rules in this area required only a mastery of fractions and the solution of simple linear equations. There is, however, no evidence that any other mathematical subject, even practical geometry or algebra, was taught at this level. Certainly, in an attempt to show the community the utility of their subject, many writers on algebra tried to include sections in their books on the application of algebra to the laws of inheritance. For example, al Khwārizmī mentions inheritances and legacies as the first of the applications of his algebra,[11] but not everyone was taken in by this. Thus Ibn Khaldūn writes:

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[10]This must be understood broadly, not simply in a narrow economic sense. Thus, A. Heinen points out in "Islamic Values and Scientific Methods Imported from Abroad" (unpublished) that al Khāzinī is interested in balance as the type of justice in the community, not simply justice in commercial relations, but conceived as "the union of all knowledge and accomplished action."

[11]"That fondness for science, by which God has distinguished the Imam al-Ma'mun .. has encouraged me to compose a short work on Calculating by .. Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, ..." Rosenthal's translation.

> Religious scholars in the Muslim cities have paid much attention to them (the laws of inheritance). Some authors are inclined to exaggerate the mathematical side ... and to pose problems requiring for their solution various branches of arithmetic such as algebra, the use of roots, and similar things. It is of no practical use in inheritance matters, because it deals with unusual and rare cases.

Turning from elementary instruction, where it appears that arithmetic was the sole mathematical component, we find that the only options for advanced instruction were private tutoring or the *madrasah*, many of which had excellent libraries and permanent chairs. In general, however, as was required for *waqf* institutions, the chairs in the religious schools were in the religious disciplines, mainly law, and only rarely offered any mathematical in-struction.[12]

In essence, if one wanted to learn mathematics beyond arithmetic, one got a tutor. For example, the famous twelfth-century algebraist al Samaw'al says in his autobiography that after he finished his religious studies at the age of thirteen (at that time he would still have been an adherent of Judaism) his father hired Ibn al Daskarī to introduce him to the study of Indian arithmetic and the solution of equations. After a further study of algebra, accounting, and surveying, he studied Euclid with Ibn al Daskarī and Ibn al Naqqāsh (the epithet of an astrolabist?). When he came to advanced books, such as the latter parts of Euclid's *Elements* and works by the algebraist al Karajī, he records that he found no one in twelfth-century Baghdad able to tutor him in such things. Ibn Sīnā specifically mentions the man with whom he read Euclid and Ptolemy. He also says that he learned Hindu arithmetic only when Ismā'īlī missionaries arrived in Bukhārā and his father sent him to a greengrocer for instruction in Indian arithmetic.[13] Indeed, so common was tutoring that it gave rise to stories such as the wandering scholar who overheard a tutor making mistakes while instructing his charge on Euclid. The stranger interrupted the lesson to correct the tutor and, when the boy realized that the stranger was right, the wanderer was hired.

One body of mathematical knowledge which we know was taught is found in a collection of treatises known as the Middle Books.[14] These writ-

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[12]Sonya Brentjes has found cases where there was even a Chair in the ancient sciences. But it must be remembered that (1) in such cases we do not know if the instruction centered mainly on philosophical and logical points and (2) there were no *madrasah*s before the eleventh century, by which time much of the foreign sciences had been assimilated.

[13]Taken from A. J. Arberry, *Aspects of Islamic Civilization* (London: G. Allen and Unwin, 1964), 136.

[14]So-called (by Arab authors) for their middle place in instruction between the *Elements* and the *Almagest*. See J. L. Berggren, "Spheres on Influence: The Transmission of the Science of Spherics from the Greek to the Islamic World," in the *Actes du Colloque pour l'histoire des Mathématiques Arabes* (Paris: Institute du Monde Arab, 1990), forthcoming in 1992.

ings deal with spherical geometry and its application to astronomy. This is undoubtedly due to the recognition that such knowledge was essential to thefound in a collection of treatises known as the Middle Books.[15] These writ-study of astronomy which, in turn, was demonstrably useful to the Muslims' religious life. Without such expertise, how could Muslims determine the time for prayer, the beginning and ending of Ramadan, and the direction of Makkah? Muslim scientists quickly pointed out that such problems could be solved accurately only with the aid of the mathematical sciences, which then became acceptable due to the benefits they conferred on the Muslims.

There were, however, two important qualifications to the advantages which astronomy and mathematics enjoyed as an ancilla to the religious sciences. One was the strong competition it received from its traditional Islamic counterpart, which was based on the Qur'an and the Sunnah. The other was its association with astrology and the occult.

