On Multiple M2-brane Model(s) and Its $\mathcal{N} = 8$ Superspace Formulation(s)

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Abstract

We give a brief review of Bagger-Lambert-Gustavsson (BLG) model, with emphasis on its version invariant under the volume preserving diffeomorphisms (SDiff$_3$) symmetry. We describe the on-shell superfield formulation of this SDiff$_3$ BLG model in standard $\mathcal{N} = 8$, $d = 3$ superspace, as well as its superfield action in the pure spinor $\mathcal{N} = 8$ superspace. We also briefly address the Aharony-Bergman-Jafferis-Maldacena (ABJM/ABJ) model invariant under $SU(M)_k \times SU(N)_{-k}$ gauge symmetry, and discuss the possible form of their $\mathcal{N} = 6$ and, for the case of Chern-Simons level $k = 1, 2$, $\mathcal{N} = 8$ superfield equations.

1 Introduction

In the fall of 2007, motivated by the search for a low-energy description of the multiple M2-brane system, Bagger, Lambert and Gustavsson [1, 2, 3] proposed a $\mathcal{N} = 8$ supersymmetric superconformal $d = 3$ model based on Filippov three algebra [4] instead of Lie algebra.

1.1 3-algebras

Lie algebras are defined with the use of antisymmetric brackets $[X, Y] = -[Y, X]$ of two elements, $X = \sum_a X^a T_a$ and $Y = \sum_a Y^a T_a$, called Lie brackets or commutator. The brackets of two Lie algebra generators, $[T_a, T_b] = f_{ab}{}^c T_c$, are characterized by antisymmetric structure constants $f_{ab}{}^c = -f_{ab}{}^c = f_{[ab]}{}^c$ which obey the Jacobi identity $f_{[ab}{}^d f_{c]d}{}^e = 0 \Leftrightarrow [T_a, [T_b, T_c]] + [T_c, [T_a, T_b]] + [T_b, [T_c, T_a]] = 0$.

In contrast, the general Filippov 3-algebra is defined by *3-brackets*

$$\{T_a, T_b, T_c\} = f_{abc}{}^d T_d \, , \qquad f_{abc}{}^d = f_{[abc]}{}^d \qquad (1)$$

which are antisymmetric and obey the so-called 'fundamental identity'

$$\begin{aligned} &\{T_a, T_b, \{T_{c_1}, T_{c_2}, T_{c_3}\}\} = \\ &3\{\{T_a, T_b, T_{[c_1}\}, T_{c_2}, T_{c_3]}\}\} \, . \end{aligned} \qquad (2)$$

To write an action for some 3-algebra valued field theory, one needs as well to introduce an invariant inner product or metric

$$h_{ab} = < T_a, T_b > \, . \qquad (3)$$

Then for the metric 3-algebra the structure constants obey $f_{abcd} := f_{abc}{}^e h_{ed} = f_{[abcd]}$.

An example of infinite dimensional 3-algebra is defined by the Nambu brackets (NB) [5] of functions on a 3-dimensional manifold M^3

$$\begin{aligned} &\{\Phi, \Xi, \Omega\} = \epsilon^{ijk} \, \partial_i \Phi \, \partial_j \Xi \, \partial_k \Omega \, , \\ &\partial_i := \partial / \partial y^i \, , \qquad i = 1, 2, 3 \, . \end{aligned} \qquad (4)$$

Here $y^i = (y^1, y^2, y^3)$ are local coordinates on M^3, $\Phi = \Phi(y)$, $\Xi = \Xi(y)$ and $\Omega = \Omega(y)$ are functions on M^3, and ϵ^{ijk} is the Levi-Cevita symbol (it is convenient to define NB using a constant scalar density e [6], but this is not important for our present discussion here and we simplify the notation by setting $e = 1$). These brackets are invariant with respect to the *volume preserving diffeomorphisms of* M_3, which we call *SDiff*$_3$ *transformations.* In practical applications one needs to assume compactness of M^3. For our discussion here it is sufficient to assume that M^3 has the topology of sphere S^3.

Another example of 3-algebra, which was present already in the first paper by Bagger and Lambert [1] is $\mathcal{A}_4$ realized by generators T_a, $a = 1, 2, 3, 4$ obeying

$$\{T_a, T_b, T_c\} = \epsilon_{abcd} \, T_d \, , \qquad a, b, c, d = 1, 2, 3, 4 \, . \qquad (5)$$

These are related to the 6 generators M_{ab} of $SO(4)$ as Euclidean $d = 4$ Dirac matrices are related to the $Spin(4) = SU(2) \times SU(2)$ generators, $T_a \leftrightarrow \gamma_a$, $M_{ab} \leftrightarrow 1/2\gamma_{ab} := 1/4(\gamma_a\gamma_b - \gamma_b\gamma_a)$.

A more general type of 3-algebras with not completely antisymmetric structure constants were discussed *e.g.* in [7], [8] and [9]. In particular, as it was shown in [8], the Aharony-Bergman-Jafferis-Maldacena (ABJM) model [10] is based on a particular 'hermitian 3-algebra' the 3-brackets of which can be defined on two $M \times N$ (complex) matrices $\mathbb{Z}^i$, $\mathbb{Z}^j$ and an $N \times M$ (complex) matrix $\mathbb{Z}^\dagger_k$ by [8]

$$[\mathbb{Z}^i, \mathbb{Z}^j; \mathbb{Z}^\dagger_k]^{M\times N} = \mathbb{Z}^i \mathbb{Z}^\dagger_k \mathbb{Z}^j - \mathbb{Z}^j \mathbb{Z}^\dagger_k \mathbb{Z}^i \, . \qquad (6)$$

Contribution to the *Selected Topics in Mathematical and Particle Physics Prague 2009.* Procs. of the Conference on the occasion of Prof. Jiri Niederle's 70th birthday.

