# Experimental and Analytical Study of Bending Stresses and Deflections in Curved Beam Made of Laminated Composite Material 

Assma Hassan Ismail<br>Department of Mechanical Engineering / University of Baghdad<br>Email:asma.sawaf@yahoo.com

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#### Abstract

Theoretical and experimental methodologies were assessed to test curved beam made of layered composite material. The maximum stress and maximum deflection were computed for each layer and the effect of radius of curvature and curve shape on them. Because of the increase of the use of composite materials in aircraft structures and the renewed interest in these types of problems, the presented theoretical assessment was made using three different approaches: curved beam theory and an approximate 2D strength of material equations and finite element method (FEM) analysis by ANSYS 14.5 program for twelve cases of multi-layered cylindrical shell panel differs in fiber orientations and number of layers. One case of E-glass composite material was experimentally made and tested to verify the relation between applied load and maximum deflection and four models were made of poly carbonyl to determine stresses under bending loads in polar scope, all results were compared with each other, the percentage accuracy was very good. The curved beam theory and strength of material equation formulas results were reasonable for the bottom surface, while it seems not enough for the top surfaces. Also, results explained positions and cases more affected by delaminating and the most preferred part of ellipse shape beam in resisting loads.


Keywords: Curved beam theory, composite material, strength of materials, finite element method.

## 1. Introduction

Material made into fiber forms can achieve significantly. better mechanical properties than their bulk counter-parts Fibers alone are not suitable for structural application to utilize the superior properties of fibers; they are embedded in matrix material that holds the fibers together to form a solid body capable of carrying complex loads. Matrix materials that are currently used for forming composites include three major categories: polymers, metals and ceramics. Fiber composites are stiff, strong and light and are thus most suitable for aircraft wing structures. They are often used in the form of laminates that consist of a number of unidirectional laminate with different fiber orientation. They provide multi directional load capacity composite laminates with excellent fatigue life, damage tolerance and corrosion
resistance. Curved beam structures made of composite materials have found many useful applications in aerospace engineering, civil engineering and the automobile industry .One of the active research fields is the modeling of composite rotor blades used on helicopters and tilt rotor aircrafts. Numerous models have been developed for thin-walled composite beams over the last several decades [4]. A review and analysis of various theories can be obtained in papers of; [4] when the variation-asymptotic method has been applied to develop an asymptotically correct model for initially curved and twisted, thin-walled, composite beams of arbitrary cross-sectional shapes and arbitrary anisotropic materials. In a two-step asymptotic reduction procedure, the three-dimensional strain energy is asymptotically reduced first to a two-dimensional shell strain energy and then to a one-dimensional beam strain
energy. This is a new attempt where initially curved and twisted, thin-walled, composite beams, with open or closed sections, have been modeled in an asymptotically correct unified framework.[9], analyzed an efficient procedure naturally curved and twisted beams with general cross-sectional shapes using naturally curved and twisted beam; St.Venant tensional warping function; generalized coordinate for warping; the minimum potential energy principle take into account the effects of torsion-related warping as well as transverse shear deformations, solutions can be used to calculate various internal forces, stresses, strains and displacements of the beams [10] developed modeling and dynamic response to blast loadings of doubly-curved sandwich panels with laminated face sheets. The implications of the panel curvature, of anisotropy and stacking sequence of face sheets, of transverse orthotropic of the core and of structural damping on dynamic response to time-dependent loads are highlighted [11], applied the finite element-based beam analysis for anisotropic beams with arbitrary-shaped crosssections with the aid of a formal asymptotic expansion method. From the equilibrium equations of the linear three-dimensional (3D) elasticity, a set of the microscopic 2D and macroscopic 1D equation are systematically derived by introducing the virtual work concept. Displacements at each order are split into two parts, such as fundamental and warping solutions, The numerical results are compared to those reported in literature as well as 3D FEM solutions [13] ,performed a curved beam element for the analysis of large deformation of flexible multi-body systems using the absolute nodal coordinate formulation. Using the GreenLagrange strain tensor as a volume element, locking phenomenon associated with the shear and cross-section deformation leads to erroneously stiffer bending characteristics, Numerical examples are presented in order to demonstrate the performance of the curved beam element developed in this investigation [7], presented a theory of space curved beams with arbitrary crosssections and an associated finite element formulation. Applying the isoperimetric concept, the kinematic quantities are approximated using Lagrangian interpolation functions. Alternative discretizations of thin-walled cross-sections with shell elements showed good agreement between the different models. Thus, the derived element can effectively be used to analyze the load-carrying capacities of special beam structures. The object of this research is comparing between results of experimentally and theoretical methods of maximum stress and deflection for composite
laminated curved thin beam (quarter of circle) with one fixed end and study the effect of changing the fiber orientation arrangement, symmetry about the middle surface plane. Theoretical methods are strength of material equation, curved beam theory and finite element method for twelve different cases. One experimentally model is manufactured to measure the maximum beam deflection for verification. Then using ANSYS graphs to obtain the change of the maximum stress positions as the mentioned parameters change, and then taking ellipse shape beam with same previous length and base radius in ANSYS to show the change in maximum stress and deflection and how much it is better.

