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# A New Fractal Printed Dipole Antenna Based on Tent Transformations for Wireless Communication Applications

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## Abstract

In this paper, a compact multiband printed dipole antenna is presented as a candidate for use in wireless communication applications. The proposed fractal antenna design is based on the second level tent transformation. The space-filling property of this fractal geometry permits producing longer lengths in a more compact size. Theoretical performance of this antenna has been calculated using the commercially available software IE3D from Zeland Software Inc. This electromagnetic simulator is based on the method of moments (MoM). The proposed dipole antenna has been found to possess a considerable size reduction compared with the conventional printed or wire dipole antenna designed at the same design frequency and using the same substrate specifications. Results have shown that the proposed design possesses a multi-band resonant behavior with adequate radiation performance with VSWR ≤ 2 (return loss ≤-10 dB) throughout the resonating bands. This makes the presented antenna (or its monopole counterpart) suitable for use in the modern multi-functions compact communication systems.

***Keywords:*** Fractal antenna, antenna miniaturization, multi-band antenna, printed dipole antenna, IFS (iteration function system).

## 1. Introduction

The word fractal comes from Latin fractus, which means broken lines, and Mandelbrot [1] first used it. Mandelbrot defined fractal as a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Euclidean geometries are limited to points, lines, sheets, and volumes and assigns an integer number to describe the dimension of each of these geometries; where the dimension of a point is zero, and 1, 2, and 3 are the dimensions of the line, sheet and volume respectively. Fractal geometry describes objects in nature by dimensions, which are not conditionally integer numbers as the Euclidean geometry implies. Euclidean geometries can be special cases from the more general fractal geometries.

Fractals can be either random or deterministic. Most fractal objects found in nature are random, that have been produced randomly from a set of non-determined steps. Fractals that have been produced as a result of an iterative algorithm, generated by successive dilations and translations of an initial set, are deterministic.

Fractals are characterized by the self-similarity, the fractional dimension and space-filling properties. The concept of a fractal is most often related with geometrical objects satisfying the criteria of self-similarity. Self-similarity means that an object is composed of sub-units and sub-sub-units on multiple levels that statistically resemble the structure of the whole object. These substructures are exactly of the shape as the original but it may be flipped, rotated, or stretched depending on the generation process producing the fractal shape. Figs (1) and (2) demonstrate this property through the generation process of well-known fractal geometries; Peano and Hilbert fractals.