1.2 BLG action

The BLG model on general 3-algebra is described in terms of an octet of 3-algebra valued scalar fields in vector representation of SO(8), $\phi^I(x) = \phi^{Ia}(x)T_a$, an octet of 3-algebra valued spinor fields in spinor (say, s-spinor) representation of SO(8), $\psi_{\alpha A}(x) = \psi_{\alpha A}{}^a(x)T_a$, and the vector gauge field A_μ^{ab} in the bi-fundamental representation of the 3-algebra. The BLG Lagrangian reads

$$\mathcal{L}_{BLG} = Tr\,\left[-\frac{1}{2}\,|\mathcal{D}\phi|^2 - \frac{g^2}{12}\left\{\phi^I,\phi^J,\phi^K\right\}^2 - \frac{i}{2}\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi + \frac{ig}{4}\left\{\phi^I\,,\phi^J\,,\bar{\psi}\right\}\rho^{IJ}\psi\right] + \frac{1}{2g}\mathcal{L}_{CS}\,, \quad I = 1,\ldots,8\,. \tag{7}$$

where g is a real dimensionless parameter, $\mathcal{L}_{CS}$ is the Chern-Simons (CS term) for the gauge potential $A_{\mu b}{}^a = A_\mu^{cd} f_{dcb}{}^a$ which is also used to define the covariant derivatives of the scalar and spinor fields. The $Spin(8)$ indices are suppressed in (7); $\rho^I := \rho^I_{A\dot{B}}$ are the 8×8 $Spin(8)$ 'sigma' matrices (Klebsh-Gordan coefficients relating the vector $\mathbf{8}_v$ and two spinor, $\mathbf{8}_s$ and $\mathbf{8}_c$, representations of $SO(8)$). These obey $\rho^I\tilde{\rho}^J + \rho^I\tilde{\rho}^J = 2\delta^{IJ}\,I$ with their transpose $\tilde{\rho}^I := \tilde{\rho}^I_{\dot{A}B}$; notice that $\rho^{IJ} := (\rho^{[I}\tilde{\rho}^{J]})_{AB}$ and $\tilde{\rho}^{IJ} := (\tilde{\rho}^{[I}\rho^{J]})_{\dot{A}\dot{B}}$ are *antisymmetric* in their spinor indices.

This model possesses $\mathcal{N} = 8$ supersymmetry and superconformal symmetries the set of which includes 8 special conformal supersymmetries. Hence the total number of supersymmetry parameters is $2\times 8 + 2\times 8 = 32$. This coincides with the number of supersymmetries possessed by M2-brane [11] and the conformal symmetry was expected for *infrared fixed point* (low energy approximation) of the multiple M2-brane system [12]. Thus, action (7) was expected to play for the multiple M2-brane system the same rôle as it is played by the $U(N)$ SYM action for the multiple Dp-brane system [13] (with N Dp-branes).

However, if this were the case, the number of generators of the Filippov 3-algebra would be related somehow to the number of M2-branes composing the system the low energy limit of which is described by the action (7). This expectation enters in conflict with the relatively poor structure of the set of finite dimensional Filippov 3-algebras with positively definite metric (3): this set was proved to contain the direct sums of $\mathcal{A}_4$ and trivial one-dimensional 3-algebras only (see [14, 15] as well as [16] and refs therein).

A very useful rôle in searching for resolution of this paradox was played by the analysis by Raamsdock [17], who reformulated the $\mathcal{A}_4$ BLG model in matrix notation. This was used by Aharony, Bergman, Jafferis and Maldacena [10] to formulate an $SU(N)_k\times SU(N)_{-k}$ and then [26] $SU(M)_k\times SU(N)_{-k}$ gauge invariant CS plus matter models, which are believed to describe the low energy multiple M2-brane dynamics. The subscript k denotes the so-called *CS level*, this is to say the integer coefficient in front of the CS term in the action of the CS plus matter models. In the dual description of the ABJM model by M-theory on the $AdS_4\times S^7/\mathbb{Z}_k$ [10] the same integer k characterizes the quotient of the 7-sphere.

The ABJM/ABJ model possesses only $\mathcal{N} = 6$ manifest supersymmetries, which is natural for $k > 2$, as the $AdS_4\times S^7/\mathbb{Z}_k$ backgrounds with $k > 2$ preserve only 24 of 32 M-theory supersymmetries in these cases. The nonperturbative restoration of $\mathcal{N} = 8$ supersymmetry for $k = 1, 2$ cases was conjectured already in [10]. Recently this enhancement of supersymmetry was studied in [9], where its relation with some special 'identities' (which we propose to call GR-identities or Gustavsson-Rey identities) conjectured to be true due to the properties of monopole operators specific for $k = 1, 2$ is proposed. We shortly discuss the ABJM/ABJ model in the concluding part of this paper.

1.3 NB BLG action

Coming back to the 3-algebra BLG models, we notice that inside their set there are clear candidates for the $N\to\infty$ limit of the multiple M2-brane system, which one can view as describing possible 'condensates' of coincident planar M2-branes. These are the BLG theories in which the Filippov 3-algebra is realized by the Nambu-bracket (4) of functions defined on some 3-manifold M_3. This model was conjectured [18, 19] to be related with the M5-brane [20, 21, 22] wrapped over M_3 (see [6] and recent [23] for further study of this proposal) and was put in a general context of SDiff$_3$ gauge theories in [24].

It is described in terms of $Spin(8)$ $\mathbf{8}_v$-plet of real scalar fields ϕ^I ($I = 1,\ldots 8$), and a $Spin(8)$ $\mathbf{8}_s$-plet of Majorana anticommuting $Sl(2;\mathbb{R})$ spinor fields ψ_A ($A = 1,\ldots,8$), both on the Cartesian product of 3-dimensional Minkowski spacetime with some 3-dimensional closed manifold without boundary, M_3. These fields transforms as scalars with respect to SDiff$_3$: $\delta_\xi\phi = -\xi^i\partial_i\phi\,,\, \delta_\xi\psi = -\xi^i\partial_i\psi$, where $\xi^i = \xi^i(y)$ is a divergenceless SDiff$_3$ parameter.

