

# Accuracy of Determining Stress Intensity Factors in Some Numerical Programs

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*At present, there are many programs for numerical analysis of cracks, in particular for determining stress intensity factors. Analyses of a single-edge cracked beam and flat plate with a semielliptical surface crack are presented in this study to examine the accuracy and applicability of the Franc2d and Franc3d programs. Further numerical computations of the MARC program and analytical solutions of stress intensity factors were included to compare the results with each other. For this purpose MARC was equipped with special user procedures. The influence of mesh fineness on the results was also investigated in all programs. The distributions of the stress intensity factors show good agreement in quality. The maximum deviations from the analytical solutions are 9.7%. With greater numbers of elements programs Franc2d and Franc3d showed some instability, which currently reduces the usefulness and reliability of these promising tools for engineering applications.*

*Keywords: fracture mechanics, stress intensity factors, numerical programs.*

## 1 Introduction

Stress intensity factors can be determined by experimental, numerical or analytical methods. However, with complicated component and crack geometry or under complex loading only numerical procedures are applicable. Many programs have been designed recently to deal with fracture phenomena (e. g., *Franc2d*, *Franc/FAM*, *Franc3d* and *AFGROW*), which, in spite of great programming efforts, still show some deficiencies in range of functionality, operation comfort and reliability. Little evidence has yet been provided about the accuracy and suitability of such programs for solving engineering problems [1, 2]. Present applications cover illustrative examples and simple problems [3, 4, 5].

Proven multi-purpose numerical programs such as *MARC* and *ABAQUS* usually possess no routines to find out stress intensity factors. To take advantage of the programs' extensive functions and high reliability a number of user subroutines for solving stress intensity factors need to be programmed.

As regards the accuracy of stress intensity factors, only single programs have been compared with analytical solutions so far, and no comparison of the programs with each other has been presented. The objective of this study was therefore to confront the results of *Franc2d*, *Franc3d* and *MARC* with each other and for a better view also to verify their deviations from the analytical solutions. The analyses were conducted on simple models while also observing the influence of mesh fineness, usability of available solvers and the overall performance of the programs.

## 2 Programs and models

The tested programs were *Franc2d* Version 2.7 [6], *Franc3d* [7] Version 1.15, and *MARC* Version 2000 [8]. The first two come from Cornell University, New York, and as freeware they can be freely distributed. *Franc2d* (two-dimensional FRACTure ANALYSIS Code) is based on the finite element method and enables analyses of two-dimensional problems with arbitrary component and crack geometries. Several methods are implemented for calculating stress intensity factors, from which the J-Integral Method [9] was chosen for the purposes of this study. *Franc3d* uses the boundary element method and was

designed for solving three-dimensional fracture problems. Also here, arbitrary component and crack geometries can be analysed. Stress intensity factors are determined by the Displacement Correlation Method [10]. Both *Franc2d* and *Franc3d* possess further important functions for modeling various fracture phenomena, such as fatigue crack growth. Currently, a new version of *Franc3d* is being developed, which is based on the finite element method and offers a greater functional range. The finite element system *MARC* is suitable for analyses of general problems of engineering mechanics. To determine stress intensity factors the Displacement Correlation Method with a linear extrapolation from two nodes at each crack face [11, 12] has been implemented in the user procedures.

As analytical solutions are known for only certain probe types, a single-edge cracked beam subjected to three-point bending and a flat plate with a semielliptical surface crack (Fig. 1) were chosen for the tests in this study. The analytical solution for the single-edge cracked beam is given by the equation

$$K_I = \frac{3F_s}{2BW^2} \sqrt{a} \frac{1.99 - \frac{a}{W} \left(1 - \frac{a}{W}\right) \left[215 - 3.93 \frac{a}{W} + 27 \left(\frac{a}{W}\right)^2\right]}{\left(1 + 2 \frac{a}{W}\right) \left(1 - \frac{a}{W}\right)^{\frac{3}{2}}} \quad (1)$$

with the conditions

$$\frac{s}{W} = 4, \quad \text{and} \quad 0 \leq \frac{a}{W} \leq 0.6.$$

The distribution of stress intensity factors along the crack front in a flat plate is expressed by the relation

$$K_I = \frac{1}{\Phi} \sigma_n \sqrt{\pi c} Y \left( \psi, \frac{c}{a}, \frac{c}{W}, \frac{a}{b} \right) \quad (2)$$

with the conditions

$$0 < \frac{c}{a} \leq 0.5, \quad 0 \leq \frac{c}{W} < 1.0 \quad \text{and} \quad 0 \leq \psi \leq \pi,$$

where  $\Phi$  is a complete elliptic integral of the second kind and  $Y$  is a geometric function. However, the function  $Y$  is not based on theoretical examinations but on experimental studies [13].

