Fuzzy Concepts in the Detection of Unexpected Situations

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This paper establishes three essential classes of unexpected situations (UX^1, UX^2, UX^3), and concentrates attention on UX^3 detection. The concepts of a special Model of System of Situations (MSS) and a Model of a System of Faults (MSF) are introduced. An original method is proposed for detecting unexpected situations indicating a violation of a proper invariant of MSS (MSF). The presented approach offers a promising application for starting and ending phases of complex processes, for knowledge discoveries on data and knowledge bases developed with incomplete experience, and for modeling communication processes with unknown (disguised) communication subjects. The paper also presents a way to utilize ill-separable situations for UX^3 detection. The paper deals with the conceptual background for detecting UX^3 situations, recapitulates recent results in this field and opens the ways for further research.

Keywords: unexpected situations, model of system of situations, model of system of faults, invariants, fuzzy variables, degree of unexpectedness, emergence zone, association rules, Hasse Diagram.

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1 Introduction

The approaches to *unexpected situations* (as a special class of so-called undetectable faults) come from various domains, and they are referenced, e.g., in [1], [2], [3], [4], [5], [6], [7], [8], [9]. (A detailed analysis of these sources is in [17].)

Our approach for UX^3 detection is based on the concept of a UX^3 type situation, on the concepts of a Model of a System of Situations (***MSS***) and a Model of a System of Faults (***MSF***), *and on an original method which detects a* UX^3 type unexpected situation as a *violation of a proper structural invariant* - constructed on ***MSS*** (***MSF***). The structural invariant is constructed on ***MSS*** during the so-called "cognitive phase" of ***MSS*** (***MSF***) development. In this "cognitive phase", it is considered that some "classes" of situations (faults) have already been established but some of the new situations are processed with large uncertainty. Violation of the structural invariant represents the detection of a UX^3 type situation. (A more detailed explanation is given in section 2).

Analysing our approach from the Fault Diagnosis (FDI) point of view, some important issues may be indicated.

The *first issue* is the concept of ***MSS*** (***MSF***) in the context of the Model-Based approach in FDI, and the assignment of our method to some of the known diagnostic approaches, e.g. to *abductive* diagnostics. Our approach is *model-based*, though the development and the use of models (***MSS***, ***MSF***) are different from the examples introduced, e.g., in [10]. Our models are developed as a result of data and signal analysis (not as a result of preformed knowledge about a diagnosed system, e.g., knowledge about the internal structure or about a mathematical model). As will be explained in the following sections, the phase of fault detection in an abductive diagnostic model (e.g., in [11], [12]) is a rather special case of UX^3 detection in our method.

The second *issue* is the concept of a *symptom*. Our approach concentrates on processing situation signals (data) of the following types: vectors of outputs from qualitative models, ultrasound signals (representing the internal structure of the material samples), sequences of symbols (signs) and words from monitoring processes, ECG and EEG signals. Special situations (faults) which represent an extraordinary (faulty) behavior of a monitored (diagnosed) system are spread along the run of the signals (e.g., in their morphology) and in the sets of data. It is sometimes hard to speak about symptoms (symptoms of What?).

The presented approach to UX^3 detection has been tested by the following three application cases:

- In a supervisory control system for an industrial *distillation column*, especially in a qualitative model designed for the starting phase of the distillation process (e.g., after maintenance operations), details, in [13], [14].
- In detecting unexpected faults in welds (*laser, micro-arc and electron beam* welds *of thin walled welded structures used in the aerospace industry*) in combination with neural fault detectors, [15], [16]
- Within the framework of a special supervisory and monitoring system, e.g., in [17].

2 Unexpected situations – concepts and examples

General features of unexpected situations and three basic classes of unexpected situations will be introduced in this section.

The first class (UX^1) is induced by the relativity of the unexpected situation in respect to the levels of available Data and Knowledge of a reasoning human. (We will denote these situations as UX (UX^1) – emphasizing the intuitive aspect of the detection of such situations.)

Example 2.1: Let us suppose that the extent of values measurable in an instrument (given in the instrument protocol) is u_{min}, u_{max}. The situation when we measure by this instrument a value 10 u_{max} (without problems) is a UX^1 situation. (One interpretation of such a phenomenon is that wrong information was introduced in the protocol of the instrument.)

