Anisochronic Internal Model Control Design

T. Vyhlídal, P. Zítek

The features of internal model control (IMC) design based on the first order anisochronic model are investigated in this paper. The structure of the anisochronic model is chosen in order to fit both the dominant pole and the dominant zero of the system dynamics being approximated. Thanks to its fairly plain structure, the model is suitable for use in IMC design. However, use of the anisochronic model in IMC design may result in so-called neutral dynamics of the closed loop. This phenomenon is studied in this paper via analysing the spectra of the closed loop system.

Keywords: internal model control, time delay system, dynamics analysis, system spectrum.

1 Introduction

Internal model control (IMC) is a well-known control design method in the field of control engineering, see [1, 2]. The strategy of IMC design is based on knowledge of the system model that is finally involved in the control loop. Since the structure of the resulting control algorithm arises from the structure of the system model, the IMC controller may acquire a quite complicated form. That is why the obtained IMC controller is often substituted by a classical PID controller, which approximates its features. The motivation for using the PID controller as the final control algorithm is given by the convention (PID algorithm is available in most programmable controllers). Thus, in this approach, the resulting IMC controller is utilised only in designing the parameters of the PID controller. On the one hand, this approach provides easier implementation of the control. However, on the other hand, the dynamics of the final control loop with a PID controller loses some of the merits of IMC design as the consequence of simplifying the controller. Nevertheless, thanks to progress in the hardware and software equipment of programmable controllers, it is not beyond the scope of a reasonable effort to implement (to program) the algorithm resulting from the IMC design on a programmable controller. For this purpose, the task is to search for a model of the lowest possible order to obtain an easily applicable control algorithm. However, use of the classical approach in modelling (based on describing the system dynamics by linear differential equations) often does not allow us to make a satisfactory approximation of the system dynamics using low order models. Particularly if the system involves time lags, distributed parameters or transport phenomena (note that such phenomena can be encountered, e.g., in heat transfer, chemical and biological processes) the order of the model resulting from the classical approach is as a rule high. Therefore, in such applications, it is advantageous to use the anisochronic modelling approach based on involving time delays in the linear model. In this paper, we first introduce the first order anisochronic model able to fit the dynamics of a broad class of systems. Then, analysing the closed loop spectrum, we investigate the features of the closed loop dynamics with the IMC controller derived from the anisochronic plant model.

2 First order anisochronic model

In many industrial applications, the following first order model with the input delay can be encountered

$$G(s) = \frac{y(s)}{u(s)} = \frac{K\exp(-s\tau)}{Ts+1} \tag{1}$$

where K is static gain coefficient, T is time constant and τ is input time delay, see, e.g., [3]. In fact, according to [4], see also [5], the model is adequate for approximating most industrial processes with well-damped dynamics. On the other hand, since there is only one parameter in the denominator of (1), supposedly, higher order system dynamics (damped as well as oscillatory) cannot be satisfactorily approximated by this model. In order to further extend the applicability of the first order anisochronic model, let another delay η be introduced into the model. The anisochronic model then acquires the following form

$$G(s) = \frac{y(s)}{u(s)} = \frac{K\exp(-s\tau)}{Ts+\exp(-s\eta)} \tag{2}$$

see [6]. An analogous but second order model has been used in [7]. The enhanced universality of model (2) is due to the possibility to assign its dominant pole couple arbitrarily in the

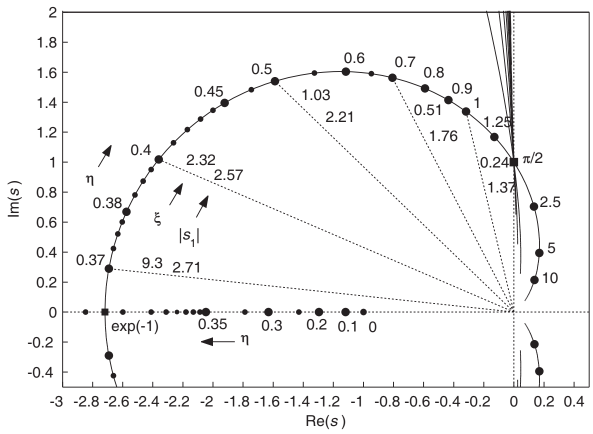

Fig. 1: The trajectory of the dominant couple of roots of the characteristic function $M(s) = 1 + \exp(-s\eta)$ with respect to η, $\xi = |\beta|/\omega$, $s_{1,2} = \beta \pm j\omega$ (the time constant T is considered here as a time-scale unit)