The action of this Nambu bracket realization of the Bagger-Lambert-Gustavsson model (NB BLG model) is

$$\mathcal{L}_{NB\;BLG} \;=\; \oint \mathrm{d}^3y\,\left[-\frac{1}{2}e\,|\mathcal{D}\phi|^2 - \frac{i}{2}e\,\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi + \frac{ig}{4}\varepsilon^{ijk}\partial_i\phi^I\partial_j\phi^J\left(\partial_k\bar{\psi}\rho^{IJ}\psi\right) - \frac{g^2}{12}e\left\{\phi^I,\phi^J,\phi^K\right\}^2\right] + \frac{1}{2g}\mathcal{L}_{CS} \tag{8}$$

In (8) the trace Tr of (7) is replaced by integral $\oint d^3y$ over M_3 and $\mathcal{L}_{CS}$ is the CS-like term involving the SDiff$_3$ gauge potential s_i and gauge pre-potential A_i

[24]. The gauge potential $s^i = \mathrm{d}x^\mu s^i_\mu$ transforms under the local SDiff$_3$ with $\xi^i = \xi^i(x,y)$ as $\delta_\xi s^i = \mathrm{d}\xi^i - \xi^j\partial_j s^i + s^j\partial_j\xi^i$ and is used to construct SDiff$_3$ covariant derivatives of scalar and spinor fields

$$\mathcal{D}\phi = d\phi + s^i\partial_i\phi\,, \mathcal{D}\psi = d\psi + s^i\partial_i\psi\,. \qquad (9)$$

As the gauge field takes values in the Lie algebra of the Lie group of gauge symmetries, and this is associated with volume preserving diffeomorphisms the infinitesimal parameter of which is a divergenceless three-vector $\xi^i(x,y)$, $\partial_i\xi^i = 0$, the SDiff$_3$ gauge field $s^i = \mathrm{d}x^\mu s^i_\mu(x,y)$ obeys

$$\partial_i s^i \equiv 0 \Leftrightarrow \partial_i s^i_\mu \equiv 0 \qquad (10)$$

which implies the possibility to express it, at least locally, in terms of gauge pre-potential one-form $A_i = \mathrm{d}x^\mu A_{\mu i}(x)$,

$$s^i = \epsilon^{ijk}\partial_j A_k \Leftrightarrow s^i_\mu = \epsilon^{ijk}\partial_j A_{\mu k}\,. \qquad (11)$$

Also the covariant field strength

$$F^i = \mathrm{d}s^i + s^j\partial_j s^i = \frac{1}{2}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu F^i_{\nu\mu}\,. \qquad (12)$$

satisfies the additional identity

$$\partial_i F^i \equiv 0 \Leftrightarrow \partial_i F^i_{\mu\nu} \equiv 0 \qquad (13)$$

and can be expressed (locally) in terms of pre-field strength,

$$F^i = \varepsilon^{ijk}\partial_j G_k \Leftrightarrow F^i_{\mu\nu} = \varepsilon^{ijk}\partial_j G_{\mu\nu\, k}\,, \qquad (14)$$

$$G_i = \mathrm{d}A_i + s^j\partial_{[j}A_{i]} = \frac{1}{2}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu G_{\nu\mu i}\,. \qquad (15)$$

The CS-like term in (8) is expressed through the gauge potential and pre-potential by

$$\mathcal{L}_{CS} = \oint \mathrm{d}^3y\, \epsilon^{\mu\nu\rho}\left[\left(\partial_\mu s^i_\nu\right)A_{\rho\, i} - \frac{1}{3}\epsilon_{ijk}s^i_\mu s^j_\nu s^k_\rho\right], \qquad (16)$$

or, in terms of differential forms, by $L_{CS} = \oint \mathrm{d}^3y\left[\mathrm{d}s^i \wedge A_i - \frac{1}{3}\epsilon_{ijk}s^i \wedge s^j \wedge s^k\right]$. The formal exterior derivative of L_{CS} can be expressed through the field strength and pre-field strength by

$$\mathrm{d}L_{CS} = \oint \mathrm{d}^3y\, F^i \wedge G_i\,. \qquad (17)$$

The Lagrangian density (8) varies into a total spacetime derivative under the following infinitesimal supersymmetry transformations with $\mathbf{8}_c$-plet constant anticommuting spinor parameter $\epsilon^\alpha_{\dot A}$ ($\dot A = 1, \ldots, 8$):

$$\delta\phi^I = i\epsilon\tilde\rho^I\psi\,, \quad \delta A_{\mu i} = -ig\left(\epsilon\gamma_\mu\tilde\rho^I\psi\right)\partial_i\phi^I\,, \qquad (18)$$
$$\delta\psi = \left[\gamma^\mu\rho^I\mathcal{D}_\mu\phi^I - \frac{g}{6}\left\{\phi^I,\phi^J,\phi^K\right\}\rho^{IJK}\right]\epsilon\,.$$

The BLG equations of motion are

$$\begin{aligned}
\mathcal{D}^\mu\mathcal{D}_\mu\phi^I &= \frac{ig}{2}\varepsilon^{ijk}\partial_i\phi^J\partial_j\bar\psi\rho^{IJ}\partial_k\psi - \\
&\quad \frac{g^2}{2}\left\{\left\{\phi^I,\phi^J,\phi^K\right\},\phi^J,\phi^K\right\}\,,\\
\gamma^\mu\mathcal{D}_\mu\psi &= -\frac{g}{2}\rho^{IJ}\left\{\phi^I,\phi^J,\psi\right\}\,, \qquad (19)\\
F^i_{\mu\nu} &= -g\,\varepsilon_{\mu\nu\rho}\varepsilon^{ijk}\left[\partial_j\phi^I\mathcal{D}^\rho\partial_k\phi^I - \frac{i}{2}\partial_j\psi\gamma^\rho\partial_k\psi\right].
\end{aligned}$$

2 NB BLG in $\mathcal{N} = 8$ superfields

The NB BLG equations of motion can be obtained from the set of superfield equations in $\mathcal{N} = 8$ superspace [30]. We will review this approach in this section.