Example 2.2: The correct representation of a process by a differential equation depends on the identified type of equation and on the precision of the identified quotients. All cases with unknown noise in the input or in the output variables, unknown drifts in the parameters and cases of so-called hidden parameters, are cases that generate UX^1 situations.

Situations of the ***second class*** (UX^2) are generated by models that are *a priori insufficient for representing* some situations in the modeled process or system. (This means, e.g., that most situations are well represented, but a small number of situations are represented incorrectly.)

Example 2.3: Situations generated as an unprovable formula in FOL (First Order Language), situations for which a Turing machine does not stop (or works too long), or situations in complex robotic production lines (which are impossible to simulate wholly), belong to this class.

Example 2.4: Situations generated by models of systems with deterministic chaos. For example, systems modeled by Duffing or Lorenz equations belong to this class.

Situations of the ***third class*** (UX^3) have causes that differ from those that induce situations of the two previous classes. One principal scenario for UX^3 emergence is shown in Fig. 1.

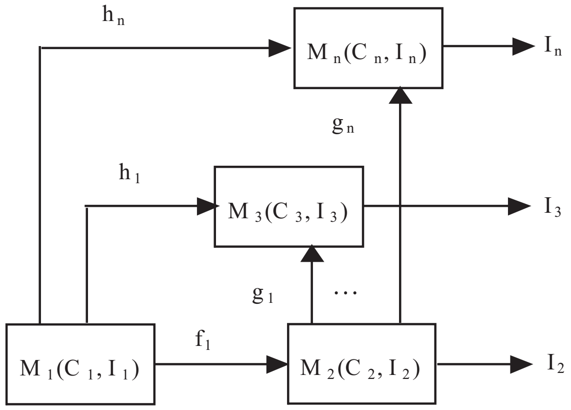

Fig. 1: Models, carriers and information

A general type of *model* is considered as a pair (C, I) which represents the synthesis of a Carrier (C) and Information (I). (A simple example of such a model is a classical photograph, where the photographic paper represents the carrier. What a human interpreter sees in the photograph is the information.)

Most cases of sign modeling work with so-called "hard" carriers equipped with resistance to the influence of the information that is encoded on them. However, sign and symbolic models with hard carriers are not resistant to the encoding and transfer of false information, (though they are very successful even in commercial terms). Such a case is described in ***Example 2.5***. The whole scenario of this example is important, including the physical and technical background. It is this background that distinguishes this case from a case of ciphering (which lies beyond the scope of our paper).

For this reason we search for additional models $M_3(C_3, I_3)$, … , $M_n(C_n, I_n)$, which enable the correctness of the model $M_2(C_2, I_2)$ to be checked with regard to $M_1(C_1, I_1)$. Such models in most cases really exist, being induced during the evolution of $M_1(C_1, I_1)$ and $M_2(C_2, I_2)$. However, it is not trivial to discover such models. The correctness of function M_2 with regard to M_1 is verified (in cases when some of models M3, …, M_n are discovered) by symbolic commutation of the transformation diagrams (from Fig. 1):

$$h_1 = g_1 o f_1, \dots \dots, h_n = g_n o f_1 \qquad (1)$$

This brief explanation of essential concepts for the theory of UX^3 situations introduced above will be extended and supplemented by a formal description and a recent application of this theory in section 3.

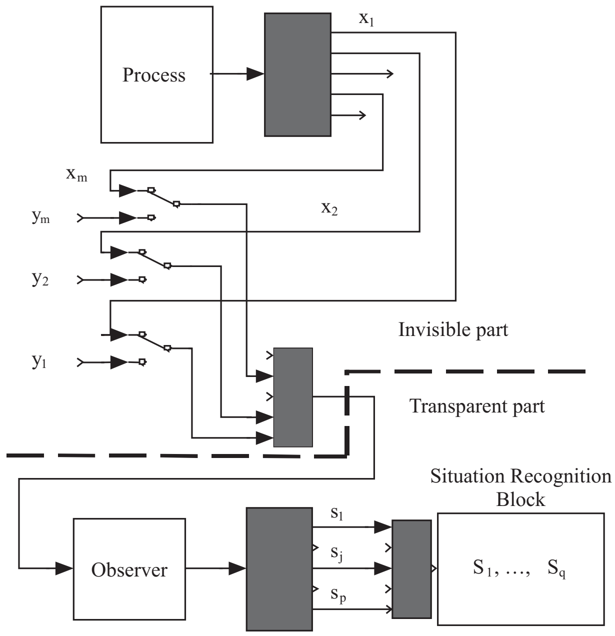

Fig. 2: Approximation of a UX^3 type situation

Example 2.5: Approximation of UX^3 type situations by a scheme with standard intentions.

