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# Vibration Characteristics Oblate Shell "With and Without Framed Structure"

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# Abstract

This paper presents the results of investigating the vibrational characteristics of oblate dish with and without framed structure . A finite element method, was applied to the dynamic analysis of oblate spheroidal shell. Different types of elements were considered in one dimension and two dimensions. It was found that the natural frequencies of oblate shells had two types of behavior against increasing the shell thickness and eccentricity, which are the membrane mode and bending mode –Since – the membrane modes natural frequencies tend to increase with the increasing the eccentricity of oblate, while the bending modes natural frequencies decrease with the increasing the eccentricity till reach the optimum eccentricity.

Experimental Validation tests have been carried out on the spheroid dish by using the model analysis technique.

Keywords: Vibration, structure.

### 1. Introduction

When the large structure such as aircrafts, bridges, ships, vehicles and tall building being constantly acted on by wave and motion, the resulting forces can introduced Vibrations at the resonance or repeated many times which may lead to structural failure. An essential requirement of an engineering structure is to sustain the loading applied to it during service life without failure. To insure that and to obtain an optimum design of structure the dynamic characteristics must be established. The dynamic characteristics of any structure are governed by its stiffness and mass properties. The most important dynamic characteristic of the structure is the natural frequencies which are a function of material constants and dimensions of structure.

It is worthy to indicate the industrial applications and important of plates and shells structure. One of important used type of elastic thin shells which has a particular interest is the oblate shells. The oblate type of shell has many practical applications, one interesting application includes the protective shell used as the housing of the early-warning scanner of the airborne warning and control system aircraft (AWACS).

The study of the dynamic analysis of plates and shells has been treated by many investigators using different methods. The oblate spheroidal shells considered extensively have been restricted to few works, Benzes and Burgin [1] were solved the problem of the free vibration of thin isotropic oblate shells using Galerkin's method. Penzs [2] was extended this work to include thin orthotropic oblate shells.

Curved blades can be modeled approximately by fact element [3]. Curved shell elements may provide a more accurate facility for the finite element modeling of curved blades. The basic equations which describe the behavior of a thin elastic shell were originally derived by Loue [4]. pawsey [5] explained the basic problems common to most shell elements, and which restrict most elements class of shells, either thin or thick, depending on the parent theory used for developing the element. Recently the concept of quasi comparison function has been introduced for the Reylegh Ritz discretization in self-adjoint eigen-value problem [6]. Babich [7] is studied the stability and natural vibrations of shells with variable geometry and mechanical parameters.

## 2. Numerical Procedure General

Large number of different theories has been derived by various Scientist, all purposing to static and dynamic analysis of shells. Differences among these theories are due to various assumptions-shells theories most incorporate bath flexural and extensional deformations.

A structure, such as a dish, may have zones with the variable thickness and construction when it is difficult to use one element type. If a Mindlin Facet element is employed for a thin structure shear looking will occur and will lead to inaccurate results. The flat or facet shell element is the appropriate and easily employed for curved shells. Two basic facet shell elements are introduced. The Kirchoff facet element based upon a combination of the 2-D plane stress element and the thin plate bending element. Thick facet element is based upon a combination of the 2-D plane-stress element and the Mindlin plate element.

### **Boundary conditions**

The dish is assumed to be clamped at two support legs and free at all other points. The boundary conditions for the clamped point may be written as follows:

 $u_i = v_i = w_i = 0$  at clamped ends.

 $\theta_{xi} = \theta_{yi} = \theta_{zi} = 0$  at clamped ends.

Another type of boundary conditions were considered which are concerned with the half oblate shell. This shell was clamped at its peripheral base. The three components of translation and three components of rotation of all nodes of the oblate shell peripheral base are assumed equal to zero.

# **Governing Equations**

The finite element procedure for the estimation of natural mode shape and natural frequency of oblate dish can be formulated by using facet element and the principle of minimum potential energy theorem.

Express the total potential of the element in terms of nodal, displacements, if the minimum total potential energy theorem is used, then the equilibrium\* equations for the clement can be established. The total potential energy for the element can be defined as follows [8]:-

$$\mathbf{X} = \mathbf{U} - \mathbf{W}$$

Where X is the total potential energy of the element, U is the strain energy of the element, W is the work done by external force F:

 $W{=}F.\delta$ 

Where  $\delta$  is the nodal deformation. It can be deduced that

$$\underline{K}_{L} = \iiint \int_{-t/2}^{t/2} (B_{m}^{t} - z B_{b}^{t}) D(\underline{B}_{m} - z B_{b}) dz dxdy$$
...(1)

Then it can be proved that:

$$\underline{K}_{L} = \underline{K}_{m} + \underline{K}_{b}$$
$$\underline{K}_{m} = \iint t \, \underline{B}'_{m} \, D\underline{B}_{m} \, dx dy$$
$$\underline{K}_{b} = \iint t \, \underline{B}'_{b} \, D\underline{B}_{b} \, dx dy$$

m: hold for membrane
b: hold for bending
K<sub>L</sub>: local stiffness matrix of the element
Similarly, mass matrix can be deduced that:

$$M = \underline{M}_{m} + \underline{M}_{b}$$
where  $\underline{M}_{m} = \iint N_{m}^{t} D_{m} N_{m} dxdy$  ...(2)
$$\underline{M}_{b} = \iint N_{b}^{t} D_{b} N_{b} dxdy$$

Where N is the shape function.

