Inverse Kinematics Analysis and Simulation of a 5 DOF Robotic Arm using MATLAB

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Abstract

Kinematics is the mechanics branch which deals with the movement of the bodies without taking the force into account. In robots, the forward kinematics and inverse kinematics are important in determining the position and orientation of the end-effector to perform multi-tasks. This paper presented the inverse kinematics analysis for a 5 DOF robotic arm using the robotics toolbox of MATLAB and the Denavit-Hartenberg (D-H) parameters were used to represent the links and joints of the robotic arm. A geometric approach was used in the inverse kinematics solution to determine the joints angles of the robotic arm and the path of the robotic arm was divided into successive lines to accomplish the required tasks of the robotic arm. Therefore, this method can be adopted for engineering applications. MATLAB (Graphical User Interface) program was used to simulate the movement of the robotic arm in 3D. Also, MATLAB (GUI) has been used to view the position of each joint. The results showed that the maximum error in the x, y, and z coordinates of the end-effector were 0.0251 %, 0.0239 %, and 0.1085 % respectively.

Keywords: Robotics, forward kinematics, inverse kinematics, D-H parameters, MATLAB.

1. Introduction

Recently, the demands to use robots are increasing in many fields such as space, medical, industrial application, etc.. But the real challenge is the kinematics analysis of robot with higher DOF [1]. The kinematic analysis contains two components: forward kinematics and inverse kinematics to study the movement of the robots. The forward kinematics is used to determine the position of the end-effector of a robot using the angles of the joints as inputs, while the position is used as input to determine the joint angles in inverse kinematics [2]. The simplest method to represent the robot links and joints to solve the forward kinematics is a Denavid-Hartenberg (D-H) method [3]. On the other hand, there are many methods that are being used with inverse kinematics such as geometric approach, algebraic solution, and iterative solution [4]. A MATLAB program is one of the most programs used in many applications such as image processing, optimization, matrices, technical computing, etc.. Robotic toolbox in MATLAB is used in determining the position of the end effector of the robotic arm and in simulating the movement of this robotic arm depending on the DH parameters [5, 6].

2. Related literature

Wathik I. M. AL-Tameemi and Wael M. H. Hadi [7] studied the forward and inverse Kinematic analysis of a 5 DOF robotic arm (Lab-Volt 5250); where the DH parameters were used

to solve the forward kinematics and the analytical solution was used with inverse kinematics analysis. MATLAB was used in both forward and inverse kinematics. The results showed that the maximum error in position was (0.5 %).

Tarun P. Singh et al [8] presented the kinematic analysis (forward and inverse) of 6 DOF manipulator arm. The Analysis was performed using MATLAB, the DH parameters were used with forward kinematics and an iterative solution was used with inverse kinematics. An acceptable error was obtained from the comparison between the analytical results and software results.

Tahseen F. Abaas and Hind H. Abdulridha [9] presented a modeling of a 5 DOF robotic arm (Lab-Volt 5150) using the DH parameters and simulated the movement of the robotic arm using the MATLAB program. The comparison between MATLAB outputs and RoboCIM outputs was performed to know the acceptance of the modeling of the robotic arm.

Alla N. Barakat et al [10] studied the 3D simulation of a robotic arm using a MATLAB program depending on DH parameters. The results explained the way of achieving the desired goal of the work which was drawing sinewave on board.

3. Kinematics Analysis

kinematics analysis of robots has defined the relationship between the links and joints with the position and orientation of the robot. The kinematics analysis is divided into forward kinematics and inverse kinematics.

3.1 Forward Kinematics

Forward kinematics is used to determine the position and orientation of the end-effector of the robotic arm from the specified joints angles. The DH method is one of the most methods used in forward kinematics which represents the relationship of the joint coordinate between two links as shown in figure (1).
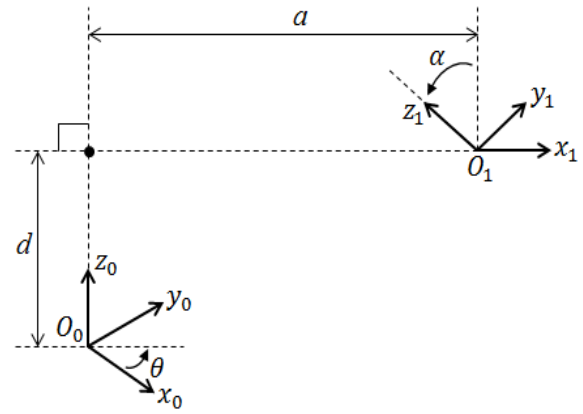

Fig. 1. The DH frame.

Where**:**
a_i: Link length.
α_i: Link twist.
d_i: Link offset.
θ_i: Joint angle.

The robotic arm used has 5 DOF and the DH parameters of this robotic are listed in Table (1).

Table 1,
DH parameters for the robotic arm.

