Neural Network Based Identification of Material Model Parameters to Capture Experimental Load-deflection Curve

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A new approach is presented for identifying material model parameters. The approach is based on coupling stochastic nonlinear analysis and an artificial neural network. The model parameters play the role of random variables. The Monte Carlo type simulation method is used for training the neural network. The feasibility of the presented approach is demonstrated using examples of high performance concrete for prestressed railway sleepers and an example of a shear wall failure.

Keywords: Neural network, nonlinear fracture mechanics, Latin Hypercube Sampling, identification.

1 Introduction

Utilization of an appropriate material model for realistic modeling of concrete is essential in order to capture both experimental results and real structures. Generally, the more sophisticated the model we deal with, the greater the number of model parameters that have to be considered. In better cases, basic parameters such as compressive strength, modulus of elasticity, etc. are known. In worse cases, practically nothing is known. Some parameters can be estimated using recommended formulas from the literature, but in most cases these formulas can be used only as a first approximation of the parameters. If an experimental load-deflection curve is to be captured, e.g., by. a nonlinear fracture mechanics model, the first calculation using an initial set of material model parameters usually deviates from desired set. Then it is necessary to make some correction of the parameters using a trial-and-error method. The parameters are changed step by step, the numerical calculations have to repeated many times and the numerical simulation results are compared with the experimental results. Such a classical approach is not very efficient, especially if a complex material model with many parameters is used. That is why different alternatives of identification algorithms have been proposed in the literature, and they are now becoming increasingly attractive (e.g. [1], [2], [3]).

The aim of this paper is to present a new approach for identifying material model parameters. The proposed approach is based on coupling stochastic nonlinear fracture mechanics analysis and an artificial neural network. The identification parameters play the role of basic random variables, with the scatter reflecting the physical range of possible values. The efficient Monte Carlo type simulation method Latin Hypercube Sampling (LHS) is used. The statistical simulation provides the set of data, “a bundle” of numerically simulated load-deflection curves. The generated basic random variables and the subsequently calculated load-deflection curves are used for training a suitable type of neural network. Once the network is trained it represents an approximation which can be utilized in a reverse way: for a given experimental load-deflection curve to provide the best possible set of material model parameters.

Several software tools had to be combined in order to make the identification possible. First, ATENA [4] nonlinear fracture mechanics softwaand FREET [5] probabilistic software package – these can be combined under the SARA software shell [6], [7]. Then DLNNET, new neural network software was developed [8].

The results of the approach were recently presented [9]. The similar concept of using Latin Hypercube Sampling statistical simulation for stochastic training of a neural network was used to estimate microplane model parameters [10].

The methodology is demonstrated on selected numerical examples of identifyinf a material model for concrete, called SBETA (the classical often used model available in ATENA software): a notched specimen under three-point bending of high-strength concrete used for railway sleepers and shear wall experiments.

2 Fundamental difficulty of nonlinear fracture mechanics modeling

For realistic modeling of structural failure from quasi-brittle materials, an advanced computational analysis should utilize nonlinear fracture mechanics. ATENA software [4] is an efficient tool for analysis of concrete, reinforced concrete and prestressed structures. The software employs a set of advanced material models for realistic calculation of the structural response. The well-known SBETA material model was verified during long development of the software, and it reflects all important aspects of concrete behaviour in both tension and compression. A fundamental difficulty naturally exists when an experimental load-deflection curve is to be captured by numerical simulation. Such a virtual experiment needs good material data in order to reproduce the load-deflection curve properly. In the case of the SBETA material model the main parameters are: modulus of elasticity E, tensile strength f_t, compressive strength f_c, fracture energy G_F, critical compressive displacement w_d, compressive strain in the uniaxial compressive test ε_c. A typical situation is that a set of preliminary parameters is first used for modeling, and in most cases only poor agreement with the experimental