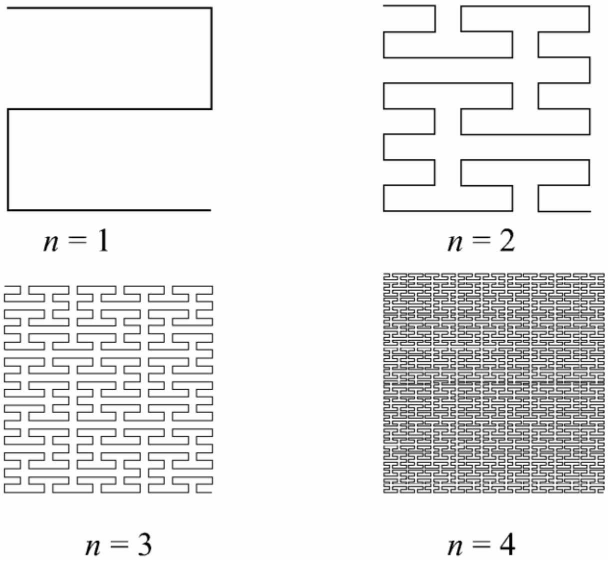


**Fig.1. The First Four Iteration Levels to Generate the Peano Pre-Fractal Curve.**

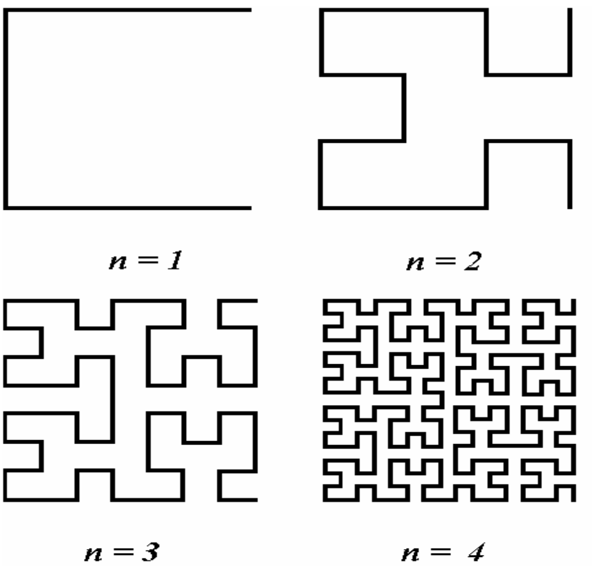


**Fig.2. The First Four Iteration Levels to Generate the Hilbert Pre-Fractal Curve.**

The second concept for a fractal is a fractional dimension. This requirement distinguishes fractals from the Euclidean geometries, which have integer dimensions. The common intuitive idea of dimension is referred to as topological dimension. A point, a line segment, a square and a cube have topological dimensions zero, one, two and three, respectively. This intuitive dimension is always expressed as an integer.

In [1, 2] the Hausdroff-Besicovich dimension is referred to as the fractional dimension, and it is defined as, a real number that precisely measures the object's complexity. Mandelbrot defines a fractal as a set for which the Hausdroff-Besicovich dimension strictly exceeds the topological dimension. He refers to this dimension as the fractal dimension of a set. Fractional dimension is related to self-similarity in that; the easiest way to create a figure that has fractional dimension is through self-similarity. The character of non-integer dimension causes the fractal dimension to be useful in measurement, analysis and classification of many fractal shapes, for example, the fractal dimension provides a way to measure how rough fractal curves are. In addition, the fractal dimension can describe how much a fractal curve fills the space.

Fractal structures have found increasing applications in different aspects of science and arts. They are successfully used in the fields of physics, chemistry, biology, architecture, etc… [3].

The research in the field of electrodynamics began soon after the scientists discovered the practical aspects of the fractal geometry. Most efforts had been devoted to understand the physical process and mathematical background of the interaction between electromagnetic waves and fractal structures [4-6]

In passive microwave circuits design, such in the design of the different types of filters, fractals have been used widely and extraordinary results were obtained. The space-filling property of fractals had led to producing miniaturized sizes of passive microwave circuits for compact wireless communication systems.

In microwave antenna design, size miniaturization of the normal printed dipole antenna can be accomplished either by the use of high dielectric constant substrates instead of air or some foam materials with dielectric constant nearly like that of air, by the modification of the basic dipole shape, or by a combination of these two techniques [7].

Employing high dielectric constant substrates is the simplest solution, but it exhibits narrow bandwidth, high loss and poor efficiency due to surface wave excitation [7]. Fractals are supposed to be considered in the second category, i.e., antenna shape modification. In this sense, the space-filling property of the fractal antenna offers the required compact size, while its self-similarity makes it resonates in more than one frequency band, due to the many resonating substructures it consists of in the whole structure [8].

The use of fractals in microwave antenna design has dramatically increased in the recent years, where miniaturized and multiband antennas have to meet the challenges imposed upon the modern communication systems to be compact and multi-functional.

In this paper a fractal printed dipole antenna based on the second level ($n$ =2) tent transformation has been presented as a candidate for use in modern compact and multi-function communication systems. The proposed antenna dimensions can be optimized to satisfy the compact size and the required radiation characteristics for the specified applications operation.

## 2. Fractal Dipole Antennas

Since the application of the fractal concept on electrodynamics, much work has been devoted to antenna design [9-17]. The first reported small fractal antenna is the Koch dipole [9]. In this work, some of the classical features such bandwidth, resonance frequency, and radiation resistance had been improved. Later, different fractal geometries, such as Hilbert, Peano, Minkowski, Sierpinski etc..., have been applied to dipole antenna design [10-17]. The reported designs offered astonishing results of antenna performance, whether in the compact size gained or in the multi-resonant behavior they possess. In Figs (1) and (2), Hilbert and Peano fractal curves up to the fourth iteration level, $(n = 4)$ are depicted, for the sake of comparison with the presented tent curve fractal. These fractals have been widely used in dipole antenna design.