Link	a_i (mm)	α_i (degree)	d_i (mm)	θ_i (degree)
1	0	90	105	θ_1
2	105	0	0	θ_2
3	100	0	0	θ_3
4	0	90	0	θ_4
5	0	0	150	θ_5

Also, the transformation matrix between two successive links can be obtained using the DH frame as follow:

$$T_i = R_{z,\theta_i}\, T_{z,d_i}\, T_{x,a_i}\, R_{x,\alpha_i}$$

$$T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_i = \begin{bmatrix} c\theta_i & -s\theta_i\, c\alpha_i & s\theta_i\, s\alpha_i & a_i\, c\theta_i \\ s\theta_i & c\theta_i\, c\alpha_i & -c\theta_i\, s\alpha_i & a_i\, s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

…(1)

Where:
$c\theta_i = cos(\theta_i)$

$s\theta_i = sin(\theta_i)$
$c\alpha_i = cos(\alpha_i)$
$s\alpha_i = sin(\alpha_i)$

The individual matrices and the global matrix of the 5 DOF robotic arm can be obtained by substituting the DH parameters in Table (1) in the equation (1), as showed in figure (2):
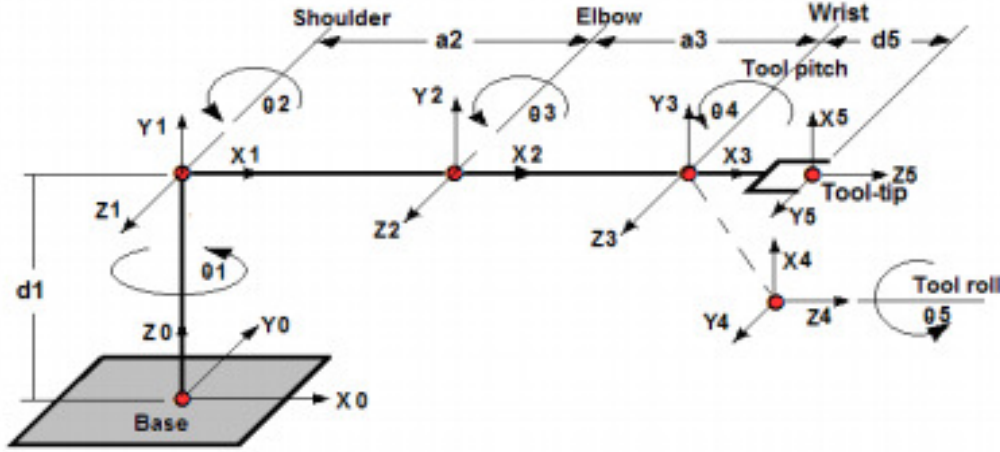

Fig. 2. The coordinate frame of the robotic arm.

$A_1^0 = R_{z,\theta_1} \, T_{z,d_1} \, T_{x,a_1} \, R_{x,\alpha_1}$

$$A_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(2)$$

$$A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(3)$$

$$A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(4)$$

$$A_4^3 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(5)$$

$$A_5^4 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(6)$$

And the global matrix A_5^0:

$A_5^0 = A_1^0 \, A_2^1 \, A_3^2 A_4^3 A_5^4$

$$A_5^0 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(7)$$

$m_{11} = c_{12345} + s_{15}$
$m_{12} = -s_5c_{1234} + s_1c_5$
$m_{13} = c_1s_{234}$
$m_{14} = c_1(d_5s_{234} + a_3c_{23} + a_2c_2)$
$m_{21} = s_1c_{2345} - c_1s_5$
$m_{22} = -s_{15}c_{234} - c_{15}$
$m_{23} = s_1s_{234}$
$m_{24} = s_1(d_5s_{234} + a_3c_{23} + a_2c_2)$
$m_{31} = c_5s_{234}$
$m_{32} = -s_5s_{234}$
$m_{33} = -c_{234}$
$m_{34} = -d_5c_{234} + a_3s_{23} + a_2s_2 + d_1$

Where:

$c_{12345} = cos(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)$
$s_{15} = sin(\theta_1 + \theta_5)$

