Color Image Denoising Using Stationary Wavelet Transform and Adaptive Wiener Filter

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Abstract

The denoising of a natural image corrupted by Gaussian noise is a problem in signal or image processing. Much work has been done in the field of wavelet thresholding but most of it was focused on statistical modeling of wavelet coefficients and the optimal choice of thresholds. This paper describes a new method for the suppression of noise in image by fusing the stationary wavelet denoising technique with adaptive wiener filter. The wiener filter is applied to the reconstructed image for the approximation coefficients only, while the thresholding technique is applied to the details coefficients of the transform, then get the final denoised image is obtained by combining the two results. The proposed method was applied by using MATLAB R2010a with color images contaminated by white Gaussian noise. Compared with stationary wavelet and wiener filter algorithms, the experimental results show that the proposed method provides better subjective and objective quality, and obtain up to 3.5 dB PSNR improvement.

Keywords: *Stationary wavelet transform (SWT), Adaptive Wiener filter, Thresholding.*

1. Introduction

Images acquired through sensors [charge-coupled device (CCD)] cameras may be contaminated by noise sources. Image processing technique also corrupts image with noise, leading to significant reduction in quality. Traditionally, linear filters (mean, median, and wiener filter) are used for removing noise from images, but it blurs data [1]. It is well known that wavelet transform is a signal processing technique which can display the signals on in both time and frequency domain. Wavelet transform is superior approach to other time-frequency analysis tools because its time scale width of the window can be stretched to match the original signal, especially in image processing studies. This makes it particularly useful for nonstationary signal analysis, such as noises and transients. For a discrete signal, a fast algorithm of discrete wavelet transform (DWT) is multiresolution analysis, which is a nonredundant decomposition [2]. One of the most popular method consists of thresholding the wavelet coefficients (using Hard threshold or the soft threshold) as introduced by Donoho [3]. Elyasi and Zarmehi [4] proposed several methods of noise removal from degraded images with Gaussian noise by using adaptive wavelet threshold (Bayes Shrink, Modified Bayes Shrink and Normal Shrink). Jacob and Martin [5] performed wiener filtering on the wavelet coefficients to denoise an image degraded by an Additive White Gaussian Noise (AWGN).

Jin et. al. [6] considered the adaptive wiener filtering of noisy images and image sequences. They began by using an adaptive weighted averaging (AWA) approach to estimate the second- order statistics required by the wiener filter and extended the AWA concept to the wavelet domain and that gained 0.5 dB over traditional wavelet wiener filter.

The drawback of nonredundant transform is their noninvariance in time/space; i.e., the coefficients of a delayed signal are not a time-shifted version those of the original signal. The stationary wavelet transform (SWT) was introduced in 1996 to make the wavelet decomposition time invariant [7]. This improves

the power of wavelet in signal de-noising. This paper exploits the benefits of stationary wavelet transform in suppressing noise at high frequencies and wiener filter to suppress noise in low frequency bands. The proposed algorithm is divided into two steps. After taking SWT to the noisy image, soft thresholding method is applied to the details subbands; then a transformed image is generated from approximation subband only while the other subbands are made equal to zero, applying inverse SWT to the generated 2-D array, then applying the adaptive wiener filter, to remove the residual noise in the low frequency band. After that the approximation band is returned by applying SWT to the denoised signal, the resulted approximation subband is grouped with the thresholded subbands, applying inverse SWT to obtain the denoised image. The proposed method is compared with the other two traditional denoising methods, namely SWT and Wiener filter, to validate the denoised characteristics of this method.

2. Stationary wavelet method

This section presents the basic principals of the SWT method. In summary, the SWT method can be described as follows. At each level, when the high-pass and low-pass filters are applied to the data, the two new sequences have the same length as the original sequences. To do this, the original data is not decimated. However, the filters at each level are modified by padding them out with zeros.