Let us consider the scheme in Fig. 2. The scheme consists of an unavailable ("invisible") part and a transparent part. The invisible part contains: *a process* (for which we have no model), *process observer variables* (x_1, …, x_n), *non-process external variables* (y_1, …, y_m) and *switches*. The transparent part contains: an *observer* and a *situation recognition block* with classes of situations (S_1, …, S_q) and with the developed ***MSS***.

The situation recognition block (which contains ***MSS***) is developed during a standard operation of the process as a result of the work of the observer. The process is "represented" (for the observer) by the process observer variables (x_1, …, x_n). Let us consider that after a period of successful function, the structure of the situation recognition block is accepted as stable. Now let us assume that some of the *switches* are suddenly switched to variables (y_1, …, y_m) which represent another external reality, but they are formally the same as (x_1, …, x_n). As a result of this action – the assignment of the new situations coming into classes (S_1, …, S_q) will work incorrectly. Such a change is undetectable using standard methods, and it requires a special detection approach.

Example 2.6: Special UX^3 situations emerge in cases when inappropriate *intentions* (in Frege's sense, e.g., in [18]) are used to describe a process. (Usual intentions are *propositions*, *quantities* or *properties*.) Quantities, for example, are sometimes wrongly used for complex processes (or systems with complicated behavior) which are poorly measurable, and representing them by time series or by complicated signals introduces further difficulties in processing and interpretation. Typical examples of such cases are ECG, HRV (Heart Rate Variability) signals. (These facts are known, and they have been published (e.g., in the Journal of Cardio-Vascular Research from 1996), and have been presented in our research (e.g., in [28].))

3 Formal models for detecting UX^3

The method proposed in this paper for detecting UX^3 uses a Model of a System of Situations (***MSS***) or a Model of a System of Faults (***MSF***). Both these systems are developed in the *cognitive phase* (as was mentioned in the Introduction) during operations and experiments with the observed process or with the FDI system. The goal of the "cognitive phase" is to form structural invariants. (A few types of such invariants will be introduced in subsections 3.2–3.4.) In our method, a *violation* of a structural invariant is a means for *detecting* a UX^3 type situation. (An investigation of the necessary statistical parameters of the "cognitive phase" (e.g., "How many situations need to be analysed and in which classes can they be searched for", etc.) was made in [26].) In this paper we assume that the "cognitive phase" satisfies the necessary statistical and modeling standards.)

Model ***MSS*** has in general the following form:

$$\boldsymbol{MSS} = \left\langle \boldsymbol{S}, \left\langle \boldsymbol{\Gamma}_1(\boldsymbol{S}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S}) \right\rangle, \left\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \right\rangle \right\rangle, \quad (2)$$

where $\boldsymbol{S}$ represents a basic set of situations, $\left\langle \boldsymbol{\Gamma}_1(\boldsymbol{S}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S}) \right\rangle$ are structures on $\boldsymbol{S}$ considered as relevant for UX^3 detection and $\left\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \right\rangle$ are invariants on some $\boldsymbol{\Gamma}_1(\boldsymbol{S}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S})$ $(i, p \in \{1, \ldots, n\})$ for UX^3 detection.

Model ***MSF*** has in general the following form:

$$\boldsymbol{MSF} = \left\langle \left\langle \boldsymbol{S}, \boldsymbol{F} \right\rangle, \left\langle \boldsymbol{\Gamma}_1(\boldsymbol{S}, \boldsymbol{F}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S}, \boldsymbol{F}) \right\rangle, \left\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \right\rangle \right\rangle, \quad (3)$$

where $\langle \boldsymbol{S}, \boldsymbol{F} \rangle$ represent basic sets of situations and faults, $\left\langle \boldsymbol{\Gamma}_1(\boldsymbol{S}, \boldsymbol{F}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S}, \boldsymbol{F}) \right\rangle$ are structures on $(\boldsymbol{S}, \boldsymbol{F})$ considered as relevant for UX^3 detection and $\left\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \right\rangle$ are invariants on some $\boldsymbol{\Gamma}_1(\boldsymbol{S}), \ldots, \boldsymbol{\Gamma}_n(\boldsymbol{S})$ for UX^3 detection.