left half of the complex plane. The sposition of the dominant pole of (2) with respect to the values of T and η can be estimated from Fig. 1. The crucial values of the ratio η/T are following: $\eta/T = \exp(-1)$ for which the model has a double real pole and $\eta/T = \pi/2$ for which the dominant couple lays on the imaginary axis, which is the last value for which model (2) is stable. For $\eta/T \le \exp(-1)$ the poles of the dominant couple are real, while for $\eta/T > \exp(-1)$ they are complex conjugate. For more details about the distribution of the poles of (2), see [6].

The dynamics of the introduced first order anisochronic models (1) and (2) are determined by the system poles only. Involving a dynamic term into the numerator of the model transfer function, i.e., introducing the zeros into the model dynamics, further extends the applicability of the first order anisochronic model. One possible way of involving a zero-effect in the system dynamics consists in using the model

$$G(s) = \frac{K\left(Ls + \exp(-s\chi)\right)\exp(-s\tau)}{Ts + \exp(-s\eta)}, \tag{3}$$

where the ratio χ/L determines the distribution of the roots of $Ls + \exp(-s\chi) = 0$ (the positions of the zeros of (3)) in the same way as the ratio η/T determines the distribution of the poles. Using (3) is effective only in case of the requirement to involve the zeros that are located on the left half of the complex plane. Model (3) cannot be used to approximate the dynamics with a positive real zero because the equation $Ls + \exp(-s\chi) = 0$ does not have positive real roots for any χ/L, provided that $L > 0$, $\chi > 0$. If the zero is positive real and single given by $\mu = 1/L$, it can be added to the model dynamics simply by using $-Ls + 1$ instead of $Ls + \exp(-s\chi)$ (using $-Ls + \exp(-s\chi)$ does not bring about considerable merits because its dominant root is positive real for any $\chi > 0$). A more difficult task is to involve dominant complex zeros with positive real parts. Theoretically, it is possible to use the term $Ls + \exp(-s\chi)$, but as $|\mathrm{Re}(\mu)|/\mathrm{Im}(\mu) > 0.5$ (where m is a dominant zero of the system) the ratio χ/L becomes very large, which is not convenient from the numerical point of view. Another problem arising from the use of model (3) is that the degree of the numerator is equal to the degree of the denominator, i.e., there is a direct input-output link in the model. To avoid such an inconvenient model structure, instead of the first order anisochronic model (3), the following second order model may be used

$$G(s) = \frac{K\left(Ls + \exp(-s\chi)\right)\exp(-s\tau)}{(T_1 + 1)\left(Ts + \exp(-s\eta)\right)} \tag{4}$$

with the additional mode with time constant T_1, see [7]. An alternative way of involving the zeros into the first order anisochronic model that does not have the drawback of equal degrees of the numerator and the denominator consists in using the following model

$$G(s) = \frac{K\left(1 - a\exp(-s\chi)\right)\exp(-s\tau)}{Ts + \exp(-s\eta)} \tag{5}$$

in which instead of the quasipolynomial, the exponential polynomial is used in the numerator, see [8]. The zeros of system (5) are the roots of the following equation

$$N(s) = 1 - a\exp(-s\chi) = 0 \tag{6}$$

Considering variable s as a complex variable, i.e., $s = \beta + j\omega$, the complex roots of (6) are the solutions of the equations

$$\mathrm{Re}\left(N(s)\right) = 1 - a\exp(-\chi\beta)\cos(\chi\omega) = 0 \tag{7}$$

$$\mathrm{Im}\left(N(s)\right) = a\exp(-\chi\beta)\sin(\chi\omega) = 0 \tag{8}$$

which result from splitting equation (6) into real and imaginary parts. Separating the exponential term from (7)

$$\exp(-\beta\chi) = \frac{1}{a\cos(\omega\chi)} \tag{9}$$

and substituting (9) into (8), the following expression results

$$\tan(\chi\omega) = 0 \tag{10}$$

Since (10) is satisfied for $\omega = k\pi/\chi$ and the right-hand side of (9) has to be positive to obtain real β, the roots $s = \beta + j\omega$ of (6), i.e., the zeros of (5) are given by

$$\beta = -\frac{1}{\chi}\ln\left|\frac{1}{a}\right| \tag{11}$$

$$\begin{matrix} \omega = \pm 2k\pi/\chi & \text{if } a > 0 \\ \omega = \pm(2k+1)\pi/\chi & \text{if } a < 0 \end{matrix}, \quad k = 0, 1, 2, \ldots\,. \tag{12}$$