Supposing a function $f(x)$ is projected at each step j on the subset $V_j (.... \subset V_3 \subset V_2 \subset V_1 \subset V_0)$. This projection is defined by the scalar product $c_{j,k}$ of $f(x)$ with the scaling function $\phi(x)$ which is dilated and translated.

$$c_{j,k} =< f(x), \phi_{j,k}(x) > \qquad ... (1)$$

$$\phi_{j,k}(x) = 2^{-j}\phi(2^{-j}x - k) \qquad ... (2)$$

where $\phi(x)$ is the scaling function, which is a low-pass filter. $c_{j,k}$ is also called a discrete approximation at the resolution 2^j. If $\varphi(x)$ is the wavelet function, the wavelet coefficients are obtained by

$$\omega_{j,k} =< f(x), 2^{-j}\varphi(2^{-j}x - k) > \qquad ...(3)$$

$\omega_{j,k}$ is called the discrete detail signal at the resolution 2^j . As the scaling function $\phi(x)$ has the following property:

$$\frac{1}{2}\phi(\frac{x}{2}) = \sum_n h(n)\phi(x-n)$$

$h (n)$ represents the scaling coefficients.
$c_{j+1,k}$ can be obtained by direct computation from $c_{j,k}$.

$$c_{j+1,k} = \sum_n h(n-2k)c_{j,n} \, and$$
$$\frac{1}{2}\varphi(\frac{x}{2}) = \sum_n g(n)\phi(x-n) \qquad ...(4)$$

$g (n)$ represents the wavelet coefficients.

The scalar products

$$\omega_{j,k} =< f(x), 2^{-(j+1)}\varphi(2^{-(j+1)}x - k) >$$

are computed with

$$\omega_{j+1,k} = \sum_n g(n-2k)c_{j,n} \qquad ...(5)$$

Equations (4) and (5) are the multiresolution algorithm of the traditional DWT. In this transform, a downsampling algorithm is used to perform the transformation. That is, one point out of two is kept during transformation. Therefore, the whole length of the function will be reduced by half after the transformation. This process continues until the length of the function becomes one. However, for stationary or redundant transform, instead of downsampling, an upsampling procedure is carried out before performing filter convolution at each scale. The distance between samples is increased by a factor of two from scale to the next. $c_{j+1,k}$ is obtained by

$$c_{j+1,k} = \sum_l h(l)c_{j,k+2^j l} \qquad ...(6)$$

And the discrete wavelet coefficients

$$\omega_{j+1,k} = \sum_l g(l)c_{j,k+2^j l} \qquad ...(7)$$

The redundancy of this transform facilitates the identification of salient features in a signal, especially for recognizing the noises. This is the transform for one-dimensional signal. For a two- dimensional image, we separate the variables x and y and have the following wavelets.

— Vertical wavelet: $\varphi^1(x, y) = \phi(x)\varphi(y)$

— Horizontal wavelet: $\varphi^2(x, y) = \varphi(x)\phi(y)$

— Diagonal wavelet: $\varphi^3(x, y) = \varphi(x)\phi(y)$

Thus, the detail signal is contained in three subimages [8].

$$\omega^1_{j+1}(k_x, k_y) = \sum_{l_x=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} g(l_x)h(l_y)c_{j,k+2^j}(l_x,l_y) \quad ..(8)$$

$$\omega^2_{j+1}(k_x, k_y) = \sum_{l_x=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} h(l_x)g(l_y)c_{j,k+2^j}(l_x,l_y) \quad ...(9)$$

$$\omega^3_{j+1}(k_x, k_y) = \sum_{l_x=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} g(l_x)g(l_y)c_{j,k+2^j}(l_x,l_y) \quad ...(10)$$

3.   Implementation of the proposed algorithm

This section, describes the method for computing the various parameters used to compute the threshold and the image denoising algorithm. The SWT is used for the recovery of the corrupted image with adaptive wiener filter. Wiener filter is a minimum mean square error filter. It has capabilities of handling both the degradation function as well as the noise. The Wiener Filter in the Fourier domain is given by the expression:

$$F(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}} \right] G(u,v) \quad ... (11)$$

where
$H(u, v)$ is the degraded function

$H^*(u,v)$ is the complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ is the power spectrum of the noise.

$S_f(u,v) = |F(u,v)|^2$ is the power spectral density (PSD) of the undegraded image.
$G(u,v)$ is the Fourier transform of the degraded image.