Let us introduce $\mathbf{8}_v$-plet of scalar, and SDiff$_3$-scalar, superfields ϕ^I, the lowest component of which (also denoted by ϕ^I) may be identified with the BLG scalar fields, and impose on it the following *superembedding-like* equation [30][1]

$$\boxed{\mathbb{D}_{\alpha\dot A}\phi^I = i\tilde\rho^I_{\dot A B}\psi_{\alpha B}.} \qquad (20)$$

The SDiff$_3$-covariant spinorial derivatives on $\mathcal{N} = 8$ superspace, entering (20),

$$\mathbb{D}_{\alpha\dot A} = D_{\alpha\dot A} + \varsigma_{\alpha\dot A}{}^i\partial_i\,, \qquad (21)$$

are constrained to obey the following algebra [30]

$$\begin{aligned}
[\mathbb{D}_{\alpha\dot A}, \mathbb{D}_{\beta\dot B}]_+ &= 2i\delta_{\dot A\dot B}(C\gamma^\mu)_{\alpha\beta}\mathcal{D}_\mu + \qquad (22)\\
&\quad 2i\epsilon_{\alpha\beta}W_{\dot A\dot B}{}^i\,\partial_i\,,
\end{aligned}$$

where $\mathcal{D}_\mu = \partial_\mu + is^i_\mu\partial_i$ is the 3-vector covariant derivative which obeys

$$\left[\mathbb{D}_{\alpha\dot A}, \mathcal{D}_\mu\right] = F_{\alpha\dot A\,\mu}{}^i\partial_i\,, \quad [\mathcal{D}_\mu, \mathcal{D}_\nu] = F_{\mu\nu}{}^i\,\partial_i\,. \qquad (23)$$

Eqs. (22), (23) are equivalent to the Ricci identity $\mathcal{D}\mathcal{D} = F^i\partial_i$ for the covariant exterior derivative $\mathcal{D} := d + s^i\partial_i = E^{\alpha\dot A}\mathbb{D}_{\alpha\dot A} + E^\mu\mathcal{D}_\mu$, plus the constraint $F^i_{\alpha\dot A\,\beta\dot B} = 2iC_{\alpha\beta}\,W_{\dot A\dot B}{}^i$.

The basic SDiff$_3$ gauge superfield strength $W_{\dot A\dot B}{}^i$ is antisymmetric on c-spinor indices (this is to say $W_{\dot A\dot B}{}^i$ is in the **28** of SO(8)); it is also divergence-free, so

$$W_{\dot A\dot B}{}^i = -W_{\dot B\dot A}{}^i\,, \qquad \partial_i W_{\dot A\dot B}{}^i = 0\,. \qquad (24)$$

Using the Bianchi identity $DF^i = 0$, one finds that

[1] The name comes from the observation that (20) can be obtained from the *superembedding equation* for a single M2-brane [25] by first linearizing with respect to the dynamical fields in the static gauge, and then covariantizing the result with respect to SDiff$_3$.

$$F_{\alpha\dot{A}\,\mu}{}^{i} = i\left(\gamma_\mu W_{\dot{A}}{}^{i}\right)_\alpha\,, \quad W_{\alpha\dot{B}}{}^{i} := \frac{i}{7}\mathbb{D}_{\alpha\dot{A}}W_{\dot{A}\dot{B}}{}^{i}\,, \quad (25)$$
$$F_{\mu\nu}{}^{i} = \frac{1}{16}\epsilon_{\mu\nu\rho}\mathbb{D}_{\dot{A}}\gamma^\rho W_{\dot{A}}{}^{i}\,,$$

and that

$$\mathbb{D}_{\alpha(\dot{A}}W_{\dot{B})\dot{C}}{}^{i} = iW_{\alpha\dot{D}}{}^{i}\left(\delta_{\dot{D}(\dot{A}}\delta_{\dot{B})\dot{C}} - \delta_{\dot{D}\dot{C}}\delta_{\dot{A}\dot{B}}\right), \quad (26)$$
$$\mathbb{D}_{\dot{A}\alpha}W_{\beta\dot{B}}{}^{i} = (C\gamma^\mu)_{\alpha\beta}\left(\mathcal{D}_\mu W_{\dot{A}\dot{B}}{}^{i} - 4\delta_{\dot{A}\dot{B}}W_\mu{}^{i}\right). \quad (27)$$

We see that the SDiff field strength supermultiplet includes a scalar **28** ($W_{\dot{A}\dot{B}}{}^{i}$), a spinor $\mathbf{8}_c$ ($W_{\alpha\dot{A}}{}^{i}$) and a singlet divergence-free vector ($W^{\mu i} = \mathbb{D}_{\dot{A}}\gamma^\rho W_{\dot{A}}{}^{i}$). There are many other independent components, but these become dependent on-shell as far as we are searching for a description of Chern-Simons (CS) rather than the Yang-Mills one. The relevant super-Chern-Simons (super-CS) system superfield equation *in the absence of 'matter' supermutiplets* is obviously $W_{\dot{A}\dot{B}}{}^{i} = 0$, since this sets to zero all SDiff_3 field strengths; in particular it implies $F^i_{\mu\nu} = 0$. In the presence of matter, the super-CS equation may get a nonvanishing right hand side.

Indeed, acting on the superembedding-like equation (20) with an SDiff_3-covariant spinor derivative, and making use of the anticommutation relation (22), one finds that $\mathbb{D}_{\alpha[\dot{A}}\tilde{\rho}^{I}{}_{\dot{B}]C}\psi^\alpha_C = 2W_{\dot{A}\dot{B}}{}^{i}\partial_i\phi^I$ which is solved by the 'super-CS' equation [30]

$$\boxed{W_{\dot{A}\dot{B}}{}^{i} = 2g\varepsilon^{ijk}\partial_i\phi^I\partial_j\phi^J\tilde{\rho}^{IJ}_{\dot{A}\dot{B}}\,.} \quad (28)$$

It was shown in [30] that the two $\mathcal{N} = 8$ superfield equations (20) and (28) imply the Nambu-bracket BLG equations (19).