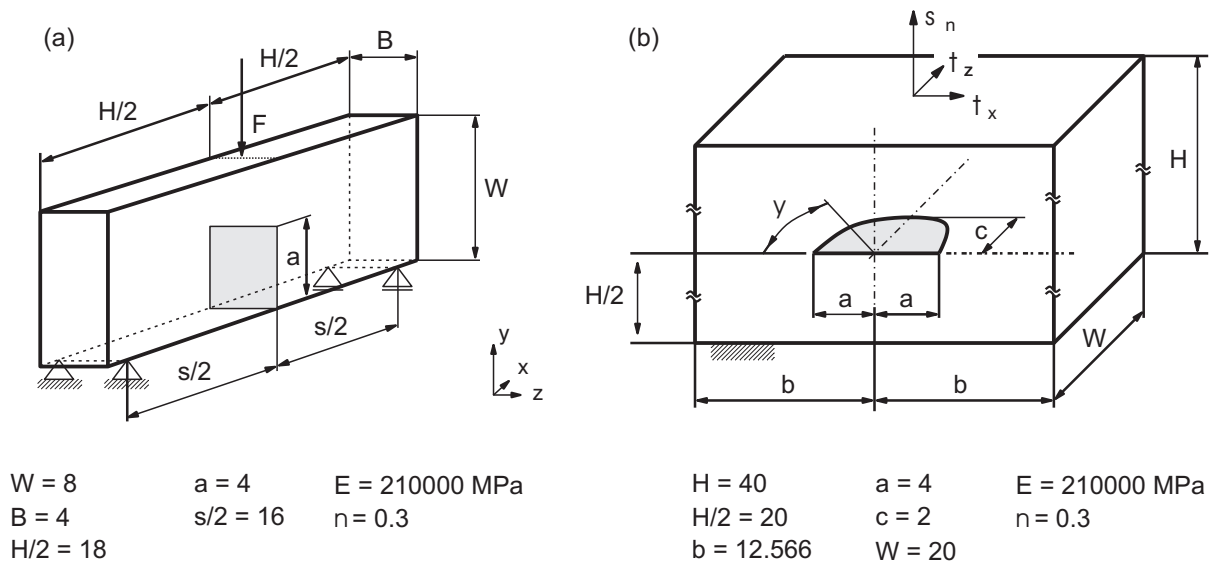


Fig. 1: A single-edge cracked beam (a) and a flat plate with a semielliptical surface crack (b). Length measures in mm.

The meshes of the analysed models were generated with the relevant preprocessors (*Casca*, *OSM*, *Mentat*) in three mesh densities at a time, in order to observe the influence of mesh fineness on stress intensity factors. Thus, from each examined probe type there were models with a coarse, medium fine and fine mesh in each program (Table 1). Although in general it is appropriate to take advantage of a model symmetry, here, with respect to future studies, complete models were created (Fig. 2). The basic elements were taken linear in *Franc3d*, and

quadratic in *Franc2d* and *MARC*. The crack front (or crack tip in two-dimensional cases) formed in all programs collapsed quarter-point quadratic elements [14], the number of which varied in three-dimensional cases from 16 to 48 along the crack front. The rosette consisted of 6 to 8 collapsed elements. The two dimensional models were considered as plain strain problems. The meshes of *Mentat* were generated by a newly introduced parametric modeling function. The load of the single-edge cracked beam models consisted of a sin-

Table 1: Numbers of elements and nodes of the analysed models

	Elements			Nodes		
	coarse	medium fine	fine	coarse	medium fine	fine
<b>Single-Edge Cracked Beam</b>						
Franc2d/Casca	556	1636	2168	1645	4525	6395
Franc3d/OSM	936	2252	4020	902	2148	3876
MARC/Mentat	2760	8400	14080	13233	37939	62115
<b>Centre Cracked Plate</b>						
Franc3d/OSM	1206	2014	3084	1001	1918	2780
MARC/Mentat	7396	12880	17608	33143	55979	76341