Models for UX^3 detection have the forms

$$\boldsymbol{MD}(\mathrm{UX}^3) = \left\langle \boldsymbol{MSS}, \boldsymbol{COND}_{\vee \boldsymbol{Inv}} \right\rangle$$

or (4)

$$\boldsymbol{MD}(\mathrm{UX}^3) = \left\langle \boldsymbol{MSF}, \boldsymbol{COND}_{\vee \boldsymbol{Inv}} \right\rangle,$$

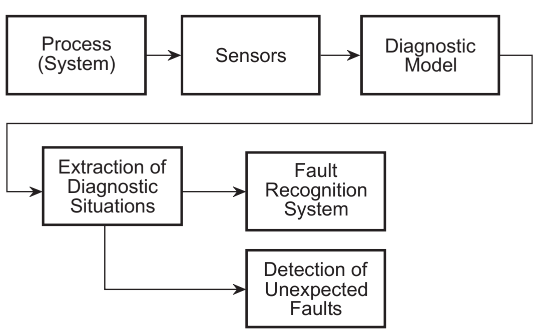

Fig. 3: Position of Models for UX^3 detection in a rough block scheme of the FDI system

where $\boldsymbol{COND}_{\vee \boldsymbol{Inv}}$ represents the conditions of violation of ***MSS*** (***MSF***) invariants. (These conditions are analysed in the process of UX^3 detection.)

Fig. 3 illustrates the position of the Models for UX^3 detection in a block scheme of the FDI system respecting one of many possible structures of the FDI system. The figure expresses only the fact that ***MSF*** and ***MD***(UX^3) work as a parallel block with a Fault Recognition System (which is usually understood as an ending member of an FDI system).

3.1 A general type of structural invariants $\boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p)$

The general type of invariants $\boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p)$ is connected with the commutation of diagrams (1) and is limited in this paper to the form of morphisms h_i, g_i, for $i = 1, \ldots, n$:

$$M_{i+2}(C_{i+2}, I_{i+2}) = g_i(M_2(C_2, I_2)), \ldots\ldots, M_{i+2}(C_{i+2}, I_{i+2}) = h_i(M_1(C_1, I_1)). \quad (5)$$

The concrete form of the morphisms depends on the type of models $M_1(C_1, I_1)$ and $M_2(C_2, I_2)$.

The following subsections introduce three examples of ***MSS*** and ***MSF*** and ***MD***(UX^3).

3.2 MD(UX^3) with an emergence zone

MSS has (in this case) the form (6)

$$\boldsymbol{MSS} = \left\langle \boldsymbol{S}, \left\langle \boldsymbol{M}(\boldsymbol{S}), \aleph, \Im \right\rangle, \left\langle (\forall s \in \aleph, \Im(\rho(s, \aleph) \le \lambda)) \right\rangle \right\rangle, \quad (6)$$

where $\boldsymbol{S}$ represents a basic set of situations, $\boldsymbol{M}(\boldsymbol{S})$ is a matroid constructed on this set of situations, $\aleph$ is a Basis on $\boldsymbol{M}(\boldsymbol{S})$, $\Im$ represents a Cover on $\boldsymbol{M}(\boldsymbol{S})$ (a subset of the matroid closure), $\rho(s, \aleph)$ is a metric function and λ is a positive real number. In addition to $\aleph, \Im$ there is the so-called emergence zone $\Re$. In this zone there are elements that can not be constructed from the elements in $\aleph, \Im$, but these elements are relevant to MSS and they are not in zone ***Outside***, see Fig. 4. With regard to the fact that "*regular*" situations can be assigned in $\aleph$ or in $\Im$ or they are classified in zone *Outside* (law of the excluded third alternative), elements of $\Re$ are considered as extraordinary situations and they represent (in the context of our paper) UX^3 situations.

In this case ***MD***(UX^3) has the form (7)

$$\boldsymbol{MD}(UX^3) = \left\langle \boldsymbol{MSS}, (\rho(\mathrm{s}^*, \aleph) \in (\lambda, U_p)) \right\rangle$$

or (7)

$$\boldsymbol{MD}\,(UX^3) = \left\langle \boldsymbol{MSS}, (\mathrm{s}^* \in \Re) \right\rangle$$

where s* is an unexpected situation and U_p is a real number (Upper bound).