## 2. Analytical Equations

The tangential and radial stresses of curved beam Figure1. according to advanced strength of material (generally) are:


Fig. 1. Section of curved beam.
$\sigma \theta=\frac{M(r n-r)}{A e r}$
$\sigma r=\frac{M}{b A e r 1}\left[r n \int \frac{d A}{r}-A 1\right]$
$r n=\frac{A}{\int \frac{d A}{r}}$
For rectangular section [1]:
$\int \frac{d A}{r}=b \operatorname{Ln}\left(\frac{r o}{r 1}\right)$
For composite material these stresses are changed
$\sigma \theta=\frac{M(r n-r)}{A e r} \frac{E \theta}{E^{*} \theta}$
$\sigma r=\frac{M}{b A e r 1}\left[r n \int \frac{d A}{r}-A 1\right] \frac{E^{*} \theta}{E \theta}$

Where $E^{*} \theta$ is the change in the elastic modulus according to the number of layer and fiber orientation,The equations of elastic modulus in fiber direction or perpendicular to fiber orientation. According to maxwell reciprocal theorem are [2];
$E_{1}=E_{f} * V_{f}+E_{m} * V_{m}$
$E_{2}=\frac{E_{f} * E_{m}}{E_{f} * V_{m}+E_{m} * V_{f}}$
$v_{12}=v_{f} * V_{f}+v_{m} * V_{m}$
$\frac{v_{12}}{E_{1}}=\frac{v_{21}}{E_{2}}$

## 3. Curved Beam Theory

The differential equations of equilibrium in polar coordinates could be simplified to [5]:
$\frac{\partial}{\partial \theta} N(\theta)+Q(\theta)=0$
$\frac{\partial}{\partial \theta} Q(\theta)-N(\theta)=0$
$\frac{\partial}{\partial \theta} M(\theta)-r Q(\theta)=0$
The force and moment resultants are integrals of the stresses over the beam thickness (h) are:
$\left[\begin{array}{l}N \\ M \\ F\end{array}\right]=\begin{gathered}h / 2 \\ -h / 2\end{gathered}\left[\begin{array}{l}\sigma \\ \sigma \\ z \\ \tau\end{array}\right]$
$\left[\begin{array}{l}N \\ M \\ F\end{array}\right]=\left[\begin{array}{ccc}A_{11} & B_{11} & 0 \\ B{ }_{11} & D_{11} & 0 \\ 0 & 0 & A_{55}\end{array}\right]\left[\begin{array}{l}\varepsilon_{\theta} \\ k \\ \gamma\end{array}\right]$
Determining EO every layer which differs from n to 1 , then;

$$
\begin{align*}
& Q_{11}=\frac{E \theta}{1-v_{12} v_{21}}  \tag{10}\\
& A_{11}=b \sum_{l=1}^{n} Q_{11} \int_{l}^{h_{l-1}^{n}}{ }^{R+Z} d z \\
& B_{11}=b \sum_{l=1}^{n} Q_{11} \int_{l}^{h_{l-1}} \frac{R Z}{R+Z} d z
\end{align*}
$$

$$
\begin{align*}
& D_{11}=b \sum^{n} Q_{11} \int^{k} \quad R Z^{2} d \\
& k=1 \quad h_{k-1} R+Z \\
& A_{55}=b \sum_{k=1}^{n} Q_{11} \int_{k-1}^{h} 1-\frac{4 Z^{2}}{h^{2}} d z \tag{12}
\end{align*}
$$