When Wiener filtering is performed on small blocks of an image at a time; the method is called Local Wiener Filtering. In this method, the PSD of the undegraded image is estimated for each

block. This calculated PSD is then used in the expression of the Wiener filter. Thus, the local statistics are also accounted for in the calculation of the Wiener filtered image. Images with many edges are handled much well by the local Wiener Filter than the global Wiener filter. We used a window of size 3 x 3 in the calculation of the local [9]

The threshold value (T_{NS}), which is adaptive to different subband characteristics, is calculated by normal shrink [4].

$$T_{NS} = \frac{\beta \hat{\sigma}_n^2}{\hat{\sigma}_y} \quad ...(12)$$

where scale parameter β is calculated once for each scale using

$$\beta = \sqrt{\log(L_k / J)} \quad ...(13)$$

where L_k is the length of the subband at the k_{th} scale, J is the total number of decompositions and $\hat{\sigma}_y$ is the standard deviation of the subband.

Noise variance $\hat{\sigma}_n^2$ is estimated in equation (14), using the robust and accurate median estimator of the suddand .

$$\hat{\sigma}_n^2 = \frac{median(|Y_i|)}{0.6745} \quad ...(14)$$

$Y_i \in$ each subband
where 0.6745 is the experiential value [10].
We have actually applied wiener filter for the image obtained from (LL subband only while maintaining other subbands equal to zero) in the spatial domain by applying inverse stationary wavelet transform in order to remove the residual noise in the low frequency subband in addition for applying softthreshold for LH, HL, HH subbands, so it offered superior results in denoising.
soft thresholding   deletes the coefficients under the threshold, but scales the ones that are left. There are different ways of scaling. The general soft shrinkage rule is defined by:

$$\delta_\lambda(w) = sgn(w)(|w| - \lambda)_+ \quad ...(15)$$

Where λ is the threshold and the plus sign indicates only the coefficients that are above the threshold are considered.

The proposed algorithm is described in this section; it is applied for each (Red, Green, and Blue) band separately.

It consists of the following steps:

1- Read the noisy color image (contaminated with white Gaussian noise)
2- For each (R, G, B) band
- Apply SWT (one level of decomposition)
- Estimate the noise variance in the noisy image using equation (14)
- Calculate the scale parameter β using equation (13).
- For each detail subband, compute the standard deviation and threshold T_{NS} using equation (12).
- Apply soft threshold to the subbands LH1, HL1, HH1.
3- Reconstruct a new image from LL1 subband only while making other subbands equal to