3.2 Inverse Kinematics

Inverse Kinematics (IK) is used to determine the required joints angles of the robotic arm to achieve the specified position and orientation of the end-effector of the robotic arm. In this work, the geometric approach was used to solve the inverse kinematics of the 5 DOF robotic arm:
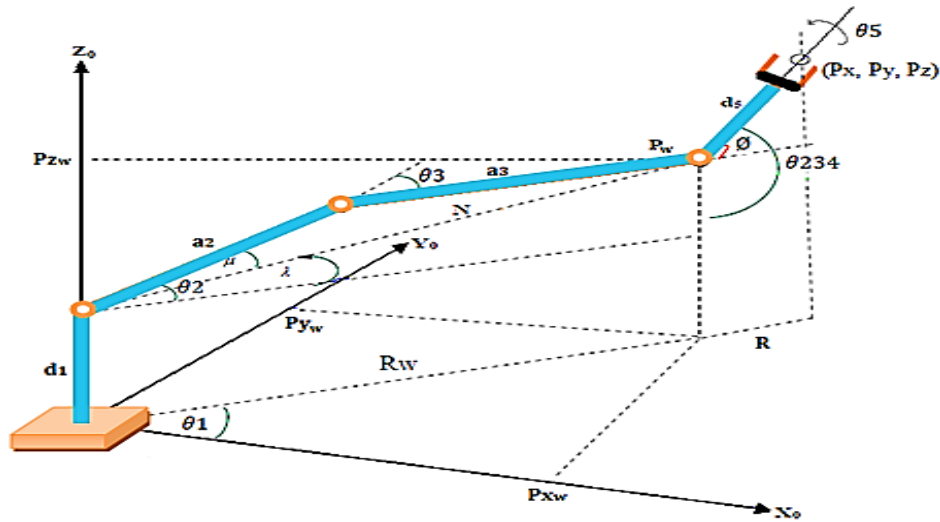
Fig. 3. The 4 link articulated robot [9].

From figure (3), the wrist angle relative to the reference coordinate (θ_{234}) represents the relation between (θ_2, θ_3, and θ_4):

$$\theta_{234} = \theta_2 + \theta_3 + \theta_4 \qquad \text{...(8)}$$

where θ_{234} can be calculated based on pitch wrist orientation angle $\emptyset$

$$90 - \theta_{234} = \pm\emptyset \qquad \text{...(9)}$$
$$R = d_5 * \cos\emptyset \qquad \text{...(10)}$$
$$P_{x_w} = P_x - R\cos\theta_1 = X_w \qquad \text{...(11)}$$
$$P_{y_w} = P_y - R\sin\theta_1 = Y_w \qquad \text{...(12)}$$
$$P_{z_w} = P_z + d_5\sin\emptyset = Z_w \qquad \text{...(13)}$$
$$R_w = \sqrt{P_x^2 + P_y^2} - R = \sqrt{P_{x_w}^2 + P_{y_w}^2} \qquad \text{...(14)}$$
$$N = \sqrt{\left(P_{z_w} - d_1\right)^2 + R_w^2} \qquad \text{...(15)}$$
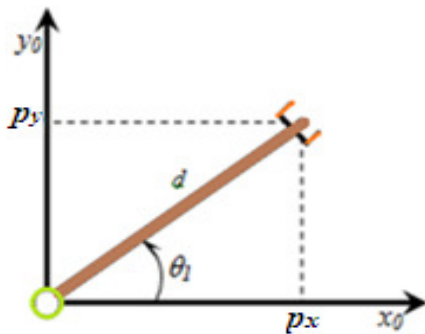

Fig. 4. Top View of the Robotic arm.

- **Solution for θ_1**: base angle can be calculated from Figure 4 as follows:

$$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right) \qquad \text{...(16)}$$

The other solution will be:
$$\hat{\theta}_1 = \theta_1 + 180 \qquad \text{...(17)}$$

- **Solution for θ_2**: by using the law of cosines:
$$a_3^2 = N^2 + a_2^2 - 2a_2 N\cos(\mu) \Rightarrow \qquad \mu = \cos^{-1}\left(\frac{N^2 + a_2^2 - a_3^2}{2a_2 N}\right) \qquad \text{...(18)}$$
$$\lambda = \tan^{-1}\left(\frac{P_{z_w} - d_1}{R_w}\right) \qquad \text{...(19)}$$
$$\theta_2 = \lambda \mp \mu \qquad \text{...(20)}$$

- **Solution for θ_3**:
$$N = \sqrt{\left(a_2^2 + a_3^2 - 2a_2a_3\cos(\pi - \theta_3)\right)} \qquad \text{...(21)}$$
$$\theta_3 = \pm\cos^{-1}\left(\frac{N^2 - a_2^2 - a_3^2}{2a_2a_3}\right) \qquad \text{...(22)}$$

- **Solution for θ_4**:
$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 \qquad \text{...(23)}$$

4. Simulation

MATLAB Graphical User Interface (GUI) was used in this work to perform many functions as shown in figure (5). It illustrated the inputs to the program was the position of the end-effector of the robotic arm while the outputs were the joints angles of the robotic arm and the position of each joint. Also, a 3D simulation of the robotic arm movement was presented in GUI as an output. The program was performed as in the following steps:

- Setting the position (P_x, P_y, and P_z) of the goal relative to the base of the robotic arm as inputs.

- A function used to determine the joints angles of the robotic arm using the equations (8) to (23).
- A function used to determine the position of each joint of the robotic arm using the equations from (2) to (6).