3 NB BLG in pure-spinor superspace

An $\mathcal{N} = 8$ superfield action for the abstract BLG model, *i.e.* for the BLG model based on a finite dimensional 3-algebra, which in practical terms implies $\mathcal{A}_4$ or the direct sum of several $\mathcal{A}_4$ and trivial 3-algebras, was proposed by Cederwall [28]. Its generalization for the case of NB BLG model invariant under infinite dimensional SDiff_3 gauge symmetry, constructed in [24], will be reviewed in this section.

The pure-spinor superspace of [28] is parametrized by the standard $\mathcal{N} = 8$ $D = 3$ superspace coordinates $(x^\mu, \theta^\alpha_{\dot{A}})$ together with additional pure spinor coordinates $\lambda^\alpha_{\dot{A}}$. These are described by the $\mathbf{8}_c$-plet of complex commuting $D = 3$ spinors satisfying the 'pure spinor' constraint

$$\lambda\gamma^\mu\lambda := \lambda^\alpha_{\dot{A}}\gamma^\mu_{\alpha\beta}\lambda^\beta_{\dot{A}} = 0\,. \quad (29)$$

This is a variant of the $D = 10$ pure-spinor superspace first proposed by Howe [31] (see [32] for earlier attempt to use pure spinors in the SYM and supergravity context). From a more general perspective, the approach of [28] can be considered as a realization of the harmonic superspace programme of [33] (although one cannot state that the algebra of all the symmetries of the superfield action of [28] are closed off shell, *i.e.* without the use of equations of motion). The $D = 10$ pure spinors are also the central element of the Berkovits approach to covariant description of quantum superstring [34]. In this approach the pure spinors are considered to be the ghosts of a local fermionic gauge symmetry related to the κ-symmetry of the standard Green-Schwarz formulation. This 'ghost nature' may be considered as a justification for that the pure-spinor superfields are assumed (in [28, 24] and here) to be *analytic* functions of λ that can be expanded as a Taylor series in powers of λ. To discuss the BLG model, we allow all the pure spinor superfields to depend also on the local coordinates y^i of the auxiliary compact 3-dimensional manifold M_3.

Following [28], we define the BRST-type operator (*cf.* [34])

$$Q := \lambda^\alpha_{\dot{A}}D_{\alpha\dot{A}}\,, \quad (30)$$

which satisfies $Q^2 \equiv 0$ as a consequence of the pure spinor constraint (29). We now introduce the $\mathbf{8}_v$-plet of complex scalar $\mathcal{N} = 8$ 'matter' superfields Φ^I, with SDiff_3 transformation

$$\delta\Phi^I = \Xi^i\partial_i\Phi^I \quad (31)$$

characterized by the commuting M_3-vector parameter $\Xi^i = \Xi^i(y)$.

We allow these superfields to be complex because they may depend on the complex pure-spinor λ but, to make contact with the spacetime BLG model, we assume that the leading term in its decomposition in power series on complex λ

$$\Phi^I = \phi^I + \mathcal{O}(\lambda)\,, \quad (32)$$

is given by a *real* $\mathbf{8}_v$-plet of 'standard' $\mathcal{N} = 8$ scalar superfields, like the basic objects in Sec. 2.

Let us consider (complex and anticommuting) Lagrangian density

$$\mathbb{L}^0_{mat} = \frac{1}{2}M_{IJ}\oint d^3y\, e\Phi^I Q\Phi^J\,, \quad (33)$$

where $M_{IJ} = \lambda^\alpha_{\dot{A}}\tilde{\rho}^{IJ}_{\dot{A}\dot{B}}\lambda_{\alpha\dot{B}}$ is one of the two nonvanishing analytic pure spinor bilinears

$$M_{IJ} := \lambda^\alpha\,\tilde{\rho}^{IJ}\lambda_\alpha\,, \qquad N^\mu_{IJKL} := \lambda\gamma^\mu\tilde{\rho}^{IJKL}\lambda\,. \quad (34)$$

It is important that, due to (29), these obey the identities (see [24] for a detailed proof)

$$M_{IJ}\,\tilde{\rho}^J\lambda \equiv 0\,, \qquad M_{[IJ}M_{KL]} = 0\,, \quad (35)$$
$$N_{PQ[IJ}\cdot N_{KL]PQ} \equiv 0\,.$$

To construct the $\mathcal{N} = 8$ supersymmetric action with the use of the Lagrangian (33) one needs to specify an adequate superspace integration measure. We refer to [29] for details on such a measure, which has the crucial property of allowing us to discard a BRST-exact terms when varying with respect Φ^I. Then, as a consequence of this and also of the identities (35), the action is invariant under the gauge symmetries $\delta\Phi^I = \lambda^\alpha_{\dot{A}}\tilde{\rho}^I_{\dot{A}B}\zeta_{\alpha B} + QK^I$ for arbitrary pure-spinor-superfield parameters ζ_α and K^I.

The variation with respect to Φ^I yields the superfield equation

$$M_{IJ}Q\Phi^J = 0\,, \tag{36}$$

which implies, as a consequence of the pure-spinor identities, that

$$Q\Phi^I = \lambda\tilde{\rho}^I\Theta \tag{37}$$

for some $\mathbf{8}_s$-plet of complex spinor superfields $\Theta_{\alpha\dot{A}}$. The first nontrivial ($\sim\lambda$) term in the λ-expansion of this equation is precisely the free field limit of the on-shell superspace constraint (20), $D_{\alpha\dot{A}}\phi^I = i\tilde{\rho}^I{}_{\dot{A}B}\psi_{\alpha B}$, with $\psi = \Theta|_{\lambda=0}$. [2] In the light of the results of Sec. 2, this implies that the free field ($g \mapsto 0$) limit of the NB BLG field equations (19) can be obtained from the pure spinor superspace action (33).