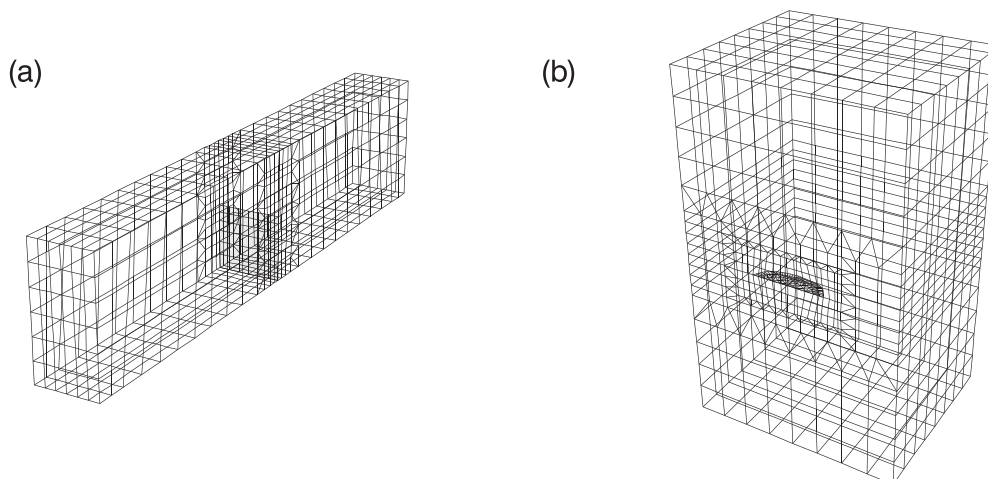


Fig. 2: An example of the used *Franc3d* meshes; (a) single-edge cracked beam, (b) flat plate

gle force ( $F = 500$  N) with two-dimensional models, or of a uniformly distributed force ( $f = 125$  Nmm<sup>-1</sup>) in three-dimensional models. The flat plate was loaded with normal stress ( $\sigma_n = 200$  MPa) on the upper surface of the model.

### 3 Strategy and results

A single-edge cracked beam was analysed in *Franc2d*, *Franc3d* and *MARC*. The flat plate was analysed as a shear three-dimensional problem only in *Franc3d* and *MARC*. The analytical solutions were carried out for both probe types. After analysis there were three results (coarse, medium fine

The dependences of the stress intensity factors on the mesh fineness are displayed in Figs. 4 and 5. The optimum solutions of stress intensity factors are compared in the following diagrams, Fig. 6:

- stress intensity factors in a single-edge cracked beam under single force or distributed force loading; three numerical solutions and one analytical solution;  $F = 500$  N or  $f = 125$  Nmm<sup>-1</sup> respectively;
- stress intensity factors in a flat plate under normal stress loading; two numerical solutions and one analytical solution;  $\sigma_n = 200$  MPa.

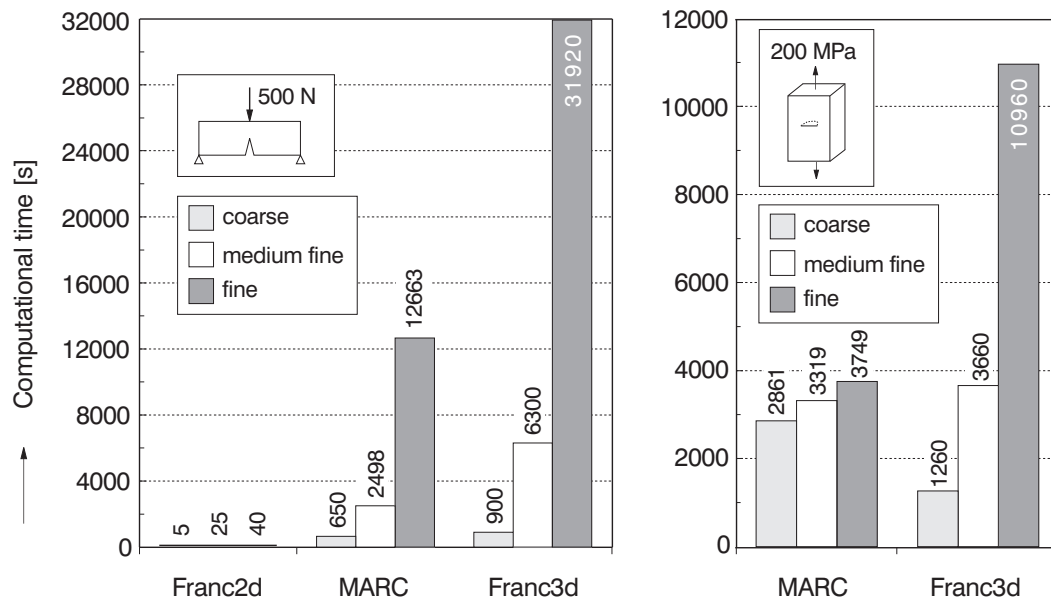


Fig. 3: Computational time of the single-edge cracked beam (left) and flat plate (right)