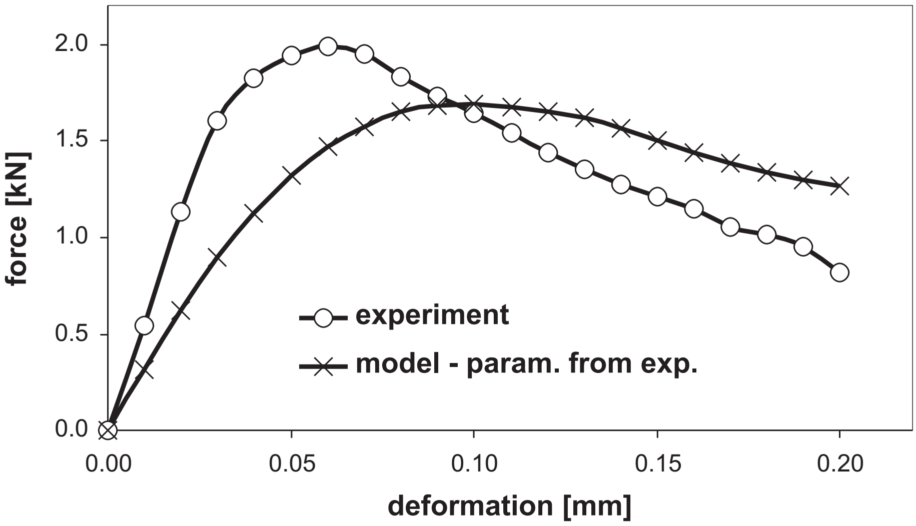

Fig. 1: Load-deflection curves – experiment and initial numerical simulation

data is achieved. A heuristic user-based iteration has to be performed.

For illustration, Fig. 1 shows an experimental load-deflection curve and a result of numerical simulation with the first set of preliminary parameters of a notched beam. This example will be discussed later in this paper. Although most of the material parameters were known (the relevant experiments had been done), the disagreement between the two curves is significant. It is clear that the parameters were not determined satisfactorily (errors, assumptions, contaminated experiments, influence of size effect, etc.), and the parameters need to be modified.

3 Identification of material model parameters

3.1 General concept

The new identification technique is based on a combination of statistical simulation and training the neural network. Several software tools had to be combined to make the identification possible. The whole procedure can be itemized as follows (the software relevant to the individual steps is referenced):

1. First, a computational model has to be developed using the appropriate FEM software, which enables modeling of both pre-peak and post-peak behaviour. The initial calculation uses a set of initial material model parameters. Software: ATENA [4].
2. The parameters of the material model to be identified are considered as random variables described by a probability distribution. Rectangular distribution is a "natural choice" as the lower and upper limits represent the bounded range of physical existence. However distributions can also be used, e.g. Gaussian (in spite of the fact that it is not bounded). These parameters are simulated randomly based on a Monte Carlo type simulation, and LHS small-sample simulation is recommended. The statistical correlation between some parameters can be taken into account.
Software: FREET [5].
3. A multiple calculation of a deterministic computational model using random realizations of the material model parameters is performed, resulting in "a bundle" of load-deflection curves (usually overlapping experimental curve).
Software: SARA [6], [7].
4. The random load-deflection curves serve as the basis for training an appropriate neural network. Such training can be called stochastic training, due to the stochastic origin of the load-deflection curves. After training, the neural network is ready to answer the referse task: to select the material model parameters which can capture the experimental load-deflection curve as closely as possible. Software: DLNNET [8].
5. The final calculation using the identified material model parameters should verify how well the parameters were identified (ATENA).

The complex program communication and the necessary interfaces are schematically shown in Fig. 2.