An interesting point of comparison in this context is the total length of the fractal in each iteration level as a function of the side length, $L$, of the area containing it. This factor acquires its importance from the fact that it mainly determines the lowest resonance frequency of the multi-band fractal dipole, and hence the reduction in size gained in comparison with the classical dipole antenna or other fractally designed dipoles.

For the Hilbert fractal curve, the total length, $S_n$, in the $n^{th}$ order generation level is given in [18] by:

$$S_n = (2^n + 1)L \qquad \dots(1)$$

where, $L$ is the side length.
While for the Peano fractal curve, the total length, $S_n$ is given by [13] as:

$$S_n = (3^n + 1)L \qquad \dots(2)$$

where, $S_n$, $n$, and $L$ are as defined earlier.

It is obvious from Equs. (1), and (2) that, the total length of the curve offered by Peano fractal is greater than that offered by Hilbert fractal of the same generation order with the same side length. This means that Peano fractal curve presents better antenna miniaturization than Hilbert fractal does, when it is used in the design of a fractal dipole.

## 3. Fractal Tent Transformation

The generation process of the fractal curve based on tent transformations is more complicated than those of the Hilbert and Peano fractals.

The presented fractal curve is constructed by applying geometrical transformations of a unit square with a side length $L$, representing the well-known tent function, Fig.(3a) using the transformation algorithm, which is called *multiple reduction copy machine* (MRCM) as proposed by [19]. This MRCM provides a good metaphor for what is known as *deterministic iterated function systems* (IFS) in mathematics. The MRCM generates a dynamical iterated function system (IFS), Fig. (3b), [19]. Using such an IFS, it is possible to produce a generation level in which all line segments join up to form a single path. As it is clear from Fig (3b), the IFS constructs such a curve with five transformations, and the space-filling property follows from the invariance of the initial square, the tent function, under the IFS. These five transformations, labeled as A, B, C, D, and E, which produce any fractal level from its preceding one, are summarized in Table1. In each transformation, more than one operation has to be performed on the original tent function, such stretching, flipping, and/or rotation. Figs (4a-d) show the generated tent fractal curve up to the fourth order ($n$ = 4).

**Table 1**
**Summary of Steps to Generate a Fractal Tent Transformation.**

| Step | Width Stretched by | Height Stretched by | Flipping | Rotation (deg.) |
|---|---|---|---|---|
| A | 2/3 | 1/3 | horizontal | none |
| B | 1/3 | 2/3 | horizontal | none |
| C | 1/3 | 2/3 | horizontal | 90 |
| D | 2/3 | 1/3 | none | -90 |
| E | 2/3 | 2/3 | none | -90 |