For thin beams the middle surface strain and curvature k changes are:
$\varepsilon_{\theta}=\frac{u}{r}+\frac{1}{r} \frac{\partial w}{\partial \theta}$
$k=\frac{1}{r^{2}}\left\{\frac{\partial u}{\partial \theta}-\frac{\partial^{2} w}{\partial \theta^{2}}\right.$
Finding $\mathcal{E} \theta$ from equations (10), then tangential stress is determined from;

$$
\begin{equation*}
\sigma \theta=Q_{11} \varepsilon \theta \tag{14}
\end{equation*}
$$

In this case, when a quarter circle beams with one fixed end is subjected to bending force in the other end will have;

$$
\begin{align*}
& M=-P R \operatorname{COS} \theta \\
& N=P \operatorname{COS} \theta \\
& F=P \operatorname{SIN} \theta \tag{15}
\end{align*}
$$

The deflections in x and y directions derived from strain and displacement equations;

$$
\begin{align*}
& W=\frac{-\varepsilon_{\theta}}{2} r_{i} \sin \theta-\frac{k r_{i}^{2}}{2} \cos \theta-\varepsilon_{\theta} r \theta+\frac{k r_{i}^{2}}{2}  \tag{16}\\
& U=\varepsilon_{\theta} r_{i} \cos \theta-\overline{\bar{W}}
\end{align*}
$$

## 4. Finite Element Method

ANSYS, The ultimate purpose of a finite element analysis is to re-create mathematically the behavior of an actual engineering system. In other words, the analysis must be an accurate mathematical model of a physical prototype. In the broadest sense, the model comprises all the, material properties, real constants, boundary conditions and the other features that used to represent the physical system. In ANSYS14
terminology, the term model generation usually takes on the narrower meaning of generating the nodes and elements that represent the special volume and connectivity of the actual system. Thus, model generation in this study will mean the process of defining the geometric configuration of the model's nodes and elements, SHELL281.

Figure 2 element is used in this study. The element is suitable for analyzing thin to moderately-thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the $\mathrm{x}, \mathrm{y}$, and z axes, and rotations about the $x, y$, and $z$-axes. It may be used for layered applications for modeling composite shells or sandwich construction Figure 3 [Mouveni].


Fig. 2. Shell 281 elements.


Fig. 3. Curved beam mesh.

## 5. Manufacturing Steps

Make the block from wood in the selected dimensions with small clearance, varnish the inside surface of block by anti-adhesion, stuff the glass fibers were cut in pieces in the selected length carefully, fill the prepared epoxy on the first layer in the specified weight, repeat the same until three layers completion, leave the filled block for hours until it stick together, then take
the composed material out of the block carefully, put the material on a prepared circular surface with the specified radius and varnished, use the anti- bubble roller to prevent creating of bubbles ,the specimen will be ready after two or three days and needs only filing by file to fit the dimensions by virneir.

## 6. Experimental Test

Composite laminated curved (quarter of circle) thin beam is (hand layout) manufactured properties mentioned in table.1. in three layers Fig. 4. 0/90/0, fiber angles the dimensions are; $\mathrm{R}=15.7 \mathrm{~cm}$, width $=2.54 \mathrm{~cm}$, thickness $=3 \mathrm{~mm}$ each layer is one millimeter.


Fig. 4. Manufactured beams.


Fig. 5. Polar scope.

Table 1,
Properties of material used.

| Property | E-glass fiber | Epoxy |
| :--- | :--- | :--- |
| Young modulus <br> GP | 74 | 4.5 |
| Density kg/m3 | 2600 | 1200 |
| Poisson ratio ט | 0.25 | 0.4 |

## 7. Photo Elastic Test

Four models, three different positions were selected from one ellipse which had the same bigger radius with the previous manufactured curved beam with one quarter in order to indicate the best model in resisting the loads applied in all directions; the small radius was 95.4 mm fig.5.by polar scope fig.6. The photo elastic material is poly carbonyl, the fringe coefficient is $6.9 \mathrm{~N} / \mathrm{mm} /$ fringe and the used equation is;

$$
\begin{equation*}
\sigma_{1}-\sigma_{2}=\frac{f . n}{t} \tag{17}
\end{equation*}
$$

Where f is fringe coefficient and n is number of fringes and $t$ is poly carbonyl thickness which is 3.5 mm .


Fig. 6 . Deflection tester.