and fine mesh) for one probe type from each used program. In particular, these were three mesh qualities  $\times$  three programs from the single-edge cracked beam, and three mesh qualities  $\times$  two programs from the flat plate. From each three results the optimum solutions were then chosen. The criteria for this were minimum deviation from the analytical solution on the one hand, and the lowest possible computational time on the other. In the end the programs' optimum solutions were compared together with the analytical solutions in one diagram for each probe. The stress intensity factors were evaluated as a single value at a crack tip or as a course of values at a crack front.

The *Franc2d* models were solved with an implicit direct solver, which required very low computational times (Fig. 3). There was no point in using more sophisticated solvers, as the problem was entirely linear. The *Franc3d* jobs were processed by the boundary element solver *BES*, which includes four different schemes. In this study the iterative scheme with out-of-core element storage proved best: the direct scheme could not be applied to larger models, as the programs crashed after the stiffness matrix assembly, and the other schemes turned out to run slightly more slowly. The *MARC* analyses were carried out with the iterative solver (in-core element storage, incomplete Choleski preconditioner). All computations were performed on an SGI Origin 2000 computer.

### 4 Discussion

With simple crack configurations, such as that of the single-edge cracked beam, mesh quality seems to have only a little influence on the values of the stress intensity factors (Fig. 4). The smallest differences can be observed for *Franc2d*. However, mesh quality appears to be significant in the case of more complicated crack forms, such as that of a semielliptical crack (Fig. 5). The stress intensity factors differ with *MARC* especially at the crack edges; *Franc3d* shows consistent values from a certain mesh fineness.

The overall comparison of stress intensity factors for a single-edge cracked beam was performed in the middle of the crack front, as these show more important evidence than the values at the crack ends. However, this was not the case for the semielliptical crack in the flat plate, where the stress intensity factors vary considerably along the whole crack front. The values of *Franc3d* are always somewhat higher than the analytical solution (deviation up to 9.7 %, Fig. 6), whereas *MARC* delivers somewhat lower values (deviation up to 8.4 %). On the one hand, the deviations can be traced to the methods used (boundary element vs. finite element method) and elements (linear vs. quadratic), but on the other hand, they may also result from the different methods of calculating the stress intensity factors. The deviations of the *Franc2d* values from the analytical solution are the lowest, which agrees with the