For thick facet element, the transverse shear strain was considered in addition to the membrane and bending strain.

The nodal dynamic equation for oblate shell is

$$M \ddot{\delta}(t) + C \dot{\delta}(t) + K \delta(t) = F(t) \dots$$

and for undraped free vibration the equation of motion become.

$$M\ddot{\delta}(t) + K\delta(t) = 0 \qquad \dots (3)$$

Fig.(1) illustrated the flow chart of the above equations. Fig.(2) shows the used mesh of the oblate dish.



Fig.1. Flow Chart of the Finite Element Analysis.



Fig.2. Mesh of the Oblate Shell.

# The standard solution of dynamic problem

The major problem in structural analysis using the FEM is the solution of a set of simultaneous algebraic equations.

Iterative methods are those which start with an initial approximation and which by applying a suitable chosen algorithm, Leads to successively better approximation.

When the process converges good approximate solution is expected to be obtained.

The accuracy and the rate of convergence of iterative methods Vary with the algorithm chosen, the common approaches used for iterative solution are simple iteration method, subspace operation technique with the static and dynamic condensation, the basic steps of simple iteration and the subspace iteration algorithm can be found in [9, 10]

# 3. Experimental Model

Experimental values were obtained for two models which were constructed from steel. The dimensions for the two models are given in table(1).

Table 1, Specifications of Tested Models

Basically, each one of the models was constructed as follows. For each model a flat sheet of plate was cut to proper dimensions as shown in Fig.(3). After fraction of the circular plate on the bench of the pressing machine and by using a suitable press die moving up and down to impact the circular plate several times get on the dish end as shown in Fig.(3.b). Repeat the pressing operation with another dies to obtain the final half shell member as shown in Fig.(3.c).

Locate the position of legs and make two holes in one half shell and then use the welding method to fix the two legs in its position. Fit, precisely the edges of the two half shell then welded together.

The welding was ground flat on the outer surface of shell to obtain on the two models as shown in Fig.(3).

Fig.(4) shows a block diagram for different equipments and instruments used in testing the models. The experimental work was carried out to measure the natural frequencies of the two oblate dishes. The results of experimental and theoretical approaches as listed in table (2) and table (3).

A natural frequency was distinguished by observing the sharp increase in maximum amplitudes of the output signal and by the intensity of the acoustic tone emitted with phase angle.

> Discrepancy % 2.15942 2.59366 2.222222 2.092308

> > 2.359551

2.725275

3.14059 3.641208 2 3.297609

| Table 2, |           |
|----------|-----------|
| Natural  | Frequenci |

| Specifications of 1            | ested Models                |                             | Natural F | requencies (Hz         | z) of Dish Mod       | lel I |
|--------------------------------|-----------------------------|-----------------------------|-----------|------------------------|----------------------|-------|
| Parameter                      | Model (1)<br>Values & units | Model (2)<br>Values & units | Mode      | Exp. of<br>Frequencies | Theo.<br>Frequencies | Disc  |
| Radius of shell curvature (R)  | 650 mm                      | 650 mm                      | 1         | 69                     | 67.51                | 2.1   |
| Radius of dish                 | -                           | 75 mm.                      | 2         | 173.5                  | 169                  | 2.:   |
| end (r)                        |                             |                             | 3         | 180                    | 176                  | 2.2   |
| Height of dish<br>end (h1)     | 92 mm.                      | 135 mm.                     | 4         | 325                    | 318.2                | 2.0   |
| The extended                   | -                           | 20 mm.                      | 5         | 356                    | 347.6                | 2.3   |
| edge height (h2)               |                             |                             | 6         | 455                    | 442.6                | 2.7   |
| Total height of half shell (H) | 92 mm.                      | 155 mm.                     | 7         | 882                    | 854.3                | 3.    |
| Modulus of                     | 208 GPa                     | 208 GB <sub>1</sub>         | 8         | 1126                   | 1085                 | 3.6   |
| Elasticity (E)                 |                             |                             | 9         | 1150                   | 1127                 |       |
| Density ( $\rho$ )             | 7850 kg $m^3$               | 7850 kg\m <sup>3</sup>      | 10        | 1213                   | 1173                 | 3.2   |
| Poisons ratio ( $V$ )          | 0.3 N. D                    | 0.3 N. D                    |           |                        |                      |       |



Fig. 3. Types of Models.



Fig. 4 Block Diagram of Dynamic Measurement System.