- 3D simulation to the movement of the robotic arm was applied using the Robotics toolbox in MATLAB depending on the DH parameters.
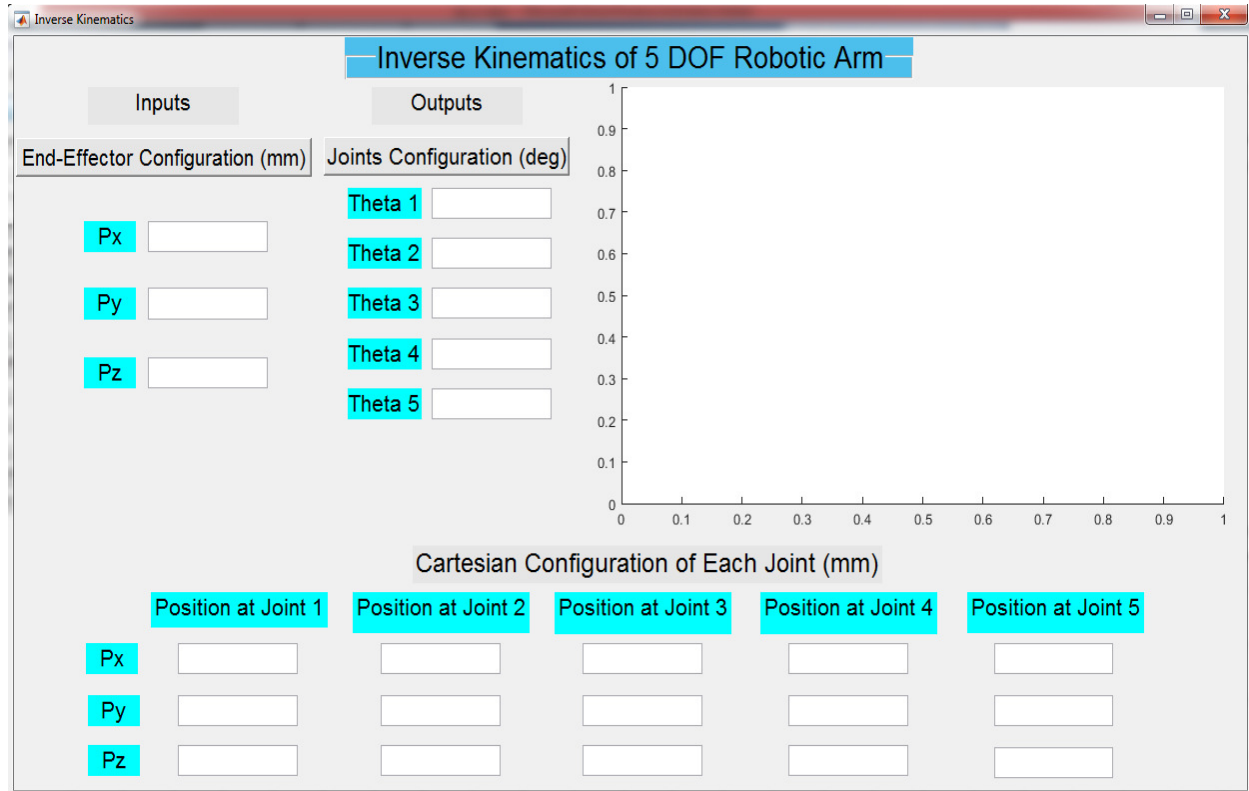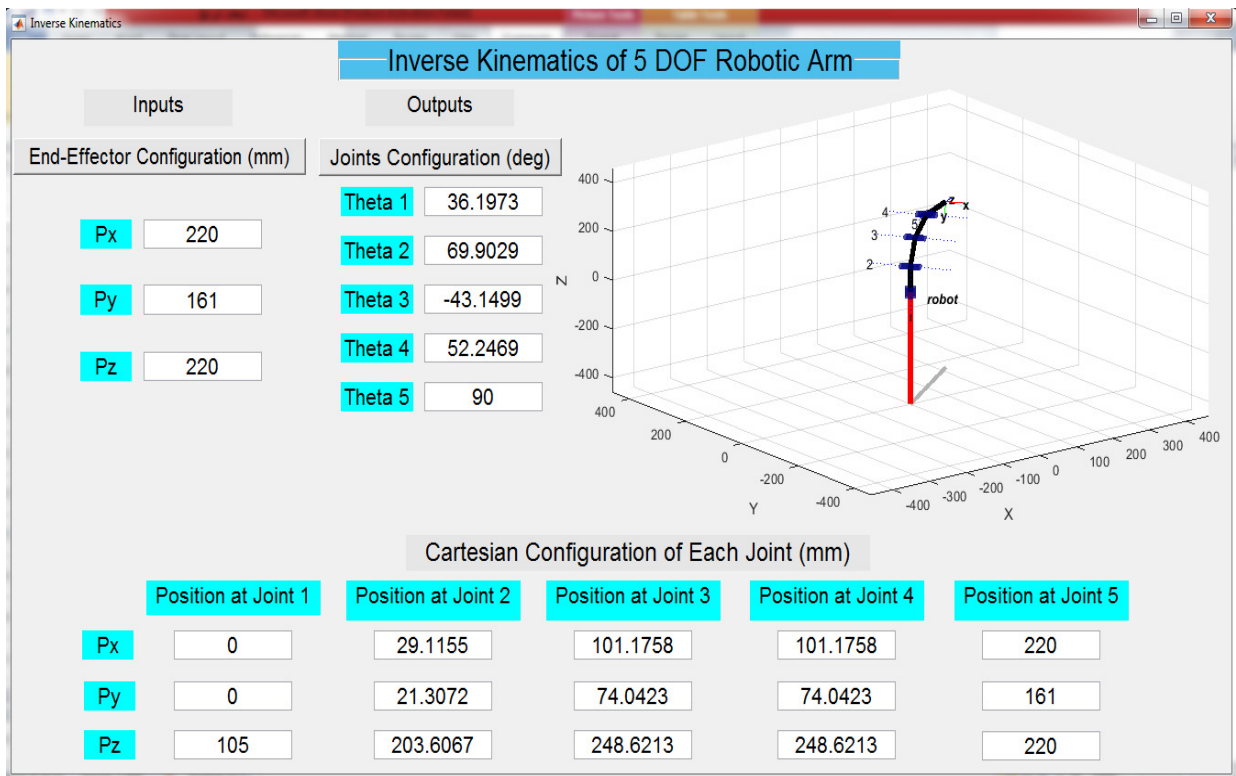