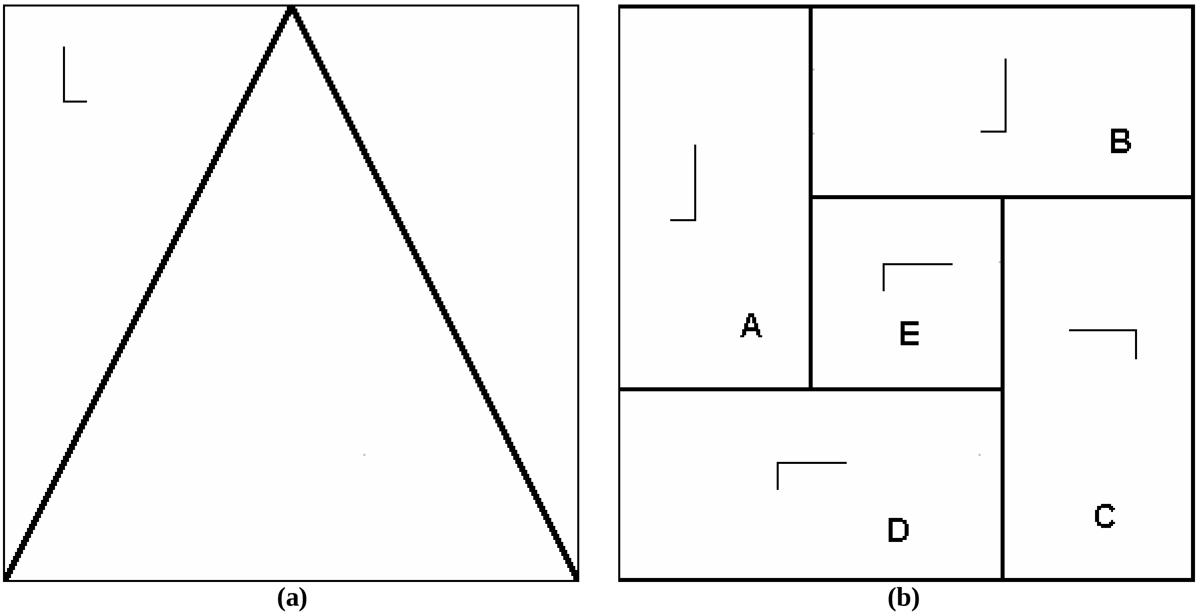


**Fig.3. (a); The Starting Tent Function as the Initiator Structure, and (b); the Iteration Function System Used to Generate the Tent Fractal Curve at the Different Iteration Levels [19].**

As shown in Figs (4a-d), the constructed curve in a certain generation level ($n$) is simply a collage of the five transformations of the previous level $(n-1)$. Because the initial tent function has a suitable symmetry, one can easily be misled when applying the IFS. The IFS uses the unit square with the inscribed letter L as an indication of the orientation as the initial square, Fig (3a).