Indigenous Competitors to the Foreign Sciences

As to the first, studies by King and Heinen have explored the religious sciences corresponding to the foreign sciences and have shown that there was a pervasive parallelism between the two. Islamic prophetic medicine, as exemplified by Ibn al Sunnī's tenth-century C.E. treatise on the subject, corresponds to the medical sciences. The Islamic cosmology of such writers as al Suyūṭī contrasts with that of Ptolemy, while the mathematical methods of determining the local azimuth of Makkah corresponded to the traditions of a sacred geography.

The tradition of folk astronomy, sanctioned by religion, was also very strong. For an eleventh-century C.E. writer like al Khatīb al Baghdādī, a contemporary of 'Umar al Khayyām, Qur'anic texts made it plain that it was star lore and its use for the essentials of life that defined the legitimate part of astronomy. In the thirteenth century C.E., the Yemeni legal scholar al Asbahī wrote: "The times of prayer are to be found by observation with one's eyes. They are not to be found by the markings on an astrolabe or by calculation using the science of the astronomers .... The astronomers took their knowledge from Euclid, the Indian astronomical tradition recorded by the authors of the Sindhind, as well as Aristotle and other philosophers, and

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[15]So-called (by Arab authors) for their middle place in instruction between the *Elements* and the *Almagest*. See J. L. Berggren, "Spheres on Influence: The Transmission of the Science of Spherics from the Greek to the Islamic World," in the *Actes du Colloque pour l'histoire des Mathématiques Arabes* (Paris: Institute du Monde Arab, 1990), forthcoming in 1992.

all of them were infidels."[16] In other words, parts of the mathematical sciences might find an institutional home in the office of the *muwaqqit* (the one who determines the time), but there was a price to be paid: scientific autonomy. To be sure, one must balance the strictures of al Asbahī with the achievements of Ibn al Shāṭir, but I would posit that Ibn al Shāṭir and al Khalīlī are famous not because they were typical *muwaqqit*s, but because they were exceptional.

There was even an alternative Muslim science of arithmetic, one that al Jāḥiẓ advocated be taught to schoolchildren in place of the Hindu system. It is important to look at this system in some detail, for this oft-ignored science can tell us how medieval Islamic mathematics looked to its contemporaries.

We have seen that although arithmetic was taught in the schools, both Ibn Sīnā and al Samaw'al learned Hindu arithmetic from tutors. What arithmetic, then, did Ibn Sīnā learn in school? It would hardly have been that of the astronomers, that sexagesimal arithmetic inherited from the Greeks which formed the astronomers' most commonly used alternative to Hindu arithmetic.

A hint as to the answer comes from a report by Ḥamd Allāh Muṣtafī, who attributes to Ibn Sīnā the invention of calculation by fingers and, thereby, the liberation of accountants from the use of counters. This system, variously known as "hand reckoning," "joints reckoning," "digit reckoning," or even "mental reckoning," was certainly not Ibn Sīnā's invention: it was already well-established by the early ninth century C.E. when al Jāḥiẓ, in his *Book of Teachers,* urged his readers to instruct their charges in "joint reckoning" as opposed to the system of the Hindus. In fact, he goes so far as to argue that it is divinely sanctioned and quotes in support of his claim verses from the Qur'an referring to reckoning the days and nights.[17]

Thus furnished with a divine pedigree, finger reckoning became the mathematics of the scribes and merchants. According to the early tenth-century C.E. writer al Sūlī, scribes preferred this system over the use of written numerals. Later in the century, Abū al Wafā', one of the great scientists of his age, devoted an entire book to the subject. This included an arithmetic of fractions based on the canonical set of unit fractions with denominators less than eleven. The ostensible reason for this was that these, alone among all fractions, could be expressed in one Arabic word, a clear indication of Arabic's importance. His work dealt with such topics as the mental extraction of

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[16]This was an extreme attitude, and another religious scholar, the *mutakallim* al Jāhiz, writes in a diatribe directed against the scientists of his day: "Here are the books we have in common, such as the book of Euclid, the book of Galen, the Almagest, which was procured by al Hajjāj, and many books not considered for general publication."

[17]Interestingly, in the same century that Muhammad was teaching these revelations to his followers, the Venerable Bede was teaching his Northumbrian monks finger reckoning for the same purpose.

square and cube roots, the closeness of approximation of fractions by canonical fractions, and the use of negative numbers in arithmetic (the first time such numbers are mentioned in Islamic mathematical writings).