Thus, by means of parameters a and χ, we can assign the horizontal chain of the roots arbitrarily in the complex plane. Prescribing the real parts of the roots β yields

$$|a| = \exp(\beta\chi) \tag{13}$$

and parameter χ results from

$$\chi = \frac{2\pi}{\omega_p}, \tag{14}$$

where ω_p prescribes the spacing of the imaginary parts of the roots. If a is chosen positive, equation (6) has one real root. The closest complex root (of the horizontal chain) to the real one has an imaginary part equal to ω_p. If a is chosen negative, equation (6) does not have a real root. In this case, the roots of the chain closest to the real axis have the imaginary parts equal to $\pm\omega_p/2$. To sum up, if $a > 0$, the roots are given as $s_1 = \beta$ and $s_{2k,2k+1} = \beta \pm j\left(k\omega_p\right)$, $k = 1, 2, \ldots$ and if $a < 0$, the roots are givesn as $s_{2k+1,2(k+1)} = \beta \pm j\left((2k+1)\omega_p/2\right)$, $k = 0, 1, \ldots$ Thus, by means of involving exponential polynomial (6) we can assign either one real dominant zero or the pair of complex conjugate dominant zeros.

3 IMC design based on a universal first order anisochronic model

In [7, 9, 6], the features of internal model control (IMC) design based on low order models with time delays are studied. The scheme of IMC is shown in Fig. 2, where $R^*(s)$ is the controller, $P(s)$ denotes the dynamics of the plant which is being controlled, and $G(s)$ is the model of the plant.

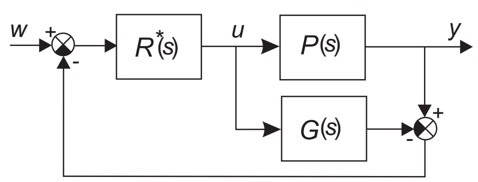

Fig. 2: Internal model control (IMC) scheme

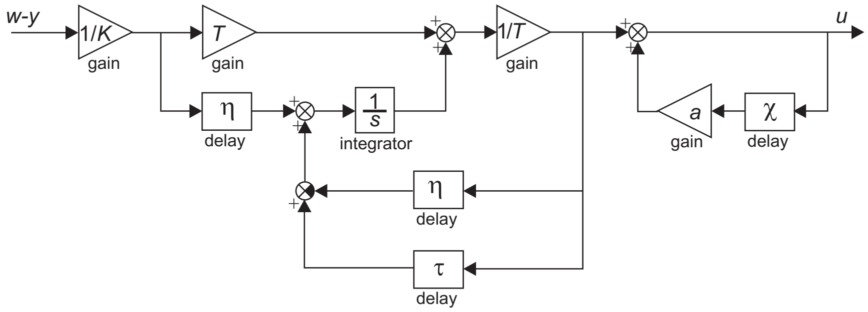

Fig. 3: Scheme of the controller (16)

Using universal first order anisochronic model (5) and provided that the system being approximated does not have positive zeros, the transfer function of the controller is given by

$$R^*(s) = \frac{1}{G_i(s)} F(s) = \\ = \frac{Ts + \exp(-s\eta)}{K(1 - a\exp(-s\chi))} \frac{1}{T_f s + \exp(-s\eta_f)}, \quad (15)$$

where $G_i(s)$ is the invertible part of model (5) (the only uninvertible part of model (5) is the term corresponding to the input delay, which is separated) and $F(s)$ is the first order anisochronic filter with $F(0) = 1$. The transfer function of the inner control loop (which is the controller transfer function if the classical control loop is considered) is given by

$$R(s) = R^*(s) = \frac{R^*(s)}{1 - R^*(s)\,G(s)} = \\ = \frac{Ts + \exp(-s\eta)}{K(1 - a\exp(-s\chi))(T_f s + \exp(-s\eta_f) - \exp(-s\tau))} \quad (16)$$

see its block diagram in Fig. 3.