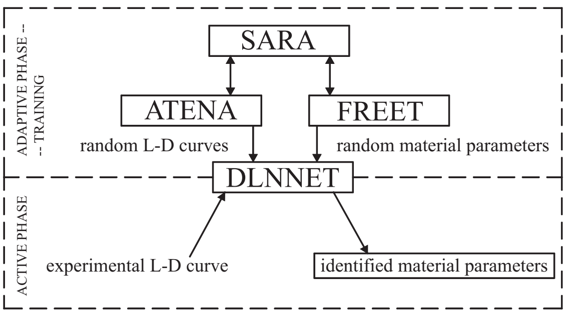

Fig. 2: Identification software communication scheme

3.2 Statistical simulation

In order to prepare the set of random load-deflection curves for training the neural network, a proper efficient Monte Carlo type simulation has to be performed. The SARA system originally developed for statistical and reliability analysis of concrete structures consists of two major parts – the FREET statistical and reliability package and the ATENA nonlinear finite element simulation.

The stochastic part of the SARA system is the FREET – Feasible Reliability Engineering Efficient Tool – probabilistic program. This probabilistic software for statistical, sensitivity and reliability analysis of engineering problems was designed with its focus on computationally intensive problems, which do not allow thousands of samples to be performed [5], [11].

A special type of numerical probabilistic simulation, LHS, makes it possible to use only a small number of Monte Carlo simulations for a good estimation of the first and second moments of the response function. LHS uses stratification of the theoretical cumulative probability distribution function (CPDF) of the input random variables. CPDFs for all random variables are divided into N equivalent no overlapping intervals, where N is the number of simulations. The representative parameters of the variables are selected randomly on the basis of random permutations of integers 1, 2, …, j, …, N. Every interval of each variable must be used only once during the simulation.

ATENA nonlinear finite element software is well-established for realistic computer simulation of damage and failure of concrete and reinforced concrete structures in a deterministic way [12], [13]. The constitutive relation at a material point (constitutive model) plays the most crucial role in the finite element analysis and decides how the structural model represents reality. Since concrete is a complex material with a strongly nonlinear response even under service load conditions, special constitutive models for the finite element analysis of concrete structures are employed [4].

The SARA system can easily be used for stochastic training of a neural network. The parameters for identification are simulated as random variables with prescribed variability. The resulting random load-deflection curves with random realizations of the parameters serve for training the network. Such a "bundle" of curves is shown in Fig. 5.

3.3 Artificial neural networks

The basic idea of an artificial neural network was to provide a numerical model of processes in the brain. Nowadays this approach is used in various fields of technical practice, mainly for classification problems [14]. In our identification technique a multilayered neural network is used. All neurons in one layer are connected with all neurons in the following layer. The connecting paths among the neurons are weighted, which models their conductivity. At the level of the neuron, the bias is added to the sum of the weighted impulses from each neuron of the previous layer. Then the transfer function is applied. Three types of transfer functions can be used: hard-limit, linear and nonlinear (e.g. sigmoid) transfer functions. The synaptic weights, biases and transfer functions determine the behavior of neurons and the whole neural network.

The output from a single neuron (Fig. 3a) can be calculated as:

$$y = f(x) = f\left(\sum_k (w_k \cdot p_k) + b\right), \qquad (1)$$

where: k – number of input impulses (1, …, K), w_k – synaptic weight of the connecting path from the k-th neuron of the previous layer, p_k – impulses from the k-th neuron of the previous layer, b – bias of the neuron and f – transfer function of the neuron.

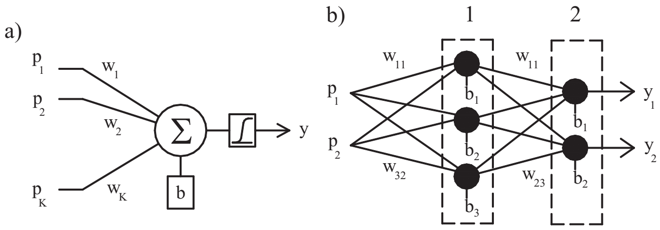

Fig. 3: a) Scheme of single neuron, b) scheme of neural network with two layers

If the output vector of the whole neural network is required, the output vectors have to be calculated layer by layer from the input layer to the output layer of the network. Output of the u-th layer of the network is:

$$y_k^u = f^{\,u}\left(\sum_{j=1}^{J} (w_{kj}^u \cdot y_j^{u-1}) + b_k^u\right), \qquad (2)$$

where: k – number of components in the output vector in the u-th layer (1, …, K = number of neurons in the u-th layer), j – number of components in the output vector in the (u–1)-th layer (1, …, J = number of neurons in the (u–1)-th layer), y_k^u – one component of the output vector, w_{kj}^u – synaptic weight – this connects the k-th neuron of the u-th layer with the j-th in the (u–1)-th layer, y_j^{u-1} – one component of the output vector in the previous layer, b_k^u – bias of the k-th neuron in the u-th layer and $f^{\,u}$ – transfer function of the neurons in the u-th layer. If u is the number of the last layer, then $\mathbf{y^u}$ is the output vector of the network.

An artificial neural network works in two phases – active and adaptive. In the active phase the signal passes through the connecting paths from the input layer to the output layer of the network. To obtain correct results of that process, the weights and biases must have appropriate values. To assign these values, an adaptive phase must be used. This process is called training the neural network. For network training a set of training parameters is needed. This set consists of ordered pairs [$\mathbf{p}_i$, $\mathbf{y}_i$], where $\mathbf{y}_i$ are the expected output vectors (in our case random realizations of the model parameters selected for identification), which are yielded by simulation of the network with input vectors $\mathbf{p}_i$ (in our case points on the load-deflection curves). The main aim during training is to minimize the following criterion:

$$E = \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K} (y_{ik}^{\nu} - y_{ik}^{*})^2, \qquad (3)$$

where: N – number of ordered pairs input – output in the training set, y^{*}_{ik} – required output value of the k-th output neuron at the i-th input and y^{ν}_{ik} – real output value (at the same input).

In order to minimize criterion E some optimization technique is used. A description of these techniques can be found in, e.g. [15], [8].

4 Numerical examples

4.1 Notched specimen of high-strength concrete used for railway sleepers

A classical experiment involving three-point bending of a notched plane concrete beam was performed in order to determine fracture parameters of concrete for mass production of railway sleepers. Six specimens 80×80×480 mm were tested with a notch 25 mm in depth. The fracture-mechanical parameters were determined on the basis of the recommendation of RILEM [16] and improvements according to Elices [17], Stibor [18] and Veselý [19]. Based on this experiment, the following parameters were obtained: modulus of elasticity $E_c = 32.4$ GPa, fracture energy $G_F = 188.7$ N/m, compressive strength $f_c = 75.0$ MPa and estimation of tensile strength $f_t = 4.0$ MPa.

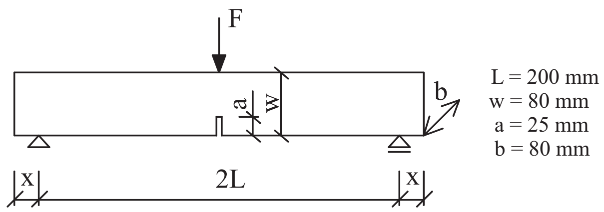

Fig. 4: Notched beam under three-point bending

The mean values of the parameters used for stochastic simulation and consequent identification are as follows: modulus of elasticity $E_c = 60$ GPa, tensile strength $f_t = 5$ MPa, compressive strength $f_c = 75$ MPa, fracture energy $G_F = 170$ N/m