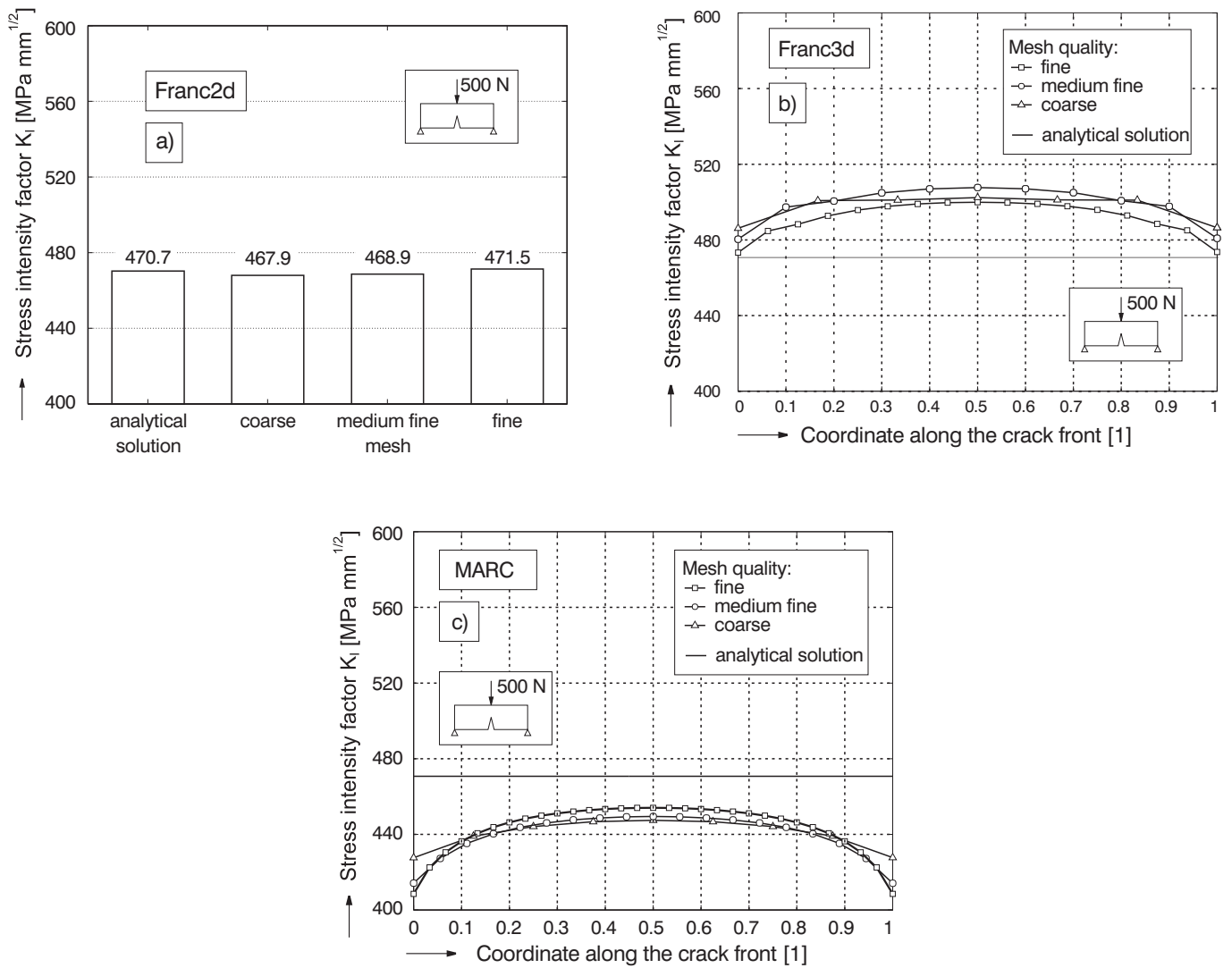


Fig. 4: Influence of mesh fineness on stress intensity factors  $K_I$  for a single-edge cracked beam using *Franc2d* (a), *Franc3d* (b) and *MARC* (c)

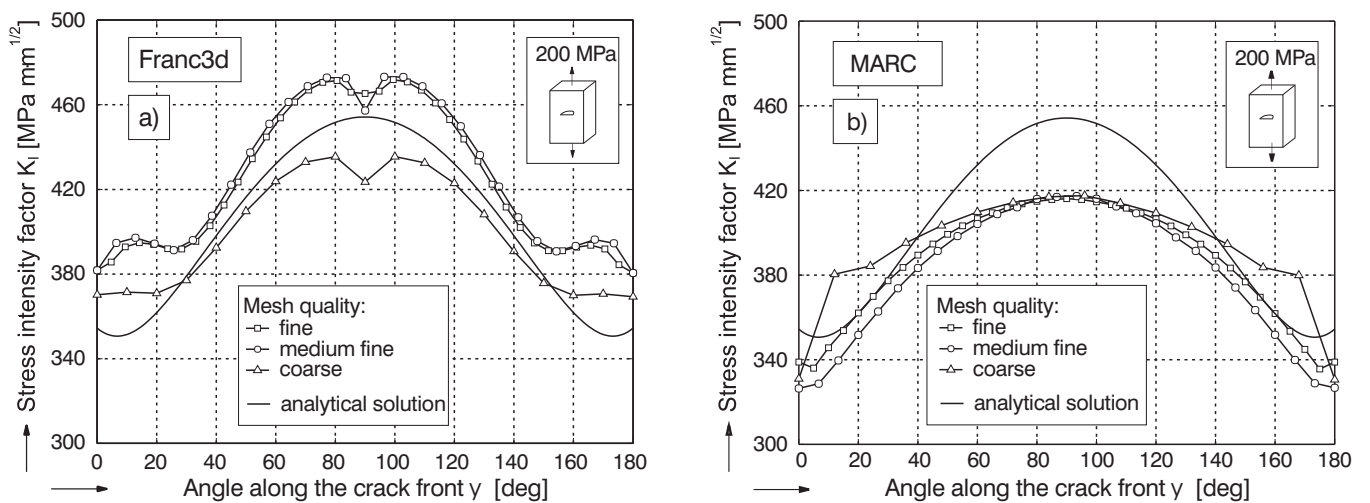


Fig. 5: Influence of mesh fineness on stress intensity factors  $K_I$  at the flat plate using *Franc3d* (a) and *MARC* (b)