Now, as the free field limit is reproduced, to construct the pure spinor superspace description of the NB BLG system we need to describe its gauge field (Chern-Simons) sector and to use it to gauge the SDiff$_3$ invariance. To this end, we introduce an M_3-vector-valued complex *anticommuting* scalar Ψ^i with the SDiff$_3$ gauge transformations

$$\delta\Psi^i = Q\Xi^i + \Psi^j\partial_j\,\Xi^i - \Xi^j\partial_j\Psi^i\,, \qquad \partial_i\Xi^i = 0 \tag{38}$$

involving the commuting M_3-vector parameter $\Xi^i = \Xi^i(x,\theta,\lambda;y^j)$ and its derivatives. In the present context, Ψ^i will play the role of the SDiff$_3$ gauge potential. We require that $\partial_i\Psi^i = 0$ so that, locally on M_3,

$$\Psi^i = \varepsilon^{ijk}\partial_j\,\Pi_k\,, \tag{39}$$

where Π_i is the complex *anticommuting*, and spacetime scalar, pre-gauge potential of this formalism.

Using Ψ^i we can define an SDiff$_3$-covariant extension of $Q\Phi^I$ by

$$\mathbb{Q}\Phi^I := Q\Phi^I + \Psi^i\partial_i\Phi^I \tag{40}$$

and construct the generalization of (33) invariant under *local* SDiff$_3$ symmetry (31), (38):

$$\begin{aligned}\mathbb{L}_{mat} &= \frac{1}{2}M_{IJ}\oint d^3y\, e\Phi^I\mathbb{Q}\Phi^J\,,\\ M_{IJ} &= \lambda\,\tilde{\rho}^{IJ}\epsilon\lambda\,.\end{aligned} \tag{41}$$

Next we have to construct the (complex and fermionic) Lagrangian density $\mathbb{L}_{CS}$ describing the (Chern-Simons) dynamics of the gauge potential Ψ^i. To this end we introduce the field-strength superfield

$$\mathcal{F}^i := Q\Psi^i + \Psi^j\partial_j\Psi^i = \varepsilon^{ijk}\partial_j\mathcal{G}_k\,, \tag{42}$$

where the last equality is valid locally on M_3 and

$$\mathcal{G}_i := Q\Pi_i + \Psi^j\partial_j\Psi_i \tag{43}$$

is the pre-field-strength superfield of this formalism. Both $\mathcal{F}^i$ and $\mathcal{G}_i$ are SDiff$_3$ covariant, so $\mathcal{F}^i\mathcal{G}_i$ is an SDiff$_3$ scalar. Furthermore, the integral of this density over M_3 is Q-exact, in the sense that

$$\int d^3y\, e\,\mathcal{F}^i\mathcal{G}_i = Q\,\mathbb{L}_{CS}\,, \tag{44}$$

where

$$\mathbb{L}_{CS} = \int d^3\sigma\, e\left(\Pi_i\, Q\Psi^i - \frac{1}{3}\epsilon_{ijk}\Psi^i\Psi^j\Psi^k\right) \tag{45}$$

is the complex and anti-commuting CS-type Lagrangian density [24] which can be used, together with $\mathbb{L}_{mat}$ of (41), to construct the candidate Lagrangian density of the NB BLG model,

$$\mathbb{L} = \mathbb{L}_{mat} - \frac{1}{g}\mathbb{L}_{CS}\,. \tag{46}$$

The Π_i equation of motion of this combined Lagrangian is

$$\mathcal{F}^i = \frac{g}{2e}M_{IJ}\epsilon^{ijk}\partial_j\Phi^I\partial_k\Phi^J\,. \tag{47}$$

At this stage it is important to assume that Ψ^i has 'ghost number one' [28], which means that it is a power series in λ with vanishing zeroth order term (and similarly for its pre-potential Π_i). In other words

$$\Psi^i = \lambda^\alpha_{\dot{A}}\varsigma^i_{\alpha\dot{A}}\,, \tag{48}$$

where ς^i is an M_3-vector-valued $\mathbf{8}_c$-plet of arbitrary anticommuting spinors. Its zeroth component in the λ-expansion is the fermionic SDiff$_3$ potential introduced, with the *same symbol*, in (21). With this 'ghost number' assumption, (47) produces at lowest nontrivial order ($\sim\lambda^2$) the superspace constraints (22) for the 'ghost number zero' contribution $\varsigma^i|_{\lambda=0}$ to the pure spinor superfield ς^i in (48), accompanied by the super CS equation (28) for the field strength $W_{\dot{A}\dot{B}}$ constructed from this potential.

An heuristic justification of the assumption (48), so crucial to obtain the correct super-CS equations, can be found in that with this form of Ψ^i the covariantized BRST operator in (40) does not contain a contribution

[2]Notice that the above mentioned gauge symmetry $\delta\Phi^I = \lambda^\alpha_{\dot{A}}\tilde{\rho}^I_{\dot{A}B}\zeta_{\alpha B}$ of the action (33) contributes to $\delta(Q\Phi^I)$ the terms of at least the second order in λ. Then the induced transformation of the pure spinor superfield $\Theta_{\alpha\dot{A}}$ in (37) is of the first order in λ so that $\psi_{\alpha\dot{A}} = \Theta_{\alpha\dot{A}}|_{\lambda=0}$, entering the superembedding-like equation (20), is inert under those transformations.

of ghost number zero, *i.e.* it has the form of (30), $\mathbb{Q} = \lambda_{\dot{A}}{}^{\alpha}\, \mathbb{D}_{\alpha\dot{A}}$, but with the SDiff$_3$ covariant Grassmann derivative $\mathbb{D}_{\alpha\dot{A}} = D_{\alpha\dot{A}} + \xi^i_{\alpha\dot{A}}\partial_i$.

Varying the interacting action with respect to Φ^I results in SDiff$_3$ gauge invariant generalization of Eqs. (36),

$$M_{IJ}\mathbb{Q}\Phi^J = 0\,, \tag{49}$$

which contains, as the first nontrivial ($\sim (\lambda)^3$) term in the λ-expansion, precisely the superembedding-like equation (20) with $\psi = \Theta|_{\lambda=0}$.