zero, by applying inverse stationary wavelet transform.
4- Apply an adaptive wiener filter to the reconstructed image in spatial domain.
5- Reapplying SWT to the resultant image of step 4.
6- Extract the approximation subband from step 5 denoted by LL2, then grouping it with the thresholded subbands denoted by ($\hat{LH}$, $\hat{HL}$, and $\hat{HH}$) of step 2 as LL, LH, HL, and HH subbands.
7- Apply inverse stationary wavelet transform, to get the denoised image.
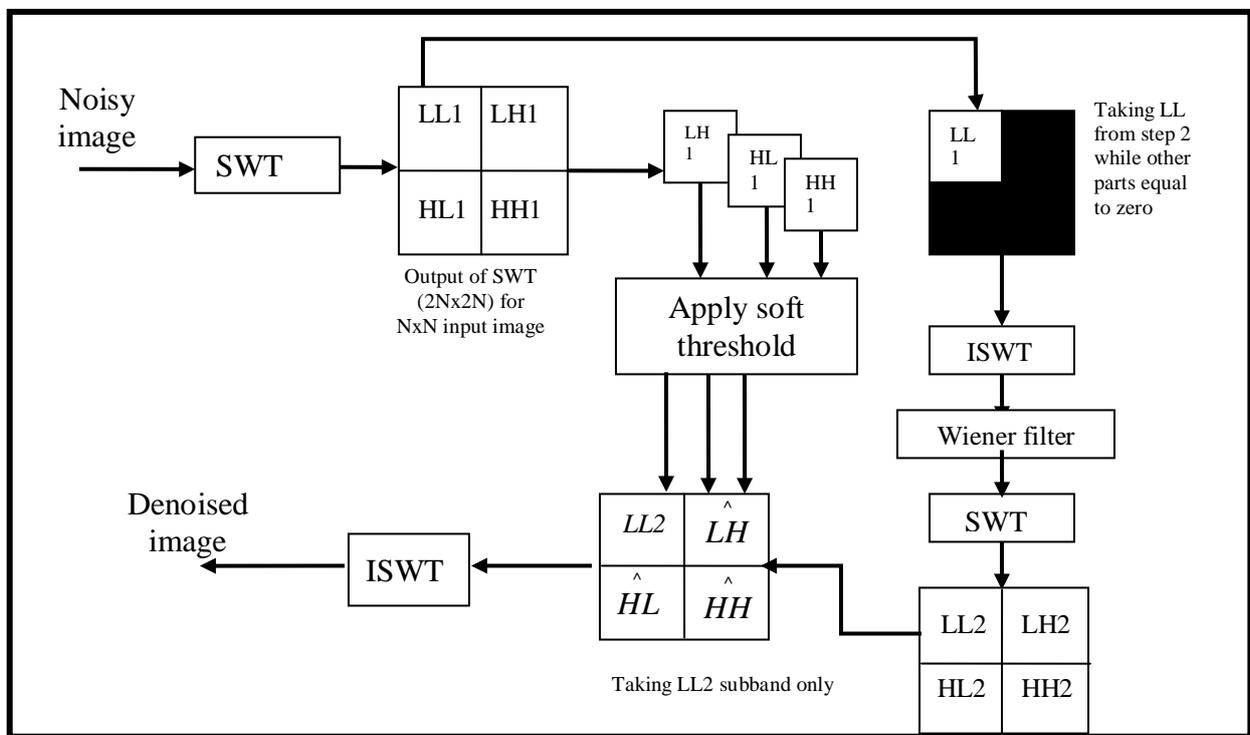Figure (1) shows the schematic of this approach

Fig.1. A schematic of the Proposed Denoising Algorithm.

4. Experimental results

The experimental evaluation is performed on color images of size 256 * 256 pixels at different white Gaussian noise levels shown in Figure (2). The objective quality of the reconstructed image is in terms of the PSNR of the three color components $X, X \in \{R, G, B\}$, which is defined as [9]:

$$PSNR = 10\log_{10}\frac{255^2}{mse(x)} \qquad ... (16)$$

where mse *(x)* is the mean-square-error of the original color component and the estimated one..

The overall *PSNR* is obtained as

$$PSNR = 10\log_{10}\frac{255^2}{mse(R) + mse(G) + mse(B)}$$
$$... (17)$$

The proposed method was implemented using Matalab R2010a. The PSNR results are shown in Table (1).

Fig.2. Test Color Images.

Table 1,
 PSNR of Various Noisy Color Images and Denoised Ones for SWT, Wiener, and the Proposed Algorithm.

Test images	Noise variance	Noisy Image PSNR (dB)	SWT PSNR (dB)	Wiener PSNR (dB)	Proposed method PSNR (dB)
Baboon	0.003	30.06	32.53	32.83	30.28
	0.007	26.43	30.84	31.19	29.95
	0.010	24.92	29.95	30.27	29.76
	0.030	20.38	26.57	26.81	28.64
	0.060	17.79	24.26	24.81	27.29
	0.080	16.82	23.30	23.72	26.65
	0.100	16.17	22.67	23.08	26.07
Airplane	0.003	30.01	34.91	35.71	33.14
	0.007	26.42	32.21	33.09	32.43
	0.010	24.93	31.04	31.09	32.04
	0.030	20.76	27.35	27.71	30.27
	0.060	18.29	24.84	25.06	28.19
	0.080	17.28	23.75	24.00	27.21
	0.100	16.56	22.99	23.25	26.35
House	0.003	30.01	34.46	35.55	34.06
	0.007	26.38	32.21	32.94	33.11
	0.010	24.88	31.04	31.70	32.63
	0.030	20.44	27.35	27.44	30.48
	0.060	17.86	24.84	24.97	28.64
	0.080	16.90	23.75	24.00	27.70
	0.100	16.20	22.99	23.31	26.95
Lena	0.003	30.04	35.55	36.25	34.87
	0.007	26.48	32.72	32.72	34.04
	0.010	24.97	31.44	31.44	33.49
	0.030	20.54	27.24	27.24	31.04
	0.060	17.96	24.72	24.72	28.81
	0.080	16.99	23.69	23.69	27.71
	0.100	16.25	22.92	22.92	26.96
Pepper	0.003	30.14	35.50	36.44	34.82
	0.007	26.57	32.63	32.63	33.67
	0.010	25.05	31.26	31.26	33.07