Nor was Abū al Wafā' the only mathematician of note to write on finger reckoning. No less an algebraist than al Karajī wrote a separate treatise on it: his *Satisfying Book of Reckoning*. It seems that astronomers also used it in their computations, since its techniques were adapted to working with base-60 fractions, i.e., "the astronomers' fractions." This is most likely the system used by the Timurid king Ulūgh Beg to compute the astronomical date, i.e., the longitude of the sun, correct to degrees and minutes, while on his horse.[18]

The variety of patterns composing the fabric of Islamic reckoning is indicated by the contents of al Baghdādī's eleventh-century treatise, *Compendium on Reckoning,* which included hand reckoning, the systems of the Hindus and of the astronomers, and even the theoretical arithmetic of the Greeks. It is also doubtless true, as recent literature suggests,[19] that the number of such treatises represented a response to the needs of the officials in the numerous *dīwāns* spawned by the disintegration of the 'Abbāsid Empire into a number of petty principalities.[20] It is less clear, despite what has also been claimed, that one result of this response was an effort to understand what the different systems had in common and the realization that the choice of a base and the nature of the operations were the essentials of arithmetic. On the contrary, I believe that the mathematicians did not need to wait for the 'Abbāsid Empire to break up before understanding that using the letters of the Arabic alphabet or adapting the forms of the Hindu numerals did not affect the arithmetic.

Suspect Sciences Associated with the Foreign Sciences

The other qualification to the advantage enjoyed by mathematics from its association with astronomy was that, even if such sciences were of some use, they kept such bad company. Thus astronomy went hand-in-glove with astrol-

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[18]In later centuries, finger arithmetic was reduced to little more than a collection of rules for mental arithmetic. The important scientists no longer wrote treatises on it, and when al Kāshī in fifteenth-century Samarqand and Bahā' al Dīn in seventeenth-century Maghrib wrote their great works on reckoning, they both featured the decimal system and made no mention of finger arithmetic. In fact, by the time of Bahā' al Dīn, even the sexagesimal system was no longer mentioned and the arithmetic of his audience, very much the audience of practitioners that Abū al Wafā' had in mind, was evidently now the same system which, nine hundred years earlier, al Jāhiz had recommended teachers ignore.

[19]R. Rashed, "Recommencements de l'Algèbre," reprinted in *Entre Arithmetique et Algèbre* (Paris, 1984).

[20]And this would be a further effect of the political factors we discussed earlier in this paper.

ogy, a subject as much denounced by religious Muslims as it was supported by some of the royalty. Mathematics was linked to the occult sciences in other ways. For example, medieval Islamic scholars investigated many remarkable properties of magic squares at a time in the history of these arrays of numbers when they really were associated with magic. As a result, from the time of Thābit (ninth century C.E.) to that of al Zanjānī (early thirteenth century C.E.),[21] there appeared a series of mathematical treatises on magic squares instructing people not only how to write down magic squares of any order greater than two but also giving information about such elegant variations as magic squares that were bordered so that the larger square was also magic. That the magical side of these squares affected mathematics is shown by al Zanjānī when, at the end of his treatise, he shows how to construct a magic square from any name. It should, however, be added that orthodox Muslims objected to these sciences not because they were false, but because, as Ibn Khaldūn said, they " .. are forbidden by religious law because they .. require (their practitioners) to direct themselves to beings other than God, such as stars and other things."

Further Critiques of Mathematics

These then were some of the cultural factors affecting the Islamic acquisition of mathematics. Naturally, such conditions as those described above fostered an atmosphere in which scientists could be credibly criticized from a much wider variety of standpoints than is possible today. This means that mathematics was created within the context of an interdisciplinary dialogue.

The possibility of such a dialogue rested on the fact that the parties concerned began with very similar backgrounds: both scientists and religious scholars had their formative years devoted to religious instruction. In addition, al Jāḥiẓ's statement, "Here are the books we have in common, such as the book of Euclid, the book of Galen, the *Almagest,* which was procured by al Ḥajjāj, and many books not considered for general publication," shows that, at least for some critics, the argument was not about the use of "foreign sciences" but rather the way in which they were used. Other writings of al Jāḥiẓ, quoted by Heinen,[22] suggest that the *mutakallimūn*[23] criticized the sci-

[21]For a study of this tradition, see J. Sesiano, "Herstellungsverfahren magischer Quadrate aus islamischer Zeit (I) [(2)]," *Sudhoffs Archiv* 64, no. 2, 187-196 [65(3):251-65].

[22]A. Heinen, "Mutakallimun and Mathematicians: Traces of a Controversy with Lasting Consequences," *Der Islam* 55 (1978): 57-73.

[23]*Kalām* was the Islamic science that used a wide knowledge of the religious and philosophical sciences to frame arguments designed to protect the faith from doubt and heresy. A master of this activity is called a *mutakallim*.

entists not because of their use of such texts, but rather because of their canonization of them. Thus he criticizes physicians for being overly reliant on ancient authorities and too little used to employing their own observations.