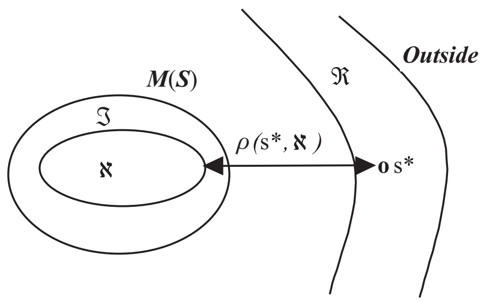

Fig. 4: Basis, cover and emergence zone

The ***MSS*** described above was used in real conditions for UX^3 detection in the starting phase of a deisobutanisation process [13], [14].

3.3 *MD*(UX^3) with bipartite graphs

In this case, ***MSF*** has the form (8)

$$\boldsymbol{MSF} = \left\langle \langle \boldsymbol{S}, \boldsymbol{F} \rangle, \langle \boldsymbol{G} \rangle, \left\langle \{ \boldsymbol{G}_1, \boldsymbol{G}_2, \ldots, \boldsymbol{G}_m \}, \text{»} \right\rangle \right\rangle, \tag{8}$$

where $\langle \boldsymbol{S}, \boldsymbol{F} \rangle$ represent basic sets of situations and faults, ***G*** is a *bipartite graph* (Situation → Faults), $\left\langle \{ \boldsymbol{G}_1, \boldsymbol{G}_2, \ldots, \boldsymbol{G}_m \}, \text{»} \right\rangle$ is a special Dulmage-Mendelsohn decomposition, [19], [21]. ($\boldsymbol{G}_1, \boldsymbol{G}_2, \ldots, \boldsymbol{G}_m$ are irreducible sub-graphs, "»" is a tree ordering on the set of the sub-graphs.) The detection of UX^3 has been represented by a violation of ordering "»" and has been indicated by the following conditions discovered for a situation s*.

In this case, ***MD***(UX^3) has the form (9)

$$\boldsymbol{MD}(\text{UX3}) = \Big\langle \boldsymbol{MSF}, (((\boldsymbol{ACC}(\text{G}_i, \text{s}^*)) \text{AND} (\text{not } \boldsymbol{ACC}(\text{G}_j, \text{s}^*))), \text{ for some } j(\text{G}_j \text{»} \text{G}_i)) \Big\rangle, \tag{9}$$

where s* is an UX^3 related to $\left\langle \{ \boldsymbol{G}_1, \boldsymbol{G}_2, \ldots, \boldsymbol{G}_m \}, \text{»} \right\rangle$.

Note 3.1: Expression ***ACC***(G, s) denotes "situation s is ACCepted by bipartite graph ***G***".

This ***MD***(UX^3) model has been used, e.g., in [15] and [16]. (However, taking into account the well-studied concept of DM-irreducibility (e.g., in [20], page 63, 64) we are aware of the limited applicability of DMD.)

3.4 *MD*(UX^3) with the association rules

The background for ***MD***(UX^3) model presented in this section continues in the line started in [22],[23] and nowadays utilises formulations, e.g., from [24], [25].

In this case, ***MSF*** has the form (10)

$$\boldsymbol{MSF} = \langle \langle \boldsymbol{S}, \boldsymbol{F} \rangle, \langle \boldsymbol{M}_\mathbf{G}, \boldsymbol{HD} \rangle, \langle \boldsymbol{ER} \rangle \rangle, \tag{10}$$

where $\langle \boldsymbol{S}, \boldsymbol{F} \rangle$ represent basic sets of situations and faults, $\boldsymbol{M}_\mathbf{G}$ is a *qualitative matrix* with data acquired from the cognitive phase of FDI system operation (the concept "cognitive phase" was introduced and explained in section 1.), ***HD*** is a Hasse Diagram [25] derived from $\boldsymbol{M}_\mathbf{G}$ and ***ER*** is a set of Evaluated Association Rules extracted from ***HD*** (each rule evaluated by quantities of Supp (Rule Support) and Conf (Rule Confidence), [24].

Note 3.2: The Hasse Diagram facilitates the process of extracting the rules from matrix $\boldsymbol{M}_\mathbf{G}$, but its use is not obligatory for the formation of ***ER***.

MD(UX^3) has (in this case) the form (11)

$$\boldsymbol{MD}(\text{UX}^3) = \left\langle \boldsymbol{MSF}, (\exists\, \mathbf{r} \in \boldsymbol{ER}, \text{not } \boldsymbol{ACC}(\mathbf{r}, \text{s}^*)) \right\rangle \tag{11}$$

where s* is a UX^3 situation.