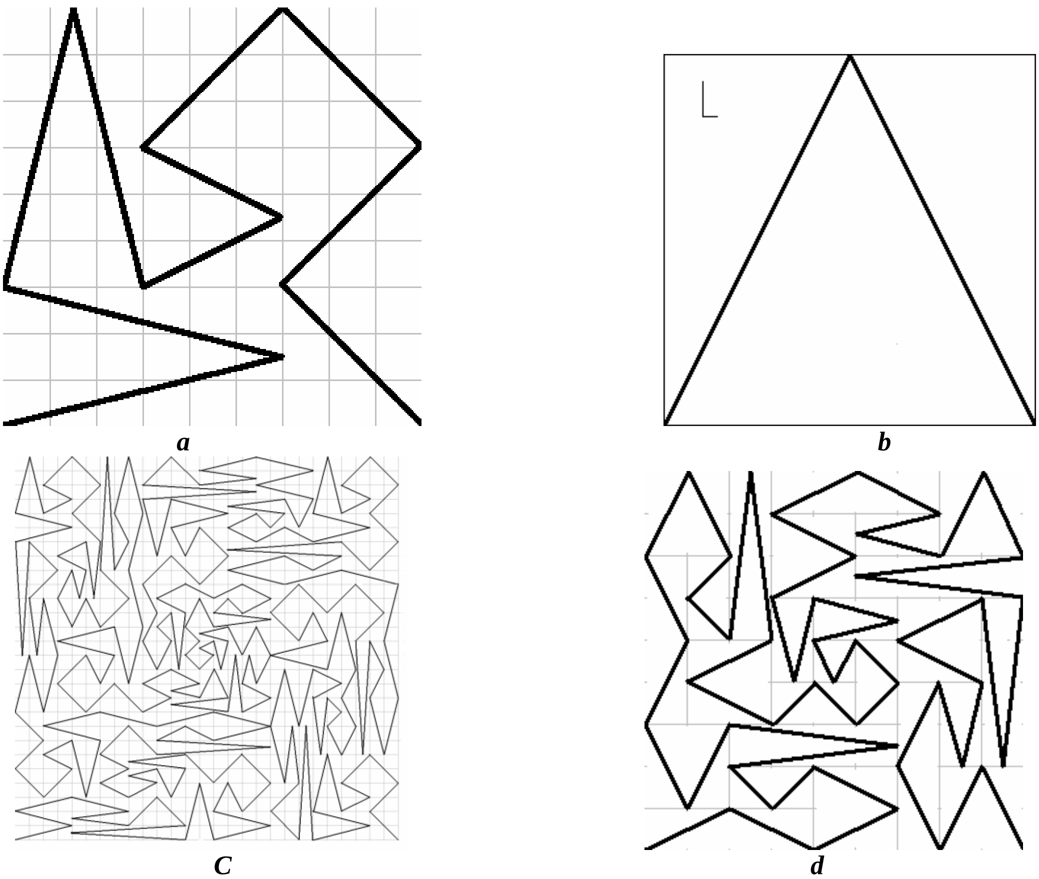

**Fig.4. The Details of the Generation Steps of the Tent Fractal Curve. Structures from (a) to (d) Correspond to the First Four Generation Levels.**

It has been found that the total length $S_n$, of the tent fractal curve at the $n^{th}$ generation, is:

$$S_n = (\frac{7}{3})^{n-1} a_n L \qquad \ldots(3)$$

where $a_n$ is a constant depending on the starting angle $\theta$, of the initial tent function.

However, the value of this angle is bounded by an upper limit of $\theta$ = 63.435°; at which all the vertices of the triangle touches the square, as shown in Fig (5), and a lower limit of $\theta$ = 0°, at which the tent function is considered as a straight line of length equals to the side length, $L$ of the square containing it.

From Fig (5), the tent function is given as:

$$f(x) = \begin{cases} ax, & x \le 0.5L \\ a(1-x), & x > 0.5L \end{cases} \qquad \ldots(4)$$

And the angle $\theta$ is defined as:

$$\theta = \tan^{-1} a$$

Thus:

$$0^\circ \le \theta \le 63.435^\circ$$

For which:

$$0 \le a \le L$$

It has also been found that $a_n$, in Equ (3), is varied as:

$$1 \le a_n \le 2.236$$

for:

$$0^\circ \le \theta \le 63.435^\circ$$

It is worth to note that for $\theta$ = 63.435°, the tent curve has no longer be a fractal after the 3rd generation step, since at the 4th generation step the resulting curve is not self-avoiding . Fig (6) shows an enlarged copy of Fig (4d). The two circles indicate that the same two points in the space have been visited twice. Nevertheless, the fractal curve can be used at this value of $\theta$, up to the 3rd generation, since a maximum space-filling is gained according to Eqn. (3), and it is still self-avoiding.

Practically, if fractal curves are applied, few numbers of iterations are enough to model an antenna [8,11,14]. However, to generate a tent fractal self-avoiding curve with higher generation levels, the starting angle must be reduced.

A comparison of Equs(1), (2), and (3) shows that the presented fractal curve has the best space-filling property than both Hilbert and Peano fractal curves at the same generation order and the same side length $L$. For example, at $n$ = 2, the total fractal curve length $S_n$, is given as:

$S_2 = 5L$, for Hilbert fractal curve

$S_2 = 10L$, for Peano fractal curve

$S_2 = 12.173L$, for tent fractal curve

This means that, the tent fractal curve will offer the best dipole antenna miniaturization as compared with the other two.

## 4. Antenna Design and Performance Evaluation

Up to the author's knowledge, the only published work about the use of tent fractal dipole antenna in the UHF band is that of Hödlmayr [20]. In that work, a wire dipole antenna has been designed and operated at this band. The concentration there is focused from practical point of view. In the present work, a 2nd iteration tent fractal structure has been modeled as a dipole antenna with two of such a structure composing its two arms as shown in Fig.7. The dipole is supposed to be printed on a material with a relative dielectric constant of nearly one, or just built in free space. This will directly permit frequency scaling of the modeled dipole to make it resonating at any desired frequency, since no need of material scaling is required. On the other hand, using a substrate with a dielectric constant greater than one, for the antenna to be printed on, results in reduced antenna efficiency due to the associated losses. In such an antenna with a wide multiband behavior, material scaling becomes an impossible task over this wide frequency range.