## 8. Results

Table (1) and Figure (7) indicates the experimentally results of maximum deflection test by dial gage fig.6. for increasing the load from 0.5 N to 3 N is very coincident to ANSYS values, Figures ( $8,9,10$ ) for one layer ( $0,45,90$ angles) indicates that the stress distributions are the same the difference in stress value is small but obvious in deflection the deflection decreased with increasing of fiber angle from 0 to 90 .For two layers figures $(13,12)$ for $0 / 90$ and $0 / 45$ the shear stress is obviously increased exposing the layers to delamination ,this increase arising from the asymmetry of fiber orientation about the middle surface, the point of maximum shear stress is at angle 45 from fixed end .For figures $(14,11)$, three layers (0/0/90, 45/45/0), two layers (0/90) also the same note because of asymmetry. While table(4) of ANSYS results indicates the higher value of
deflection was at third case ( one layer 90 fiber angle because of the low value of hoop elastic modulus ,the lower value was at case ( $0 / 90 / 0$ ) because of the symmetry of fibers about the middle surface, higher stress was in case of two layers $0 / 90,0 / 45$,three layers $45 / 45 / 0$ obviously also because of asymmetry the less stress and deflection was at cases $0,0 / 0,0 / 0 / 0$.Figures 15 to 18 , and Table (5), gave the photo elastic test results which gave the good coincidence with ANSYS results, the most suitable position in ellipse was the third model as it indicated in table because the stress became less obviously, Figures $(19,20,21)$ indicates the stress distribution in the ellipse shape beam for the same length and same base circle radius which is smaller $32 \%$ in stress and $90 \%$ Table (6) of strength of material equation gave the same as in ANSYS where the less stress and less deflection was at zero angle ,higher stresses in 0/90 and 45/45/0.The curved beam theory method gave also the higher values of stresses in $0 / 90$ and $0 / 45$, less values at zero angles as in Table (6).The increase in no. of layers from 1 to 3 increase the stress and deflection little as Figures $(23,24)$, and the effect of increasing fiber angle from 0 to 90 indicated in figures $(24,25)$. Figures $(26,27)$ indicates the difference in values of deflections and stresses in twelve cases between three theoretical methods STM,CBT,FEM. The radius of curvature increase gave increase in strength by lowering the stress induced, and decreases the delamination possibility also because the shear stresses decrease as Figure $(28,29)$. That means ellipse shape beam is more suitable shape for structures.

## 9. List of Graphs

Table 2,
Experiment deflection results compared with ANSYS for 0/90/0 case.

| Load N | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Deflection <br> cm | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| ANSYS <br> results cm | 0.1 | 0.21 | 0.32 | 0.42 | 0.53 | 0.64 |



Fig. 7. Experimentally change of deflection with load.


Fig. 8. Stress in $y$ direction in one layer 0 fiber angle.


Fig. 9. Stress in y direction in one layer for 45 fiber angle.


Fig.10. Stress in $y$ direction in one layer 90 fiber angle.


Fig. 11. Shear stress in xy direction in 45/45/0 fiber angle in second layer.


Fig. 12. Shear stress in $x y$ direction in first layer for two 0/90 layer.


Fig. 11. Shear stress in $x z$ direction in second $0 / 45$ fiber angle for two layers.


Fig. 13. Shear stress in $x y$ direction in first layer for three layers 0/0/90 fiber angle.


Fig. 15. First beam in polar scope under 2.25 N load.


Fig. 16. Forth beam in polar scope under 2.25 N load.


Fig. 17. Second beam in polar scope under 2.25N load.


Fig.18. Third ellipse shape beam in polar scope under 2.25 N load.


Fig. 19. Second ellipse shape beam in ANSYS.


Fig. 20. Fourth ellipse shape beam in ANSYS.


Fig. 21.Third ellipse shape beam in ANSYS.


Fig. 22. Change of deflection in $Y$ direction with no.of layers.


Fig. 23. Change of stress $Y$ direction with no. of layers.


Fig. 24.Change of stress in $Y$ direction with fiber angle.


Fig. 25. Change of stress in $Y$ direction with fiber.


Fig. 26. Comparison in deflection values between three methods for twelve cases.


Fig. 27. Comparison in stress values between three methods for twelve cases.


Fig. 28. Change of stress with radius of curvature.


Fig. 29. Change of deflection with radius of curvature.


Fig. 30.Change of deflection with mesh degree.