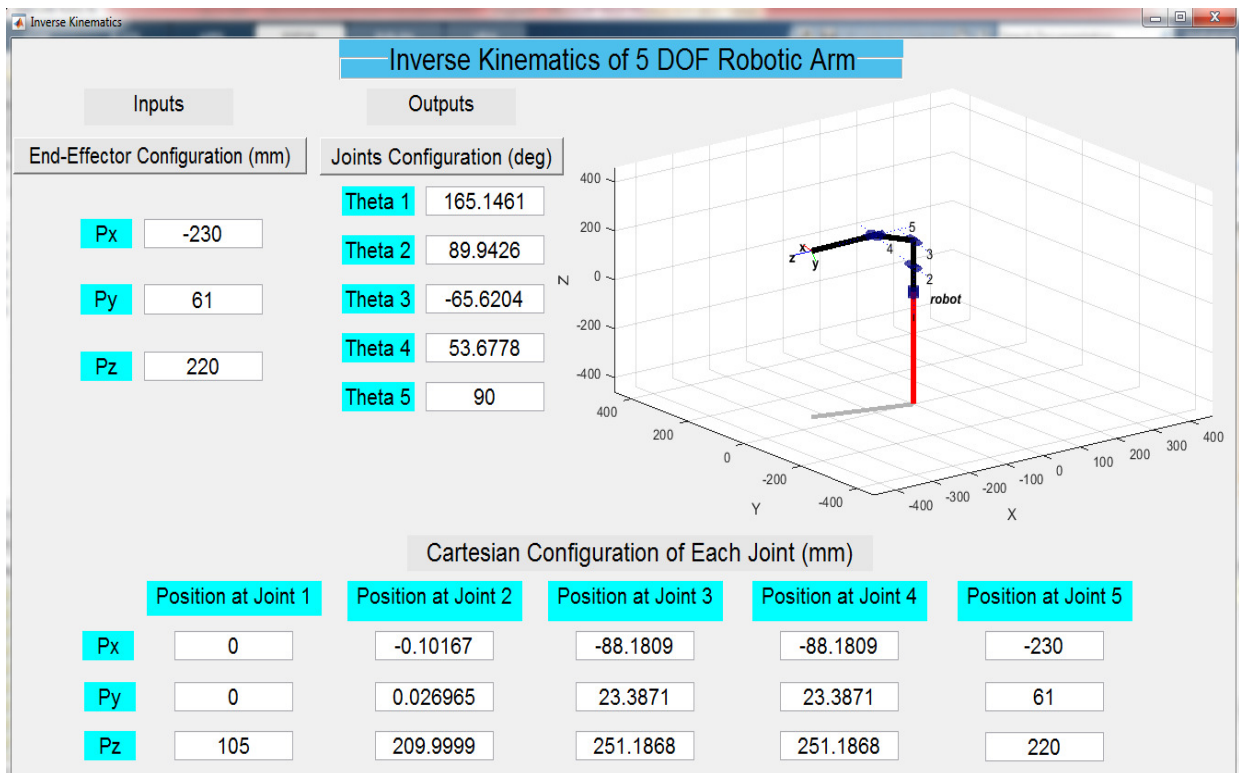**Fig. 5. The window of the GUI program.**