The antenna is fed with a coaxial cable of 50Ω characteristic impedance. The width of the dipole trace has been chosen to be 0.5% the dipole length [12, 21]. The spacing between the two arms constituting the dipole is found to be of less than 5% the dipole length.

Theoretical calculations of the antenna performance of the antenna and electromagnetic simulation are carried out using IE3D package, from Zeland Software Inc. This EM simulator performs performance calculations of 3*D* electromagnetic structures using the method of moments, (MoM).

At first, a dipole has been modeled with a side length of 15 mm. The return loss response of this model shows an obvious multiband behavior with first resonance frequency at 7 GHz. This initial structure is then frequency scaled to the desired frequency, 2.45 GHz. The resulting dipole antenna has been found to have a side length of 43.75 mm. The corresponding return loss response of this antenna is depicted in Fig.8. The first resonance takes place at a frequency of approximately 2.45 GHz, while other six resonances occur at nearly regular intervals in the swept frequency range shown in the figure. For demonstration purposes, some of the computed antenna parameters such as the radiation efficiency, the antenna efficiency, the gain (*G*), and the directivity (*D*), for many frequency points around the first resonance frequency are listed in Table 2.

**Table 2**
**Some Antenna Parameters around the First Resonant Frequency.**

| Antenna Parameters / Frequency (GHz) | Rad. Eff. (%) | Ant. Eff. ( %) | G ( dBi) | D ( dBi) |
|---|---|---|---|---|
| 2.100 | 95.18 | 76.32 | 1.151 | 2.325 |
| 2.214 | 96.21 | 90.69 | 1.379 | 1.804 |
| 2.328 | 95.59 | 93.57 | 1.659 | 1.948 |
| 2.442 | 95.82 | 95.32 | 2.142 | 2.350 |
| 2.557 | 97.81 | 95.76 | 2.474 | 2.662 |
| 2.671 | 98.59 | 94.42 | 2.533 | 2.782 |
| 2.785 | 99.18 | 91.70 | 2.334 | 2.710 |
| 2.900 | 99.56 | 91.21 | 2.136 | 2.535 |

Fig.9 shows the elevation directivity patterns at the first resonance frequency at $\phi = 0°$ and $\phi = 90°$. Fig.10 shows the three dimension radiation pattern of the modeled dipole at the same frequency. It is clear that the antenna parameters and its resulting radiation characteristics are acceptable for the proposed applications.

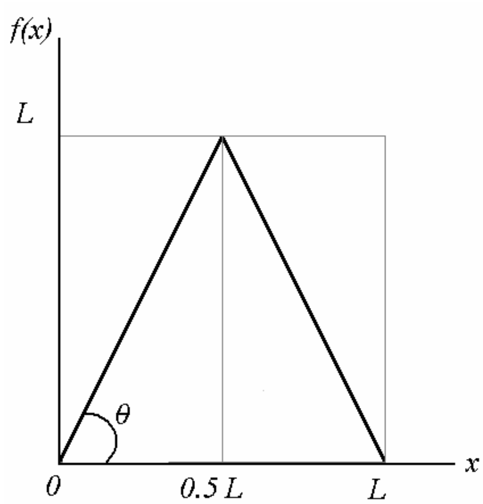


**Fig.5. The Tent FUNCTION with Side Length, *L* and the Starting Angle, *θ*.**

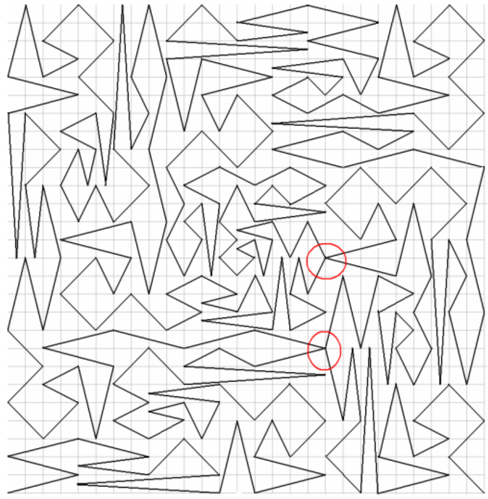

**Fig.6. An Enlarged Copy of Fig. 4d. The Two Circles Shown Indicate that, at the 4th Iteration Level the Resulting Structure is not a Fractal Anymore, Since Two Same Points in Space have been Visited Twice.**