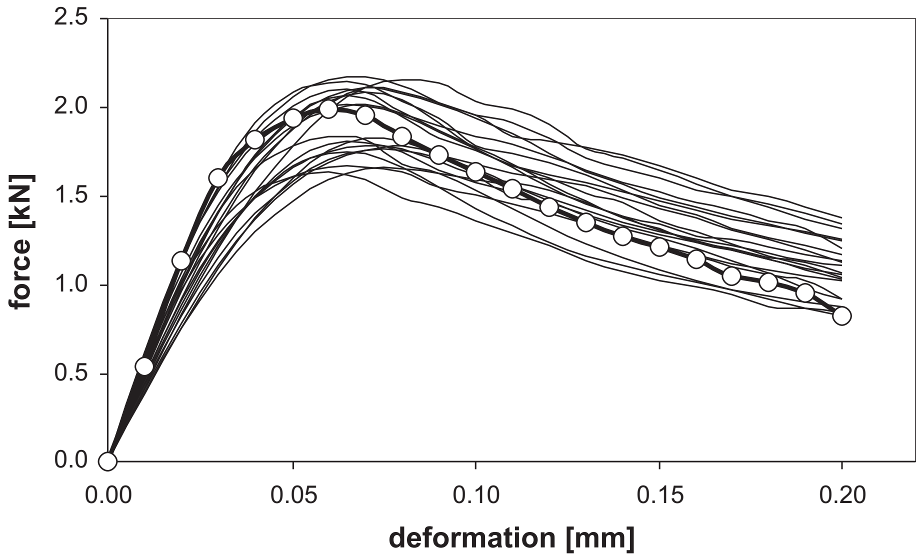

Fig. 5: Random load-deflection curve realizations – 20 simulations of LHS

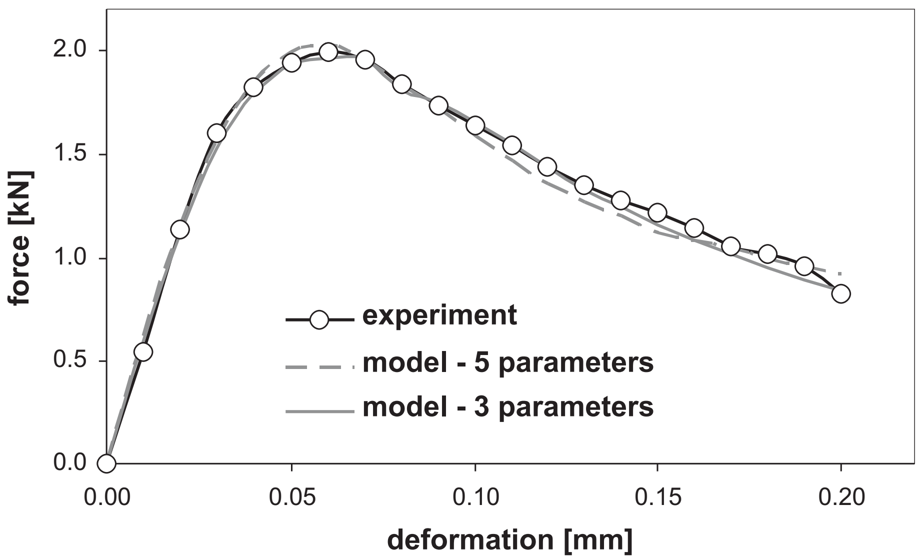

Fig. 6: Load deflection curves – experiment and numerical simulation using identified parameters

and compressive strain at compressive strength in the uniaxial compressive test $\varepsilon_c = 0.003$.

For stochastic training, randomness was introduced using the same coefficient of variation 0.15 and rectangular probability distribution for all random variables. Twenty simulations of LHS resulted in the load-deflection curves presented in Fig. 5. Note that none of these random curves captured the experiment well. This input-output information serves for training the selected neural network: a network with 20 inputs (20 points on the load-deflection curve for each simulation is utilized for training), one hidden layer consisting of 15 neurons with a nonlinear transfer function and one output layer of 5 neurons with a linear transfer function. Instead of 5 neurons, a second alternative with only 3 output neurons was also used. The original number of parameters (5) could be decreased (to 3) as the sensitivity analysis showed the dominating and nondominating random variables. The trained neural network provided the material model parameters (for 3 or 5 considered parameters): $E_c = 70.3$ and 73.2 GPa, $f_t = 5.5$ and 5.3 MPa, $f_c = 75$ (mean value) and 103.22 MPa, $G_F = 128$ and 141 N/m, $\varepsilon_c = 0.003$ (mean value) and 0.004.