If $G(s) = P(s)$, the closed loop dynamics are given by the first order anisochronic model

$$G_{wy}(s) = \frac{\exp(-s\tau)}{T_f s + \exp(-s\eta_f)} \quad (17)$$

and the dynamics of the closed loop can be chosen by the ratio of parameters T_f and η_f. However, the model approximates only a part of the dynamics as a rule. Therefore, let us study the closed loop dynamics for the case $G(s) \neq P(s)$. Let the filter be $F(s) = 1/F_f(s)$, ($F_f(s) = T_f s + \exp(-s\eta_f)$), the model $G(s) = K\,N(s)/M(s)\exp(-s\tau)$, ($N(s) = 1 - a\exp(-s\chi)$, $M(s) = Ts + \exp(-\eta s)$) and the true plant model $P(s) = Q(s)/S(s)$, then the controller transfer function is given by

$$R(s) = \frac{M(s)}{KN(s)(F_f(s) - \exp(-s\tau))} \quad (18)$$

and the transfer function of the closed loop is the following

$$G_{wy}(s) = \frac{M(s)Q(s)}{S(s)KN(s)(F_f(s) - \exp(-s\tau)) + M(s)Q(s)} \quad (19)$$

Thanks to $N(s)$, obviously, there are delayed terms of the highest derivative of $y(t)$ in the model of the closed loop. Thus the closed loop is of neutral dynamics, [10]. In general, this fact rather restricts the class of models that may be used in the described anisochronic IMC design, see [13]. This is due to the fact that the asymptotic features of the spectrum of the poles of (19) are determined by the roots of the exponential polynomial $N(s)$. The problem is that the spectrum of $N(s)$ may be very sensitive to the changes in the delays involved in $N(s)$, see [8]. However, if $N(s)$ involves only one delay, as in (6), the distribution of the root spectrum of $N(s)$ and also the distribution of the pole spectrum of (19) are rather insensitive with respect to small changes in the delay. The basic features of such IMC design are shown in the following example.

4 Application example, spectrum based analysis of the dynamics

In order to demonstrate the outstanding approximation features of a first order anisochronic model, consider that the plant is originally described by the tenth order model with the transfer function

$$P(s) = \frac{20s + 1}{(2s + 1)^{10}} \quad (20)$$

with the multiple pole $\lambda_{1\ldots10} = -0.5$ and the single dominant zero $\mu = -0.05$. Let us find the parameters of model (5) that approximates model (20) in the low frequency range. Assessing $\tau = 8$ and $\chi = 10$, which implies $a = 0.607$ (according to (13), $\beta = -0.05$) and $K = 1/(1 - a) = 2.54$, the dead time of the system and the rising part of the response are approximated quite well, see Fig.4.

The remaining two parameters of model (5), i.e., $T = 13.1$ and $\eta = 5.5$, have been assessed using the least squares method to approximate two points of the frequency response of (20).

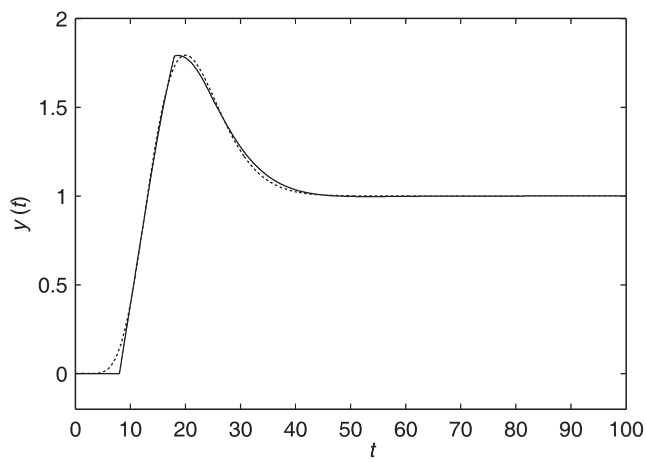

Fig. 4: Step responses of system (20) (dashed) and of its approximation (5) (solid)

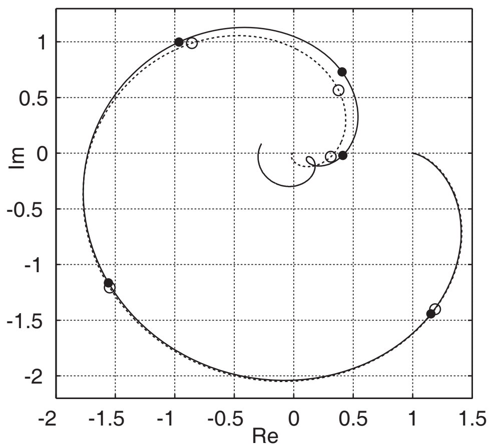

Fig. 5: Frequency responses of system (20) (dashed) and of its approximation (5) (solid)