Table 3, Natural Frequencies ( Hz) of Dish Model II

| Mode | Exp. of<br>frequencies | Theo.<br>frequencies | Discrepancy<br>% |
|------|------------------------|----------------------|------------------|
| 1    | 45                     | 39.47                | 12.28889         |
| 2    | 141                    | 131.5                | 6.737589         |
| 3    | 181                    | 175.6                | 2.983425         |
| 4    | 211                    | 202.5                | 4.028436         |
| 5    | 218                    | 223                  | -2.29358         |
| 6    | 229                    | 224.4                | 2.008734         |
| 7    | 232                    | 226.9                | 2.198276         |
| 8    | 305                    | 297.5                | 2.459016         |
| 9    | 424                    | 413                  | 2.59434          |
| 10   | 431                    | 422.7                | 1.925754         |

### 4. Discussion and conclusion

In the foregoing work it is demonstrated how the finite element method and experimental model analysis, can be employed to determine the free vibration frequencies and mode shapes of simplified representation of oblate dish. The comparison of predicted and measured natural frequencies are written acceptable engineering agreement for the system tested. Two types of results are recorded in tables (1) and (3).

Figure (5) and figure (6) show the variation of natural frequencies with the mode number of experimental and theoretical results. Figures (7) and (8) show the natural frequencies of vibrations as function of the shell thickness once of the eccentricity value e = 0.925 once again for half oblate of eccentricity e = 0.585 clamped at its peripheral base for several modes.

Natural frequencies are seen to have two types of behavior against increasing the shell thickness. One type, which is associated with the membrane modes, remain unaffected by the thickness variations, which the other type, which is associated with the bending modes. Tends to increase with the thickness. The dynamic behaviour of oblate shells depend upon the coupling and uncoupling of membrane modes and bending modes. It was found that when the oblate increase the natural eccentricies of frequencies will be increase until reading an optimum value of eccentricities. after ; the optimum value of eccentricity, the natural frequencies will be decrease until the oblate shell becomes circular plate (e = 1) as shown in Fig (10), while Fig (9) show the effect of shell thickness on the Von Mises stresses of oblate clamped at its peripheral base.



Fig.5. Experimental Results of Natural Frequencies versus Mode for Oblate Dish with (a=325, b=92mm, ts=5mm, mat: steel alloy).



Fig.6. Theoretical Results of Natural Frequencies versus Mode for Oblate Dish with & Without Internal Structure with (a=325, b=55mm, ts=5mm, tp=3mm, mat: steel).



Fig.7. The Effect of Shell Thickness on the Frequencies of Half Oblate Shell Clamped at Peripheral Base with (a=185mm, b=70mm, e=925mm, mat: Aluminum).



Fig.8. The Effect of Shell Thickness on the Frequencies of Half Oblate Shell Clamped at Peripheral Base with (a=125mm, b=150mm, e=585mm, mat: Aluminum).



Fig.9. The Effect of Shell Thickness on the Von Misses Stress of Oblate Shell Clamped at Peripheral Base with (a=185mm, b=70mm, e=925mm, mat: Aluminum).



Fig.10. The Variation of Critical Von Misses Stress of Oblate Shell for Half Oblate Spheroids Shell Clamped at its Peripheral Base with (a=185mm,b=70mm,e=925mm,mat: Aluminum).

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# خواص الاهتزازات للصفائح المنبعجة بأضافة او عدم اضافة الاطار الهيكلى

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# الخلاصة:

في هذه الدراسة تم تحليل خواص الاهتز ازات للصحون المفلطحة بأستخدام الاطر الهيكلية او بدونها . حيث تم استخدام طريقة العناصر المحدده ( F.E.M) في التحليل النظري لتحديد الخواص الديناميكية لهذا النوع من الصفائح و غالباً ماتستخدم هذه العناصر في تحليل الخواص الميكانيكية للصفائح المنبعجة الكروية وشبة الكروية بأتجاة واحد او بأتجاهين . حيث تم حساب الترددات الطبيعية لنموذج صفيحة منبعجة طبقاً لحالتين هي زيادة سمك الصفيحة واللامركزية لها مع الاخذ بنظر الاعتبار تحديد مايكافئ في تلك الترددات من نسق الانحناء والنسق الغشائي ، حيث ان النسوق الغشائية للاهتز از ات الطبيعية تزداد بزيادة اللاستقر ارية لصفيحية المنبعجة بينما نسوق الانحناء تقل بزيادة اللامركزية للصفيحة وتستمر هذه الحالة المثالية للامركزية لها مع وذلك تم اجراء تجارب عملية لفوص الامترات من نسق الانحناء والنسق الغشائي ، حيث ان النسوق الغشائية للاهتز از ات الطبيعية تزداد بزيادة اللاستقر ارية لصفيحية المنبعجة بينما نسوق الانحناء تقل بزيادة اللامركزية للصفيحة وتستمر هذه الحالة هكذا حتى نصل بالفحص الى الحالة المثالية للامركزية . وكذلك تم اجراء تجارب عملية لفحص الاهتز از ات على نموذج تم تصنيعة ليمثل صحن دوار لطائرات الانذال المبكر (( الاواكس )) حيث تم الحصول على نتائج مثالية تم مطابقتها مع التحليلات النظرية .