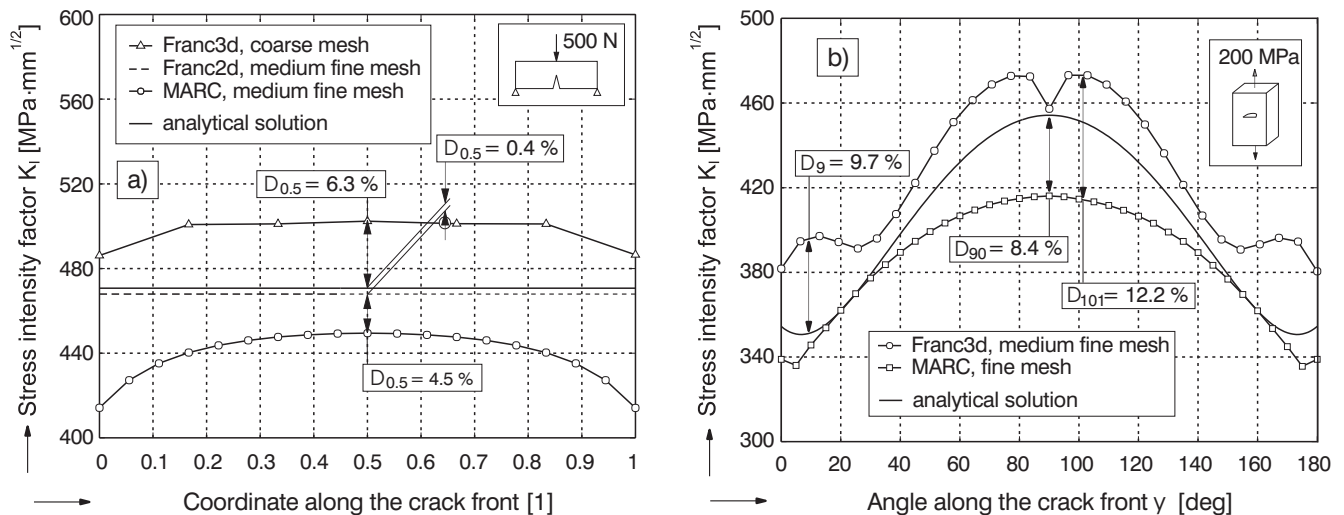


Fig. 6: Comparison of stress intensity factors  $K_I$  for a single-edge cracked beam (a) and flat plate (b)

supposition that domain integral methods (such as the J-Integral Method) are more accurate than local methods (such as the Displacement Correlation Method) [15, 16].

The stress intensity factors determined by *Franc3d* at the crack edges in the flat plate do not correspond well to the other displayed solutions (Fig. 6). Stress intensity factors at the crack edges are in general dependent especially on the geometric configuration [13], but they should not acquire such a falling form. False values at crack edges are generally caused by some unsuitable treatment of the singularities which always exist at the ends of a crack front [17]. Here, this phenomenon as well as the sudden decline of the stress intensity factors at  $\psi = 90^\circ$  may also be connected to the boundary element method that is used.

*Franc2d* and *Franc3d* show some errors during both manual operation and computational processing. These emerge mostly with larger models (from about 4000 elements) and result in disfunction of some commands (e. g., manual redefining of elements). This is probably caused by deficient memory management: during computational processing too high memory demand and falsely defined elements can lead to a program crash.

## 5 Conclusion

In this study some analyses with *Franc2d*, *Franc3d* and *MARC* were conducted to compare the accuracy of determining stress intensity factors and to examine the behavior of the programs. A single-edge cracked beam and a flat plate with a semielliptical surface crack were used as test models. *Franc2d* shows good accuracy, but it is applicable only to two-dimensional problems. *Franc3d* delivers acceptable values and appears to be a promising tool for engineering applications. To this end, reliability and function range should be improved. *MARC* with special user procedures shows lower but certainly usable values. Although this multi-purpose program shows high reliability, the programming effort to adapt it for solving fracture problems remains high.

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