Since model (5) is supposed to approximate model (20) in the low frequency range, the points of the frequency response being approximated have been chosen those with $\Phi(\omega_1) = \arg\left(P(j\omega_1)\right) = -\pi/2$ and $\Phi(\omega_2) = \arg\left(P(j\omega_2)\right) = -\pi$. As can be seen in Fig. 5, the frequency response of model (5) approximates the frequency response of (20) very well (even in the third quadrant of the frequency response). Also the approximation of the system step response is very good considering that the anisochronic model (5) is of the first order.

Let us use the given parameters of model (5) in controller (16) and let us investigate the dynamics of the closed loop. The closed loop system is of the 11$^{\text{th}}$ order with the transfer function given by (19). Choosing $T_f = 10$ and $\eta_f = 7$ the closed loop dynamics are supposed to be given by the dominant couple of poles $\tilde{\lambda}_{1,2} = -0.081 \pm 0.156\,j$ (the dominant pole of the ideal closed loop (17) with relative damping $\xi = 0.51$, see Fig. 1). Thus, let us investigate the distribution of the closed loop with the system having the dynamics described by (20) and the IMC controller based on approximation model (5). Since the closed loop characteristic function is the quasi-polynomial, a special numerical algorithm known as a mapping based rootfinder, see [11, 12, 13], is to be used to locate the characteristic function roots. The characteristic function of the closed loop (19) is given by

$$\begin{aligned} M_{wy}(s) = {} & (2s+1)^{10} K\left(1 - a\exp(-\chi s)\right) \cdot \\ & \cdot\left(T_f s + \exp(-s\eta_f) - \exp(-s\tau)\right) + \\ & +\left(Ts + \exp(-\eta s)\right)(20s+1) \end{aligned} \tag{21}$$

The first step of the mapping based rootfinder consists in substituting $s \to \beta + j\omega$ and splitting the characteristic function into real and imaginary parts, i.e., $R(\beta,\omega) = \operatorname{Re}\left[M_{wy}(\beta + j\omega)\right]$ and $I(\beta,\omega) = \operatorname{Im}\left[M_{wy}(\beta + j\omega)\right]$. Then, the implicit functions $R(\beta,\omega) = 0$ and $I(\beta,\omega) = 0$ are mapped in the s-plane and their intersection points are located determining the positions of the roots of (21). The result of the mapping based rootfinding technique can be seen in Fig. 6, where the decisive part of the root spectrum of (21) is shown.

As it is shown in Fig. 6, the following poles $\lambda_1 = -0.05$, $\lambda_{2,3} = -0.081 + 0.176\,j$ and $\lambda_{4,5} = -0.126 + 0.089\,j$ are the closest

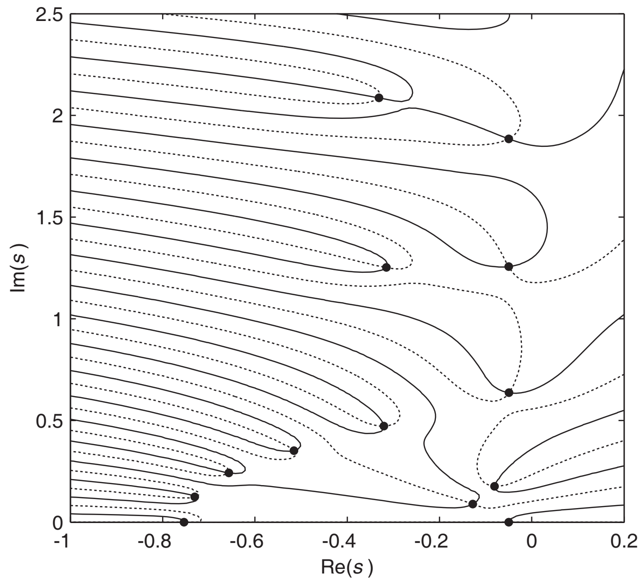

Fig. 6: Poles of the closed loop system with plant model (20) and controller (5), $\operatorname{Re}\left(M_{wy}(s)\right) = 0$ - solid, $\operatorname{Im}\left(M_{wy}(s)\right) = 0$ - dashed, $M_{wy}(s)$ - given by (21)