	0.030	20.72	27.25	27.25	30.58
	0.060	18.18	24.73	24.73	28.37
	0.080	17.17	23.65	23.65	27.20
	0.100	16.43	22.82	22.82	26.29
Mona Liza	0.003	30.33	33.23	33.91	30.14
	0.007	26.68	30.90	31.91	29.36
	0.010	25.21	29.90	30.84	28.48
	0.030	20.81	26.45	26.89	27.46
	0.060	18.26	24.11	24.32	25.99
	0.080	17.26	23.22	23.31	25.25
	0.100	16.50	22.38	22.60	24.55
Watch	0.003	30.10	36.39	35.92	35.72
	0.007	26.57	33.47	34.20	34.85
	0.010	25.10	32.11	33.25	34.25
	0.030	20.73	27.88	29.80	31.59
	0.060	18.18	25.22	27.36	29.04
	0.080	17.19	24.14	26.36	27.90
	0.100	16.47	23.26	25.49	26.92
Flower	0.003	30.24	35.67	35.33	34.59
	0.007	26.66	33.01	33.60	33.69
	0.010	25.17	31.79	32.65	33.19
	0.030	20.68	27.57	29.18	30.64
	0.060	18.11	24.91	26.77	28.39
	0.080	17.12	23.89	25.90	27.33
	0.100	16.40	23.07	25.11	26.47

From the results we can find that the PSNR of the proposed method offers superior improvement over SWT based method and Wiener filter method at middle and high values of noise with smoothing edges of the image. At low values of noise the proposed method offers poorer response because the levels of noise are very low at low band frequencies so the effect of wiener filtering for the reconstructed images from LL subband of the SWT step is not effective. At middle and high levels of noise, the LL subband of SWT step is more attacked by noise values, so the effect of wiener filtering is more noticeable.

Figure (3) shows the results of each denoising method for three testing images (Pepper, Airplane, Flower) for noise variance (0.1, 0.06. 0.08).

Also the results of the proposed algorithm, for gray scale images (Pepper and House) were compared with the results of the proposed methods of [4] in term of SNR (Signal to Noise Ratio) which was adopted in this paper.

$$SNR = 10\log 10\left\{ \frac{\sum_{x,y}[X(x,y)]^2}{\sum_{x,y}\left[X(x,y) - \hat{X}(x,y)\right]^2} \right\} \quad ...(18)$$

Where $X(x,y)$ and $\hat{X}(x,y)$ are the original image and denoised image.

Table (2) shows the comparison of the proposed method with results of [4] in which Modified Bayes Shrink was proposed.