The final calculation using ATENA resulted in a very good agreement with the experimental load-deflection curve for both alternatives, Fig. 6.

4.2 Shear wall failure

The shear wall shown in Fig. 7 was tested by Maier and Thürliman [20]. The square panel was orthogonally reinforced and provided with stiffening flanges. Loading by a vertical force was first applied representing a dead load. Then a horizontal force was applied and increased to failure. During the experiment there was extensive diagonal cracking prior to failure, followed by explosive crushing of the concrete under maximum load. The experimental failure pattern is shown in Fig. 7(a).

The analysis was done by ATENA using plane-stress isoparametric finite elements with the composite reinforced concrete material. All 10 shear wall parameters of the material models (both concrete and steel reinforcement) were

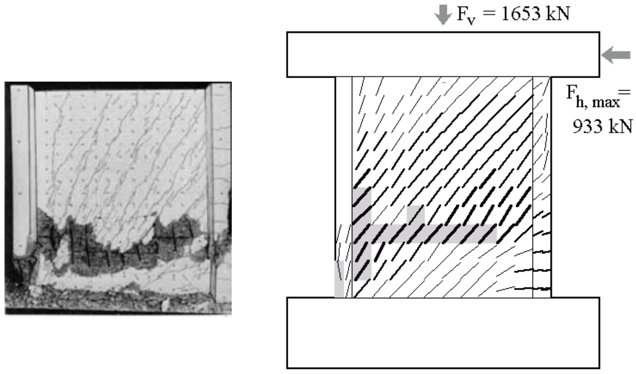

Fig. 7: Shear wall experimental failure and its ATENA simulation: (a) experimental failure, (b) failure simulated by ATENA

identified here. The mean values of the parameters used for stochastic simulation and consequent identification are as follows: for concrete (SBETA model) – modulus of elasticity $E_c = 30$ GPa, compressive strength $f_c = 35$ MPa, tensile strength $f_t = 2.5$ MPa, fracture energy $G_F = 75$ N/m, compressive strain in the uniaxial compressive test $\varepsilon_c = 0.0025$, critical compressive displacement $w_d = 0.003$ m; for steel (bilinear law) – yield strain $x_1 = 0.0027$, yield stress $f_{x1} = 574$ MPa, ultimate strain $x_2 = 0.015$ and ultimate stress $f_{x2} = 764$ MPa.

For stochastic training, randomness was intuitively introduced using coefficient of variation 0.10 for E_c, f_t and f_c, 0.2 for G_F and ε_c, 0.3 for w_d and 0.1 for all steel parameters. A rectangular probability distribution for all random variables is used. The experimental load-deflection curve and 20 simulations of LHS are presented in Fig. 8. This input-output information serves for training the selected neural network: network with 24 inputs (24 points on load-deflection curve for every simulation is utilized for training), two hidden layer consisting of 12 and 10 neurons with nonlinear transfer functions and one output layer of 10 neurons with a linear transfer