We have now shown, following [24], how the on-shell $\mathcal{N} = 8$ superfield formulation of Sec. 2, and hence all BLG field equations (19), may be extracted from the equations of motion derived from the pure spinor superspace action (46). Of course, the field content and equations of motion should be analyzed at all higher-orders in the λ-expansion. To this end, one must take into account the existence of additional gauge invariance [28, 29]

$$\begin{aligned} \delta\Phi^I &= \bar{\lambda}\tilde{\rho}^I\zeta_\alpha + (\mathbb{Q} + \Psi^j\,\partial_j)K^I\,, \\ \delta\Pi_i &= K^I\, M_{IJ}\,\partial_i\Phi^J\,, \end{aligned} \tag{50}$$

for arbitrary pure-spinor-superfield parameters ζ_α and K^I .

What one can certainly state, even without a detailed analysis of these symmetries, is that, if additional fields are present inside the pure spinor superfields of the model (46), they are decoupled from the BLG fields in the sense that they do not enter the equations of motion of the BLG fields which are obtained from the pure spinor superspace equations. This allowed us [24], following the terminology of [28], to call (46) the $\mathcal{N} = 8$ superfield action for the NB BLG model.

4 Remarks on ABJM/ABJ model

The $\mathcal{N} = 6$ pure spinor superspace action for the ABJM model [10] invariant under $SU(N)_k \times SU(N)_{-k}$ gauge symmetry, was proposed in [29][3]. One can extract the standard (not pure spinor) $\mathcal{N} = 6$ superspace equation by varying the action of [29] and fixing its gauge symmetries. It is also instructive (and probably simpler) to develop independently the on-shell $\mathcal{N} = 6$ superspace formalism for the ABJM as well as for the ABJ [26] model invariant under $SU(M)_k \times SU(N)_{-k}$ symmetry [37].

For any value of the CS-level k the starting point of the on-shell $\mathcal{N} = 6$ superfield formalism could be the following (superembedding-like) superspace equation for complex $M \times N$ matrix superfield $\mathbb{Z}^i$ [37][4]

$$\begin{aligned} &\mathbb{D}^I_\alpha \mathbb{Z}^i = \tilde{\gamma}^{Iij}\psi_{\alpha j}\,, \\ &I = 1, 2, \ldots, 6, \qquad i, j = 1, 2, 3, 4\,. \end{aligned} \tag{51}$$

Here $\tilde{\gamma}^{Iij} = \frac{1}{2}\epsilon^{ijkl}\gamma^I_{kl} = -(\gamma^I_{ij})^*$ and $\gamma^I_{ij} = -\gamma^I_{ji}$ are $SO(6)$ Klebsh-Gordan coefficients (generalized Pauli matrices), which obey $\gamma^I\tilde{\gamma}^J + \gamma^J\tilde{\gamma}^I = \delta^{IJ}$. The matrix superfield $\mathbb{Z}^i$ carries $(\mathbf{M}, \bar{\mathbf{N}})$ representation of the $SU(M)\times SU(N)$ gauge group. Its hermitian conjugate $\mathbb{Z}^\dagger_i$ is $N \times M$ matrix carrying $(\bar{\mathbf{M}}, \mathbf{N})$ representation and obeying $\mathbb{D}^I_\alpha\mathbb{Z}^\dagger_i = \gamma^I_{ij}\psi^{\dagger j}_\alpha$. Note that, although in the original ABJM model [10] $M = N$, the $N \times N$ matrix superfields $\mathbb{Z}^i$ and $\mathbb{Z}^\dagger_i$ carry different representation of $SU(N) \times SU(N)$: $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$, respectively. Here we speak in terms of the case with $M \neq N$, which is terminologically simpler, but all our arguments clearly also apply for $M = N$.

The Grassmann spinorial covariant derivatives $\mathbb{D}^I_\alpha$ in (51) includes the gauge group $SU(M)\times SU(N)$ connection and obey the algebra

$$\{\mathbb{D}^I_\alpha, \mathbb{D}^J_\beta\} = i\gamma^a{}_{\alpha\beta}\delta_{IJ}\mathcal{D}_a + i\epsilon_{\alpha\beta}W^{IJ}\,. \tag{52}$$

This algebra involves the 15-plet of the basic field strength superfields $W^{IJ} = -W^{JI}$ which can be expressed through the matter superfields by the following $\mathcal{N} = 6$ super-CS equation [37]

$$W^{IJ}_{SU(M)} = i\mathbb{Z}^i\mathbb{Z}^\dagger_j\,\gamma^{IJ}{}_i{}^j, \quad W^{IJ}_{SU(N)} = i\mathbb{Z}^\dagger_j\mathbb{Z}^i\gamma^{IJ}{}_i{}^j. \tag{53}$$

Here $W^{IJ}_{SU(M)}$ and $W^{IJ}_{SU(N)}$ are the basic field strength corresponding to $SU(M)$ and $SU(N)$ subgroups of the gauge group $SU(M)_k\times SU(N)_{-k}$. One can check that the consistency conditions for Eqs. (51) and (53) are satisfied if the matter superfield obeys the superfield equation of motion

$$\begin{aligned} \gamma^J_{ij}D^{\beta(I}D^{J)}_\beta\mathbb{Z}^j + 4i\gamma^J_{ij}[\mathbb{Z}^j, \mathbb{Z}^k; \mathbb{Z}^\dagger_k] + \\ 4i\gamma^J_{jk}[\mathbb{Z}^j, \mathbb{Z}^k; \mathbb{Z}^\dagger_i]\,, \end{aligned} \tag{54}$$

where $[\mathbb{Z}^j, \mathbb{Z}^k; \mathbb{Z}^\dagger_k]$ are hermitian 3-brackets (6). This superfield equation implies, in particular, the fermionic equations of motion [37]

$$\begin{aligned} \gamma^a_{\alpha\beta}D_a\psi^\beta_i &= -\frac{2}{3}[\psi_{\alpha j}, \mathbb{Z}^j; \mathbb{Z}^\dagger_i] + \frac{1}{6}[\psi_{\alpha i}, \mathbb{Z}^j; \mathbb{Z}^\dagger_j] + \\ &\quad \frac{1}{2}\epsilon_{ijkl}[\mathbb{Z}^j, \mathbb{Z}^k; \psi^{\dagger l}_\alpha]\,. \end{aligned} \tag{55}$$