5. Results and Discussions

To test the efficiency of the geometric approach in the inverse kinematics analysis, several cases were studied. The inputs to the GUI program, shown in figure (5), were the position of the goal $(P_x, P_y, and\ P_z)$ and the outputs were the joints angles of the robotic arm. The 3D representation of the robotic arm depending on the DH parameters is shown in figure (6) and figure (7). The 3D representation illustrates four cases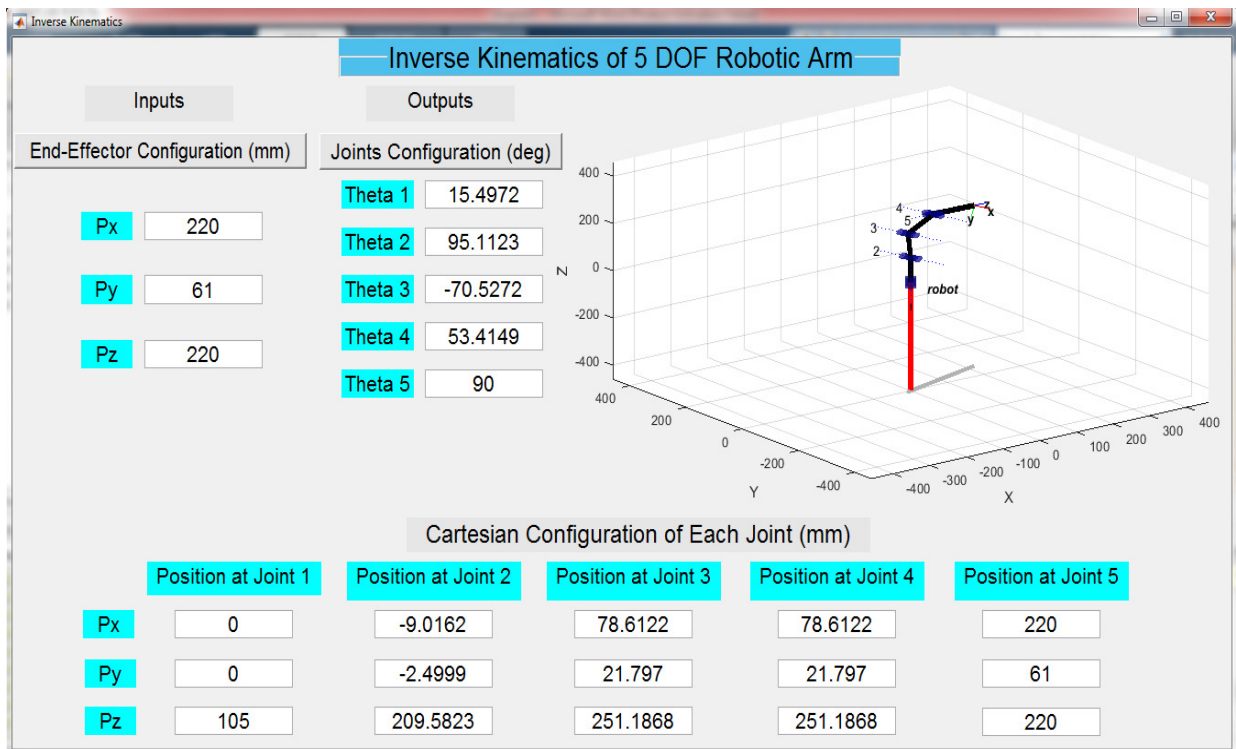. The extracted angles of joints were applied in equation (7) to determine the position of the end-effector and compare these results with the input results. The errors between them were obtained and listed in Table 2 to indicate the efficiency of the program used. The algorithm of the geometric approach was developed for the inverse kinematic analysis of the robotic arm and compared the results of this method with results extracted using an algebraic solution. There was a large match in these results, so the geometric approach method was applied.
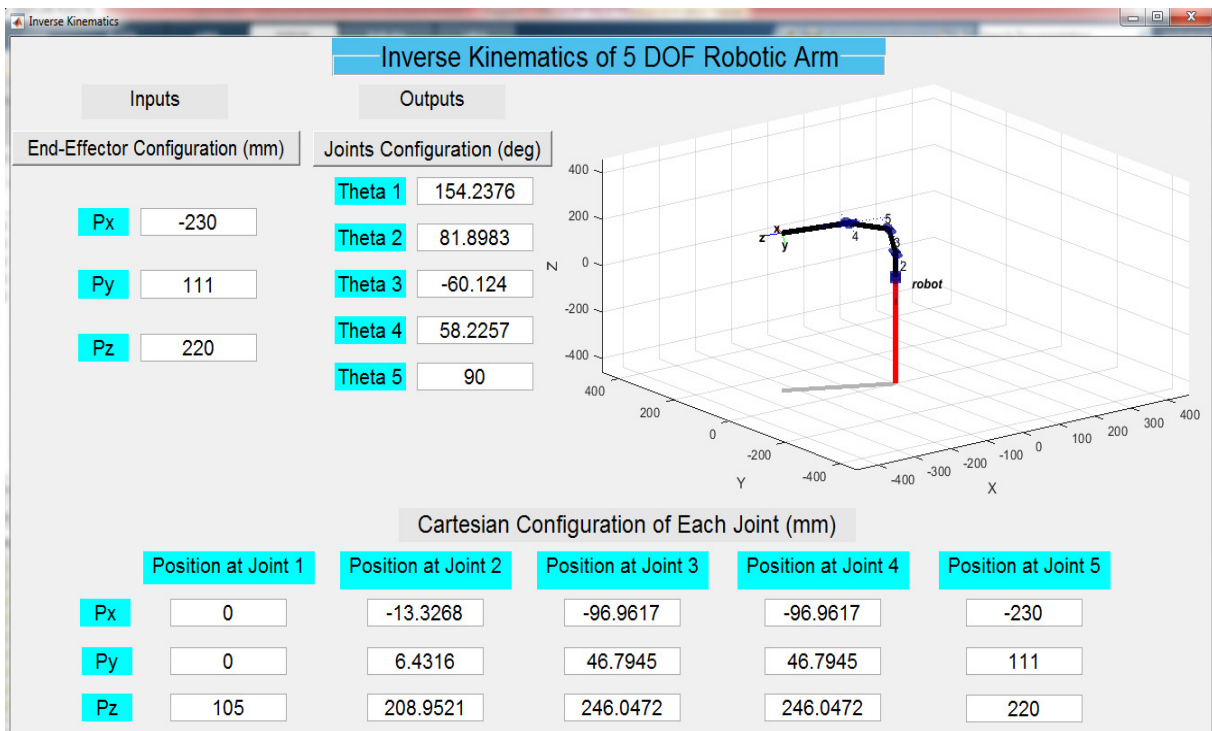
Case (1)