Note 3.3: The expression ***ACC***(**r**, s*) denotes "situation s* is ACCepted by rule **r**".

3.5 Additional important circumstances

A. The basic purpose of ***MSS*** (***MSF***) and of ***MD***(UX^3) is in the formation of *rules* that enable to distinct between "ill-separable situations" and "UX^3 situations". Such decision rules are simple:

⟨**IF** situation **s** satisfies the invariant⟩ ⇒ ⟨**THEN s** it is an ill separable situation⟩.

⟨**IF** situation **s** does not satisfy the invariant ⟩ ⇒ ⟨**THEN s** is UX^3 situations⟩.

B. When detecting a UX^3 situation, we should like to know how important the discovered UX^3 situation is in comparison with other possible UX^3 situations (which could be detected by the considered invariant). For this reason a numerical function $\boldsymbol{D}(\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \rangle, UX^3)$ has been suggested. It is called the Degree of UX^3 (the Degree of Unexpectedness) with respect to invariants $\langle \boldsymbol{Inv}(\boldsymbol{\Gamma}_i), \ldots, \boldsymbol{Inv}(\boldsymbol{\Gamma}_p) \rangle$ and depends on the following two factors:

- the ***complexity*** (the constructibility) of the applied invariant (invariants),
- the ***sensitivity*** of the invariant (invariants) to the measure ***of the violation***.

4 Degree of UX^3

The following form has been introduced for the computation of $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_\text{p}), UX^3)$ (with one invariant $\boldsymbol{Inv}(\boldsymbol{\Gamma}_\text{p})$)

$$\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3) = \frac{1}{\text{Q}_\text{cpm}} \left(\frac{1}{\sum\limits_{x_i \in \mathbf{ELM}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_\text{p}))} (\omega_i\, {}^1\mu(x_i))^2} \right)^{\frac{1}{a}} * \left[\sum_{x_i \in \mathbf{ELM}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_\text{p}))} (\omega_i ({}^1\mu(x_i) - {}^2\mu(x_i)))^2 \right]^{\frac{1}{a}}, \tag{12}$$

where Q_Cpmx is a quotient of the invariant complexity, x_i are elements of the invariant structure (from the ground set $\mathbf{Elm}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p))$, ω_i are quotients of importance for elements x_i. Quantities ${}^1\mu(x_i)$ are quotients of deployment of elements x_i before invariant violation and ${}^2\mu(x_i)$ are quotients of deploy-

Table 1

No.	Name of Structure	Q_Cpmx [1]
1	Klein group of the 4^th order	1
2	Permutation group of the 6^th order (4×4)	0.9455
3	Group of linear transformations of the ∞^th order	0.7432
4	Semi-group of binary equivalencies	0.5571
5	Semi-group of binary relations	0.1979
6	Matroid of the 2^nd order	0.1750
7	Linear regular grammar	0.1345
8	Non context grammar	0.1047
9	Structure of association rules	0.0883

ment for elements x_i after violation of the invariant by means of x_i. (Number $a = 2$ (usually) but not necessarily).

Note 4.1: If an element is not violated, it holds: ${}^1\mu(x_i) = {}^2\mu(x_i)$.

Quotient of structure complexity Q_{Cpmx} expresses the difficulty to form a model of such a structure. The complexity of the structures is compared in [26]. Some illustrative examples of Q_{Cpmx} quantities for structures with one composition operation are introduced in Table 1.

Note 4.2: The specification of structures 1–5, 7, 8 from Table 1 is known from the literature. Structure 6 is a matroid with 10 independent sets and with the cardinality of its basis equal to 2.

The quantity of $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3)$ gives a qualitative evaluation of the difficulty of the operations that follow after UX^3 detection. Usual such operations (e.g., in the FDI field) are: localisation, isolation, identification and interpretation a UX^3 situation. The higher the $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3)$ quantity is, the more difficult these operations are to execute.