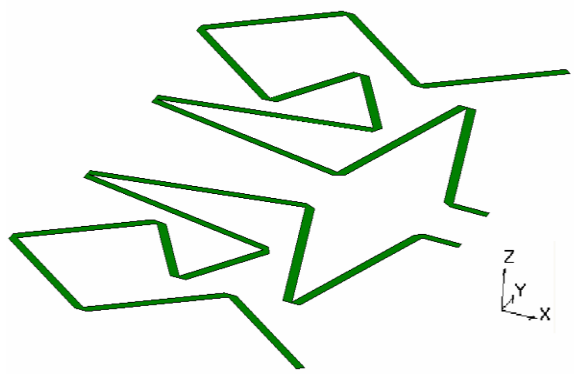


**Fig.7. The Layout of the Modeled Tent Fractal Antenna with Respect the Coordinate System.**

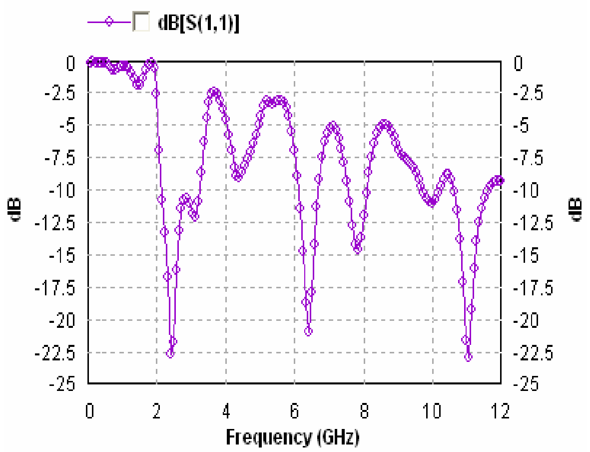


**Fig.8. The Return Loss Response of the Tent Fractal Dipole Antenna after Frequency Scaling to 2.45 GHz.**

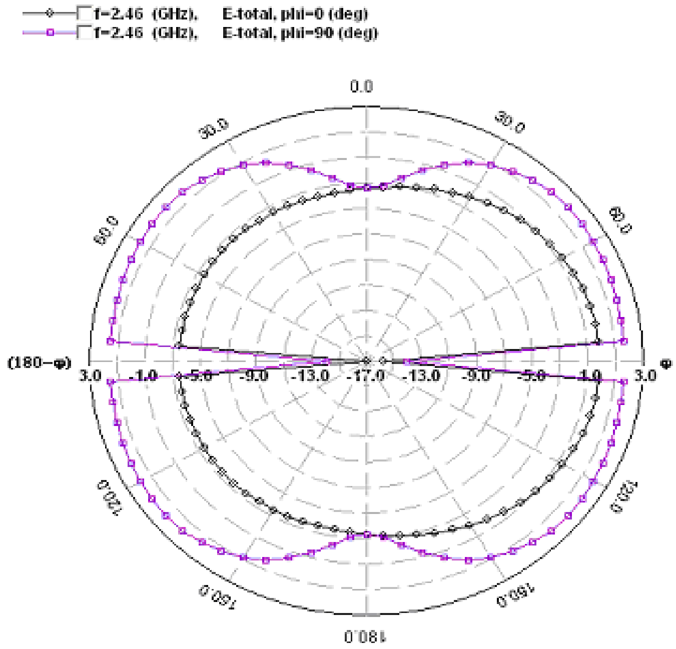


**Fig.9. The Elevation Patterns Directivity Display at a Frequency of 2.46 GHz.**

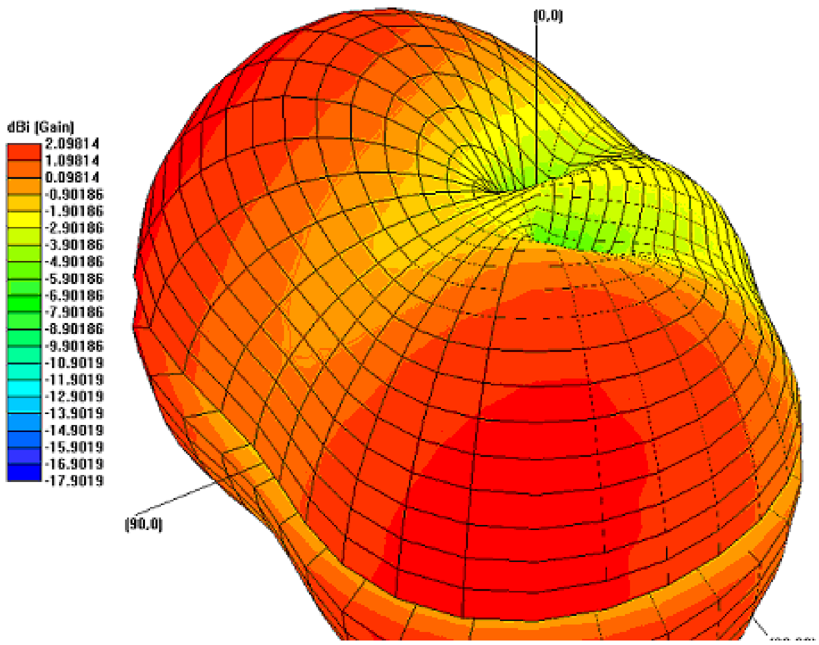


**Fig.10. The 3D Radiation Pattern at a Frequency of 2.45 GHz.**

## 5. Conclusions

The tent fractal dipole antenna has been presented in this paper, analyzed in details, and simulated. The antenna seems promising to be used in multi-function communication systems due to its good multiband response.

Simulation results assure the multiband operation of this antenna with accepted radiation characteristics for the proposed applications. Results showed that the modeled dipole antenna has reasonable antenna parameters at resonance. Much work has to be carried out to investigate its counterpart monopole antenna. The effect of substrate parameters on antenna performance has to be investigated when printing the antenna on a microstrip substrate.