Case (2)

Case (3)

Case (4)
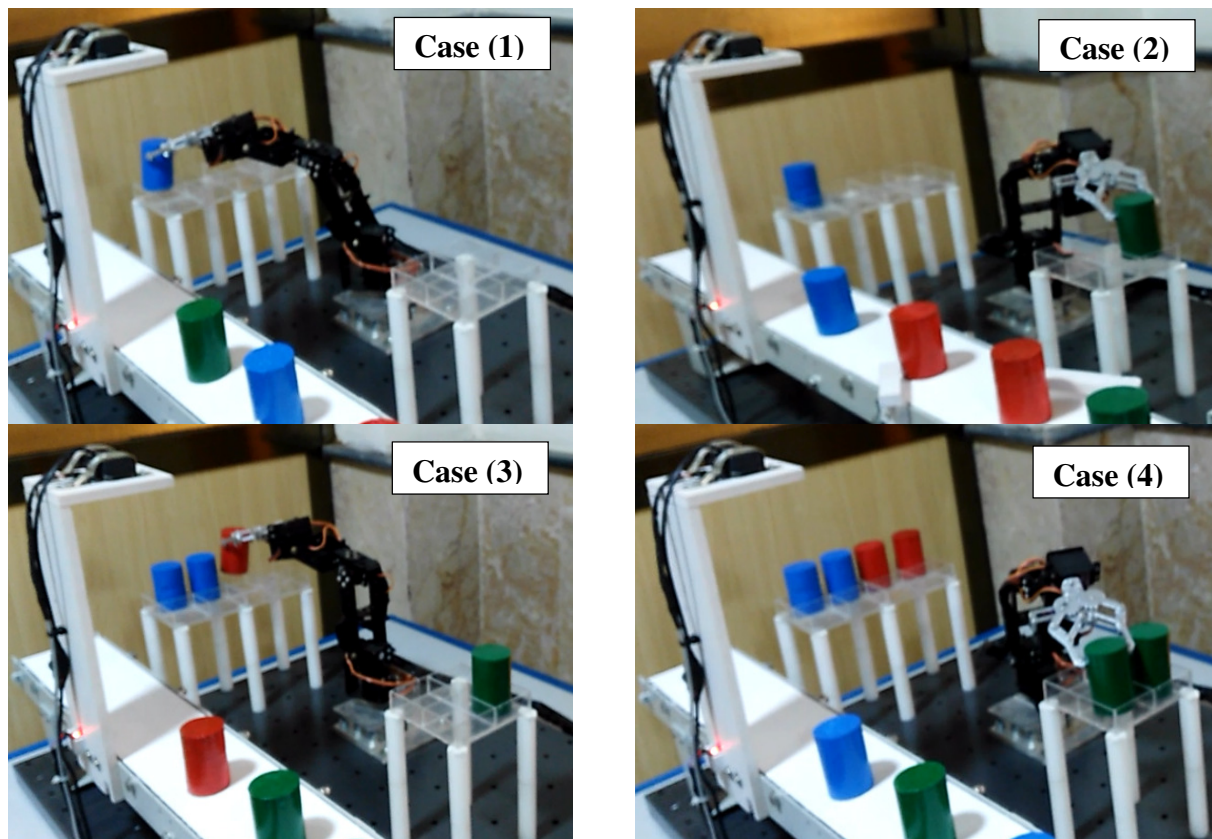
Fig. 6. GUI for four cases.