Example 4.1: Let us suppose Klein group $\boldsymbol{B} = \langle\{\mathrm{I, N, R, C}\}, \mathbf{o}\rangle$ as an invariant $\boldsymbol{Inv}(\boldsymbol{\Gamma}_1)$, and let us suppose the violation of elements N and R given by the following quantities of quotients:

$\omega_I = \omega_N = \omega_R = \omega_C = 1$, ${}^1\mu(x_I) = {}^1\mu(x_R) = {}^1\mu(x_N) = {}^1\mu(x_C) = 1$,
${}^2\mu(x_I) = {}^2\mu(x_C) = 1$, $2\mu(x_N) = 2\mu(x_C) = 0.8$ and $a = 2$.

$$D(\boldsymbol{B}, UX^3) = (0.08/4)^{1/2} = 0.1414.$$

Example 4.2: Let us suppose a set of rules $\boldsymbol{ER}$ (with 11 rules) as an invariant $\boldsymbol{Inv}(\boldsymbol{\Gamma}_1)$, and let us suppose violation of rules $\mathbf{r}_3$ and $\mathbf{r}_5$ given by the following conditions:

$\omega_1 = 0.7$, $\omega_2 = 0.7$, $\omega_3 = 1.4$, $\omega_4 = 0.9$, $\omega_5 = 1.0$, $\omega_6 = 1.2$, $\omega_7 = 0.5$, $\omega_8 = 0.8$, $\omega_9 = 0.5$, $\omega_{10} = 1.2$, $\omega_{11} = 0.7$,
${}^1\mu(\mathbf{r}_1) = {}^1\mu(\mathbf{r}_2) = \ldots = {}^1\mu(\mathbf{r}_{11}) = 1$, ${}^2\mu(\mathbf{r}_1) = {}^2\mu(\mathbf{r}_2) = 1$,
${}^2\mu(\mathbf{r}_3) = {}^2\mu(\mathbf{r}_5) = 0.0$,
${}^2\mu(\mathbf{r}_4) = {}^2\mu(\mathbf{r}_6) = {}^2\mu(\mathbf{r}_7) = {}^2\mu(\mathbf{r}_8) = {}^2\mu(\mathbf{r}_9) = {}^2\mu(\mathbf{r}_{10}) = {}^2\mu(\mathbf{r}_{11}) = 1$
and $a = 2$.

$$\boldsymbol{D}(\boldsymbol{ER}, UX^3) = \frac{1}{Q_{Cpmx}}\left(\frac{\omega_3^2 + \omega_5^2}{\sum\limits_{i=1,\ldots,11}\omega_i^2}\right)^{\frac{1}{2}} = \frac{0.565}{0.0883} = 6.4\ .$$

The examples introduced here illustrate the $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3)$ quantities for violation of two very different invariant structures. The results correspond to an intuitive understanding of variable $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3)$. The *more complicated* the structure of $\boldsymbol{Inv}(\boldsymbol{\Gamma}_p)$ is and the *higher* its violation is, the *higher* is the potential quantity of $\boldsymbol{D}(\boldsymbol{Inv}(\boldsymbol{\Gamma}_p), UX^3)$. (For illustration: $\boldsymbol{D}(\boldsymbol{B}, UX^3) \in [0, 1]$, $\boldsymbol{D}(\boldsymbol{ER}, UX^3) \in [0, 11.325]$.)

5 Conclusions

This paper demonstrates the use of a *fuzzy approach* for modeling very complex problems. Fuzzy concepts are contained in all essential conceptual constructs as UX^3, ***MSS***, ***MSF***, a *violation of an invariant*, emergence zone, Evaluated Association Rules, Hasse Diagram (and in the fuzzy values and variables).

The paper has introduced methods for UX^3 detection. It has introduced a general approach for the development of ***MSS***, ***MSF*** and ***MD***(UX^3) models, and three variants of these models have been described.

Examples of ***MD***(UX^3) with an emergence zone, with Bipartite Graphs and with Evaluated Associations rules in the role of structural invariants of ***MSS*** and (***MSF***) have been presented, e.g., in [13]–[17].)

In [17] we introduced an illustrative application of ***MD***(UX^3) with Evaluated Association Rules for the conditions of an industrial monitoring system (developed with *data support* from "Ventilation system of the Mrazovka road tunnel in Prague" in Czech Republic). The proposed method may be applied for processes with a similar formal description, e.g., in transport systems, power supply systems, and in the chemical industry.

The approach is also well applicable in special signal based cases (with no available model of the observed and detected system), where the signals are acquired from special sensors and especially when neural networks or fuzzy systems are used for processing them. (Such applications fields are described, e.g., in [16], [27], [28].)

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