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# هوائي جديد ثنائي القطب مطبوع مبني على اساس الترتيب الهندسي الجزئي لتحويلات دالة الخيمة لتطبيقات الاتصالات اللاسلكية

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**الخلاصة**

في هذا البحث ،يتم استعراض هوائي مصغر متعدّد النطاق الترددي (multiband) من النوع ثنائي القطب المطبوع printed antenna dipole على انه مرشّح للإستعمالِ في تطبيقاتِ الاتصالات اللاسلكية. إنّ تصميمَ الهوائي المُقتَرَحِ مستند على اساس الترتيب الهندسي الجزئي fractal geometry لتحويلِ دالة الخيمة (tent function) من المستوى الثاني . تَسْمحُ خاصية املاء الفراغ (الفضاء) التي يتصف بها هذا الترتيب الهندسي بإنْتاج اطوالِ اكبر في مساحات أكثر إنضغاطاً. تم حساب الأداء النظري لهذا الهوائي باستخدام الحقيبة البرمجِية المتوفرةِ تجاريا IE3D، من انتاج مؤسسة Zeland، والتي تستند في اجراء التقييم باسلوب المحاكاة على طريقةِ ايجاد العزوم (MoM) . اظهرت النتائج بأن الهوائي المقترح يوفر تخفيضا كبيرا في الحجمِ مقَارنةَ بالهوائي الثنائي القطب التقليدي المصمّمَ في نفس ترددِ التصميمِ. وعلاوة على فأنّ التصميمَ المُقتَرَحَ يمتلكَ سلوكا متعدّد الرنين مع احتفاظه بمواصفات اشعاع مناسبة في أنحاء انطقة الرَنين، وهذا يَجْعلُ الهوائي ثنائي القطب المقترح (أو نظيره الاحادي القطب monopole) مناسبا للإستعمالِ في أنظمةِ الاتصالِ متعددةِ الوظائفَ الحديثةَ ذات الاحجام المصغرة.