poles to the s-plane origin which are likely to be the dominant poles of the system. The poles of couple $\lambda_{2,3}$ are quite close to the prescribed poles $\tilde{\lambda}_{1,2}$. However, from the distribution of the other dominant poles, it is not obvious that poles $\lambda_{2,3}$ determine the dynamics of the closed loop (λ_1 is even closer to the origin of the s-plane). Since the closed loop is the neutral system we also analyse the essential spectrum of the neutral system. The essential spectrum of the system is here given by the solutions of the equation $N(s) = 0$, see [10]. The spectra of the poles (black circles), zeros (empty circles) and the essential spectrum (asterisks) corresponding to closed loop system (19) with chosen $T_f = 10$ and $\eta_f = 7$ are shown in Fig. 7. As can be seen, pole λ_1 is likely to be compensated by the real zero. Also the couple of poles $\lambda_{4,5}$ are quite close to a couple of zeros and they are also partly compensated. Consequently, the dominant mode of the set-point response is really given by the couple of poles $\lambda_{2,3}$ (see Fig. 7, the poles of the prescribed anisochronic dynamics are marked by squares). The dominant role of the couple $\lambda_{2,3}$ in the set-point response dynamics of the closed loop is confirmed by the responses of the real closed loop system (19) and the ideal closed loop system (17) shown in Fig. 8. As can be seen, the real set point response (solid) is very close to the ideal one (dashed). The characteristic feature of the class of neutral systems, i.e., some of the poles converge to the eigenvalues of the essential spectrum, can be seen in Fig. 7 and in the enlarged region in Fig. 9.

The consequence of using the numerator of form (6) to approximate the system dominant zero is that the closed loop system has infinitely many poles with real parts close to the value given by (11). Therefore, controller (16) cannot be used to control systems with zeros in the right half of the complex plane. On the other hand, if the dominant zero is negative and not too close to the imaginary axis, the neutral character of the closed loop system does not bring about any features that are risky to the dynamics. Provided that closed loop system (19) (with negative dominant zero) does not have any unstable poles close to the s-plane origin, it does not have any

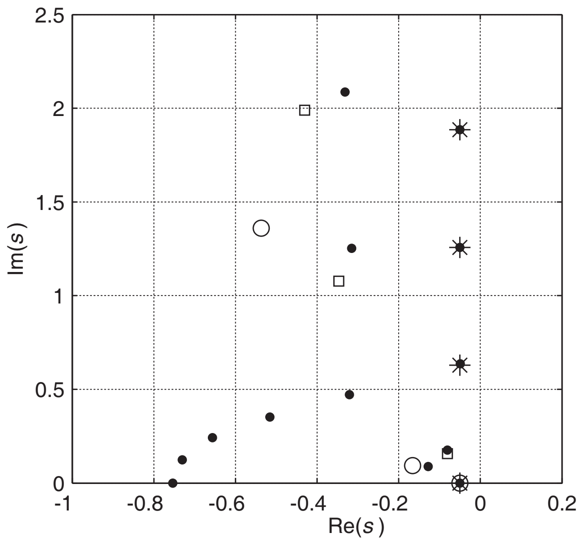

Fig. 7: Spectra of closed loop system for $T_f = 10$ and $\eta_f = 7$, detail of Fig. 9

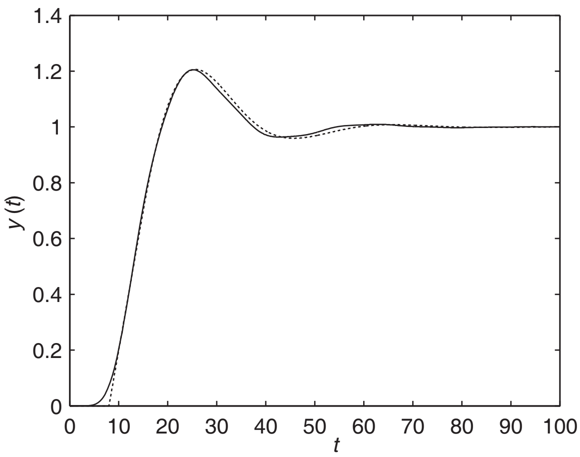

Fig. 8: Comparison of the ideal and real set-point responses, ideal - dashed, real - solid

unstable poles at all. This is given by the fact that the chain of the poles converging to the spectrum of the difference equation has the tendency to get closer to the eigenvalues of the essential spectrum as the magnitudes of the poles in the chain increase.