There was, however, controversy. One piece of evidence is the ninth-century C.E. polemic of al Jāḥiẓ, to which Heinen has drawn attention, concerning *kalām*. It is clear from the account given that the *mutakallimūn* regarded mathematics as a deficient science because it was limited to deductive arguments. In addition, its language was restricted by the requirement that each word have but one meaning which could not be modified according to new relationships that might be discovered between one thing and another at a future date. Moreover, one charge that mathematicians were evidently forced to admit is that mathematics left many of the basic concepts unexplained. *Kalām,* on the other hand, tries to go beyond the mere juggling of definitions by starting with the very basics and "to form new and more appropriate concepts whenever necessary and useful."

Nor was criticism of mathematicians on philosophical grounds limited to the early centuries of Islam, for it is also found in the late tenth-century C.E. correspondence of Abū Sahl al Kūhī with the amateur al Ṣābī. At issue is al Kūhī's derivation of a value for π, in which he uses rigorous geometric argument, and an unfortunate guess for the position of a semicircle's center of gravity based on the following chart:[24]

Figure 1: Centers of Gravity

Isosceles Triangle	1:3	Isosceles Cone	1:4
Parabola	2:5	Paraboloid	2:6
Semicircle		Hemisphere	3:8

The conjecture of al Kūhī (3:7) for the missing value was of course unavoidable—any self-respecting mathematician would have done the same! What is surprising is that he stayed with it even when he discovered that it led to a value of 3 1/9 for π, a result which fell outside of Archimedes' bounds of 3 10/71 and 3 10/70 for the value of that constant.

[24]The chart is based on the fact that the center of gravity of a body with an axis of symmetry lies on that axis and divides it into two segments, and the chart records the ratio of the shorter of the two segments to the whole axis. For a full account of this correspondence, see J. L. Berggren, "The Correspondence of Abū Sahl al Kūhī and Abū Ishāq al Ṣābī," *Journal for the History of Arabic Science* 7 (1983): 39-124.

His confidence in his finding, in the face of all criticism, does not result, in my opinion, from a fear of learning that he was wrong or, as has been suggested, from a temporary loss of reason caused by his excitement over this discovery of a value for π. In fact, it appears from his comments that a deeper reason for his attitude lay in his conviction that the technique of approximation used in *Measurement of the Circle* was fundamentally flawed. As he puts it: "... it is clear that the method [of approximation] does not lead to the truth at all ....." He also states later that "Archimedes is above seeking the measurement of the circumference of the circle by this method, and ... this is because of the greatness of Archimedes and the abasement of that method of calculation." He says repeatedly that truth comes from exact arguments built on sound premises, not from approximations. In the course of discussing values of π, al Kūhī speaks scornfully of "the physicists" (*ṭabīʿiyūn*), such as Aristotle and Galen, whose "knowledge is through opinion, dogma and likelihood."

It seems that his rejection of approximation as a valid mathematical technique stems from his desire to put as much intellectual distance between such thinkers and himself as possible. And, whatever may have been al Kūhī's own reasons for desiring this separation between himself and the philosophers, such a distance would certainly have had a cultural advantage in an intellectual climate where the value of philosophy (*falsafah*) was much more questionable than that of mathematics.

Nor did criticism of al Kūhī stop in the tenth century C.E., for, two centuries later, the physician Aḥmad al Sarī, an individual learned both in mathematics and philosophy, resumed the criticism.[25] He argued that al Kūhī had made the fundamental error, which Aristotle had exposed in his *Organon*, of using a science (mechanics) that is posterior to establish a result in one (geometry) that is anterior.

This criticism is particularly unfortunate, since it was precisely by exploiting the relationship between mechanics and geometry that Archimedes made some of his greatest discoveries. It seems that al Kūhī was heading in the same direction, since he wrote in the preface to his *Measurement of the Paraboloid* that finding the volume of the paraboloid was motivated by his desire to find its center of gravity. Therefore, I posit that philosophical criticisms such as those of al Ṣābī and al Sarī contributed to the neglect of the fruitful area opened up by al Kūhī through his use of centers of gravity to prove theorems about areas and volumes. One can only wish that al Kūhī had kept his corollary on the measurement of the circle to himself, since it very likely discredits his work on centers of gravity, work which appears to be first rate.

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[25]See the account in J. Sesiano, "Note sur trois théorèmes de Mécanique et leur conséquence," *Centaurus* 22, no. 4 (1979): 281-97.

Conclusion

To summarize, then, I have tried to suggest how cultural factors may have affected the Islamic reception and acquisition of foreign sciences. Much of what has been said in this paper is necessarily conjectural, but this does not negate the fact that Islamic mathematics was not just good Greek mathematics done by people who happened to write in Arabic. In their own eyes, Muslim scientists were responding to the needs, concerns, and criticism of a civilization profoundly different from that of classical Greece and therefore directed, as they saw fit, the growth of their science in certain directions and away from others. Thus, they approached mathematics on the terms of their own culture. If we are to understand the results of this approach, we must try to understand what those terms were.