From the comparison table we can notice that the results of the proposed method are slightly better than Modified BS in [4].
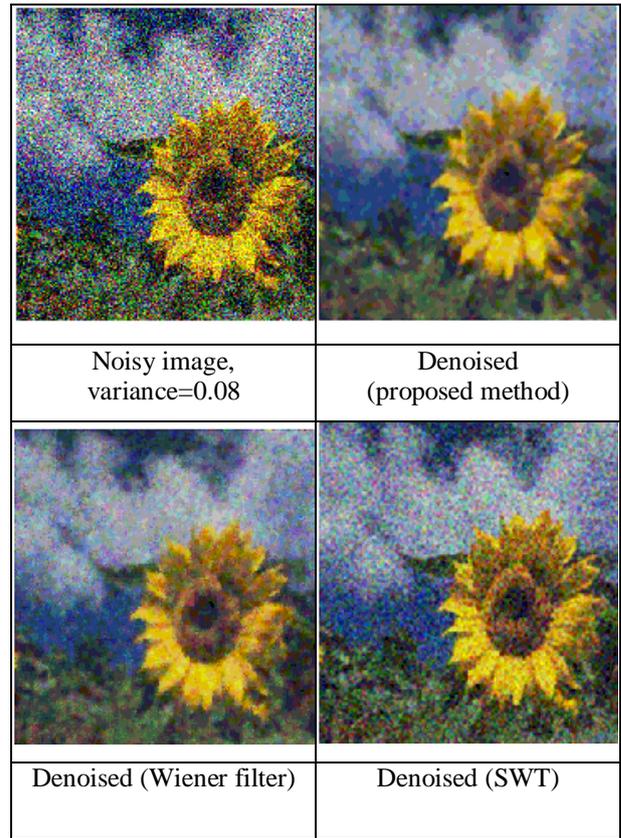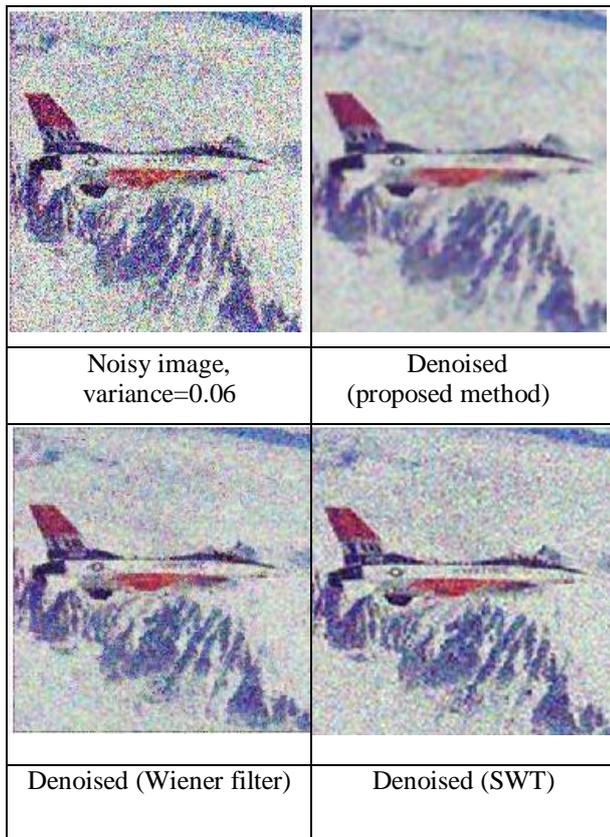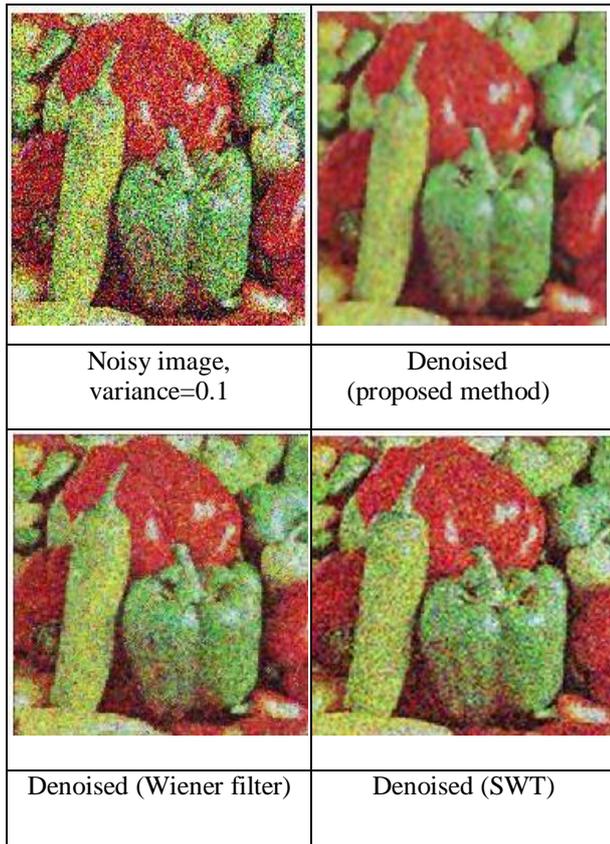
Noisy image, variance=0.1	Denoised (proposed method)
Denoised (Wiener filter)	Denoised (SWT)