Fig. 7. The movement of the robotic arm of four cases.

Table 2,
The error of the end-effector position

Case No.	End-Effector Position True (mm)	Joint Angle Measured (degree)	End-Effector Position Measured (mm)	Absolute Error $= \left\|\dfrac{True - Measured}{True}\right\| \times 100\%$
1	$P_x = 220$	$\theta_1 = 36.1973$	$P_x = 219.9986$	0.0006
		$\theta_2 = 69.9029$		
	$P_y = 161$	$\theta_3 = -43.1499$	$P_y = 161.0146$	0.009
		$\theta_4 = 52.2469$		
	$P_z = 220$	$\theta_5 = 90$	$P_z = 219.9934$	0.003
2	$P_x = -230$	$\theta_1 = 165.1461$	$P_x = -230.0103$	0.0045
		$\theta_2 = 89.9426$		
	$P_y = 61$	$\theta_3 = -65.6204$	$P_y = 60.9861$	0.023
		$\theta_4 = 53.6778$		
	$P_z = 220$	$\theta_5 = 90$	$P_z = 219.9964$	0.0016
3	$P_x = 220$	$\theta_1 = 15.4972$	$P_x = 220.0553$	0.0251
		$\theta_2 = 95.1123$		
	$P_y = 61$	$\theta_3 = -70.5272$	$P_y = 60.9854$	0.0239
		$\theta_4 = 53.4149$		
	$P_z = 220$	$\theta_5 = 90$	$P_z = 220.2386$	0.1085
4	$P_x = -230$	$\theta_1 = 154.2376$	$P_x = -230.0493$	0.0214
		$\theta_2 = 81.8983$		
	$P_y = 111$	$\theta_3 = -60.124$	$P_y = 111.0121$	0.0109
		$\theta_4 = 58.2257$		
	$P_z = 220$	$\theta_5 = 90$	$P_z = 220.1713$	0.0778

From the results of the error in Table (2), indicated the little small error occurred in the position of the end-effector of the robotic arm that confirms the effectiveness of the program used, where the maximum error in the x, y, and z coordinates of the end-effector were (0.0251 %, 0.0239 %, and 0.1085 %) respectively.

6. Conclusions

In this work, the Inverse kinematics analysis of 5 DOF robotic arm was accomplished using the MATLAB GUI program. the results indicated the used of the geometric approach in inverse kinematics analysis to determine the joints angles of the robotic arm was effectively and the position of each joint was determined in the GUI program and show it. A 3D simulation for the movement of the robotic arm was performed depending on the DH parameters in representing the links and joints of the robotic arm to help in understanding and representing the movement of the robotic arm. The curve of the second, third, fourth, and etc. degree can be adopted as the path of the end-effector of the robotic arm by dividing this curve into many small lines to enable the end-effector to perform the required curve path.

7. References

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التحليل الحركي العكسي والمحاكاة لذراع روبوت ذو خمس درجات لحرية الحركة بأستخدام MATLAB

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الخلاصة

التحليل الحركي هو الفرع الميكانيكي الذي يتعامل مع⬜ركة الأجسام دون أخذ القوة في الاعتبار. في الروبوتات، يعد التحليل الحركي الامامي والعكسي مهم في تحديد موضع الماسك واتجاهه للقيام بمهام متعددة. يستعرض هذا العمل التحليل الحركي العكسي لذراع روبوت ذو خمس درجات لحرية الحركة باستخدام robotics toolbox في برنامج MATLAB واستخدمت متغيرات (Denavit-Hartenberg) لتمثيل أذرع ومفاصل ذراع الروبوت. استخدم النهج الهندسي في ⬜ل التحليل الحركي العكسي لايجاد زوايا المفاصل لذراع الروبوت وتم تقسيم مسار الذراع الآلي إلى خطوط متتالية لإنجاز المهام المطلوبة للذراع الآلية، وبالتالي، يمكن اعتماد هذه الطريقة للتطبيقات الهندسية. يستخدم برنامج MATLAB (واجهة المستخدم الرسومية) لمحاكاة ثلاثية الأبعاد لحركة ذراع الروبوت. أيضًا، تم أستخدام واجهة الرسوم الحاسوبية ببرنامج MATLAB لعرض موقع كل مفصل. أظهرت النتائج أن الحد الأقصى للخطأ في إ⬜داثيات x، y، z للماسك هي 0,0251 ٪ ، 0,0239 ٪ ، و 0,1085 ٪ على التوالي.