## 10. List of Tables

Table 3,
ANSYS 14.5 results

|  | case | Deflection Max cm |  |  | Maximum Tangential stress Mpa |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
|  |  | First layer | Second layer | Third layer |  |  |  |
| 1 | 0 | 0.5261 | 16.96 |  |  |  |  |
| 2 | 45 | 2.386 | 15.46 |  |  |  |  |
| 3 | 90 | 1.879 | 14.81 |  |  |  |  |
| 4 | $0 / 0$ | 0.526 | 16.96 | 16.172 |  |  |  |
| 5 | $0 / 90$ | 1.096 | 23.35 | 9.41 |  |  |  |
| 6 | $0 / 45$ | 1.035 | 13.72 | 23.55 | 15.08 |  |  |
| 7 | $0 / 0 / 0$ | 0.52616 | 16.96 | 0.7306 | 14.64 |  |  |
| 8 | $0 / 90 / 0$ | 0.5059 | 17.25 | 4.398 | 10.77 |  |  |
| 9 | $0 / 0 / 90$ | 0.8605 | 22.45 | 3.369 | 20.25 |  |  |
| 10 | $45 / 0 / 45$ | 1.679 | 21.21 | 0.6457 | 19.2 |  |  |
| 11 | $45 /-45 / 45$ | 1.093 | 19.81 | 1.775 | 23.1 |  |  |
| 12 | $45 / 45 / 0$ | 1.861 | 12.17 | 6.08 |  |  |  |

Table 4,
Strength of materials equation results.

| case | Deflection Max cm |  | Maximum Tangential stress Mpa |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | First layer | Second layer | Third layer |
| 1 | 0 | 0.47947 | 9.78 |  |  |
| 2 | 45 | 1.85 | 9.78 |  |  |
| 3 | 90 | 0.5388 | 9.78 |  |  |
| 4 | $0 / 0$ | 0.5402 | 9.78 | 10.8 |  |
| 5 | $0 / 90$ | 0.67526 | 9.78 | 27. |  |
| 6 | $0 / 45$ | 0.492 | 9.78 | 12. |  |
| 7 | $0 / 0 / 0$ | 0.6752 | 9.78 | 14. |  |
| 8 | $0 / 90 / 0$ | 0.81 | 9.78 | 16.2 |  |
| 9 | $0 / 0 / 90$ | 0.5788 | 9.78 | 11.57 |  |
| 10 | $45 / 0 / 45$ | 0.6023 | 9.78 | 11.57 |  |
| 11 | $45 /-45 / 45$ | 0.6025 | 9.78 | 11. |  |
| 12 | $45 / 45 / 0$ | 0.719 | 9.87 | 22.561 |  |

Table 5,
Curved beam theory results

| case | Deflection Max cm |  | Maximum Tangential stress Mpa |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | First layer | Second layer | Third layer |
| 1 | 0 | 0.4257 | 16.578 |  |  |
| 2 | 45 | 1.752 | 14.037085 |  |  |
| 3 | 90 | 0.5389 | 16.6658 |  |  |
| 4 | $0 / 0$ | 0.5327 | 17.3876 | 11.5881 |  |
| 5 | $0 / 90$ | 0.7316 | 19.919 | 7.41 |  |
| 6 | $0 / 45$ | 0.7819 | 19.72 | 14.2355 |  |
| 7 | $0 / 0 / 0$ | 0.6433 | 17.7435 | 11.906 | 5.9547 |
| 8 | $0 / 90 / 0$ | 0.6236 | 16.4189 | 5.809 | 5.456 |
| 9 | $0 / 0 / 90$ | 0.6436 | 17.86 | 11.9034 | 5.9517 |
| 10 | $45 / 0 / 45$ | 0.4552 | 13.5391 | 10.34357 | 11.4247 |
| 11 | $45 /-45 / 45$ | .6412 | 17.7154 | 14.636 | 6.7542 |
| 12 | $45 / 45 / 0$ | 0.4595 | 13.6817 | 11.308 | 11.5441 |

Table 6,
Polari scope pictures results.

| Photo elastic <br> case | Number <br> of fringes | Principal stress <br> difference <br> $\mathbf{0 6 M p}$ |
| :--- | :--- | :--- |
| First |  |  |
| Second | 2 | 7 |
| Third | 3.5 | 4 |
| Fourth | 5 | 10 |