Noisy image, variance=0.08	Denoised (proposed method)
Denoised (Wiener filter)	Denoised (SWT)

Fig. 3. Results of Proposed, Wiener filter, and SWT denoised images.

Noisy image, variance=0.06	Denoised (proposed method)
Denoised (Wiener filter)	Denoised (SWT)

Table 2,
SNR of the Proposed Method and Modified BS for Pepper and House grey images.

Test images	Noise Variance	Noisy Image SNR	Proposed SNR	Modified BS SNR
Pepper	0.01	14.55	22.49	21.21
	0.03	10.08	20.36	18.76
	0.06	8.15	18.38	17.51
House	0.003	20.37	24.63	26.46
	0.03	10.67	21.41	21.39
	0.05	8.76	21.15	20.18

5. Conclusion

Many methods of denoising algorithms based are on wiener filtering on the wavelet coefficients. In this paper a simple and efficient algorithm for adaptive noise reduction is proposed, it combines the adaptive wiener filter in spatial domain and thresholding method in the stationary wavelet transform domain. Experimental results show that

the noise reduction method exhibits better performance in both PSNR and visual effect, for middle and high values of white Gaussian noise variances, which make the algorithm robust for images attacked by middle and large values of noise. The average increase of PSNR of the denoised image with respect to noisy one is approximately (8-9) dB, while the improvement with respect to other methods is up to 3.5 dB.

Notation

c_{jk}	Discrete approximation at the resolution 2^j
H(u,v)	Degraded function
HH	High High subband
HL	High Low subband
LL	Low Low subband
LH	Low High subband
PSNR	Peak Signal to Noise Ratio
$S_\eta(u,v)$	Power Spectrum of Noise
$S_f(u,v)$	Power Spectral Density
SNR	Signal to Noise Ratio
T_{NS}	Normal Shrink Threshold

Geek letters

β	scale parameter
$\phi(x)$	scaling function
$\varphi(x)$	wavelet function
ω_{jk}	discrete detail signal at the resolution 2^j
$\hat{\sigma}_n$	standard deviation of noise
$\hat{\sigma}_y$	standard deviation of the subband
$\hat{\sigma}_n^2$	noise variance
λ	threshold value
δ_y	soft thresholded coefficients

6. References

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إزالة الضوضاء من الصور الملونة باستعمال تحويلة المويجة المستقرة ومرشح وينر المتكيّف

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الخلاصة

إن عملية إزالة الضوضاء من الصور المتأثرة بضوضاء من نوع Gaussian هي من المشاكل في عمليات المعالجة الصورية. العديد من الدراسات اعتمدت على تطبيق تقنية العتبة على معاملات المويجة، إن معظم هذه الدراسات ركزت على التشكيل الإحصائي لمعاملات المويجة وعلى الاختيار الأمثل لقيمة العتبة. يقدم هذا البحث طريقة جديدة لإزالة الضوضاء بواسطة دمج تقنية إزالة الضوضاء باستعمال تحويلة المويجة المستقرة ومرشح Wiener ، حيث يتم تطبيق مرشح Wiener على الصورة المسترجعة من معاملات التقريب فقط بينما يتم تطبيق تقنية العتبة على قيم معاملات التفاصيل التي تم الحصول عليها بتطبيق تحويلة المويجة المستقرة ومن ثم دمج النتيجتين. لقد تم تنفيذ الطريقة المقترحة باستعمال برنامج ماتلاب R2010a على صور ملونة وملوثة بضوضاء من نوع Gaussian . أظهرت النتائج تحسين واضح للصور وصل لحد3.5dB.