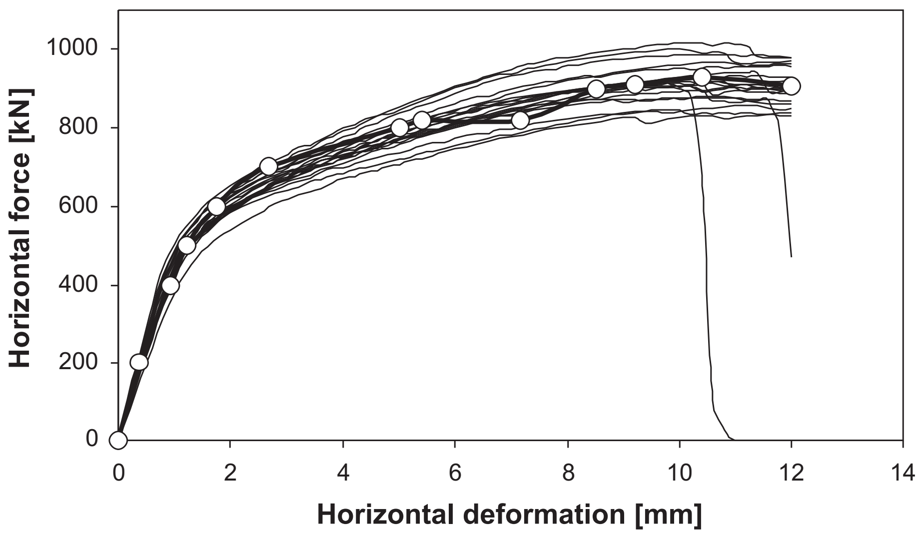

Fig. 8: Random load-deflection curve realizations – 20 simulations of LHS

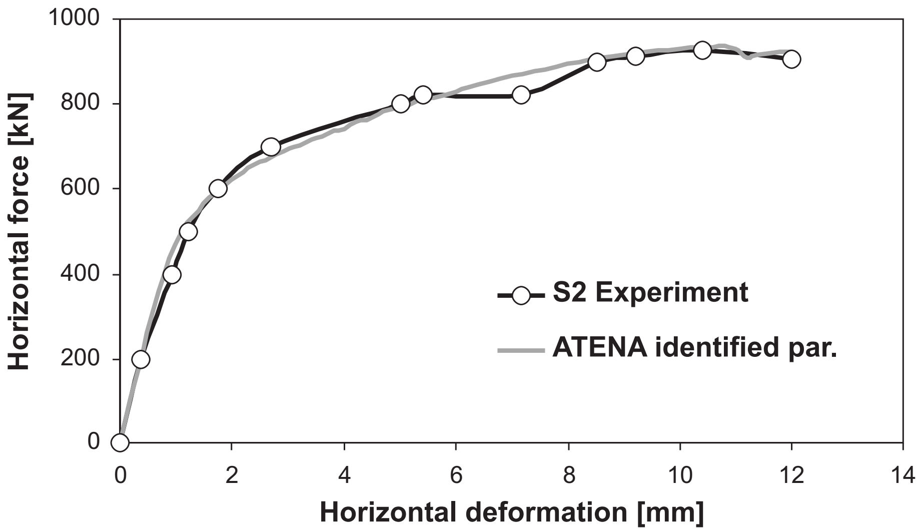

Fig. 9: Load deflection curves – experiment and simulation using identified parameters

function. The trained neural network provided the material model parameters: $E_c = 33$ GPa, $f_c = 35.3$ MPa, $f_t = 2.47$ MPa, $G_F = 77.85$ N/m, $\varepsilon_c = 0.0026$, $w_d = 0.0031$ m, $x_1 = 0.0028$, $f_{x1} = 570.7$ MPa, $x_2 = 0.0147$ and $f_{x2} = 768.8$ MPa.

The final calculation using ATENA resulted in a very good agreement with experimental load-deflection curve, Fig. 9.

5 Conclusion

Determining the material model parameters generally presents a great problem when using nonlinear analysis and sophisticated material constitutive laws. A methodology for efficient numerical identification of material model parameters is suggested. This approach utilizes stochastic computational analysis in combination with an artificial neural network. Small-sample simulation techniques are employed, which enables analysis of computationally intensive problems of nonlinear fracture mechanics. The feasibility of the approach is documented by numerical examples. Application of the concept to problems, where the experimental load-deflection curve is known could result in much better identification of the material model parameters than with a heuristic "trial and error" approach. Such well-identified parameters based on experiment can then be used for calculating a real structure. Very good results have been achieved, which indicates that efficient techniques have been combined at all three basic levels: deterministic nonlinear modeling, probabilistic stratified simulation and neural network approximation.

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