## 11. Conclusions

1. The middle layer is subjected to less stress because the moment center pass through it where the resultant stress is zero.
2. Unsummetry of layers fiber angle about the geometrical center makes greater generated stress than the same layers but in symmetrical arrangement, and high difference in stress between top and bottom layers, high x,y shear stress which make delaminating more possible because of torsion effect.
3. Theoretically increasing layers makes young modulus less than the material first young modulus that increase the generated stress, but in ANSYS the change in stress is very small for laminars from 1 to three layers, it analyze stresses in three dimensions and indicate that the value of Z direction stress which is ignored in CBT and SM methods is not zero that is one of difference reasons in results.
4. The accuracy is obvious between CBT and ANSYS for the stress in the bottom layer which equals $95 \%$ and between SM and ANSYS is $65 \%$ but the difference is clear for the top layer.
5. The experimentally founded values of deflections coincide the deflections of other methods in $95 \%$ percentage.
6. The ellipse shape beam is more suitable for aircraft structures, and third shape is the most preferred because the stress was less.

## 12. List of Symbols

| STM | Strength of materials equation |  |
| :--- | :--- | :--- |
| CBT | Curved beam theory |  |
| FEM | Finite element method |  |
| $\varepsilon \Theta$ | strain |  |
| $\sigma \Theta$ | Stress in tangential direction | $\mathrm{N} / \mathrm{m} 2$ |
| o12, 021 | Poisson ratio of total composit |  |
| v, | material |  |
| of | Poisson ratio of fiber |  |


| vm | Poisson ratio of matrix |  |
| :---: | :---: | :---: |
| W,U | Displacement in tangential and longitudinal direction | m |
| W | Second derivative of tangential displacement | m |
| Vf | Volumetric ratio of fiber |  |
| Vm | Volumetric ratio of matrix |  |
| E $\Theta$ | Young modulus of composite material in tangential direction | N/m2 |
| Er | Young modulus of composite material in radial direction | N/m2 |
| E1 | Young modulus of composite material in fiber direction | N/m2 |
| E2 | Young modulus of material in direction perpendicular to fiber | N/m2 |
| Ef | Young modulus of fibers | N/m2 |
| E m | Young modulus of matrix | N/m2 |
| k | Radius of curvature | m |
| rn | radius of beam section at center of bending stress | m |
| ro | Outside radius of beam section | m |
| A | Beam section area | m2 |
| N,P | applied force in Y direction | N |
| M | bending moment | N.m |
| F | shear force | N |
| $\Theta$ | angle of any section of beam from X-axis | degree |
| ri | inner radius | m |
| Z | thickness at any point | m |
| h | thickness | m |
| b | Width of beam | m |
| e | Difference between geometrical center and bending center | m |
| I | Moment of inertia | m4 |

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# دراسة تحليلية وعملية لأجهادات الاتحناء والتمدد في عمود مقوس مصنوع من مادة مركبة ذات طبقات 

## أسماء حسن أسماعيل

قسم الهندسة الميكانيكية/ كلية المنـدسة /جامعة بغديداد
البريد الالكتروني : asma.sawaf@yahoo.com


#### Abstract

الخلاصة

أعتمدت طرق نظرية و عملية لفحص عمود مقوس مصنوع من مادة مركبة من عدة طبقات حيث تم حساب الاجهاد و الميل الاقصى لكل طبقة مع بيان  استخدم ثلاث طرق نظرية مختلفة وه ي نظرية العمود المقوس، معادلات مقاومة المواد ثنائية الآبعاد و طريقة العنصر المحدد لاثثنثي عشرة حالة مختلفة في زو ايا الالياف و عدد الطبقات.كما تم تصنيع نموذج لحالة واحدة من الياف الزجاج والايبوكسي وفحصه لبيان علاقة الحمل بالتمدد الاقصى واربعة نماذج من مادة البوليكاربونيل لاستخدامها في جهاز المرونة الضوئية لبيان توزيع الاجهادات تحت تأثير احمال الانحناء . ، وقورنت النتائج كلها مع بعضها البعض واعطت نسبة تطابق جيدة جدا وكانت النتائج الخاصة بطريقة نظرية العمود المقوس وطريقة مقاومة المواد معقولة للطبقة السفلى بشكل واضح ولكا لانكها لم تكن كافية الدقة للطبقة العليا للعينات كما اظهرت نتائج البرنامج الصورية المو اقع والحالات الاكثر عرضة لحالة انفصال الطبقات وا لشكل العقوس الافضل من

بين المواقع المختلفة من العمود البيضوي الشكل.