5 Conclusions

The prime objective of the paper is to break the conventional concept of a PID controller by including the delayors in its structure and in this way to reduce the order of models needed for controller design. In fact, the resulting IMC controllers are somewhere between the analog and discrete principles of operation, with the specific feature that the time shifts (sample-delays) are not integer multiples of a sampling time interval. It should be noted that the delays in the controller model do not serve only for compensating some specific delays (e.g., transport delays) in the plant. The purpose of delayors is to fit the dynamic effects of various origin, including the distributed parameters of the plant. The first order example model (5) used in the paper actually represents

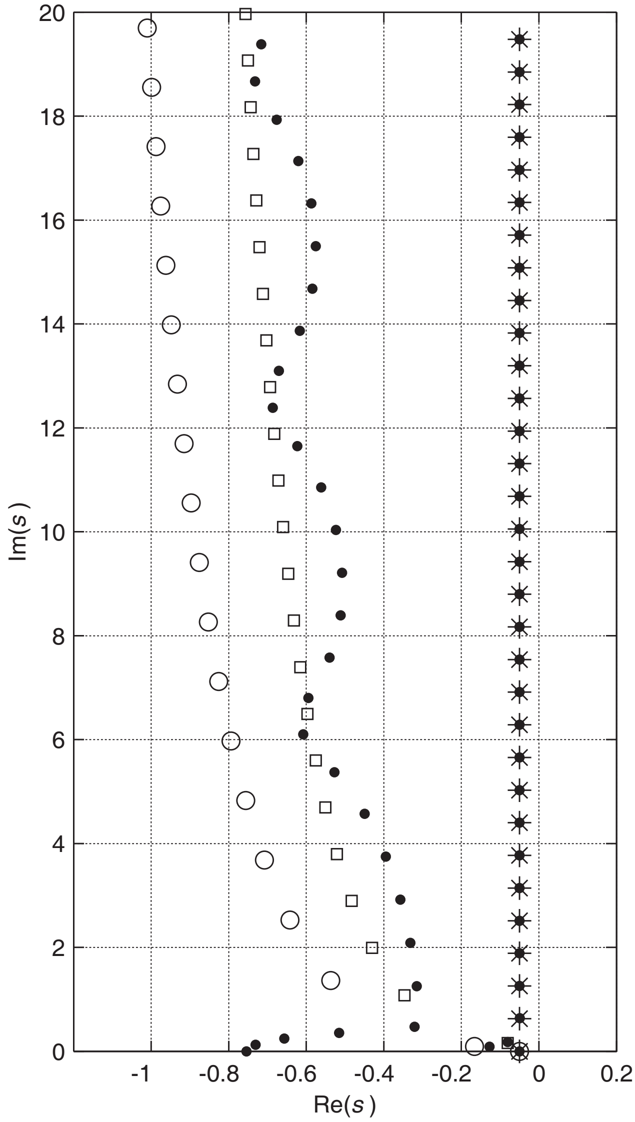

Fig. 9: Spectra of closed loop system for $T_f = 10$ and $\eta_f = 7$. Spectra of closed loop system (19): black circles – poles of (19), empty circles – zeros of (19), asterisks – roots of $N(s)$, squares – poles of system (17) (ideal closed loop).

those plants classified as "higher-order" in the usual sense of the term. Controlling these plants by a conventional PID controller is doubtful. The use of delays in the modelling results in a quite plane structure of the final control algorithm, which can easily be implemented on a programmable controller, where simultaneous application of the integrators and delayors is quite possible. As has also been shown, implementation of the IMC controller based on the anisochronic model can result in the so-called neutral character of the closed loop dynamics, which may introduce risky features into the closed loop system dynamics. However, if the controller of structure (16) is used (provided that the dominant zero being approximated is located in the left half of the complex plane) the neutral character of the closed loop does not bring any substantially negative features to the dynamics.

Acknowledgement

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Ing. Tomáš Vyhlídal, Ph.D.
e-mail: vyhlidal@fsid.cvut.cz

Prof. Ing. Pavel Zítek, DrSc.
phone: +420 224 352 564
fax: +420 233 336 414
e-mail: zitek@fsid.cvut.cz

Centre for Applied Cybernetics
Institute of Instrumentation and Control Engineering
Czech Technical University in Prague
Faculty of Mechanical Engineering
Technická 4
166 07 Praha 6, Czech Republic