We refer to [37] for further details on the $\mathcal{N} = 6$ superspace formalism of the ABJM/ABJ model, including

[3]Note the existence of the *off-shell* $\mathcal{N} = 3$ superfield formalism for the ABJM model [35] which was used to develop the quantum calculation technique in [36]

[4]Here and below we use the Latin symbols from the middle of the alphabet, $i, j, \ldots$, to denote the four-valued $SU(4)$ index, $i, j, \ldots = 1, 2, 3, 4$; we hope that this will not produce confusion with real 3-valued vector indices of M_3, see secs. 1.3, 2 and 3, as far as we do not use these in the present discussion.

for the explicit form of the bosonic equations of motion.

Searching for an $\mathcal{N}=8$ superfield formulation for the ABJM/ABJ models with CS levels $k=1,2$ it is natural to assume that the universal $\mathcal{N}=6$ sector is present as a part of $\mathcal{N}=8$ superspace formalism and, to describe two additional fermionic directions of $\mathcal{N}=8$ superspace, introduce, in addition to six $\mathbb{D}^I_\alpha$, one complex spinor Grassmann derivative $\mathbb{D}_\alpha$, and its conjugate $(\mathbb{D}_\alpha)^\dagger = -\bar{\mathbb{D}}_\alpha$ obeying

$$\{\mathbb{D}_\alpha, \bar{\mathbb{D}}_\beta\} = i\gamma^a{}_{\alpha\beta}\mathcal{D}_a + i\epsilon_{\alpha\beta}W\,, \qquad (56)$$
$$\{\mathbb{D}_\alpha, \mathbb{D}_\beta\} = 0\,, \qquad \{\bar{\mathbb{D}}_\alpha, \bar{\mathbb{D}}_\beta\} = 0\,,$$
$$\{\mathbb{D}_\alpha, \mathbb{D}^J_\beta\} = i\epsilon_{\alpha\beta}W^J\,, \qquad (57)$$
$$\{\bar{\mathbb{D}}_\alpha, \mathbb{D}^J_\beta\} = i\epsilon_{\alpha\beta}\bar{W}^J\,.$$

The structure of additional $\mathcal{N}=2$ supersymmetries proposed in [9] suggests to impose on the basic $\mathcal{N}=8$ superfields the chirality condition in the new fermionic directions [37],

$$\bar{\mathbb{D}}_\alpha \mathbb{Z}^i = 0\,, \qquad \mathbb{D}_\alpha \mathbb{Z}^\dagger_i = 0\,. \qquad (58)$$

While the natural candidate for the super-CS equation for the $SO(6)$ scalar superfield strength W is

$$W = \mathbb{Z}^i \mathbb{Z}^\dagger_i\,, \qquad (59)$$

to write a possibly consistent super-CS equation for 6 complex field strength W^J, which has to be chiral, $\mathbb{D}_\alpha W^J = 0 = \bar{\mathbb{D}}_\alpha \bar{W}^J$, to provide the consistency of the constraints (56), (57) and (52),

$$\bar{W}^J_{SU(M)} =\propto \mathbb{Z}^i \gamma^J_{ij} \tilde{\mathbb{Z}}^j\,, \quad W^J_{SU(M)} =\propto \tilde{\mathbb{Z}}^\dagger_i \tilde{\gamma}^{J\,ij} \mathbb{Z}^\dagger_j\,, \quad (60)$$

one needs to involve "non-ABJM superfields", the leading components of which are the "non-ABJM fields" of [9]. These are $N\times M$ matrix $\tilde{\mathbb{Z}}^i$ and $M\times N$ matrix $\tilde{\mathbb{Z}}^\dagger_i$ which obey

$$\bar{\mathbb{D}}_\alpha \tilde{\mathbb{Z}}^i = 0\,, \qquad \mathbb{D}_\alpha \tilde{\mathbb{Z}}^\dagger_i = 0 \qquad (61)$$

and must be related with ABJM superfields $\mathbb{Z}^i$, $\mathbb{Z}^\dagger_i$ by using the suitable monopole operators (converting $(\bar{\mathbf{M}}, \mathbf{N})$ representation into $(\mathbf{M}, \bar{\mathbf{N}})$) which exist for the case of CS levels $k=1,2$ only [9]. According to [9], the existence of these monopole operators is reflected by the 'identities' between hermitain three brackets (6) of the ABJM and non-ABJM (super)fields. The set of these 'GR-identities' includes

$$[(\ldots), \tilde{\mathbb{Z}}^\dagger_i; \tilde{\mathbb{Z}}^i] = -[(\ldots), \mathbb{Z}^i; \mathbb{Z}^\dagger_i]\,. \qquad (62)$$

The consistency of the system of $\mathcal{N}=8$ superfield equations (51)–(60) and the set of GR-identities necessary for that are presently under investigation [37].

Acknowledgement

The author thanks José de Azcárraga, Warren Siegel, Dmitri Sorokin, Paul Townsend and Linus Wulff for useful discussions. This work was partially supported by the Spanish MICINN under the project FIS2008-1980 and by the Ukrainian National Academy of Sciences and Russian RFFI grant 38/50–2008.

Notice added: After this manuscript has been finished, a paper [38] devoted to $\mathcal{N}=8$ superspace formulations of $d=3$ gauge theories appeared on the net. It contains a detailed description of the on-shell $\mathcal{N}=8$ superspace formulation of the BLG model for finite dimensional three algebras, similar to the formulation of the SDiff_3 invariant Nambu bracket BLG model in [30], and of its derivation starting from the gauge theory constraints and Bianchi identities. Also the component field content of the SYM model defined by the constraints (22) and its finite-3-algebra counterpart is discussed there.

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